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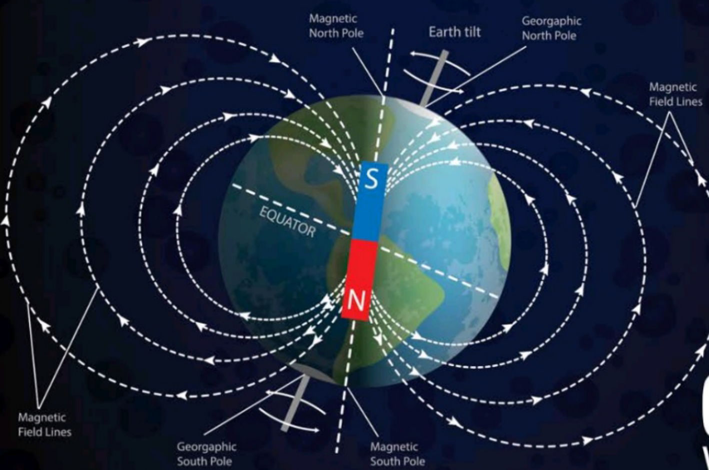

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CLASS XII

PHYSICS for you

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CONTENTS

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NEET

Phase-2 held on 14th October

SOLVED PAPER 2020

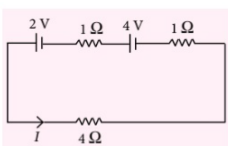
1. The mean free path l for a gas molecule depends upon diameter, d of the molecule as
(a) $l \propto \frac{1}{d}$ (b) $l \propto \frac{1}{d^2}$ (c) $l \propto d$ (d) $l \propto d^2$

2. An intrinsic semiconductor is converted into n-type extrinsic semiconductor by doping it with
(a) Germanium (b) Phosphorous
(c) Aluminium (d) Silver

3. The half life of a radioactive sample undergoing α-decay is 1.4×10^{17} s. If the number of nuclei in the sample is 2.0×10^{21}, the activity of the sample is nearly
(a) 10^3 Bq (b) 10^4 Bq (c) 10^5 Bq (d) 10^6 Bq

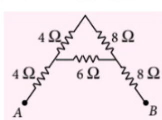
4. The E.M. wave with shortest wavelength among the following is
(a) microwaves (b) ultraviolet rays
(c) X-rays (d) gamma-rays

5. For the circuit shown in the figure, the current I will be



(a) 0.5 A (b) 0.75 A (c) 1 A (d) 1.5 A

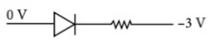
6. The equivalent resistance between A and B for the mesh shown in the figure is

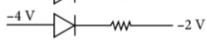


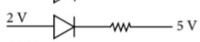
(a) 4.8 Ω
(b) 7.2 Ω
(c) 16 Ω (d) 30 Ω

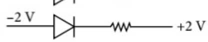
7. A wheel with 20 metallic spokes each 1 m long is rotated with a speed of 120 rpm in a plane perpendicular to a magnetic field of 0.4 G. The induced emf between the axle and rim of the wheel will be (1 G = 10^{-4} T)
(a) 2.51 V (b) 2.51×10^{-4} V
(c) 2.51×10^{-5} V (d) 4.0×10^{-5} V

8. Out of the following which one is a forward biased diode ?

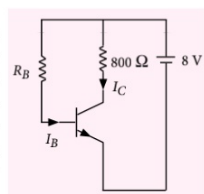
(a) 

(b) 

(c) 

(d) 

9. A n-p-n transistor is connected in common emitter configuration (see figure) in which collector voltage drop across load resistance (800 Ω) connected to the collector circuit is 0.8 V. The collector current is



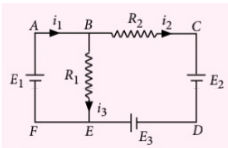
(a) 0.2 mA (b) 2 mA
(c) 0.1 mA (d) 1 mA

10. Two solid conductors are made up of same material, have same length and same resistance. One of them has a circular cross-section of area A_1 and the other one has a square cross-section of area A_2. The ratio A_1/A_2 is
(a) 2 (b) 1.5 (c) 1 (d) 0.8

11. Two coherent sources of light interfere and produce fringe pattern on a screen. For central maximum, the phase difference between the two waves will be
(a) $\pi/2$ (b) zero (c) π (d) $3\pi/2$

12. Time intervals measured by a clock give the following readings :
1.25 s, 1.24 s, 1.27 s, 1.21 s and 1.28 s
What is the percentage relative error of the observations?
(a) 1.6% (b) 2%
(c) 4% (d) 16%

13. For the circuit given below, the Kirchoff's loop rule for the loop *BCDEB* is given by the equation



(a) $-i_2R_2 + E_2 + E_3 + i_3R_1 = 0$
(b) $-i_2R_2 + E_2 - E_3 + i_3R_1 = 0$
(c) $i_2R_2 + E_2 - E_3 - i_3R_1 = 0$
(d) $i_2R_2 + E_2 + E_3 + i_3R_1 = 0$

14. An ideal gas equation can be written as $P = \frac{\rho RT}{M_0}$, where ρ and M_0 are respectively
(a) number density, mass of the gas
(b) mass density, mass of the gas
(c) number density, molar mass
(d) mass density, molar mass

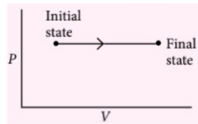
15. The magnetic flux linked with a coil (in Wb) is given by the equation
$\phi = 5t^2 + 3t + 16$
The magnitude of induced emf in the coil at the fourth second will be
(a) 10 V (b) 33 V (c) 43 V (d) 108 V

16. The length of the string of a musical instrument is 90 cm and has a fundamental frequency of 120 Hz. Where should it be pressed to produce fundamental frequency of 180 Hz ?
(a) 80 cm (b) 75 cm (c) 60 cm (d) 45 cm

17. The magnetic field in a plane electromagnetic wave is given by
$B_y = 2 \times 10^{-7} \sin(\pi \times 10^3 x + 3\pi \times 10^{11} t)$ T
Calculate the wavelength.
(a) $\pi \times 10^{-3}$ m (b) $\pi \times 10^3$ m
(c) 2×10^{-3} m (d) 2×10^3 m

18. A barometer is constructed using a liquid (density = 760 kg m^{-3}). What would be the height of the liquid column, when a mercury barometer reads 76 cm? (density of mercury = 13600 kg m^{-3})
(a) 0.76 m (b) 1.36 m (c) 13.6 m (d) 136 m

19. The *P-V* diagram for an ideal gas in a piston cylinder assembly undergoing a thermodynamic process is shown in the figure. The process is



(a) isothermal (b) adiabatic
(c) isochoric (d) isobaric

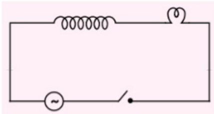
20. The efficiency of a Carnot engine depends upon
(a) the temperature of the source only
(b) the temperature of the sink only
(c) the temperatures of the source and sink
(d) the volume of the cylinder of the engine.

21. The electric field at a point on the equatorial plane at a distance *r* from the centre of a dipole having dipole moment $\vec{p}$ is given by
(r >> separation of two charges forming the dipole, ε_0-permittivity of free space)
(a) $\vec{E} = -\frac{\vec{p}}{4\pi\varepsilon_0 r^3}$ (b) $\vec{E} = \frac{\vec{p}}{4\pi\varepsilon_0 r^3}$
(c) $\vec{E} = \frac{2\vec{p}}{4\pi\varepsilon_0 r^3}$ (d) $\vec{E} = -\frac{\vec{p}}{4\pi\varepsilon_0 r^2}$

22. A liquid does not wet the solid surface if angle of contact is
(a) zero (b) equal to 45°
(c) equal to 60° (d) greater than 90°

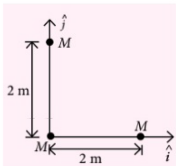
23. Three stars *A*, *B*, *C* have surface temperatures T_A, T_B, T_C respectively. Star *A* appears bluish, star *B* appears reddish and star *C* yellowish. Hence,
(a) $T_A > T_C > T_B$ (b) $T_A > T_B > T_C$
(c) $T_B > T_C > T_A$ (d) $T_C > T_B > T_A$

24. A light bulb and an inductor coil are connected to an ac source through a key as shown in the figure below. The key is closed and after sometime an iron rod is inserted into the interior of the inductor. The glow of the light bulb



(a) increases (b) decreases
(c) remains unchanged (d) will fluctuate

25. Three identical spheres, each of mass M, are placed at the corners of a right angle triangle with mutually perpendicular sides equal to 2 m (see figure). Taking the point of intersection of the two mutually perpendicular sides as the origin, find the position vector of centre of mass.



(a) $\frac{4}{3}(\hat{i}+\hat{j})$ (b) $2(\hat{i}+\hat{j})$
(c) $(\hat{i}+\hat{j})$ (d) $\frac{2}{3}(\hat{i}+\hat{j})$

26. The de Broglie wavelength of an electron moving with kinetic energy of 144 eV is nearly
(a) 102×10^{-2} nm (b) 102×10^{-3} nm
(c) 102×10^{-4} nm (d) 102×10^{-5} nm

27. The angle of I' (minute of arc) in radian is nearly equal to
(a) 1.75×10^{-2} rad (b) 2.91×10^{-4} rad
(c) 4.85×10^{-4} rad (d) 4.80×10^{-6} rad

28. The total energy of an electron in the n^{th} stationary orbit of the hydrogen atom can be obtained by
(a) $E_n = -13.6 \times n^2$ eV (b) $E_n = \frac{13.6}{n^2}$ eV
(c) $E_n = -\frac{13.6}{n^2}$ eV (d) $E_n = -\frac{1.36}{n^2}$ eV

29. A wire of length L metre carrying a current of I ampere is bent in the form of a circle. Its magnetic moment is
(a) $IL^2/4\pi$ A m^2 (b) $IL^2/4$ A m^2
(c) $I\pi L^2/4$ A m^2 (d) $2IL^2/\pi$ A m^2

30. What is the depth at which the value of acceleration due to gravity becomes $1/n$ times the value that at the surface of earth? (radius of earth = R)
(a) $\frac{R}{n}$ (b) $\frac{R}{n^2}$
(c) $\frac{R(n-1)}{n}$ (d) $\frac{Rn}{(n-1)}$

31. An object is placed on the principal axis of a concave mirror at a distance of $1.5f$ (f is the focal length). The image will be at
(a) $3f$ (b) $-3f$
(c) $1.5f$ (d) $-1.5f$

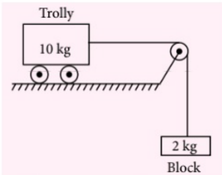
32. The angular speed of the wheel of a vehicle is increased from 360 rpm to 1200 rpm in 14 second. Its angular acceleration is
(a) 1 rad s^{-2} (b) 2π rad s^{-2}
(c) 28π rad s^{-2} (d) 120π rad s^{-2}

33. The acceleration of an electron due to the mutual attraction between the electron and a proton when they are 1.6 Å apart is
($m_e \simeq 9 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C)
$\left(\text{Take } \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2}\right)$
(a) 10^{25} m s^{-2} (b) 10^{24} m s^{-2}
(c) 10^{23} m s^{-2} (d) 10^{22} m s^{-2}

34. What happens to the mass number and atomic number of an element when it emits γ-radiation?
(a) Mass number increases by four and atomic number increases by two.
(b) Mass number decreases by four and atomic number decreases by two.
(c) Mass number and atomic number remain unchanged.
(d) Mass number remains unchanged while atomic number decreases by one.

35. If the critical angle for total internal reflection from a medium to vacuum is 45°, then velocity of light in the medium is,
(a) 3×10^8 m s^{-1} (b) 1.5×10^8 m s^{-1}
(c) $\frac{3}{\sqrt{2}}\times10^8$ m s^{-1} (d) $\sqrt{2}\times10^8$ m s^{-1}

36. Calculate the acceleration of the block and trolly system shown in the figure. The coefficient of kinetic friction between the trolly and the surface is 0.05. ($g = 10$ m s^{-2}, mass of the string is negligible and no other friction exists).

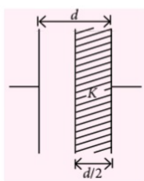


(a) 1.00 m s^{-2} (b) 1.25 m s^{-2}
(c) 1.50 m s^{-2} (d) 1.66 m s^{-2}

37. A point mass m is moved in a vertical circle of radius r with the help of a string. The velocity of the mass is $\sqrt{7gr}$ at the lowest point. The tension in the string at the lowest point is
(a) $1mg$ (b) $6mg$ (c) $7mg$ (d) $8mg$

38. A plano-convex lens of unknown material and unknown focal length is given. With the help of a spherometer we can measure the
(a) refractive index of the material
(b) focal length of the lens
(c) radius of curvature of the curved surface
(d) aperture of the lens.

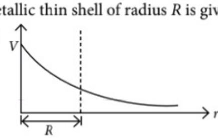
39. A parallel plate capacitor having cross-sectional area A and separation d has air in between the plates. Now an insulating slab of same area but thickness $d/2$ is inserted between the plates as shown in figure having dielectric constant $K(= 4)$.

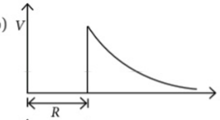


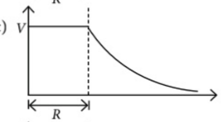
The ratio of new capacitance to its original capacitance will be
(a) 4 : 1 (b) 2 : 1 (c) 8 : 5 (d) 6 : 5

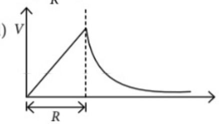
40. The power of a biconvex lens is 10 dioptre and the radius of curvature of each surface is 10 cm. Then the refractive index of the material of the lens is
(a) 3/2 (b) 4/3 (c) 9/8 (d) 5/3

41. The variation of electrostatic potential with radial distance r from the centre of a positively charged metallic thin shell of radius R is given by the graph

(a)


(b)


(c)


(d)


42. Which of the following gate is called universal gate?
(a) NOT gate (b) OR gate
(c) AND gate (d) NAND gate

43. Identify the function which represents a periodic motion.
(a) $e^{-\omega t}$ (b) $e^{\omega t}$
(c) $\log_e(\omega t)$ (d) $\sin\omega t + \cos\omega t$

44. The wave nature of electrons was experimentally verified by
(a) Davisson and Germer
(b) de Broglie (c) Hertz
(d) Einstein

45. A person sitting in the ground floor of a building notices through the window of height 1.5 m, a ball dropped from the roof of the building crosses the window in 0.1 s. What is the velocity of the ball when it is at the topmost point of the window? ($g = 10$ m s^{-2})
(a) 20 m s^{-1} (b) 15.5 m s^{-1}
(c) 14.5 m s^{-1} (d) 4.5 m s^{-1}

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(c)	**2.**	(c)	**3.**	(c)	**4.**	(a)	**5.**	(c)
6.	(d)	**7.**	(c)	**8.**	(b)	**9.**	(d)	**10.**	(a)
11.	(a)	**12.**	(d)	**13.**	(c)	**14.**	(a)	**15.**	(a)
16.	(a)	**17.**	(d)	**18.**	(b)	**19.**	(a)	**20.**	(a,c)
21.	(a,c)	**22.**	(a, d)	**23.**	(a, d)	**24.**	(7)	**25.**	(5)
26.	(2)	**27.**	(d)	**28.**	(a)	**29.**	(b)	**30.**	(c)

SOLUTIONS

1. (b) : Mean free path, $l = \frac{kT}{\sqrt{2}\,\pi d^2 P}$

$\therefore \quad l \propto \frac{1}{d^2}$

2. (b) : An intrinsic semiconductor is converted into n-type extrinsic semiconductor by doping it with group-15 elements. In this case, phosphorus is a group-15 element.

3. (b) : Given, $T_{1/2} = 1.4 \times 10^{17}$ s and $N = 2.0 \times 10^{21}$

Activity, $R = \lambda N = \frac{0.693}{T_{1/2}} N = \frac{0.693}{1.4 \times 10^{17}} \times 2 \times 10^{21}$

$= 1 \times 10^4$ Bq

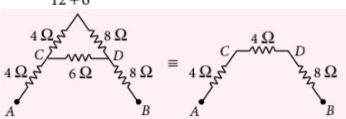
4. (d)

5. (c) : Here, total potential difference is,
$V = 4\text{ V} + 2\text{ V} = 6\text{ V}$
Total resistance $= 1\,\Omega + 1\,\Omega + 4\,\Omega = 6\,\Omega$

$\therefore$ Current, $I = \frac{V}{R} = \frac{6}{6} = 1$ A

6. (c) : The resistance between points C and D

$R_{CD} = \frac{12 \times 6}{12 + 6} = 4\,\Omega$



$\therefore$ Total resistance between points A and B is,
$R = 4\,\Omega + 4\,\Omega + 8\,\Omega = 16\,\Omega$

7. (b) : Given, $B = 0.4$ G $= 0.4 \times 10^{-4}$ T, $r = 1$ m
$\omega = 120$ rpm $= 4\pi$ rad s^{-1}

Now, induced e.m.f., $\varepsilon = \frac{1}{2} B\omega r^2$

$= \frac{1}{2} \times 0.4 \times 10^{-4} \times 4\pi \times 1^2 = 2.51 \times 10^{-4}$ V

8. (a)

9. (d) : Given, $R_L = 800\,\Omega$ and $V_C = 0.8$ V

$\therefore \quad I_C = \frac{V_C}{R_L} = \frac{0.8}{800} = 1\,\text{mA}$

10. (c) : Given, $l_1 = l_2$, $\rho_1 = \rho_2$ and $R_1 = R_2$

Resistance, $R = \frac{\rho l}{A}$

$\Rightarrow \frac{\rho_1 l_1}{A_1} = \frac{\rho_2 l_2}{A_2}$ or, $\frac{A_1}{A_2} = \frac{\rho_1 l_1}{\rho_2 l_2} = 1$

11. (b) : For constructive interference,
phase difference, $\phi = 2n\pi$ $[n = 0, 1, 2, 3, ...]$
For central bright fringe, $n = 0$, $\therefore \phi = 0$

12. (a) : Percentage error $= \frac{\overline{\Delta a}}{a_m} \times 100$

Absolute error, $a_m = \frac{1.25 + 1.24 + 1.27 + 1.21 + 1.28}{5}$

$= 1.25$ s

Mean absolute error,

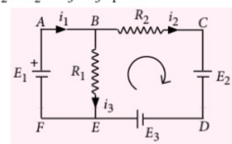
$$\overline{\Delta a} = \frac{|1.25 - 1.25| + |1.24 - 1.25| + |1.27 - 1.25| + |1.21 - 1.25| + |1.28 - 1.25|}{5}$$

$= 0.02$ s

$\therefore$ Percentage error $= \frac{0.02}{1.25} \times 100 = 1.6\%$

13. (c) : Considering the loop $BCDEB$, the algebraic sum of all the potential differences in this closed loop is zero.

$\therefore \quad i_2R_2 + E_2 - E_3 - i_3R_1 = 0$



14. (d) : From ideal gas equation, $PV = nRT$

$\therefore \quad P = \frac{1}{V} nRT = \frac{m}{M_o} \frac{1}{V} RT = \left(\frac{m}{V}\right)\left(\frac{RT}{M_o}\right) = \frac{\rho RT}{M_o}$

$\therefore \quad \rho = \frac{m}{V} =$ mass density

M_o = molar mass

15. (c) : Given, $\phi = 5t^2 + 3t + 16$

Induced emf, $\varepsilon = \frac{d\phi}{dt} = 10t + 3$

At $t = 4$ s, $\varepsilon = 40 + 3 = 43$ V

16. (c) : Given initial length of string, $l = 90$ cm and fundamental frequency, $\upsilon = 120$ Hz

i.e., $\frac{v}{2l} = 120\text{ Hz} \Rightarrow \frac{v}{2} = 120 \times 90$ ...(i)

Now in order to have a fundamental frequency of 180 Hz, let the new length be l'.

$\therefore \quad \frac{v}{2l'} = 180 \Rightarrow l' = \frac{v}{2 \times 180} = \frac{120 \times 90 \times 2}{2 \times 180}$ (Using (i))

$\therefore \quad l' = 60$ cm

17. **(c)** : Given, $B_y = 2 \times 10^{-7}\sin(\pi \times 10^3 x + 3\pi \times 10^{11} t)$T
Comparing it with standard equation,
$B = B_0\sin(kx + \omega t)$ T

We have, $\omega = 2\pi\upsilon = 3\times10^{11}\, t$ or $\upsilon = \frac{3\times10^{11}}{2}$ s^{-1}

Now since, $\frac{c}{\upsilon} = \lambda$

$$\Rightarrow \lambda = \frac{3\times10^8}{(3/2)\times10^{11}} = 2\times10^{-3} \text{ m}$$

18. **(c)** : Given, density of liquid ρ_l = 760 kg m^{-3}, density of mercury, ρ_m = 13600 kg m^{-3} and length of mercury h_m = 76 cm
Height of the liquid column, h = ?
Since $P_l = P_m$, $\rho_l g h = \rho_m g h_m$

$$\Rightarrow h = \frac{\rho_m h_m}{\rho_l} = \frac{13600\times76}{760} = 1360 \text{ cm} = 13.6 \text{ m}$$

19. **(d)** : Since the pressure remains constant, this is an isobaric process.

20. **(c)** : Efficiency, $\eta = 1 - \frac{T_2}{T_1}$

where, T_1 = temperature of the source
and T_2 = temperature of the sink

21. **(a)**

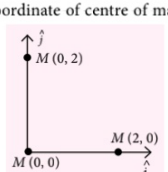
22. **(d)** : If the angle of contact is an obtuse angle $(\theta > 90°)$, then liquid will not wet the solid surface.

23. **(a)**

24. **(b)** : When iron rod is inserted into the inductor, its inductance (L) increases, thereby increasing the inductive reactance $X_L = \omega L$

Therefore, current $\left(I = \frac{V}{X_L}\right)$ decreases which results decrease in the brightness of the bulb.

25. **(d)** : Here, coordinate of centre of mass,



$$r_{cm} = \sum_{i=1}^{n} \frac{m_i r_i}{\sum m_i} = \frac{M(0) + M(2)\hat{j} + M(2)\hat{i}}{3M} = \frac{2}{3}(\hat{i} + \hat{j})$$

26. **(b)** : Given, kinnetic energy K = 144 eV

Then, the de-Broglie wave length,

$$l' = \lambda = \frac{h}{\sqrt{2mK}} = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times144\times1.6\times10^{-19}}}$$

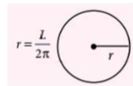
= 0.102 nm or 102 × 10^{-3} nm

27. **(b)** : $1' = \left(\frac{1}{60}\right)^\circ = \frac{1}{60}\times\frac{\pi}{180}$ radian $= 2.91\times10^{-4}$ rad

28. **(c)**

29. **(a)** : Since Magnetic moment, $M = NIA$

$\therefore$ $M = IA$ ($\because N = 1$)



Area, $A = \pi r^2 = \frac{\pi L^2}{4\pi^2} = \frac{L^2}{4\pi}$

$\therefore$ $M = IA = \frac{IL^2}{4\pi}$ A m^2

30. **(c)** : Acceleration due to gravity at a depth, d from Earth's surface,

$$g(d) = g\left(1 - \frac{d}{R}\right)$$

Given, $g(d) = \frac{1}{n} g$

$\therefore$ $g\left(1 - \frac{d}{R}\right) = \frac{1}{n} g$ or $d = R\frac{(n-1)}{n}$

31. **(b)** : Given $u = -1.5f$, $f = -f$
From mirror formula,

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ or $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

or $\frac{1}{v} = \frac{1}{-f} + \frac{1}{1.5f}$ $\Rightarrow v = -3f$

32. **(b)** : Given, ω_0 = 360 rpm

$\therefore$ $\omega_0 = 360 \times \frac{2\pi}{60} = 12\pi$ rad s^{-1}

$\omega = 1200 \text{ rpm} = 1200 \times \frac{2\pi}{60} = 40\pi$ rad s^{-1}

Now, $\omega = \omega_0 + \alpha t$

$\therefore$ $\alpha = \frac{\omega - \omega_0}{t} = \frac{40\pi - 12\pi}{14} = \frac{28\pi}{14} = 2\pi$ rad s^{-2}

33. **(d)** : Given, r = 1.6 × 10^{-10} m, m_e = 9 × 10^{-31} kg, e = 1.6 × 10^{-19} C
Force between the electron and proton,

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_p q_e}{r^2} = m_e a_e$$

$\therefore \quad a_e = \frac{F_e}{m_e} = \frac{1}{m_e}\left[\frac{1}{4\pi\varepsilon_0}\frac{q_p q_e}{r^2}\right]$

or $a_e = \frac{1}{9\times10^{-31}}\left[9\times10^9\times\frac{1.6\times10^{-19}\times(1.6\times10^{-19})}{(1.6\times10^{-10})^2}\right]$

$\therefore \quad a_e = 10^{22}\text{ m s}^{-2}$

34. (c)

35. (c) : Given, critical angle of medium, $C = 45°$

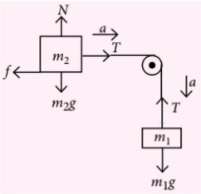
Since, $\mu = \frac{1}{\sin C}$

$\Rightarrow \frac{\text{velocity of light in vacuum, } c}{\text{velocity of light in medium, } v} = \frac{1}{\sin C}$

or $v = c\sin C = 3\times10^8\times\sin 45° = 3\times10^8\times\frac{1}{\sqrt{2}}$

or $v = \frac{3}{\sqrt{2}}\times10^8\text{ m s}^{-1}$

36. (b) :



From FBD, $m_1g - T = m_1a$ ...(i)

and $T - \mu_k m_2 g = m_2 a$ ...(ii)

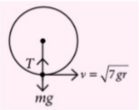
Solving eqn's (i) and (ii), we have,

$a = \frac{(m_1 - \mu_k m_2)g}{m_1 + m_2} = \frac{(2 - 0.05\times10)10}{10+2} = 1.25\text{ m s}^{-2}$

37. (d) : Here, $T - mg = \frac{mv^2}{r}$

or $T - mg = \frac{m}{r}(\sqrt{7gr})^2$

or $T = 7mg + mg$
$= 8mg$



38. (c) : Spherometer is used to measure the radius of curvature of the curved surfaces.

39. (c) : Initially, the capacitance of the capacitor,

$C_0 = \frac{A\varepsilon_0}{d}$

When a dielectric slab of thickness $\frac{d}{2}$ is introduced, new capacitance is,

$C_1 = \frac{A\varepsilon_0}{\left(d - t + \frac{t}{K}\right)} = \frac{A\varepsilon_0}{\left(d - \frac{d}{2} + \frac{d}{8}\right)} \qquad [\because K = 4]$

or $C_1 = \frac{8A\varepsilon_0}{5d}$

$\therefore \frac{C_1}{C_0} = \frac{8}{5}$

40. (a) : Given, $P = 10$ D and $R_1 = R_2 = 10$ cm

$\therefore f = \frac{100}{P}\text{ cm} = \frac{100}{10} = 10\text{ cm}$

From lens maker's formula,

$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \Rightarrow \frac{1}{10} = (\mu-1)\left[\frac{1}{10}\left(-\frac{1}{10}\right)\right]$

$\Rightarrow \mu = \frac{3}{2}$

41. (c) : For a positively charged spherical shell,

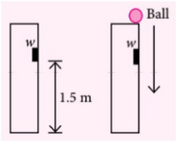
$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\ (r > R) \Rightarrow V \propto \frac{1}{r}$

and $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}(r \le R) \Rightarrow V = \text{constant}$

42. (d) : NAND gate is called the universal gate.

43. (d) **44. (a)**

45. (c) : The ball crosses the window in 0.1 s.



The length of the window = 1.5 m.

Then from equation of motion,

$S = ut + \frac{1}{2}at^2$

$1.5 = u\times0.1 + \frac{1}{2}\times10\times(0.1)^2$

$\Rightarrow u = 14.5\text{ m s}^{-1}$

SOLVED PAPER

Held on
5th Sep, Evening Shift

JEE MAIN 2020

1. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where $v(t)$ is its instantaneous velocity. The insta-ntaneous acceleration of the satellite is

(a) $-bv^3(t)$ (b) $-\frac{bv^3(t)}{M(t)}$

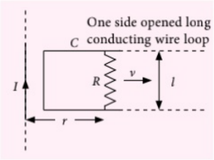
(c) $-\frac{2bv^3(t)}{M(t)}$ (d) $-\frac{bv^3(t)}{2M(t)}$

2. Ten charges are placed on the circumference of a circle of radius R with constant angular separation between successive charges. Alternate charges 1, 3, 5, 7, 9 have charge $(+q)$ each, while 2, 4, 6, 8, 10 have charge $(-q)$ each. The potential V and the electric field E at the centre of the circle are respectively
(Take $V = 0$ at infinity.)

(a) $V = \frac{10q}{4\pi\varepsilon_0 R}; E = 0$ (b) $V = 0; E = \frac{10q}{4\pi\varepsilon_0 R^2}$

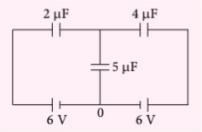
(c) $V = 0; E = 0$ (d) $V = \frac{10q}{4\pi\varepsilon_0 R}; E = \frac{10q}{4\pi\varepsilon_0 R^2}$

3. An infinitely long straight wire carrying current I, one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown in the figure. The connector has length l and resistance R. It slides to the right with a velocity v. The resistance of the conductor and the self inductance of the loop are negligible. The induced current in the loop, as a function of separation r, between the connector and the straight wire is



(a) $\frac{\mu_0}{4\pi}\frac{Ivl}{Rr}$ (b) $\frac{\mu_0}{\pi}\frac{Ivl}{Rr}$ (c) $\frac{2\mu_0}{\pi}\frac{Ivl}{Rr}$ (d) $\frac{\mu_0}{2\pi}\frac{Ivl}{Rr}$

4. In the circuit shown, charge on the 5 μF capacitor is



(a) 18.00 μC (b) 10.90 μC
(c) 16.36 μC (d) 5.45 μC

5. A galvanometer is used in laboratory for detecting, the null point in electrical experiments. If, on passing a current of 6 mA it produces a deflection of 2°, its figure of merit is close to

(a) 333° A div^{-1} (b) 6×10^{-3} A div^{-1}
(c) 666° A div^{-1} (d) 3×10^{-3} A div^{-1}

6. The correct match between the entries in column I and column II are

I Radiation	II Wavelength
(A) Microwave	(i) 100 m
(B) Gamma rays	(ii) 10^{-15} m
(C) A.M. radio waves	(iii) 10^{-10} m
(D) X-rays	(iv) 10^{-3} m

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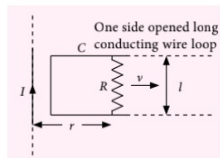

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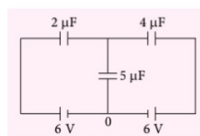

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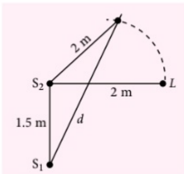
(a) (A)-(ii), (B)-(i), (C)-(iv), (D)-(iii)
(b) (A)-(i), (B)-(iii), (C)-(iv), (D)-(ii)
(c) (A)-(iii), (B)-(ii), (C)-(i), (D)-(iv)
(d) (A)-(iv), (B)-(ii), (C)-(i), (D)-(iii)

7. Two coherent sources of sound S_1 and S_2, produce sound waves of the same wavelength, $\lambda = 1$ m, in phase. S_1 and S_2 are placed 1.5 m apart (see figure). A listener, located at L, directly in front of S_2 finds that the intensity is at a minimum when he is 2 m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed when the listener is at a distance d from S_1. Then, d is



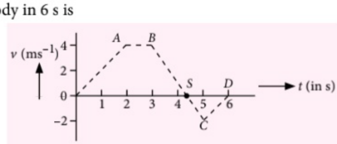
(a) 12 m (b) 5 m (c) 2 m (d) 3 m

8. Two different wires having lengths L_1 and L_2 and respective temperature coefficient of linear expansion α_1 and α_2 are joined end-to-end. Then the effective temperature coefficient of linear expansion is

(a) $\dfrac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$ (b) $2\sqrt{\alpha_1 \alpha_2}$

(c) $\dfrac{\alpha_1 + \alpha_2}{2}$ (d) $4\dfrac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}\dfrac{L_2 L_1}{(L_2 + L_1)^2}$

9. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6 s is



(a) $\dfrac{37}{3}$ m (b) 12 m (c) 11 m (d) $\dfrac{49}{4}$ m

10. In an experiment to verify Stoke's law, a small spherical ball of radius r and density ρ falls under gravity through a distance h in air before entering a tank of water. If the terminal velocity of the ball inside water is same as its velocity just before entering the water surface, then the value of h is proportional to (ignore viscosity of air)

(a) r^4 (b) r (c) r^3 (d) r^2

11. A parallel plate capacitor has plate of length 'l', width 'w' and separation of plates is 'd'. It is connected to a battery of emf V. A dielectric slab of the same thickness 'd' and of dielectric constant $K = 4$ is being inserted between the plates of the capacitor. At what length of the slab inside plates, will the energy stored in the capacitor be two times the initial energy stored?

(a) $2l/3$ (b) $l/3$ (c) $l/4$ (d) $l/2$

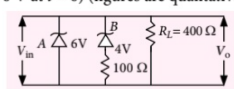
12. A driver in a car, approaching a vertical wall notices that the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 m s^{-1}, then the speed of the car is

(a) 54 km hr^{-1} (b) 36 km hr^{-1}
(c) 18 km hr^{-1} (d) 24 km hr^{-1}

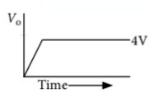
13. A ring is hung on a nail. It can oscillate, without slipping or sliding (i) in its plane with a time period T_1 and, (ii) back and forth in a direction perpendicular to its plane, with a period T_2. The ratio $\dfrac{T_1}{T_2}$ will be

(a) $\dfrac{2}{\sqrt{3}}$ (b) $\dfrac{2}{3}$ (c) $\dfrac{3}{\sqrt{2}}$ (d) $\dfrac{\sqrt{2}}{3}$

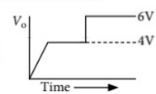
14. Two Zener diodes (A and B) having breakdown voltages of 6 V and 4 V respectively, are connected as shown in the circuit. The output voltage V_o variation with input voltage linearly increasing with time, is given by ($V_{input} = 0$ V at $t = 0$) (figures are qualitative)



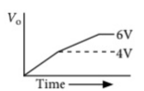
(a)



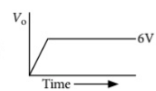
(b)



(c)



(d)



15. A radioactive nucleus decays by two different processes. The half life for the first process is 10 s and that for the second is 100 s. The effective half life of the nucleus is close to

(a) 9 s (b) 6 s (c) 55 s (d) 12 s

16. The quantities $x=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$, $y=\frac{E}{B}$ and $z=\frac{l}{CR}$ are defined where C-capacitance, R-resistance, l-length, E-electric field, B-magnetic field and ε_0, μ_0, –free space permittivity and permeability respectively. Then

(a) x, y and z have the same dimension
(b) only x and z have the same dimension
(c) only x and y have the same dimension
(d) only y and z have the same dimension.

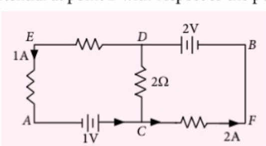
17. In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is

(a) 32 (b) 326 (c) 128 (d) $\frac{1}{32}$

18. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is ($h << R$, where R is the radius of the earth)

(a) $\frac{R^2\omega^2}{2g}$ (b) $\frac{R^2\omega^2}{g}$ (c) $\frac{R^2\omega^2}{4g}$ (d) $\frac{R^2\omega^2}{8g}$

19. In the circuit, given in the figure currents in different branches and value of one resistor are shown. Then potential at point B with respect to the point A is



(a) +2 V (b) –2V (c) –1 V (d) +1 V

20. An iron rod of volume 10^{-3} m^3 and relative permeability 1000 is placed as core in a solenoid with 10 turns cm^{-1}. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be

(a) 50×10^2 Am^2 (b) 5×10^2 Am^2
(c) 500×10^2 Am^2 (d) 0.5×10^2 Am^2

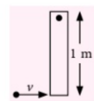
21. A body of mass 2 kg is driven by an engine delivering a constant power of 1 J s^{-1}. The body starts from rest and moves in a straight line. After 9 seconds, the body has moved a distance (in m)________ .

22. Nitrogen gas is at 300°C temperature. The temperature in (K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is __________ . (Molar mass of N_2 gas is 28 g)

23. A prism of angle $A = 1°$ has a refractive index $\mu = 1.5$. A good estimate for the minimum angle of deviation (in degrees) is close to $N/10$. Value of N is __________.

24. The surface of a metal is illuminated alternately with photons of energies E_1 = 4 eV and E_2 = 2.5 eV respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal in (eV) is __________.

25. A thin rod of mass 0.9 kg and length 1 m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of move 0.1 kg moving in a straight line with velocity 80 m s^{-1} hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad s^{-1}) of the rod immediately after the collision will be __________.



SOLUTIONS

1. **(b) :** Force acting on the spaceship,

$$F=\frac{d}{dt}(p)=\frac{d}{dt}(Mv)$$

or $F=M(t)\frac{dv}{dt}+v\frac{dM(t)}{dt}$

or $F=M(t)\frac{dv}{dt}+bv^3(t)$

Since, F = 0,

$\therefore \quad 0=M(t)\frac{dv}{dt}+bv^3(t)$

or $M(t)\frac{dv}{dt}=-bv^3(t)$ or $\frac{dv}{dt}=a=\frac{-bv^3(t)}{M(t)}$

2. **(c) :** $\sum Q=0$

Therefore, electric field and potential at the centre of the circle is zero. *i.e.*, E = 0; V = 0

3. **(d) :** Magnetic field due to the current carrying wire is

$$\vec{B}=\frac{\mu_0}{2\pi}\cdot\frac{I}{r}\otimes$$

where r is the separation between the wire and the connector.

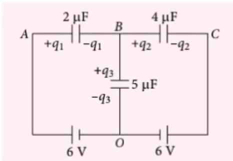
As B is same for the entire length of the connector. Thus, motional emf can be given as,

$$\varepsilon = \int (\vec{v} \times \vec{B}) \cdot \overline{dl} = Bvl$$

Therefore, induced current, $i = \frac{\varepsilon}{R}$

or $\quad i = \frac{Bvl}{R} = \frac{\mu_0}{2\pi} \frac{Ivl}{Rr}$

4. (c) :



Potential at point A, $V_A = 6$ V

Potential at point C, $V_C = 6$ V

Suppose, potential at point B, $V_B = x$

Now, at point B,

$-q_1 + q_2 + q_3 = 0$

or $\quad -2(V_A - V_B) + 4(V_B - V_C) + 5(V_B - V_O)$

or $\quad -2(6 - x) + 4(x - 6) + 5x = 0$

or $\quad -12 + 2x + 4x - 24 + 5x = 0$ or $11x = 36$

or $\quad x = \frac{36}{11}$ V

Now, charge across 5 μF capacitor, $q_3 = 5 \times (V_B - V_O)$

or $\quad q_3 = 5 \times \frac{36}{11} = 16.36\,\mu\text{C}$

5. (d) : Current sensitivity of a galvanometre $= \frac{\phi}{I}$

where ϕ is the deflection angle and I is the current. Figure of merit of a gavanometer is defined as the reciprocal of current sensititvity.

$\therefore \quad$ Figure of merit, $G = \frac{I}{\phi} = \frac{6 \times 10^{-3}\text{ A}}{2\text{ div}}$

$G = 3 \times 10^{-3}$ A div^{-1}

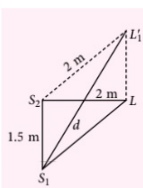
6. (d) : Microwave – 10^{-3} m

Gamma rays – 10^{-15} m

A.M radiowaves – 100 m

X-rays – 10^{-10} m

7. (d) : For coherent sound waves, minimum intensity is obtained when the path difference,



$$\Delta x = (2n + 1)\ \frac{\lambda}{2}$$

Here, $\Delta x = (S_1L) - (S_2L)$

$= \sqrt{(2)^2 + (1.5)^2} - 2$

$= 2.5 - 2 = 0.5$ m

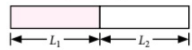
$0.5 = (2n + 1)\ \frac{\lambda}{2}$ or $n = 0$

For maximum intensity, $\Delta x = n\lambda$

$S_1L' - S_2L' = 1$

$d - 2 = 1$ or $d = 3$ m

8. (a) : Let the coefficient of linear expansion of two rods be α_1 and α_2 respectively.



For first rod, $\alpha_1 = \frac{\Delta L_1}{L_1 \times \Delta t} \quad \therefore\ \Delta L_1 = \alpha_1 L_1 \Delta t \qquad$ ...(i)

For second rod, $\alpha_2 = \frac{\Delta L_2}{L_2 \times \Delta t} \quad \therefore\ \Delta L_2 = \alpha_2 L_2\, \Delta t$ ...(ii)

For the series combination of the two rods, the equivalent temperature coefficient of linear expansion is

$$\alpha = \frac{\Delta L_1 + \Delta L_2}{(L_1 + L_2) \times \Delta t}$$

Putting the values of ΔL_1 and ΔL_2 from eqns. (i) and (ii), we have

$$\alpha = \frac{\alpha_1 L_1 \Delta t + \alpha_2 L_2 \Delta t}{(L_1 + L_2) \times \Delta t} \quad \text{or} \quad \alpha = \frac{L_1\alpha_1 + L_2\alpha_2}{L_1 + L_2}$$

9. (a) : Total distance covered = area under the $v - t$ graph

$$\therefore \quad S = \frac{1}{2}\left(1 + \frac{13}{3}\right) \times 4 + \frac{1}{2} \times \frac{5}{3} \times 2 \Rightarrow S = \frac{37}{3}\text{ m}$$

10. (a) : Velocity attained by the spherical ball while falling under gravity through a distance h, $v = \sqrt{2gh}$. Velocity attained by the spherical ball after entering into the water tank,

$$v_T = \frac{2}{9} \frac{r^2(\rho - \sigma)g}{\eta}$$

where $\quad \rho$ = density of the spherical ball

r = radius of the spherical ball

σ = density of fluid (water)

and η = coefficient of viscosity

According to the question, $v = v_T$

i.e., $\sqrt{2gh} = \frac{2}{9} \frac{r^2(\rho - \sigma)g}{\eta}$ or $h = \frac{2r^4(\rho - \sigma)^2 g}{81\eta^2}$

$\therefore \quad h$ is proportional to r^4.

11. (b) : Energy stored by a capacitor, $U = \frac{1}{2}CV^2$

Given that, $\frac{U_1}{U_2} = 2 \Rightarrow \frac{C_1}{C_2} = 2$

or $\frac{Kx+(l-x)}{l}=2$ or $4x + l - x = 2l$ $(\because K = 4)$

or $x=\frac{l}{3}$

12. (a) : Here, velocity of sound v = 345 m s^{-1}
Frequency of the horn, υ = 440 Hz
Frequency after reflection, υ' = 480 Hz
Let v_c be the speed of the car.

$\therefore\ \upsilon'=\upsilon\left(\frac{v+v_c}{v-v_c}\right) \Rightarrow 480=440\left(\frac{345+v_c}{345-v_c}\right)$

or $\frac{12}{11}=\frac{345+v_c}{345-v_c}$

$12 \times 345 - 12\ v_c = 11 \times 345 + 11\ v_c$
$4140 - 12\ v_c = 3795 + 11\ v_c$
$23\ v_c = 345 \Rightarrow v_c = 15$ m s^{-1}

or $v_c = 15\times\frac{18}{5} = 54$ km h^{-1}

13. (a) : Since the ring is oscillating in its plane, the moment of inertia

$I_1 = MR^2 + MR^2 = 2MR^2$

where M is the mass of the ring and R is its radius.
Now when the ring is oscillating perpendicular to its plane, the moment of inertia

$I_2=\frac{MR^2}{2}+MR^2=\frac{3}{2}MR^2$

Now the time period of oscillation, $T \propto \sqrt{I}$

$\therefore\ \frac{T_1}{T_2}=\sqrt{\frac{I_1}{I_2}}=\sqrt{\frac{2MR^2}{\frac{3}{2}MR^2}}=\sqrt{\frac{4}{3}}=\frac{2}{\sqrt{3}}$

14. (c)

15. (a) : Let decay constant for first process and second process be λ_1 and λ_2 respectively.

$\therefore\ \lambda_1=\frac{0.693}{t_1}$ and $\lambda_2=\frac{0.693}{t_2}$

where t_1 and t_2 be the half-life for first and second process respectively.

Total rate of decay, $-\frac{dN}{dt}=\lambda_1 N+\lambda_2 N$

or $-\frac{dN}{dt}=(\lambda_1+\lambda_2)N$ or $\lambda_{eq} = (\lambda_1 + \lambda_2)$

or $\frac{0.693}{t}=\frac{0.693}{t_1}+\frac{0.693}{t_2}$ or $\frac{1}{t}=\frac{1}{10}+\frac{1}{100}$

$\therefore\ t=\frac{100}{11}\approx 9$ sec

16. (a) : $x=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ = velocity of light (c) = [LT^{-1}]

$y=\frac{E}{B}$ = velocity of light (c) = [LT^{-1}]

$z=\frac{l}{CR}=\frac{l}{\text{Time constant}}=[\text{LT}^{-1}]$

17. (c) : For an adiabatic process, PV^γ = constant

$\therefore\ P_1V_1^\gamma = P_2V_2^\gamma$

or $\frac{P_2}{P_1}=\left(\frac{V_1}{V_2}\right)^\gamma$

or $\frac{P_2}{P_1}=\left(\frac{\rho_2}{\rho_1}\right)^\gamma$ $(\because$ density, $\rho=\frac{m}{V}$ and mass m is constant)

Given that, $\rho_2 = 32\rho_1$ and $P_2 = nP_1$

and $\gamma=\frac{7}{5}$ for diatomic gas

$\therefore\ n = (32)^{7/5} \Rightarrow n = (2)^7 = 128$

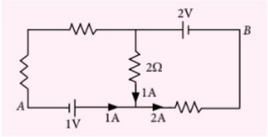
18. (a) : Acceleration due to gravity at equator,
$g_A = g - \omega^2 R$
Acceleration due to gravity at a height h aboves the poles,

$g_B=g\left(1-\frac{2h}{R}\right)$

Given that, $g_A = g_B$

$\therefore\ g-\omega^2R=g\left(1-\frac{2h}{R}\right)$ or $h=\frac{R^2\omega^2}{2g}$

19. (d) :



$V_B + 2 - 2 \times 1 - 1 = V_A$
$\therefore\ V_B - V_A = 1$V

20. (b) : Here, volume, $V = 10^{-3}$ m^3
Relative permeability, μ_r = 1000, Number of turns = 10
Current, i = 0.5 A
Magnetic moment of the rod, $M = (\mu_r - 1)\ NiA$

$M=(\mu_r-1)\,Ni\cdot\frac{V}{l}$ or $M=(\mu_r-1)\,\frac{N}{l}iV$

$M=999\times\frac{10}{10^{-2}}\times 0.5\times 10^{-3}$

or $M = 499.5 \approx 5.0 \times 10^2$ A m^2

21. (18) : Here, $P = 1$ J s^{-1}, $m = 2$ kg

$Pt = W = \Delta K$ or $Pt=\frac{1}{2}mv^2$

or $t = v^2 \Rightarrow v = \sqrt{t} \Rightarrow \frac{ds}{dt} = \sqrt{t}$ or $\int_0^s ds = \int_0^9 \sqrt{t}\, dt$

or $s = \frac{2}{3}\left[(t)^{3/2}\right]_0^9$ or $s = \frac{2}{3} \times 27 = 18$ m

22. (41.0) : Temperature of Nitrogen gas, $T_N = 300°C$ $= (300 + 273) = 573$ K

Molecular weight of Nitrogen $M_N = 14$

Molecular weight of Hydrogen $M_H = 1$

Now, rms speed of N_2 molecule, $r_N = \sqrt{\frac{3RT_N}{M_N}}$

rms speed of H_2 molecule, $r_H = \sqrt{\frac{3RT_H}{M_H}}$

Given that, $r_N = r_H$

$\therefore \sqrt{\frac{3RT_N}{M_N}} = \sqrt{\frac{3RT_H}{M_H}}$ or $\frac{T_N}{M_N} = \frac{T_H}{M_H}$ or $\frac{573}{14} = \frac{T_H}{1}$

or $T_H = 40.93 \text{ K} \approx 41.0$ K

23. (5) : For minimum deviation from a prism,

$\delta = (\mu - 1) A$; $\delta = (1.5 - 1) \times 1$

$\delta = 0.5 = \frac{5}{10}$ $\therefore N = 5$

24. (2.00) : According to Einstein's photoelectric equation,

$E - \frac{1}{2}mv^2 = \phi$

For first photon,

$4 - \frac{1}{2}mv_1^2 = \phi$ or $\frac{1}{2}mv_1^2 = 4 - \phi$ ...(i)

For second photon,

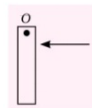
$2.5 - \frac{1}{2}mv_2^2 = \phi$; $\frac{1}{2}mv_2^2 = 2.5 - \phi$ ...(ii)

From eq. (i) and (ii), $\left(\frac{v_1}{v_2}\right)^2 = \frac{4 - \phi}{2.5 - \phi}$

$10 - 4\phi = 4 - \phi$

$3\phi = 6 \Rightarrow \phi = 2.00$ eV

25. (20) : Angular momentum will remain conserved about the pivotal point O,



$\therefore (0.1 \times 80 \times 1) = \left[\left(\frac{1}{10} \times 1\right) + \left(\frac{9}{10} \times \frac{1}{3}\right)\right]\omega$

or $\omega = \frac{8 \times 10}{4} = 20$ rad s^{-1}

CLASS-XI

BRUSH UP for

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Unit 5

GRAVITATION

THE UNIVERSAL LAW OF GRAVITATION

- According to Newton's law of gravitation, each body attracts other body with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
- Let m_1 and m_2 be the masses of two bodies and r be the separation between them.

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\Rightarrow \quad F = \frac{G m_1 m_2}{r^2}$$

Here, G is the constant of proportionality which is called universal gravitational constant. The value of G is 6.67×10^{-11} N m^2 kg^{-2}.

The direction of the force F is along the line joining the two particles.

- The gravitational force between two particles is independent of the presence of other bodies or the properties of the intervening medium.
- Gravitational force is a conservative force therefore work done in displacing a body from one place to another is independent of the path followed. It depends only on the initial and final positions.
- The gravitational force obeys Newton's third law *i.e.* $F_{12} = -F_{21}$
- Principle of superposition of gravitation : It states that the resultant gravitational force $\vec{F}$ acting on a particle due to number of other particles is equal to vector sum of the gravitational forces exerted by individual particle on the given particle.

i.e., $\vec{F} = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + ... + \vec{F}_{0n}$

$$= \sum_{i=1}^{n} \vec{F}_{0i}$$

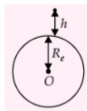
where $\vec{F}_{01}, \vec{F}_{02}, \vec{F}_{03}, ..., \vec{F}_{0n}$ are the gravitational forces on a particle of mass m_0 due to particles of masses $m_1, m_2, ..., m_n$ respectively.

GRAVITY

- It is defined as the force of attraction exerted by the earth towards its centre on a body lying on or near the surface of the earth.
- It is merely a special case of gravitation and is also called as earth's gravitational pull.
- It is the measure of weight of the body. The weight of the body
= mass (m) × acceleration due to gravity (g) = mg.
- The unit of weight of the body will be the same as that of force. It is a vector quantity. It is always directed towards the centre of the earth.

☞ VARIATION OF ACCELERATION DUE TO GRAVITY

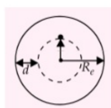
- Acceleration due to gravity on the surface of the earth is given by, $g = \frac{GM_e}{R_e^2}$
- **Effect of altitude :** Now, consider the body at a height h above the surface of the earth, then the acceleration due to gravity at height h given by



$$g_h = \frac{GM_e}{(R_e+h)^2} = g\left(1+\frac{h}{R_e}\right)^{-2}$$

$$\simeq g\left(1-\frac{2h}{R_e}\right) \text{when } h << R_e$$

  - ➤ The decrease in the value of g at the height h $= g - g_h = \frac{2gh}{R_e}$.
  - ➤ Then percentage decrease in the value of g $= \frac{g-g_h}{g}\times 100 = \frac{2h}{R_e}\times 100\%$
- **Effect of depth :** The gravitational pull on the surface is equal to its weight *i.e.*



$$mg = \frac{GM_e m}{R_e^2}$$

$$\therefore\ mg = \frac{G\times\frac{4}{3}\pi R_e^3 \rho m}{R_e^2}$$

$$\text{or } g = \frac{4}{3}\pi G R_e \rho \quad ...(i)$$

When the body is taken to a depth d, the mass of the sphere of radius $(R_e - d)$ will only be effective for the gravitational pull and the outward shell will have no resultant effect on the mass. If the acceleration due to gravity on the surface of the solid sphere is g_d, then

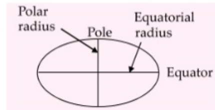
$$g_d = \frac{4}{3}\pi G(R_e - d)\rho \quad ...(ii)$$

By dividing equation (ii) by equation (i), we get

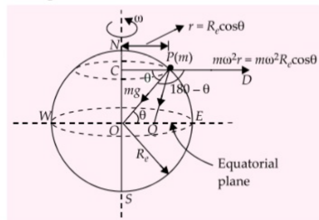
$$\Rightarrow g_d = g\left(1-\frac{d}{R_e}\right)$$

- **Effect of the position on the earth's surface :** The equatorial radius is about 21 km longer than its polar radius.

We know, $g = \frac{GM_e}{R_e^2}$, hence $g_{pole} > g_{equator}$. The weight of the body increases as the body taken from the equator to the pole.



- **Effect of rotation of the earth :** The earth rotates about its axis with angular velocity ω. Consider a particle of mass m at latitude θ. The angular velocity of the particle is also ω.



According to parallelogram law of vector addition, the resultant force acting on mass m along PQ is

$$F = [(mg)^2 + (m\omega^2 R_e\cos\theta)^2 + \{2mg\times m\omega^2 R_e\cos\theta\}\cos(180°-\theta)]^{1/2}$$

$$= [(mg)^2 + (m\omega^2 R_e\cos\theta)^2 - (2m^2 g\omega^2 R_e\cos\theta)\cos\theta]^{1/2}$$

$$= mg\left[1+\left(\frac{R_e\omega^2}{g}\right)^2\cos^2\theta - 2\frac{R_e\omega^2}{g}\cos^2\theta\right]^{1/2}$$

- ➤ At pole $\theta = 90° \Rightarrow g_{pole} = g$,
- ➤ At equator $\theta = 0°$

  $\Rightarrow g_{equator} = g\left[1-\frac{R_e\omega^2}{g}\right]$.

  Hence $g_{pole} > g_{equator}$
- ➤ If the body is taken from pole to the equator, then change in acceleration due to gravity

  $\Delta g = \frac{R_e\omega^2}{g}$
- ➤ Hence % change in weight of a body

$$= \frac{mg - mg\left(1-\frac{R_e\omega^2}{g}\right)}{mg}\times 100 = \frac{mR_e\omega^2}{mg}\times 100$$

$$= \frac{R_e\omega^2}{g}\times 100$$

KEPLER'S LAWS OF PLANETARY MOTION

- **First law (law of orbits) :** All planets move in elliptical orbits with the sun situated at one of the foci of the ellipse.
- **Second law (law of areas) :** The radius vector drawn from the sun to the planet sweeps out equal areas in equal intervals of time *i.e.* the areal velocity of the planet (or the area swept out by the planet per unit time) around the sun is constant *i.e.*, areal velocity $= \frac{d\vec{A}}{dt} =$ a constant, for a planet.
  - Angular momentum $(\vec{L})$ of a planet is related with areal velocity $\left(\frac{d\vec{A}}{dt}\right)$ by the relation $\vec{L} = 2m\left(\frac{d\vec{A}}{dt}\right)$
  - Kepler's second law follows from the law of conservation of angular momentum.
  - The area covered by the radius vector in dt seconds $= \frac{1}{2}r^2\,d\theta$.
    The areal velocity $= \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}r^2\omega = \frac{1}{2}rv.$
  - According to Kepler's second law, the speed of the planet is maximum, when it is closest to the sun and is minimum when the planet is farthest from the sun.
- **Third law (law of periods) :** The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semi major axis of the elliptical orbit *i.e.* $T^2 \propto a^3$ where a is the semi major axis of the elliptical orbit of the planet around the sun.

GRAVITATIONAL FIELD

- The space around a material body in which its gravitational pull can be experienced is called its gravitational field.
- The intensity of the gravitational field of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field. It is denoted by symbol E.
- The intensity of gravitational field at a point due to a body of mass M, at a distance r from the centre of the body is, $E = -\frac{GM}{r^2}$

  where negative sign shows that the gravitational intensity is of attractive force.
- Intensity of gravitational field is a vector quantity. Its dimensional formula is $[M^0LT^{-2}]$.
- Unit of intensity of gravitational field in SI system is N kg^{-1} and in CGS system is dyne g^{-1}.

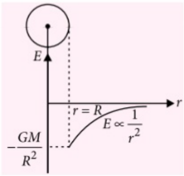
GRAVITATIONAL POTENTIAL

- The gravitational potential at a point in the gravitational field of a body is defined as the amount of work done in bringing a unit mass from infinity to that point. It is denoted by symbol V.
- The gravitational potential at a point in the gravitational field due to a body of mass M at a distance r from the centre of the body is given by
  $V = -\frac{GM}{r}$
- Gravitational potential is a scalar quantity. Its dimensional formula is $[M^0L^2T^{-2}]$.
- Unit of gravitational potential in SI system is J kg^{-1} and in CGS system is erg g^{-1}.
- Gravitational potential (V) is related with gravitational field intensity (E) by a relation
  $E = -\frac{dV}{dr}$

GRAVITATIONAL FIELD AND POTENTIAL OF SOME CONTINUOUS MASS DISTRIBUTIONS

- Uniform ring of mass M and radius R
  - Gravitational field on the axis,
    $$E = -\frac{GMx}{(R^2+x^2)^{3/2}}$$
  - Gravitational potential on the axis,
    $$V = -\frac{GM}{(R^2+x^2)^{1/2}}$$
- Uniform disc of mass M and radius R
  - Gravitational field on the axis,
    $$E = -\frac{2GM}{R^2}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]$$
  - Gravitational potential on the axis,
    $$V = -\frac{2GM}{R^2}[x-\sqrt{R^2+x^2}]$$

- Thin spherical shell of mass M and radius R
  - Gravitational field at a distance r from centre:

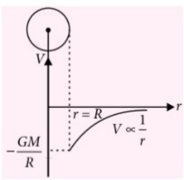


(i) Inside the shell, $E(r < R) = 0$

(ii) On the surface of the shell, $E(r = R) = -\frac{GM}{R^2}$

(iii) Outside the shell, $E(r > R) = -\frac{GM}{r^2}$

  - Gravitational potential at a distance r from centre:



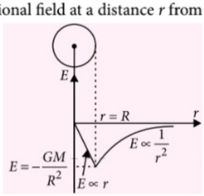
(i) Inside the shell, $V(r < R) = -\frac{GM}{R}$

(ii) On the surface of shell, $V(r = R) = -\frac{GM}{R}$

(iii) Outside the shell, $V(r > R) = -\frac{GM}{r}$

Note that field intensity inside the shell is zero. Field intensity and potential on the surface or outside points can be calculated by assuming the entire mass of the shell to be concentrated at its centre

- A solid sphere of mass M and radius R, with uniform mass density
  - Gravitational field at a distance r from centre:



(i) Inside the solid sphere, $E(r < R) = -\frac{GMr}{R^2}$

(ii) On the surface of the sphere,

$$E(r = R) = -\frac{GM}{R^2}$$

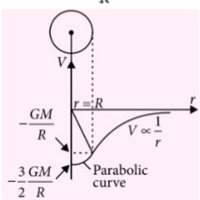
(iii) Outside the sphere, $E(r > R) = -\frac{GM}{r^2}$

  - Gravitational potential at a distance r from the centre:

(i) Inside the sphere, $V(r < R) = -\frac{GM}{2R^3}(3R^2 - r^2)$

(ii) On the surface of the sphere,

$$V(r = R) = -\frac{GM}{R}$$



(iii) Outside the sphere, $V(r > R) = -\frac{GM}{r}$

(iv) At the centre of the sphere,

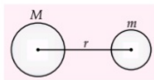
$$V(r = 0) = -\frac{3}{2}\frac{GM}{R}$$

☞ GRAVITATIONAL POTENTIAL ENERGY

- The gravitational potential energy of a body at a point in a gravitational field of another body is defined as the amount of work done in bringing the given body from infinity to that point.

  Gravitational potential energy
  = Gravitational potential × mass of the body

- The gravitational potential energy of mass m in the gravitational field of mass M at a distance r from it is



  $$U = -\frac{GMm}{r}$$

  where, r is the distance between M and m.

- The gravitational potential energy of a mass m at a distance r (> R_e) from the centre of the earth is

  $$U = mV = -\frac{GM_e m}{r}$$

- Gravitational potential energy of a mass at infinite distance from the earth is zero.
- Gravitational potential energy is a scalar quantity. Its dimensional formula is $[ML^2T^{-2}]$ and SI unit is J.
- Gravitational potential energy of a body of mass m at height h above the earth's surface is given by

  $$U_h = \frac{-GM_e m}{(R_e + h)}$$

- Gravitational potential energy of a body of mass m on the earth's surface is given by

  $$U_s = \frac{-GM_e m}{R_e}$$

- The change in potential energy when a body of mass m is moved vertically upwards through a height h from the earth's surface is given by

  $$\Delta U = U_h - U_s = GM_e m\left[\frac{1}{R_e} - \frac{1}{R_e + h}\right]$$

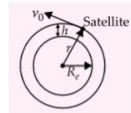
  $$= \frac{GM_e mh}{R_e^2\left(1 + \frac{h}{R_e}\right)} = \frac{mgh}{\left(1 + \frac{h}{R_e}\right)} \quad \left(\because g = \frac{GM_e}{R_e^2}\right)$$

  ➢ For $h << R_e$, $\Delta U = mgh$.

☞ SATELLITE

- Satellite is natural or artificial body describing orbit around a planet under its gravitational attraction. Moon is a natural satellite while INSAT-1B is an artificial satellite of the earth.

- Orbital speed of the satellite, when it is revolving around the earth at a height h is given by



  $$v_o = \sqrt{\frac{GM_e}{r}} = \sqrt{\frac{GM_e}{R_e + h}}$$

  $$= R_e\sqrt{\frac{g}{R_e + h}} \qquad \left(\text{As } g = \frac{GM_e}{R_e^2}\right)$$

  ➢ When the satellite is orbiting close to the earth's surface, *i.e.*, $h << R_e$, then

  $$v_o = R_e\sqrt{\frac{g}{R_e}} = \sqrt{gR_e}$$

  $$v_o = \sqrt{9.8 \times 6.4 \times 10^6} = 7.92 \times 10^3 \text{ m s}^{-1}$$

  $$\approx 8 \text{ km s}^{-1}$$

  ➢ The orbital speed of the satellite is independent of the mass of the satellite.

  ➢ The orbital speed of the satellite depends upon the mass and radius of the earth/planet around which the revolution of satellite is taking place.

  ➢ The direction of orbital speed of the satellite at an instant is along the tangent to the orbital path of satellite at that instant.

- **Time period of a satellite :** It is the time taken by satellite to complete one revolution around the earth and it is given by

  $$T = \frac{2\pi r}{v_o} = 2\pi\sqrt{\frac{r^3}{GM_e}} = 2\pi\sqrt{\frac{(R_e + h)^3}{GM_e}}$$

  $$= \frac{2\pi}{R_e}\sqrt{\frac{(R_e + h)^3}{g}}$$

  ➢ For a satellite orbiting close to the earth's surface *i.e.* $h << R_e$

  $$T = 2\pi\sqrt{\frac{R_e}{g}} = 84.6 \text{ min}.$$

  ➢ The period of revolution of the satellite depends upon its height above earth's surface. Larger is the height of the satellite, the greater will be its time period of revolution.

- Height of satellite above the earth's surface

  $$h = \left(\frac{T^2 R_e^2 g}{4\pi^2}\right)^{1/3} - R_e$$

- Kinetic energy of a satellite

$$K = \frac{1}{2}mv_o^2 = \frac{1}{2}\frac{GM_em}{r} = \frac{1}{2}\frac{GM_em}{(R_e + h)}$$

- Potential energy of a satellite

$$U = -\frac{GM_em}{r} = -\frac{GM_em}{R_e + h}$$

- Total energy (mechanical) of a satellite

$$E = K + U = -\frac{GM_em}{2r} = -\frac{GM_em}{2(R_e + h)}$$

  - For satellite orbiting very close to the surface of earth *i.e.*, $h << R_e$ then $E = -\frac{GM_em}{2R_e}$.

- Kinetic energy of a satellite is equal to negative of total energy while potential energy is equal to twice the total energy.
  i.e. $K = -E$, $U = 2E$
- Binding energy of a satellite

$$E_B = -E = \frac{GM_em}{2r} = \frac{GM_em}{2(R_e + h)}.$$

- Angular momentum of a satellite

$$L = mv_or = mr\sqrt{\frac{GM_e}{r}} = [m^2rGM_e]^{1/2}$$

  - Angular momentum of a satellite depends on both, mass of the satellite (m) and mass of the earth (M_e). It also depends upon the radius of the orbit (r) of the satellite.
  - Angular momentum is conserved in the motion of satellite.

ESCAPE SPEED

- The escape speed on earth (or any planet) is defined as the minimum speed with which a body has to be projected vertically upwards from the surface of earth (or any other planet) so that it just crosses the gravitational field of earth (or of that planet) and never returns on its own. Escape speed v_e is given by

$$v_e = \sqrt{\frac{2GM}{R}}$$

where M = Mass of the earth/planet
R = Radius of the earth/planet

$$v_e = \sqrt{\frac{2G \times \text{volume} \times \text{density}}{R}}$$

or $$v_e = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3\rho} = \sqrt{\frac{8\pi\rho GR^2}{3}}$$

For earth, $v_e = 11.2$ km s^{-1}.

- The escape speed depends upon the mass and radius of the earth/planet from the surface of which the body is to be projected.
- The escape speed is independent of the mass and direction of projection of the body from the surface of earth/planet.
- For a point close to earth's surface the escape speed and orbital speed are related as
  $v_e = \sqrt{2}\, v_o$
- A given planet will have atmosphere if the root mean square speed of molecules in its atmosphere (*i.e.*, $v_{rms} = \sqrt{3RT / M}$) is smaller than the escape speed for that planet.
- Moon has no atmosphere because the r.m.s. speed of gas molecules there, are greater than the escape speed of moon.

WRAP it up!

1. A satellite is moving in a circular orbit at a certain height above the earth's surface. It takes 5.26×10^3 s to complete one revolution with a centripetal acceleration equal to 9.32 m s^{-2}. The height of the satellite orbit above the earth's surface is (Radius of earth = 6.37×10^6 m)

(a) 70 km (b) 160 km
(c) 190 km (d) 220 km

2. A synchronous satellite goes around the earth once in every 24 h. What is the radius of orbit of the synchronous satellite in terms of the earth's radius? (Given mass of the earth, $M_e = 5.98 \times 10^{24}$ kg, radius of the earth, $R_e = 6.37 \times 10^6$ m, universal constant of gravitation, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}).

(a) 2.4 R_e (b) 3.6 R_e
(c) 4.8 R_e (d) 6.6 R_e

3. A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The gravitational potential at a point situated at distance $\frac{a}{2}$ from the centre, will be

(a) $-\frac{3GM}{a}$ (b) $-\frac{2GM}{a}$

(c) $-\frac{GM}{a}$ (d) $-\frac{4GM}{a}$

4. In the solar system, sun is in the focus of system for sun-earth binding system. Then the binding energy for the system will be
(Given that the radius of the earth orbit round the sun is 1.5×10^{11} m, mass of the earth is 6×10^{24} kg, mass of the sun is 10^{30} kg)

(a) 2.7×10^{33} J (b) 1.3×10^{33} J

(c) 2.7×10^{30} J (d) 1.3×10^{30} J

5. A saturn year is 29.5 times the earth year. How far is the saturn from the sun if the earth is 1.5×10^8 km away from the sun?

(a) 1.2×10^9 km (b) 1.3×10^9 km

(c) 1.4×10^9 km (d) 1.5×10^9 km

6. Two satellites of earth, S_1 and S_2 are moving in the same orbit. The mass of S_1 is four times the mass of S_2. Which one of the following statements is true?

(a) The potential energies of earth and satellite in the two cases are equal.

(b) S_1 and S_2 are moving with the same speed.

(c) The kinetic energies of the two satellites are equal.

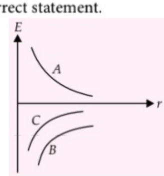
(d) The time period of S_1 is four times that of S_2.

7. The escape velocity of a body from the surface of earth is 11.2 km s^{-1}. A body is projected with a velocity of 22.4 km s^{-1}. Velocity of the body at infinite distance from the centre of the earth would be

(a) 11.2 km s^{-1} (b) zero

(c) $11.2\sqrt{3}$ km s^{-1} (d) $11\sqrt{2}$ km s^{-1}

8. Figure shows the variation of energy E with the orbital radius r of a satellite in a circular motion. Mark the correct statement.



(a) A shows the kinetic energy, B shows the total energy and C the potential energy of the satellite.

(b) A and B are the kinetic energy and potential energy respectively and C the total energy of the satellite.

(c) A and B are the potential energy and kinetic energy respectively and C the total energy of the satellite.

(d) C and A are the kinetic and potential energies and B the total energy of the satellite.

9. A ball is thrown vertically upwards with a velocity equal to half the escape velocity from the surface of the earth. The ball rises to a height h above the surface of the earth. If the radius of the earth is R_e, then the ratio $\frac{h}{R_e}$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 2 (d) 3

10. If r denotes the distance between the sun and the earth, then the angular momentum of the earth around the sun is proportional to

(a) $r^{3/2}$ (b) r (c) $\sqrt{r}$ (d) r^2

11. Two satellites of masses m_1 and m_2 ($m_1 > m_2$) are revolving around the earth in a circular orbit of radii r_1 and r_2 ($r_1 > r_2$) respectively. Which of the following statements is true regarding their speeds v_1 and v_2?

(a) $v_1 = v_2$ (b) $v_1 > v_2$

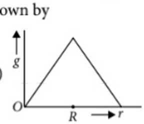
(c) $v_1 < v_2$ (d) $\frac{v_1}{r_1} = \frac{v_2}{r_2}$

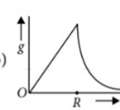
12. Four particles each of mass M, are located at the vertices of a square with side L. The gravitational potential due to this at the centre of the square is

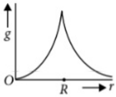
(a) $-\sqrt{32}\frac{GM}{L}$ (b) $-\sqrt{64}\frac{GM}{L^2}$

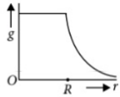
(c) zero (d) $\sqrt{32}\frac{GM}{L}$

13. Starting from the centre of the earth having radius R, the variation of g(acceleration due to gravity) is shown by

(a) 

(b) 

(c) 

(d) 

14. A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0 the value of acceleration due to gravity at the earth's surface is

(a) $\frac{mg_0R^2}{2(R+h)}$ (b) $-\frac{mg_0R^2}{2(R+h)}$

(c) $\frac{2mg_0R^2}{R+h}$ (d) $-\frac{2mg_0R^2}{R+h}$

15. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is

(a) $1:4$ (b) $1:\sqrt{2}$ (c) $1:2$ (d) $1:2\sqrt{2}$

16. A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass = 5.98×10^{24} kg) have to be compressed to be a black hole?

(a) 10^{-9} m (b) 10^{-6} m (c) 10^{-2} m (d) 100 m

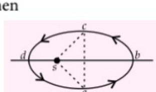
17. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; $h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.)

(a) $\sqrt{2gR}$ (b) $\sqrt{gR}$

(c) $\sqrt{gR/2}$ (d) $\sqrt{gR}\,(\sqrt{2}-1)$

18. Figure shows elliptical path $abcd$ of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (See figure) With db as the semi-major axis, and ca as the semi-minor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then

(a) $t_1 = 4t_2$
(b) $t_1 = 2t_2$
(c) $t_1 = 3t_2$
(d) $t_1 = t_2$



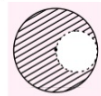
19. An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is

(a) Zero since astronaut feels weightless

(b) $\frac{GMm}{(R+h)^2} < F_G < \frac{GMm}{R^2}$

(c) $F_G = \frac{GMm}{(R+h)^2}$ (d) $0 < F_G < \frac{GMm}{R^2}$

20. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is (G = gravitational constant)



(a) $\frac{-2GM}{3R}$ (b) $\frac{-2GM}{R}$

(c) $\frac{-GM}{2R}$ (d) $\frac{-GM}{R}$

SOLUTIONS

1. (b): Time period of revolution of satellite

$$T = 2\pi\sqrt{\frac{(R_e+h)^3}{GM_e}}$$

or $$\frac{T^2}{4\pi^2} = \frac{(R_e+h)^3}{GM_e} \quad ...(i)$$

Centripetal acceleration, $a = \frac{GM_e}{(R_e+h)^2}$

or $$\frac{(R_e+h)^2}{GM_e} = \frac{1}{a} \quad ...(ii)$$

Divide (i) by (ii), we get

$$(R_e+h) = \frac{T^2}{4\pi^2}\times a = \left(\frac{5.26\times10^3}{2\pi}\right)^2 \times 9.32$$

$R_e + h = 6.53 \times 10^6$ m

$h = 6.53 \times 10^6 \text{ m} - 6.37 \times 10^6 \text{ m} = 0.16 \times 10^6$ m

$= 160 \times 10^3$ m $= 160$ km

2. (d): Time period of revolution of satellite

$$T = 2\pi\sqrt{\frac{R^3}{GM_e}}$$

Also, $$g = \frac{GM_e}{R_e^2} \quad ...(i)$$

$$\therefore \quad T^2 = \frac{4\pi^2R^3}{gR_e^2} \quad \text{(Using (i))}$$

Substituting the given values in above equation, we get

$$(24\times60\times60)^2 = \frac{4\times(3.14)^2R^3}{9.8R_e^2}$$

$R = 4.22\times10^7$ m

$$\frac{R}{R_e} = \frac{4.22\times10^7}{6.37\times10^6} = 6.6 \text{ or } R = 6.6R_e$$

3. **(a) :** Gravitational potential due to the shell of radius a at any point inside it $= -\frac{GM}{a}$

Gravitational potential due to the particle at the centre at a point P distant $\frac{a}{2}$ from the centre

$$= -\frac{GM}{a/2} = -\frac{2GM}{a}$$

$\therefore$ Net gravitational potential at P

$$= -\frac{GM}{a} - \frac{2GM}{a} = -\frac{3GM}{a}$$

4. **(b) :** Binding energy = – total energy of system

$$= \frac{GM_sM_e}{2R}$$

Mass of sun, $M_s = 10^{30}$ kg

Mass of earth, $M_e = 6 \times 10^{24}$ kg

Radius, $R = 1.5 \times 10^{11}$ m

Binding energy of the system

$$= \frac{6.67\times10^{-11}\times10^{30}\times6\times10^{24}}{2\times1.5\times10^{11}} = 1.3\times10^{33} \text{ J}$$

5. **(c) :** Here, $T_s = 29.5\ T_e$ and $R_e = 1.5 \times 10^8$ km

According to Kepler's third law, $T^2 \propto R^3$

$$\therefore \frac{T_s^2}{T_e^2} = \frac{R_s^3}{R_e^3} \text{ or } R_s = R_e\left(\frac{T_s}{T_e}\right)^{2/3}$$

$$R_s = 1.5\times10^8\left(\frac{29.5\,T_e}{T_e}\right)^{2/3} = 1.4 \times 10^9 \text{ km}$$

6. **(b) :** Both, orbital speed of satellite $v_o = \sqrt{GM_e/r}$ and time period of revolution of satellite, $T = \left[\frac{4\pi^2 r^3}{GM_e}\right]^{1/2}$ are independent of mass of satellite. Therefore orbital speed and time period of revolution of both the satellites are same.

Hence option (b) is correct.

The kinetic energy of a satellite, $K = \frac{GM_e m}{2r}$ and potential energy of a satellite, $U = -\frac{GM_e m}{r}$ both depend on the mass of satellite.

7. **(c) :** Total energy at earth's surface = Energy at infinity

$$\frac{1}{2}mv_i^2 - \frac{GM_e m}{R_e} = \frac{1}{2}mv_f^2$$

If v is the velocity of the body at infinite distance from the centre of the earth and u is the velocity of projection of body, then

$$\frac{1}{2}mu^2 - \frac{1}{2}mv_e^2 = \frac{1}{2}mv^2$$

$$v^2 = u^2 - v_e^2$$

$$\text{or } v = \sqrt{u^2 - v_e^2} = \sqrt{(22.4)^2 - (11.2)^2}$$

$$= 11.2\sqrt{3} \text{ km s}^{-1}$$

8. **(b) :** $\text{K.E} = \frac{GMm}{2r}$; $\text{P.E.} = -\frac{GMm}{r}$; $\text{T.E.} = -\frac{GMm}{2r}$

$\therefore$ K.E. is always positive and $\text{K.E.} \propto \frac{1}{r}$

P.E. is always negative and $\text{P.E.} \propto \frac{1}{r}$

T.E. is also negative and $\text{T.E.} \propto \frac{1}{r}$

Also T.E. < P.E.

Thus the curve A represents K.E., curve B represents P.E. and curve C represents T.E. of the satellite.

9. **(b) :** Here, $\frac{1}{2}mv^2 - \frac{GM_e m}{R_e} = -\frac{GM_e m}{(R_e + h)}$

$$\text{or } v^2 = \frac{2GM_e}{R_e}\left(\frac{h}{R_e + h}\right) \quad \text{...(i)}$$

The escape velocity, $v_e = \left(\frac{2GM_e}{R_e}\right)^{1/2}$

and $v = \frac{v_e}{2}$ (given)

Using these in (i), we get

$$\frac{1}{4}\frac{2GM_e}{R_e} = \frac{2GM_e}{R_e}\left(\frac{h}{R_e+h}\right) \text{ or } h = \frac{R_e}{3} \text{ or } \frac{h}{R_e} = \frac{1}{3}$$

10. **(c) :** Angular momentum of the earth around the sun is

$L = M_e v_o r$

$$= M_e\sqrt{\frac{GM_s}{r}}\,r \qquad \left(\because v_o = \sqrt{\frac{GM_s}{r}}\right)$$

$$\therefore L = \left[M_e^2 GM_s r\right]^{1/2}$$

where, M_e = mass of the earth

M_s = mass of the sun

r = distance between the sun and the earth

$\therefore \quad L \propto \sqrt{r}$

11. **(c) :** The speed of a satellite of mass m revolving around the earth in a circular orbit of radius r is given by

$v = \sqrt{\frac{GM_e}{r}}$ where M_e is the mass of the earth.

It does not depend upon the mass of the satellite.

Since,

$$v \propto \frac{1}{\sqrt{r}} \quad \therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}}$$

$$\text{As } r_1 > r_2 \ \therefore \ \frac{v_1}{v_2} < 1 \text{ or } v_1 < v_2$$

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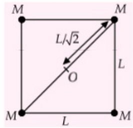
12. (a) : Gravitational potential at the centre is

$$U = -4\left(\frac{GM}{L/\sqrt{2}}\right)$$

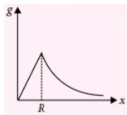
$$= -\frac{4\sqrt{2}\,GM}{L}$$

$$= -\sqrt{2\times16}\,\frac{GM}{L} = -\sqrt{32}\,\frac{GM}{L}$$



13. (b): Acceleration due to gravity

$$g = \begin{cases} \frac{GM}{R^3}x & ; \ x < R \\ \frac{GM}{x^2} & ; \ x \ge R \end{cases}$$



14. (b): Total energy of satellite at height h from the earth surface,

$$E = \text{PE} + \text{KE}$$

$$= -\frac{GMm}{(R+h)} + \frac{1}{2}mv^2 \quad \text{...(i)}$$

Also, $\frac{mv^2}{(R+h)} = \frac{GMm}{(R+h)^2}$

or, $v^2 = \frac{GM}{R+h}$ ...(ii)

From eqns. (i) and (ii),

$$E = -\frac{GMm}{(R+h)} + \frac{1}{2}\frac{GMm}{(R+h)} = -\frac{1}{2}\frac{GMm}{(R+h)}$$

$$= -\frac{1}{2}\frac{GM}{R^2}\times\frac{mR^2}{(R+h)} = -\frac{mg_0R^2}{2(R+h)} \quad \left(\because \ g_0 = \frac{GM}{R^2}\right)$$

15. (d): As escape velocity,

$$v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}\cdot\frac{4\pi R^3}{3}\rho} = R\sqrt{\frac{8\pi G}{3}\rho}$$

$$\therefore \quad \frac{v_e}{v_p} = \frac{R_e}{R_p}\times\sqrt{\frac{\rho_e}{\rho_p}}$$

$$= \frac{1}{2}\times\sqrt{\frac{1}{2}} = \frac{1}{2\sqrt{2}} \quad (\because R_p = 2R_e \text{ and } \rho_p = 2\rho_e)$$

16. (c) : The earth will become black hole if the escape velocity on earth is equal to the velocity of light.

i.e. $v_e = c$

or $\sqrt{\frac{2GM}{R}} = c$ or $R = \frac{2GM}{c^2}$

$$R = \frac{2\times6.67\times10^{-11}\ \text{N m}^2\ \text{kg}^{-2}\times5.98\times10^{24}\,\text{kg}}{(3\times10^8\ \text{m s}^{-1})^2}$$

$= 8.86\times10^{-3}$ m $\approx 10^{-2}$ m

17. (d): Orbital velocity of the satellite,

$$v_o = \sqrt{\frac{GM}{R+h}}, \ v_o \approx \sqrt{\frac{GM}{R}} \quad (\because h << R)$$

Let v_e be the minimum velocity required by the satellite to escape from its orbit.

$$\therefore \quad \frac{1}{2}mv_e^2 = \frac{GmM}{R+h}$$

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM}{R+h}} \approx \sqrt{\frac{2GM}{R}} \quad (\because h << R)$$

so, required increment in the orbital velocity

$$= v_e - v_o = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}}$$

$$= \sqrt{\frac{GM}{R}}(\sqrt{2}-1) = \sqrt{gR}(\sqrt{2}-1)$$

18. (c) : Let the area of the ellipse be A.

As per Kepler's 2nd law, areal velocity of a planet around the sun is constant, *i.e.*, $\frac{dA}{dt}$ = constant.

$$\therefore \quad \frac{t_1}{t_2} = \frac{\text{Area of } abcsa}{\text{Area of } adcsa} = \frac{\frac{A}{2}+\frac{A}{4}}{\frac{A}{2}-\frac{A}{4}} = \frac{\frac{3A}{4}}{\frac{A}{4}} = 3$$

$\Rightarrow \quad t_1 = 3t_2$

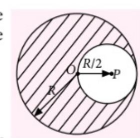
Note : Here db is the major axis of the ellipse, not semi-major axis and ca is the minor axis of the ellipse, not semi-minor axis.

19. (c) : Gravitational pull on the astronaut

$$F_G = \frac{GMm}{(R+h)^2}$$

Net force on the astronaunt is zero.

20. (d): Potential at point P (centre of cavity) before removing the spherical portion,



$$V_1 = \frac{-GM}{2R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right)$$

$$= \frac{-GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = \frac{-11GM}{8R}$$

Mass of spherical portion to be removed, $M' = \frac{MV'}{V}$

$$= \frac{M\frac{4\pi}{3}\left(\frac{R}{2}\right)^3}{\frac{4\pi}{3}R^3} = \frac{M}{8}$$

Potential at point P due to spherical portion to be removed

$$V_2 = \frac{-3GM'}{2R'} = \frac{-3G(M/8)}{2(R/2)} = \frac{-3GM}{8R}$$

$\therefore$ Potential at the centre of cavity formed

$$V_P = V_1 - V_2 = \frac{-11GM}{8R} - \left(\frac{-3GM}{8R}\right) = \frac{-GM}{R}$$

CBSE warm-up!

CLASS-XI

Practice questions for CBSE Exams as per the reduced syllabus, latest pattern and marking scheme issued by CBSE for the academic session 2020-21.

Series 3

CHAPTERWISE PRACTICE PAPER :
System of Particles and Rotational Motion | Gravitation

Time Allowed : 3 hours
Maximum Marks : 70

GENERAL INSTRUCTIONS

(1) All questions are compulsory. There are 33 questions in all.
(2) This question paper has five sections: Section A, Section B, Section C, Section D and Section E.
(3) Section A contains ten very short answer questions and four assertion reasoning MCQs of 1 mark each, Section B has two case based questions of 4 marks each, Section C contains nine short answer questions of 2 marks each, Section D contains five short answer questions of 3 marks each and Section E contains three long answer questions of 5 marks each.
(4) There is no overall choice. However internal choice is provided. You have to attempt only one of the choices in such questions.

Section - A

1. Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius, about an axis passing through their axes of symmetry ?

2. Why are space rockets usually launched from west to east in the equatorial plane ?

OR

What would be the weight of the body inside the earth if it were a hollow sphere ?

3. What is the angular velocity of the second's hand of the clock?

4. Does the centre of mass of a solid necessarily lie within the body? If not, give an example.

OR

Why do we prefer to use a wrench of longer arm?

5. If the earth were hollow, but still had the same mass and radius, would your weight be different?

6. Why do different planets have different escape velocity?

7. What is a parking orbit?

OR

Where should a geostationary satellite be launched?

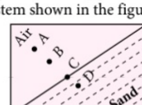
8. Show that centre of mass of an isolated system moves with a uniform velocity along a straight line path.

OR

State right hand rule to find the direction of angular momentum.

9. The escape velocity for a satellite is 11.2 km s^{-1}. If the satellite is launched at an angle of 60° with the vertical, what will be the escape velocity?

10. Which of the points is likely position of the centre of mass of the system shown in the figure ?



For question numbers 11, 12, 13 and 14, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false and R is also false

11. **Assertion (A) :** The earth is slowing down and as a result the moon is coming nearer to it.
Reason (R) : The angular momentum of the earth moon system is not conserved.

12. **Assertion (A) :** Orbit of a satellite is within the gravitational field of earth whereas escaping is beyond the gravitational field of earth.
Reason (R) : Orbital velocity of a satellite is greater than its escape velocity.

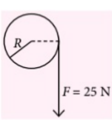
13. **Assertion (A) :** Value of radius of gyration of a body depends on axis of rotation.
Reason (R) : Radius of gyration is root mean square distance of particle of the body from the axis of rotation.

14. **Assertion (A) :** An astronaut experience weightlessness in a space satellite.
Reason (R) : When a body falls freely it does not experience gravity.

Section-B

Questions 15 and 16 are case study based questions and are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

15. A cord of negligible mass is wound round the rim of a fly wheel of mass 20 kg and radius 20 cm. A steady pull of 25 N is applied on the cord as shown in figure. The flywheel is mounted on a horizontal axle with frictionless bearings.



(i) The torque on the wheel is
(a) 4 Nm (b) 5 Nm (c) 2 Nm (d) 8 Nm

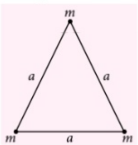
(ii) The moment of inertia of the wheel around its axis is
(a) 0.4 kg m^2 (b) 1.5 kg m^2
(c) 2.5 kg m^2 (d) 3.0 kg m^2

(iii) The angular acceleration of the wheel is
(a) 9.5 rad s^{-2} (b) 13.5 rad s^{-2}
(c) 12.5 rad s^{-2} (d) 15.4 rad s^{-2}

(iv) The work done by the pull when 2 m of the cord is unwound is
(a) 20 J (b) 40 J (c) 30 J (d) 50 J

(v) The kinetic energy of the wheel at this point is (Assume that the wheel starts from rest)
(a) 50 J (b) 100 J (c) 150 J (d) 160 J

16. Three equal masses, each of mass m, are placed at the three corners of an equilateral triangle of side a.



(i) If a fourth particle of equal mass is placed at the centre of triangle, then net force acting on it, is equal to
(a) $\frac{Gm^2}{a^2}$ (b) $\frac{4Gm^2}{3a^2}$
(c) $\frac{3Gm^2}{a^2}$ (d) zero

(ii) In the above problem, if fourth particle is at the mid-point of a side, then net force acting on it, is equal to
(a) $\frac{Gm^2}{a^2}$ (b) $\frac{3Gm^2}{2a}$ (c) $\frac{4Gm^2}{3a}$ (d) zero

(iii) The potential energy of the system as shown in the figure is
(a) $\frac{-Gm^2}{a}$ (b) $\frac{-2Gm^2}{a}$
(c) $\frac{-3Gm^2}{a}$ (d) $\frac{-4Gm^2}{a}$

(iv) If the given system of masses placed at three vertices of an equilateral triangle of side a are to be shifted to another equilateral triangle side $2a$, then work done on the system is
(a) $\frac{3Gm^2}{a}$ (b) $\frac{3Gm^2}{2a}$ (c) $\frac{4Gm^2}{3a}$ (d) $\frac{Gm^2}{a}$

(v) Three particles each of mass m are kept at vertices of an equilateral triangle of side a. The gravitational field at the centre due to these particles is

(a) zero (b) $\frac{3Gm}{a^2}$

(c) $\frac{9Gm}{a^2}$ (d) $\frac{12}{\sqrt{3}}\frac{Gm}{a^2}$

Section-C

17. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = 10^{-10} m).

Find the approximate location of the centre of mass of the molecule. Given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

18. Deduce the relation between torque and angular momentum.

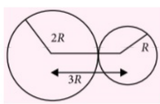
OR

Show that the angular momentum of a satellite of mass M_s revolving around the earth having mass M_e in an orbit of radius r is $L = \sqrt{GM_e M_s^2 r}$.

19. How will you weigh the sun, that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km.

OR

Two uniform solid spheres of radii R and $2R$ are at rest with their surfaces just touching. Find the force of gravitational attraction between them if density of spheres be ρ.



20. When a mass $\left(\frac{m}{2}\right)$ of sand is poured uniformly on a rotating disc of mass m. How will be the angular velocity of the disc change?

21. Obtain a formula for the orbital radius, R_G, of a geostationary satellite in terms of 'T'- the time duration of the earth's day.

22. A particle is projected upward from the surface of the earth (radius R) with kinetic energy equal to half the minimum value of energy needed for it to escape. To which height does it rise above the surface of the earth?

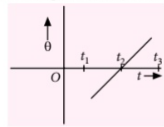
23. A solid sphere rolls down an inclined plane. Find the ratio of its rotational kinetic energy the total kinetic energy.

24. The gravitational force between a hollow spherical shell (of radius R and uniform density) and a point mass is F. Show the nature of F versus r graph where r is the distance of the point from the centre of the hollow spherical shell of uniform density.

OR

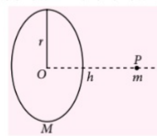
Find the work done in shifting a particle of mass m from the centre of the earth to the surface of the earth (where M is the mass of the earth and R is the radius of the earth).

25. The variation of angular position θ, of a point on a rotating rigid body, with time t is shown in the figure. Is the body rotating clock-wise or anti-clockwise?



Section-D

26. A mass m is placed at P a distance h along the normal through the centre O of a thin circular ring of mass M and radius r as shown in the following figure.



If the mass is removed further away such that OP becomes $2h$, by what factor the force of gravitation will decrease, if $h = r$?

27. What is a couple? What effect does it have on a body? Show that the moment of couple is same irrespective of the point of rotation of a body?

OR

Find the components along the x, y, z axes of the angular momentum $\vec{L}$ of a particle, whose position vector is $\vec{r}$ with components x, y, z and momentum is $\vec{p}$ with components p_x, p_y and p_z. Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.

28. Obtain an expression for escape velocity of the body projected from the surface of earth.

OR

A sphere of radius 10 cm weighs 1 kg. Calculate its moment of inertia (i) about the diameter (ii) about the tangent.

29. Suppose the rod is non-uniform and its mass per unit length (λ) varies linearly with x according to the expression $\lambda = \alpha x$, where α is a constant. Find the centre of mass as a fraction of L.

30. How will the value of g be affected if (i) the rotation of the earth stops (ii) the rotational speed of the earth is doubled (iii) the rotational speed of the earth is increased to seventeen times its present value?

Section-E

31. A rocket is fired 'vertically' from the surface of Mars with a speed of 2 km s^{-1}. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of Mars before returning to it? Mass of Mars = 6.4×10^{23} kg; radius of Mars = 3395 km; $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

OR

(a) A geostationary satellites is orbiting at a height of $6R$ above the surface of earth; R being the radius of the earth. What will be the time period of revolution of another satellite at a height of 2.5 R from the surface of earth?

(b) Give some uses of geostationary satellites.

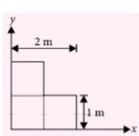
32. (a) Derive an expression for the moment of inertia of a thin uniform solid cylinder about an axis passing through its centre and perpendicular to its length.

(b) Determine the moment of inertia of a thin circular ring
(i) about its diameter
(ii) about a tangent in its plane, and
(iii) about a tangent perpendicular to its plane.

OR

Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed ω_1 and ω_2 are brought into contact face with their axes of rotation coincident. (i) What is the angular speed of the two-disc system? (ii) Show that the initial kinetic energy of the combined system is less than the sum of the kinetic energies of the two discs. How do you account of this loss in energy? Take $\omega_1 \neq \omega_2$.

33. (a) Find the centre of mass of a uniform L-shaped (a thin flat plate) with dimensions as shown in figure. The mass of lamina is 3 kg.



(b) Find centre of mass of a triangular lamina.

OR

(a) Derive an expression for torque in polar co-ordinates.

(b) A torque of 20 N m is applied on a wheel initially at rest. Calculate the angular momentum of the wheel after 3 s.

SOLUTIONS

1. We know, moment of inertia is given by, $I = \Sigma mr^2$
For hollow cylinder of radius R, all its mass (m) lies at a distance R from the axis of symmetry.
For solid sphere of radius R, most of its mass m lies at a distance smaller than R from axis of symmetry.
Hence, $I_{\text{hollow cylinder}} > I_{\text{sphere}}$.

2. We know that the earth rotates from west to east and as such all points on the earth have velocity from west to east. Moreover, this velocity is maximum in the equatorial plane as $v = R\omega$. This maximum linear velocity is added to the launching velocity of the rocket and consequently launching becomes easier.

OR

Since, the force of attraction due to a hollow spherical shell on a point mass situated inside it is zero. Therefore, the weight of the body inside the hollow earth will be zero.

3. For one complete rotation by the second's hand of the clock, angular displacement, $\theta = 2\pi$ rad
Time taken, $t = 60$ s
Therefore, angular velocity, $\omega = \dfrac{\theta}{t}$

$$\Rightarrow \omega = \frac{2\pi}{60} = \frac{2\times 3.14}{60} = 0.105 \text{ rad s}^{-1}$$

4. No. For example, the centre of mass of L-shaped rod lies in the region outside the rod.

OR

The torque applied on the nut by the wrench is equal to the force multiplied by the perpendicular distance from the axis of rotation. Hence to increase torque, a wrench of longer arm is preferred.

5. No, because g on the surface of the earth remains same, *i.e.*, $g = \dfrac{GM}{R^2}$.

6. As $v_e = \sqrt{\dfrac{2GM}{R}}$, therefore escape velocities have different values on different planets which are of different masses and different sizes.

7. Parking orbit is the orbit of geostationary satellite in which the period of revolution of a satellite is equal to the period of rotation of the earth about its axis.

OR

A geostationary satellite should be launched near the equator towards the east direction so that it will get an initial boost equal to the velocity of earth surface.

8. Suppose an external force $\vec{F}_{tot}$ acts on a system of mass M and produces an acceleration $\vec{a}_{CM}$ in its centre of mass. Then

$$\vec{F}_{tot} = M\vec{a}_{CM}$$

In the absence of any external force, $\vec{F}_{tot} = 0$,

so $M\vec{a}_{CM} = 0$ or $\vec{a}_{CM} = 0$ or $\frac{d\vec{v}_{CM}}{dt} = 0$

As the derivative of the constant is zero, so

$$\vec{v}_{CM} = \text{constant}$$

where $\vec{v}_{CM}$ is the velocity of the centre of mass. Hence in the absence of any external forces, the centre of mass of system moves with a uniform velocity.

OR

Curl the fingers of the right hand in the direction of rotation, then the thumb points in the direction of angular momentum.

9. The escape velocity does not depend on the angle of projection, therefore the escape velocity of a satellite launched at an angle of 60° with vertical is 11.2 km s^{-1}.

10. As centre of mass of body lies towards region of heavier mass. So point D is likely position of the centre of mass of the given system.

11. **(d)**: One of the basic requirement of the existence of all planetary motions is the conservation of angular momentum. If this law is violated, then no gravitational law will be able to explain the behaviour of the planets and the satellites as they do.

12. **(c)**

13. **(a)** : Radius of gyration of a body about a given axis is equal to $k = \frac{\sqrt{r_1^2 + r_2^2 + ... r_n^2}}{n}$.

It depends upon shape and size of the body, position and configuration of the axis of rotation and also on distribution of mass of body w.r.t. the axis of rotation.

14. **(b)**

15. **(i) (b)** : Torque, $\tau = FR = 25 \text{ N} \times 0.20 \text{ m} = 5.0 \text{ Nm}$

(ii) (a) : Moment of inertia of the wheel about its axis,

$$I = \frac{MR^2}{2} = \frac{20 \times (0.20)^2}{2} = 0.4 \text{ kg m}^2$$

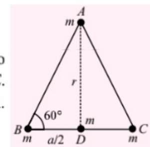
(iii) (c) : As $\tau = I\alpha$

$\therefore$ Angular acceleration, $\alpha = \frac{\tau}{I} = \frac{5.0 \text{ Nm}}{0.4 \text{ kg m}^2} = 12.5 \text{ rad s}^{-2}$.

(iv) (d) : Work done by the pull unwinding 2 m of the cord = 25 N × 2 m = 50 J

(v) (a)

16. **(i) (d)**



(ii) (c) : $r = \frac{a}{2}\tan 60° = \frac{\sqrt{3}a}{2}$

Net force on mass m at D is zero due to two masses at B and C. Only force is due to the mass at A.

$$F = \frac{Gmm}{r^2} = \frac{Gm^2}{(\sqrt{3}a/2)^2} = \frac{4Gm^2}{3a^2}$$

(iii) (c) : For N particles, there are $\frac{N(N-1)}{2}$ pairs

$\therefore$ For 3 particle system, there are $\frac{3(3-1)}{2} = 3$ pairs

Potential energy of the system, $U = \frac{-Gm^2}{a} - \frac{Gm^2}{a} - \frac{Gm^2}{a} = -\frac{3Gm^2}{a}$

(iv) (b) : Potential energy of the initial system,

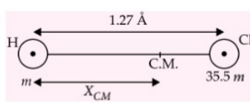
$$U_i = 3\left(\frac{-Gm^2}{a}\right)$$

Similarly, $U_f = 3\left(\frac{-Gm^2}{2a}\right)$

$$W = U_f - U_i = 3\left(\frac{-Gmm}{2a}\right) - 3\left(-\frac{Gmm}{a}\right) = \frac{3Gm^2}{2a}$$

(v) (a)

17.



Suppose hydrogen atom is at the origin *i.e.*, $x_1 = 0$

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m \times 0 + 35.5m \times (1.27 \text{ Å})}{m + 35.5m} = 1.235 \text{ Å}$$

18. Torque, $\vec{\tau} = \vec{r} \times \vec{F}$

Angular momentum, $\vec{L} = \vec{r} \times \vec{p}$

Differentiating both sides w.r.t. time t, we get

$$\frac{d\vec{L}}{dt}=\frac{d}{dt}(\vec{r}\times\vec{p})=\frac{d\vec{r}}{dt}\times\vec{p}+\vec{r}\times\frac{d\vec{p}}{dt}$$

$$=\vec{v}\times\vec{p}+\vec{r}\times\vec{F}\qquad\left[\because\frac{d\vec{p}}{dt}=\vec{F}\right]$$

$$=0+\vec{\tau}\qquad[\because\vec{v}\times\vec{p}=\vec{v}\times m\vec{v}=0]$$

$$\therefore\quad\vec{\tau}=\frac{d\vec{L}}{dt}$$

Thus the torque acting on a particle is equal to its rate of change of angular momentum.

OR

Let the satellite revolve around the earth with orbital speed v. Then,

centripetal force = gravitational force between on the satellite the earth and the satellite

or $\frac{M_s v^2}{r}=G\frac{M_e M_s}{r^2}$ or $v^2=\frac{GM_e}{r}$ or $v=\sqrt{\frac{GM_e}{r}}$

As the satellite is considered a point mass, its angular momentum is

$$L=M_s vr=M_s\sqrt{\frac{GM_e}{r}}\cdot r=\sqrt{GM_e M_s^2 r}$$

19. The gravitational force acting on the earth due to the sun is $F=\frac{GM_S M_E}{r^2}$, $r\rightarrow$ mean orbital radius of the earth around the sun.

Now, the centripetal force acting on the earth due to the sun is

$$F_c=M_E r\omega^2=M_E r\frac{4\pi^2}{T^2},\ \omega\rightarrow\text{angular velocity}$$

Since, this centripetal force is provided by the gravitational pull of the sun on the earth, so,

$$M_S=\frac{4\pi^2 r^2}{GT^2}=\frac{4\times(3.14)^2\times(1.5\times10^{11})^3}{(6.67\times10^{-11})\times(365\times24\times60\times60)^2}$$

$$\simeq 2\times10^{30}\text{ kg}$$

OR

Here, distance between the centre of two spheres is $3R$.

Also, density of spheres is ρ.

Thus, mass of the sphere of radius R,

$$M_R=\left(\frac{4}{3}\pi R^3\right)\times\rho=\frac{4\pi R^3\rho}{3}$$

and, mass of the sphere of radius $2R$,

$$M_{2R}=\left[\frac{4}{3}\pi(2R)^3\right]\rho=\frac{32\pi R^3\rho}{3}$$

Hence, force of gravitational attraction between the spheres can be calculated as $F=\frac{GM_R M_{2R}}{(3R)^2}$

On solving this, we get $F=\frac{128\pi^2 R^4\rho^2 G}{81}$

20. When mass $(m/2)$ of sand is poured uniformly on rotating disc of mass m then total mass of the system $=m+\frac{m}{2}=\frac{3m}{2}$

As we know angular momentum, $L = I\omega$,

where I is moment of inertia and ω is the angular velocity.

As we know moment of inertia is proportionally dependent on mass then if mass increases, angular velocity decreases.

21. As we know, orbital velocity,

$v_0=\sqrt{\frac{GM}{R_G}}$, $R_G\rightarrow$ radius of geostationary orbit.

Time period of geostationary satellite,

$$T_G=\frac{2\pi R_G}{v_0}=\frac{2\pi R_G}{\sqrt{GM/R_G}}=2\pi\sqrt{\frac{R_G^3}{GM}}$$

As $T_G = T$; $T=2\pi\sqrt{\frac{R_G^3}{GM}}$

$$GMT^2=4\pi^2 R_G^3\text{ or }R_G=\left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$$

22. For the particle to escape,

kinetic energy = potential energy

$$\frac{1}{2}mv_e^2=\frac{GMm}{R}\qquad\text{...(i)}$$

But supplied kinetic energy = $\frac{1}{2}\times\frac{1}{2}mv_e^2$

Suppose the particle rises to a height h, then

$$\frac{1}{2}\times\frac{1}{2}mv_e^2=\frac{GMm}{R}-\frac{GMm}{R+h}$$

or $\frac{GMm}{2R}=\frac{GMmh}{(R+h)R}$ [Using (i)]

$\therefore\quad h=R$

23. Translational kinetic energy, $E_T=\frac{1}{2}mv^2$

Rotational kinetic energy, $E_R=\frac{1}{2}I\omega^2$

Total energy, $E=E_T+E_R=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$

$$=\frac{1}{2}mv^2+\frac{1}{2}\times\frac{2}{5}mr^2\times\frac{v^2}{r^2}=\frac{1}{2}mv^2+\frac{1}{5}mv^2=\frac{7}{10}mv^2$$

$$\frac{E_R}{E_T}=\frac{\frac{1}{5}mv^2}{\frac{7}{10}mv^2}=\frac{2}{7} \Rightarrow \frac{E_R}{E_T}=\frac{2}{7}$$

24. $F=\frac{GMm}{r^2}$

R = radius of hollow sphere

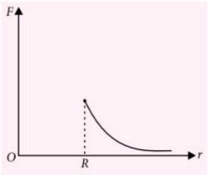
M = mass of hollow spherical shell

m = mass of point mass

$\because$ M is distributed on the surface of sphere only, then

$$F=\begin{cases}0 & \text{for } 0 \le r < R\\ \frac{GMm}{r^2} & \text{for } r \ge R\end{cases}$$

Hence, required graph,



OR

Gravitational potential energy at the centre of the earth is, $U_i=-\frac{3GMm}{2R}$.

Gravitational potential energy at the surface of the earth is, $U_f=-\frac{GMm}{R}$.

Work done, $W=U_f-U_i=-\frac{GMm}{R}-\left(-\frac{3GMm}{2R}\right)$

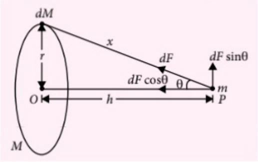
$$W=-\frac{GMm}{R}+\frac{3GMm}{2R}=\frac{GMm}{2R}$$

25. The slope of $\theta - t$ graph,

$\frac{d\theta}{dt}=\omega=(+)\text{ve}$ ($\because$ Inclination < 90°)

Positive ω stands for anti-clockwise rotation.

26. Consider a small element of the ring of mass dM.



Distance between dM and $m = x$

Also $x^2 = r^2 + h^2$

$\therefore$ Gravitational force between dM and m

$$dF=\frac{G(dM)m}{x^2}$$

Now, dF has two components as shown in figure.

$dF\cos\theta$ along PO and $dF\sin\theta$ perpendicular to PO.

Due to symmetry of ring, $\int dF\sin\theta=0$

So, net force on mass m due to ring is given by

$$F=\int dF\cos\theta=\int\frac{G(dM)m}{x^2}\cdot\frac{h}{x}$$

$$\Rightarrow F=\frac{Gm}{x^3}h\int dM=\frac{GMm\times h}{x^3} \Rightarrow F=\frac{GMmh}{(r^2+h^2)^{3/2}}$$

Now, when $h = r$,

$$F=\frac{GMmr}{(r^2+r^2)^{3/2}}=\frac{GMm}{2\sqrt{2}r^2}$$

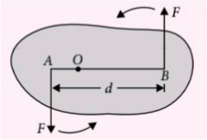
When, $h = 2r$,

$$F'=\frac{GMm(2r)}{(r^2+4r^2)^{3/2}}=\frac{2GMm}{5\sqrt{5}r^2} \quad \therefore \frac{F'}{F}=\frac{\frac{2}{5\sqrt{5}}}{\frac{1}{2\sqrt{2}}}=\frac{4\sqrt{2}}{5\sqrt{5}}$$

27. A pair of equal and opposite forces acting on a body along two different lines of action constitute a couple. A couple has a turning effect, but no resultant force acts on a body.

The moment of couple can be found by taking the moments of the two forces about any point and then adding them.

In figure, two opposite forces, each of magnitude F act at two points A and B of a rigid body, which can rotate about point O. The turning tendency of the two forces is anticlockwise.



Moment or torque of the couple about O is

$\tau = F \times AO + F \times OB = F(AO + OB) = F \times AB$

or $\tau = Fd$

Moment of a couple = Force × perpendicular distance between two forces.

Hence the moment of a couple is equal to the product of either of the forces and the perpendicular distance, called the arm of the couple, between their lines of

BRAIN MAP

MECHANICAL PROPERTIES OF SOLIDS AND FLUIDS

A solid vertical column can rest on table without external support but a liquid column cannot because it cannot withstand shear stresses

Properties of Solids

Elasticity and Plasticity

- Elasticity is the property of the body by virtue of which it tends to regain its original size and shape, when applied force is removed.
- Plasticity is the inability of a body to regain its original size and shape on the removal of the deforming forces.

Stress and Strain

- Stress is the internal restoring force acting per unit area of a deformed body.

$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}}$$

- Strain is the ratio of change in configuration to the original configuration.

$$\text{Strain} = \frac{\text{Change in configuration}}{\text{Original configuration}}$$

- According to Hooke's law, within elastic limit stress is directly proportional to strain, that is the extension produced in a wire is directly proportional to the load applied to the wire.
  i.e., Stress $\propto$ Strain or Stress = $E \times$ Strain, where E is modulus of elasticity.

Types of Modulus of Elasticity

- Young's modulus, $Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$

  or $Y = \frac{F/A}{\Delta l / l} = \frac{Fl}{A\Delta l}$
- Bulk modulus, $B = \frac{\text{Normal stress}}{\text{Volumetric strain}}$

  or $B = \frac{-F/A}{\Delta V/V} = \frac{-FV}{A\Delta V} = \frac{-PV}{\Delta V}$
- Compressibility, $k = \frac{1}{B} = \frac{-\Delta V}{PV}$.
- Modulus of rigidity, $G = \frac{\text{Shearing stress}}{\text{Shearing strain}}$

  or $G = \frac{\sigma_s}{\theta} = \frac{F/A}{\Delta x / L} = \frac{FL}{A\Delta x}$

Elastic Potential Energy

- Work done against the internal restoring forces acting between the various particles of the wire when it is stretched, is stored in the form of potential energy in the wire and is known as elastic potential energy,

  $U = \frac{1}{2}F \times \Delta L = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume of wire}$
- Potential energy stored per unit volume of stretched wire,

  $u = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times \text{Young's modulus} \times (\text{strain})^2$

Poisson's Ratio

- Poisson's ratio $(\sigma) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-\Delta R/R}{\Delta L/L} = \frac{-L\Delta R}{R\Delta L}$

Relations between Y, B, G and σ

- $Y = 3B(1-2\sigma)$
- $Y = 2G(1+\sigma)$
- $\sigma = \frac{3B-2G}{2G+6B}$
- $\frac{9}{Y} = \frac{1}{B} + \frac{3}{G}$

Properties of Fluids

Pressure

- It is the normal force or thrust exerted by a liquid at rest per unit area of the surface in contact with it. $P = F/A$
- Gauge pressure = Total pressure − atmospheric pressure

  $= P - P_0 = h\rho g$.
- The pressure is same at all points inside the liquid lying at the same depth in a horizontal plane.

Archimede's Principle

- It states that when a body is immersed wholly or partly in a liquid at rest , it loses some of its weight, which is equal to the weight of the liquid displaced by the immersed part of the body.
  Apparent weight = Actual weight −Buoyant force $= W - W'$
  (ρ' is density of the liquid) $= V\rho g - V\rho' g$

  $= V\rho g\left(1 - \frac{\rho'}{\rho}\right) = mg\left(1 - \frac{\rho'}{\rho}\right)$
- When $W > W'$, the body sinks down.
- When $W = W'$, the body floats completely immersed in the liquid.
- When $W < W'$, the body floats partly immersed.

Fluid in Motion

- **Bernoulli's Theorem :** It states that for the streamline flow of an ideal liquid, the total energy per unit mass remains constant at every cross-section throughout the liquid flow.

$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

- **Equation of continuity :**
  Av = constant
  It is a statement of conservation of mass in flow of incompressible fluids.
- **Torricelli's law :**
  Velocity of efflux is the velocity with which the liquid flows out of a narrow hole at depth h below the free surface of the liquid.

$$v = \sqrt{2gh + \frac{2(P-P_0)}{\rho}}.$$

- If the hole is open to the atmosphere, then $P = P_0$, so $v = \sqrt{2gh}$

Capillarity

- The phenomenon of rise or fall of liquid in a capillary tube is called capillarity.
- Ascent formula : Height of the liquid within the capillary tube,

$$h = \frac{2S\cos\theta}{a\rho g}$$

  where a is the radius of the capillary tube.

Viscosity

- Coefficient of viscosity of a liquid is the tangential force required to maintain a unit velocity gradient between two parallel layers of liquid, each of unit area.

$$\eta = \frac{-F}{A\left(\frac{dv}{dx}\right)}$$

  where $\frac{dv}{dx}$ is the velocity gradient between two layers of liquid.
- **Poiseuille's formula :** Volume of the liquid flowing per second through a narrow tube, $Q = \frac{\pi}{8}\frac{Pr^4}{\eta l}$.
- **Stoke's law :** Backward dragging force F on a small spherical body of radius r, moving through a fluid of coefficient of viscosity η, with velocity v is given by $F = 6\pi\eta r v$.
- Terminal velocity, $v_T = \frac{2r^2(\rho - \sigma)}{9\eta}g$
- Reynold's number is a number which determines the nature of flow of liquid through a pipe.

$$R = \frac{\rho v d}{\eta}$$

Surface Tension, Surface Energy and Angle of Contact

- **Surface tension** (S) is the property of the liquid by virtue of which the free surface of liquid at rest tends to have minimum surface area.
- **Surface energy** of given liquid surface is defined as the amount of work done against the force of surface tension in forming the liquid surface of given area at a constant temperature.
- **Angle of contact** (θ) is the angle enclosed between the tangents to the liquid surface at the point of contact and the solid surface inside the liquid. It determines whether a liquid will spread on the surface of a solid or it will form droplets on it.

Excess Pressure in Drops

- Excess pressure inside a liquid drop

  $P = \frac{2S}{R}$ (with one free surface)
- Excess pressure inside a soap bubble

  $P = \frac{4S}{R}$ (with two free surface)
- Excess pressure in an air bubble

  $P = \frac{2S}{R}$ (with one free surface) action. Note that the torque exerted by couple about O does not depend on the position of O. Hence torque or moment of a couple is independent of the choice of the fulcrum or the point of rotation.

OR

Angular momentum, $\vec{L}=\vec{r}\times\vec{p}$ ... (i)

It is a vector quantity and its direction is given by right hand rule for vector product.

In cartesian co-ordinates,

$$\left.\begin{aligned}\text{and}\quad \vec{r}&=x\hat{i}+y\hat{j}+z\hat{k}\\ \vec{p}&=p_x\hat{i}+p_y\hat{j}+p_z\hat{k}\end{aligned}\right\}\quad ...(ii)$$

From (i) and (ii), we get

$$\vec{L}=(x\hat{i}+y\hat{j}+z\hat{k})\times(p_x\hat{i}+p_y\hat{j}+p_z\hat{k})$$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ p_x & p_y & p_z\end{vmatrix}$$

or $L_x\hat{i}+L_y\hat{j}+L_z\hat{k}$

$$=\hat{i}(yp_z-zp_y)+\hat{j}(zp_x-xp_z)+\hat{k}(xp_y-yp_x)$$

On comparing, we get

$$\left.\begin{aligned}L_x&=yp_z-zp_y\\ L_y&=zp_x-xp_z\\ L_z&=xp_y-yp_x\end{aligned}\right\}\quad ...(iii)$$

Equation (iii) gives the required components of L along x, y and z axes.

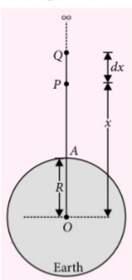
As the particle moves in x-y plane, then

$z=0$ and $p_z=0$

Hence, $\vec{L}=\hat{k}(xp_y-yp_x)=L_z\hat{k}$

Hence angular momentum has only z-component.

28. Consider the earth to be a sphere of mass M and radius R with centre O. Suppose a body of mass m lies at point P at distance x from its centre, as shown in the figure. The gravitational force of attraction on the body at P is $F=\frac{GMm}{x^2}$.



The small work done in moving the body through small distance $PQ = dx$ against the gravitational force is given by

$$dW=Fdx=\frac{GMm}{x^2}dx.$$

The total work done in moving the body from the surface of the earth ($x = R$) to a region beyond the gravitational field of the earth ($x=\infty$) will be

$$W=\int dW=\int_R^\infty\frac{GMm}{x^2}dx=GMm\int_R^\infty x^{-2}dx=GMm\left[-\frac{1}{x}\right]_R^\infty$$

$$=GMm\left[-\frac{1}{\infty}+\frac{1}{R}\right]=\frac{GMm}{R}$$

If v_e is the escape velocity of the body, then the kinetic energy $\frac{1}{2}mv_e^2$ imparted to the body at the surface of the earth will be just sufficient to perform work W.

$$\therefore\quad \frac{1}{2}mv_e^2=\frac{GMm}{R}\quad\text{or}\quad v_e^2=\frac{2GM}{R}$$

Escape velocity, $v_e=\sqrt{\frac{2GM}{R}}$ ...(i)

As $g=\frac{GM}{R^2}$ or $GM=gR^2$

$$\therefore\quad v_e=\sqrt{\frac{2gR^2}{R}}\quad\text{or}\quad v_e=\sqrt{2gR}\quad ...(ii)$$

If ρ is the mean density of the earth, then $M=\frac{4}{3}\pi R^3\rho$

From (i),

$$\therefore\quad v_e=\sqrt{\frac{2G}{R}\times\frac{4}{3}\pi R^3\rho}=\sqrt{\frac{8\pi\rho GR^2}{3}}\quad ...(iii)$$

Equations (i), (ii) and (iii) give different expressions for the escape velocity of a body.

OR

Here, mass of the sphere, $M = 1$ kg,

radius of the sphere, $R = 10$ cm $= 0.1$ m

(i) We know that moment of inertia of the sphere about its diameter

$$=\frac{2}{5}MR^2=\left[\frac{2}{5}\times1\times(0.1)^2\right]\text{kg m}^2=4\times10^{-3}\text{ kg m}^2$$

(ii) Further, moment of inertia of the sphere about its tangent

$$=\frac{7}{5}MR^2=\left[\frac{7}{5}\times1\times(0.1)^2\right]\text{kg m}^2=1.4\times10^{-2}\text{ kg m}^2$$

29. As we know, $x_{CM}=\frac{1}{M}\int_0^L x\,dm=\frac{1}{M}\int_0^L x(\lambda\,dx)$

$$= \frac{1}{M}\int_0^L x(\alpha x)dx = \frac{\alpha}{M}\int_0^L x^2 dx = \frac{\alpha}{M}\times\frac{L^3}{3} = \frac{\alpha L^3}{3M}$$

But $M = \int dm = \int_0^L \lambda\, dx = \int_0^L \alpha x\, dx = \frac{\alpha L^2}{2}$

Thus, $x_{CM} = \frac{\alpha L^3}{3(\alpha L^2/2)} = \frac{2}{3}L$ (from origin, $x = 0$)

30. (i) If the rotation of the earth stops, no centrifugal force will act on the bodies lying on it. The value of g increases to maximum value at the equator. As no centrifugal force acts on the body at poles, so the value of g is not affected.

(ii) If the rotational speed of the earth is doubled, the centrifugal force on the bodies increases. The value of g decreases maximum at the equator and is not affected at poles.

(iii) If the rotational speed of the earth is increased to seventeen times its present value, the value of g at the equator will become zero.

$g_{eq} = g - (17\,\omega)^2 R$

$$= 9.80 - (17)^2 \times \frac{4\times(3.14)^2}{(24\times3600)^2}\times 64\times10^5 \approx 0$$

At the poles, the value of g remains unchanged.

31. *Refer to answer 133, Page no. 256, 257* (MTG CBSE Champion Physics Class 11)

OR

Refer to answer 158, Page no. 260 (MTG CBSE Champion Physics Class 11)

32. *Refer to answer 108, Page no. 217, 218* (MTG CBSE Champion Physics Class 11)

OR

Refer to answer 143, Page no. 222 (MTG CBSE Champion Physics Class 11)

33. *Refer to answer 20, Page no. 207, 208* (MTG CBSE Champion Physics Class 11)

OR

Refer to answer 65, Page no. 211, 212 (MTG CBSE Champion Physics Class 11)

CLASS-XII

BRUSH UP for NEET/JEE 2021

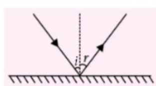
Brush up your concepts to get high rank in NEET/JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

Unit 5

OPTICS

Reflection of Light

- If the incident light after interacting with a boundary separating the two media comes back in the same medium, the phenomenon is called reflection and the boundary is known as reflector.
- The angle which the incident ray and the reflected ray make with the normal to the surface are termed as the angle of incidence (i) and reflection (r) respectively.

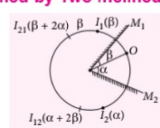


- Laws of reflection
  - The angle of incidence i equals the angle of reflection r.
  - Incident ray, the normal and the reflected ray lie in the same plane.
- The laws of reflection are valid both in case of plane and curved reflecting surfaces.
- For normal incidence *i.e.*, $\angle i = 0$, $\angle r = 0$. Hence a ray of light falling normally on a mirror retraces its path on reflection.

Reflection from Plane Surface

- The image formed by a plane mirror is at the same distance behind the mirror as the object is in front of it.
- The image formed by a plane mirror is laterally inverted. The lateral inversion means that the right side of the object appears as the left side of the image and vice-versa.
- The image formed by a plane mirror is virtual, erect w.r.t. object and of the same size as the object.
- If keeping the incident ray fixed, the plane mirror is rotated through an angle θ, the reflected ray turns through double the angle *i.e.*, 2θ in that very direction.
- If the object is fixed and the mirror moves relative to the object with a speed v, the image moves with a speed $2v$ relative to the object.
- If the mirror is fixed and the object moves relative to the mirror with a speed v, the image also moves with the same speed v relative to the mirror.
- Deviation suffered by a light ray incident at an angle i is given by
  $\delta = (180° - 2i)$

Number of Images Formed by Two Inclined Mirrors

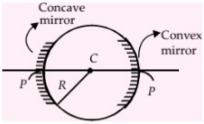


- For two inclined plane mirrors M_1, M_2
  I_1 = Image of O by M_1;
  I_2 = Image of O by M_2
  I_{21} = Image of I_2 by M_1;
  I_{12} = Image of I_1 by M_2 and so on.
- Total number of images is given by
  $\frac{360°}{\theta} = N$ with some special cases; $\theta = \alpha + \beta$

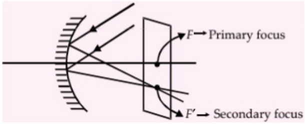
- If $\frac{360°}{\theta}$ = even number; number of images $= \frac{360°}{\theta} - 1$.
- If $\frac{360°}{\theta}$ = odd number; number of images $= \frac{360°}{\theta} - 1$ if the object is placed on the angle bisector.
- If $\frac{360°}{\theta}$ = odd number; number of images $= \frac{360°}{\theta}$, if the object is not placed on the angle bisector.
- If $\frac{360°}{\theta} \neq$ integer, then count the number of images as explained.

Spherical Mirrors

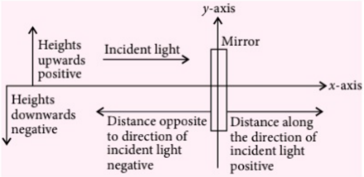
- Spherical mirror is formed by polishing one surface of a part of sphere. Depending upon which part is shining the spherical mirror is classified as



  - Concave mirror, if the side towards center of curvature is shining.
  - Convex mirror, if the side away from the center of curvature is shining.
- Important terms for spherical mirrors
  - Pole (P) is the mid point of reflecting surface.
  - Centre of curvature (C) is the centre of the sphere of which the mirror is a part.
  - Radius of curvature is the radius of the sphere of which the mirror is a part. Distance between P and C.
  - Principal axis is the straight line connecting pole P and centre of curvature C.
  - A narrow beam of rays, parallel and near to principal axis, is reflected from a mirror so that all the rays converge or appear to converge at a point on the principle axis, this point is called principle focus of mirror.
  - Focal length (f) is distance from pole to focus.
  - Aperture the diameter of the mirror is called aperture of the mirror.
  - Plane perpendicular to principal axis and passing through focus is known as focal plane.



- **Sign convention :** We follow cartesian co-ordinate system conventions according to which
  - The pole of mirror is the origin.
  - The distance measured in the direction of the incident rays is considered as positive x-axis.
  - The heights measured in the vertically up direction are positive y-axis.



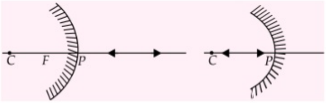
- Mirror Formula, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

  u = distance of object, v = image distance, f = focal length and $f = R/2$; R = radius of curvature.

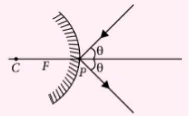
Ray Tracing

Following facts are useful in ray tracing.

- If the incident ray is parallel to the principal axis, the reflected ray passes through the focus.
- If the incident ray passes through the focus, then the reflected ray is parallel to the principal axis.
- Incident ray passing through centre of curvature will be reflected back through the centre of curvature (because it is normal to the plane.



- It is easy to make the ray tracing of a ray incident at the pole as shown below.



☞ Magnification

- Linear magnification : $m = \frac{h_2}{h_1} = -\frac{v}{u}$

  h_1 = height of the object, h_2 = height of the image. (h_1 and h_2 both are perpendicular to the principal axis of mirror)
- If the image is upright or erect with respect to the object then m is positive. And m is negative if the image is inverted with respect to the object.
- Longitudinal magnification, $m_l = \frac{v_2 - v_1}{u_2 - u_1}$
  - For very small object

    $m_l = \frac{dv}{du} = -\frac{v^2}{u^2} = -(m_t)^2$

☞ Refraction of Light

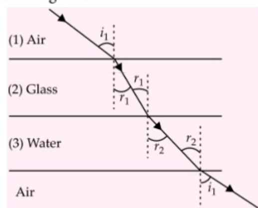
- When light passes obliquely from one transparent medium to another, the direction of its path may change at the interface of the two media. This phenomenon is known as refraction of light.
- If a ray of light passes from an optically rarer medium to a denser medium, it bends towards the normal.
- If a ray of light passes from an optically denser medium to a rarer medium, it bends away from the normal.
- A ray of light travelling along the normal passes undeflected, the incident ray and refracted ray make angle zero with normal.
- Laws of refraction
  - The incident ray, the normal to the interface at the point of incidence and the refracted ray all lie in the same plane.
  - The ratio of the sine of the angle of incidence to the sine of the angle of refraction is always a constant (a different constant for a different set of media).

    i.e., $\frac{\sin i}{\sin r} = \text{constant} = {}^1\mu_2$
  - This constant (${}^1\mu_2$) is called refractive index of medium 2 (in which refracted ray travels) with respect to medium 1 (in which incident ray travels). It is known as Snell's law and holds good for all angles of incidence.

☞ Refractive Index and its Effect on Refraction

- The refractive index of a medium for a light of given wavelength may be defined as the ratio of the speed of light in vacuum to its speed in that medium.

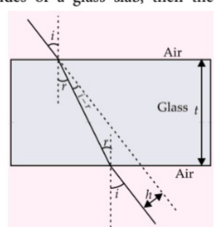
  $\mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{v}$
- Refractive index of a medium with respect to vacuum is also called absolute refractive index.
- When a light ray travels from one medium to another, its frequency remains constant but its wavelength as well as velocity changes.
- The deviation of the incident ray when it is refracted is given by an angle $\delta = |i - r|$.
- If a light ray passes through a number of parallel media and if the first and the last medium are same, the emergent ray is parallel to the incident ray as shown in figure below.



$${}^1\mu_2 = \frac{\sin i_1}{\sin r_1},\ {}^2\mu_3 = \frac{\sin r_1}{\sin r_2} \text{ and } {}^3\mu_1 = \frac{\sin r_2}{\sin i_1}$$

Hence, ${}^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = \frac{\sin i_1}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i_1} = 1$

- For principle of reversibility, ${}^1\mu_2 = \frac{1}{{}^2\mu_1}$
- **Lateral shift due to glass slab :** When the medium is same on both sides of a glass slab, then the deviation of the emergent ray is zero. That is the emergent ray is parallel to the incident ray but it does suffer lateral displacement/shift with respect to the incident ray and is given by



Lateral shift, $h = t\frac{\sin(i - r)}{\cos r}$

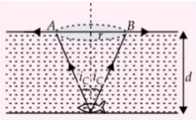
where t is the thickness of the slab.
- **Apparent depth :** An object placed in a denser medium (*e.g.* water). when viewed from a rarer medium (*e.g.* air) appears to be at a lesser depth than its real depth. This is on account of refraction of light.

$$\text{Apparent depth} = \frac{\text{Real depth}}{\text{Refractive index}}$$

- As the refractive index of any medium (other than vacuum) is greater than unity, so the apparent depth is less than the real depth.
- The height through which an object appears to be raised in a denser medium is called normal shift or apparent shift.
  Normal shift = real depth – apparent depth
  $$d = t - \frac{t}{\mu} = t\left(1 - \frac{1}{\mu}\right)$$
  Here, t is the real depth of water and μ is its refractive index.

Total Internal Reflection

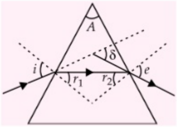
- The total internal reflection is the phenomenon in which a ray of light travelling from an optically denser into an optically rarer medium at an angle of incidence greater than the critical angle for the two media is totally reflected back into the same medium.
- Necessary conditions for total internal reflection.
  - Light is travelling from optically denser to optically rarer medium.
  - The angle of incidence at the surface is greater than the critical angle for the pair of media.
- The critical angle for the two media is the angle of incidence in the optically denser medium for which the angle of refraction is 90°. It is given by
  $$\sin i_C = \frac{1}{\mu}$$
- Let i = angle of incidence. Consider the following cases:
  - If $i < i_C$, then refraction takes place.
  - If $i = i_C$, then grazing emergence takes place.
  - If $i > i_C$, then total internal reflection takes place.
- A fish in waterat a depth d sees the world outside through a horizontal circle of radius
  $$r = d \tan i_C = \frac{d}{\sqrt{\mu^2 - 1}}$$



- Applications of total internal reflection
  - Brilliance of diamonds
  - Mirage
  - Totally reflecting glass prisms
  - Optical fibres

Refraction through a Prism

- A prism is a homogeneous, transparent medium bounded by two plane surfaces inclined at an angle A with each other. These surfaces are called the refracting surfaces and angle between them is called the refracting angle or the angle of prism A.
- The angle between the incident ray and the emergent ray is called the angle of deviation.
  For refraction through a prism it is found that



$\delta = i + e - A$ where $A = r_1 + r_2$
When A and i are small
$\therefore \quad \delta = (\mu - 1)A$

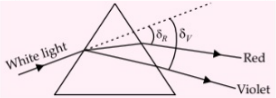
- In a position of minimum deviation $\delta = \delta_m$, $i = e$, and $r_1 = r_2 = r$

$$\therefore \; i = \left(\frac{A + \delta_m}{2}\right) \text{ and } r = \frac{A}{2}$$

- The refractive index of the material of the prism is
  $$\mu = \frac{\sin\left[\frac{(A + \delta_m)}{2}\right]}{\sin\left(\frac{A}{2}\right)}$$
  This is called prism formula.

Dispersion of Light

- It is the phenomenon of splitting of white light into its constituent colours on passing through a prism. This is because different colours have different wavelengths ($\lambda_R > \lambda_V$). According to Cauchy's formula
  $$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$
  where A, B, C are arbitrary constants. Therefore, μ of material of prism for different colours is different ($\mu_V > \mu_R$). As $\delta = (\mu - 1)A$, therefore different colours turn through different angles on passing through the prism. This is the cause of dispersion.



- Angular dispersion, $\theta = \delta_V - \delta_R = (\mu_V - \mu_R)A$
  where μ_V and μ_R are the refractive indices for violet and red light respectively.
- Dispersive power, $\omega = \dfrac{\text{angular dispersion}}{\text{mean deviation}}$

$$\omega = \frac{\mu_V - \mu_R}{(\mu - 1)}$$

where $\mu = \frac{\mu_V + \mu_R}{2}$ = mean refractive index

- It depends on the material of the prism.
- It is a unit less and dimensionless quantity.
- Dispersive power of a flint glass prism is more than that of a crown glass prism.

- When two prisms are combined together, we can get deviation without dispersion or vice versa.
- Condition for deviation without dispersion is

  $\theta + \theta' = 0$

  $(\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A' = 0$

  or $A' = -\frac{(\mu_V - \mu_R)A}{(\mu'_V - \mu'_R)}$

  where A and A' are refracting angles of two prisms respectively and μ_V, μ_R and μ'_V, μ'_R be the refractive indices of the violet and red light of the corresponding prisms.

  Negative sign shows that the refracting angles of the two prisms are in opposite direction.

  Under this condition, net deviation produced by the combination is

  $= \delta + \delta' = (\mu - 1)A + (\mu' - 1)A'$

  The prism which produces deviation without dispersion is called achromatic prism.

- Condition for dispersion without deviation is

  $\delta + \delta' = 0$

  $(\mu - 1)A + (\mu' - 1)A' = 0$ or $A' = -\frac{(\mu - 1)A}{(\mu' - 1)}$

  where μ and μ' be the refractive indices of the material of two prisms respectively.

  Negative sign shows that the refracting angles of two prisms are in opposite direction.

  Under this condition, net angular dispersion produced by the combination is

  $= (\delta_V - \delta_R) + (\delta'_V - \delta'_R) = (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A'$

  The prism which produces dispersion without deviation is called direct vision prism.

Scattering of Light

- As sunlight travels through the earth's atmosphere, it gets scattered (changes its direction) by the atmospheric particles. Light of shorter wavelengths is scattered much more than the light of longer wavelengths. The amount of scattering is inversely proportional to the fourth power of the wavelength. This is called Rayleigh scattering.
- Illustrations of scattering of light
  - Blue colour of sky
  - White colour clouds
  - The sun looks reddish at the time of sun rise and sun set

Refraction at Spherical Surfaces

- A refractive surface which forms a part of a sphere of transparent medium is called a spherical refracting surface. Spherical refracting surfaces are of two types
  - Convex spherical refracting surface
  - Concave spherical refracting surface
- For both surfaces refracting formula is given by

  $$\frac{{}^1\mu_2}{v} - \frac{1}{u} = \frac{{}^1\mu_2 - 1}{R}$$

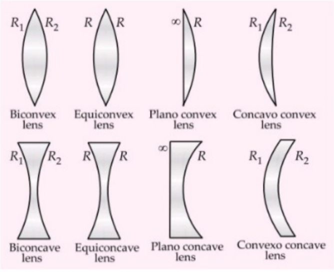
  where ${}^1\mu_2$ is refractive index of second medium with respect to first and u, v, R are the object distance, image distance and radius of curvature of the spherical surface respectively.

  - If μ_1 and μ_2 are refractive indices of first and second medium with respect to air, then

    $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Lenses

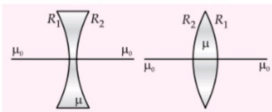
- A lens is a transparent medium bounded by two refracting surfaces such that at least one of the refracting surfaces is curved.
- If the thickness of the lens is negligibly small in comparison to the object distance or the image distance, the lens is called thin.
- Broadly, lenses are of the following types :



- The focal length (f) of a lens depends upon the refractive indices of the material of the lens and the medium in which the lens is present and the radii of curvature of both sides. The following relation giving focal length (f) is called as lens maker's formula.

$$\frac{1}{f}=\left(\frac{\mu}{\mu_0}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

where μ = refractive index of the material of the lens, μ_0 = refractive index of the medium.



- Lens formula
$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$
- The ratio of the size of the image formed by a lens to the size of the object is called linear magnification produced by the lens. It is denoted by m.
If h_1 and h_2 are the sizes of the object and image respectively, then
$$m=\frac{h_2}{h_1}=\frac{v}{u}.$$
- The power of a lens is defined as the reciprocal of the focal length in metre.
$$P=\frac{1}{f(\text{in m})}$$
  - The SI unit of power of lens is diopter (D).
  - For a convex lens, P is positive.
  - For a concave lens, P is negative.
  - When focal length (f) of lens is in cm, then
$$P=\frac{100}{f\text{ (in cm)}}\text{ dioptre.}$$
- When a number of thin lenses of focal length f_1, f_2, ... etc. are placed in contact coaxially, the equivalent focal length F of the combination is given by
$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+....$$
  - The total power of the combination is given by $P = P_1 + P_2 + P_3 + ...$
  - The total magnification of the combination is given by
$m = m_1 \times m_2 \times m_3 ....$
- When two thin lenses of focal lengths f_1 and f_2 are placed coaxially and separated by a distance d, the focal length of combination is given by
$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}.$$
  - In terms of power, $P = P_1 + P_2 - dP_1P_2$.
- **Silvering of a lens :** Let a planoconvex lens is having a curved surface of radius of curvature R and has refractive index μ.
  - If its plane surface is silvered, it behaves as a concave mirror of focal length
$$f=-\frac{R}{2(\mu-1)}.$$
  - If the curved surface of planoconvex lens is silvered then it behaves as a concave mirror of focal length
$$f=-\frac{R}{2\mu}.$$
- Displacement method is used to determine focal length of convex lens in laboratory.
  - For a convex lens, the minimum distance between the object and its real image is $4f$. If a convex lens is placed between an object and a screen fixed at a distance D such that $D \geq 4f$, there are two positions of the lens which give a sharp image on the screen.
  - The focal length of the lens is given by
$$f=\frac{D^2-d^2}{4D}$$
where D = distance between the screen and the object, d = distance between the two positions of the lens.
  - If I_1, I_2 are the two sizes of image of the object of size O, then $O=\sqrt{I_1I_2}$.

Simple Microscope

- It is also called magnifying glass or simple magnifier. It consists of a converging lens of small focal length.
- Magnifying power of a simple microscope
  - when the image is formed at infinity (far point),
$$M=\frac{D}{f}$$
where f is the focal length of convex lens.
  - When the image is formed at the least distance of distinct vision D (near point),
$$M=1+\frac{D}{f}.$$
- A magnification of about 10 is the best, after this lens aberrations begin to distort the image.

Compound Microscope

- It consists of two convergent lenses of short focal lengths and apertures arranged coaxially. Lens facing the object is called objective or field lens while the lens facing the eye, is called eye-piece or ocular. The objective has a smaller aperture and smaller focal length than eye-piece.

- Magnifying power of a compound microscope
  - When the final image is formed at infinity (normal adjustment),
    $M = -\frac{v_o}{u_o}\left(\frac{D}{f_e}\right)$.
    Length of tube, $L = v_o + f_e$.
  - When the final image is formed at least distance of distinct vision,
    $M = -\frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$

  where u_o and v_o represent the distance of object and intermediate image from the objective lens, f_e is the focal length of an eye lens.

  Length of the tube, $L = v_o + \left(\frac{f_e D}{f_e + D}\right)$.

Astronomical Telescope (Refracting Type)

- It consists of two converging lenses. The one facing the object is called objective or field lens and has large focal length and aperture while the other facing the eye is called eye-piece or ocular has small focal length and aperture.
- Magnifying power of an astronomical telescope
  - When the final image is formed at infinity (normal adjustment),
    $M = -\frac{f_o}{f_e}$.
    Length of tube, $L = f_o + f_e$
  - When the final image is formed at least distance of distinct vision,
    $M = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$.
    Length of tube, $L = f_o + \frac{f_e D}{f_e + D}$

Terrestrial Telescope

- It is used for observing far off objects on the ground. The essential requirement of such a telescope is that final image must be erect w.r.t. the object. To achieve it, an inverting convex lens (of focal length f) is used in between the objective and eye piece of astronomical telescope. This lens is known as erecting lens.
- In normal adjustment,
  Magnifying power, $M = \frac{f_o}{f_e}$
- Length of the tube, $L = f_o + 4f + f_e$

Galileo's Terrestrial Telescope

- It consists of an objective which is a convex lens of large focal length and an eye-piece which is a concave lens of short focal length ($f_o > f_e$).
- In the normal adjustment,
  Magnifying power, $M = \frac{f_o}{f_e}$
  Length of the tube, $L = f_o - f_e$

Wavefront

- A wavefront is defined as the continuous locus of all such particles of the medium which are vibrating in the same phase of any instant.
- The geometrical shape of a wavefront depends on the source of disturbance. Some of the common shapes of wavefronts are
  - **Spherical wavefront :** In the case of waves travelling in all directions from a point source, the wavefronts are spherical in shape.
  - **Cylindrical wavefront :** If the source of light is linear in shape such as a fine rectangular slit, the wavefront is cylindrical in shape.
  - **Plane wavefront :** As a spherical or cylindrical wavefront advances, its curvature decreases progressively and this wavefront behaves as a plane wavefront at infinity.

Huygen's Principle

- According to Huygen's principle, each point on a wavefront is a source of secondary waves, which add up to give a wavefront at any later time.
- Assumptions in Huygen's principle
  - The secondary wavelets spread out in all directions with the speed of light in the given medium.
  - The new wavefront at any later time is given by the forward envelope (tangential surface) in the forward direction of the secondary wavelets at that time.
  - Each point on a wavefront acts as fresh source of new disturbance called secondary waves or wavelets.

Coherent Sources

- Two sources of light which continuously emit light waves of same frequency (or wavelength) with a zero or constant phase difference between them, are called coherent sources.
- Conditions for obtaining two coherent sources of light.
  - The two sources of light must be obtained from a single source by some method.
  - The two sources must give monochromatic light.
  - The path difference between the waves arriving on the screen from the two sources must not be large.

- If two light waves of the same frequency and having zero or constant phase difference travelling in the same direction super position gets redistributed becoming maximum at some points and minimum at others. This phenomenon is called interference of light.
- **Addition of coherent waves :** If A_1, A_2 are the amplitudes of interfering waves due to two coherent sources and ϕ is constant phase difference between the two waves at any point P, then
  The resultant amplitude at P will be
  $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$
  Resultant intensity at P is
  $I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$
  - When $\phi = 2n\pi$, where $n = 0, 1, 2,....$
    then, $A = A_{max} = (A_1 + A_2)$
    $I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$
  - When $\phi = (2n - 1)\pi$, where $n = 1, 2, 3, ...$
    then, $A = A_{min} = (A_1 - A_2)$
    $I_{min} = I_1 + I_2 - 2\sqrt{I_1I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$
    $$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$
  - If the amplitudes of the two waves are equal $A_1 = A_2 = A_0$, then resultant amplitude
    $$A = \sqrt{2A_0^2 + 2A_0^2 \cos\phi} = 2A_0 \cos\left(\frac{\phi}{2}\right)$$
    Resultant intensity, $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$
    In this case, $A_{max} = 2A_0$, $I_{max} = 4I_0^2$
    $A_{min} = 0$, $I_{min} = 0$

  Note : If the sources are incoherent, $I = I_1 + I_2$.

☞ Young's Double Slit Experiment

- The phenomenon of interference of light was first observed by a British Physicist Thomas Young. Using two slits illuminated by a monochromatic light source, he obtained alternately bright and dark band on the screen. These bands are called interference fringes or interference bands.
- Constructive interference (*i.e.* formation of bright fringes)
  - For n^{th} bright fringe,
    Path difference $= n\lambda$ or $d\sin\theta = \frac{dx}{D} = n\lambda$
    where $n = 0$ for central bright fringe
    $n = 1$ for first bright fringe,
    $n = 2$ for second bright fringe and so on
    d = distance between the two slits
    D = distance of slits from the screen
  - The position of n^{th} bright fringe from the centre of the screen is given by
    $$x_n = n\lambda \frac{D}{d}$$
- Destructive interference (*i.e.*, formation of dark fringes)
  - For n^{th} dark fringe,
    path difference $= (2n-1)\frac{\lambda}{2}$
    where, $n = 1$ for first dark fringe,
    $n = 2$ for second dark fringe and so on.
  - The position of n^{th} dark fringe from the centre of the screen is given by
    $$x_n = (2n-1)\frac{\lambda}{2}\frac{D}{d}$$
- The distance between any two consecutive bright or dark fringes is called fringe width.
  Fringe width, $\beta = \frac{\lambda D}{d}$
  - Angular fringe width, $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$
- If W_1, W_2 are widths of two slits, I_1, I_2 are intensities of light coming from these two slits; A_1, A_2 are the amplitudes of light from these slits, then
  $$\frac{W_1}{W_2} = \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2}$$
- Fringe visibility, $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$
- When entire apparatus of Young's double slit experiment is immersed in a medium of refractive index μ, then fringe width becomes
  $$\beta' = \frac{\lambda' D}{d} = \frac{\lambda D}{\mu d} = \frac{\beta}{\mu}$$
- When a thin transparent plate of thickness t and refractive index μ is placed in the path of one of the interfering waves, fringe width remains unaffected but the entire pattern shifts by
  $$\Delta x = (\mu - 1)t\frac{D}{d} = (\mu - 1)t\frac{\beta}{\lambda}$$
  This shifting is towards the side in which transparent plate is introduced.
  - Number of shifted fringes,
    $$N = \frac{\Delta x}{\beta} = \frac{(\mu - 1)t}{\lambda}$$

☞ Interference in Thin Films

- A thin film of liquid (*e.g.* soap film or a layer of oil over water) appears bright or dark when viewed in monochromatic light. This effect is caused due to the interference of light reflected from the top and bottom faces of the film.
- Interference in reflected light (reflected system of light)
  - For a bright fringe,
    $2\mu t\cos r = (2n+1)\frac{\lambda}{2}$; where, n = 0, 1, 2, 3, ...
  - For a dark fringe
    $2\mu t\cos r = n\lambda$; where, n = 0, 1, 2, 3 ...
- Interference in transmitted light (transmitted system of light)
  - For a bright-fringe,
    $2\mu t\cos r = n\lambda$; where n = 0, 1, 2, 3, ...
  - For a dark fringe,
    $2\mu t\cos r = (2n+1)\frac{\lambda}{2}$; where n = 0, 1, 2, 3, ...
- The conditions for maxima and minima in the reflected system are just opposite to those for the transmitted system. Thus the reflected and transmitted systems are complimentary *i.e*, a film which appears bright by reflected light will appear dark by transmitted light and vice-versa.

☞ Diffraction of Light

- The phenomenon of bending of light around the corners of an obstacle or aperture is called diffraction of light.
- Diffraction of light is not easily noticed because the obstacles and apertures of size of wavelength of light 10^{-6} m are hardly available.
- In ray optics, we ignore diffraction and assume that light travels in straight lines. This assumption is reasonable because under ordinary conditions, diffraction (bending) of light is negligible.
- The smaller the size of the obstacle or aperture, the greater is the bending (or diffraction) of light around the corners of the obstacle or aperture and vice-versa.
- The diffraction phenomenon is generally divided into the following two classes :
  - **Fresnel diffraction :** In this case, either the source or the screen or both are at finite distances from the aperture or obstacle causing diffraction.
  - **Fraunhofer diffraction :** In this case, the source and the screen on which the pattern is observed are at infinite distances from the aperture or the obstacle causing diffraction.

☞ Diffraction due to a Single Slit

- The diffraction pattern produced by a single slit of width a consists of a central maximum bright band with alternating bright and dark bands of decreasing intensity on both sides of the central maximum.
- Condition for n^{th} secondary maximum is,
  Path difference $= a\sin\theta_n = (2n+1)\frac{\lambda}{2}$
  where n = 1, 2, 3, ...
- Condition for n^{th} secondary minimum is,
  Path difference = $a\sin\theta_n = n\lambda$
  where n = 1, 2, 3,.......
- Width of secondary maxima or minima
  $\beta = \frac{\lambda D}{a} = \frac{\lambda f}{a}$
  where
  a = width of slit
  D = distance of screen from the slit
  f = focal length of lens for diffracted light
- Width of central maximum $= \frac{2\lambda D}{a} = \frac{2f\lambda}{a}$
  The width of central maxima is also called primary fringe width.
- Angular fringe width of central maximum $= \frac{2\lambda}{a}$.
- Angular fringe width of secondary maxima or minima $= \frac{\lambda}{a}$

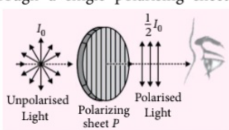
☞ Resolving Power of Optical Instruments

- Resolving power is the ability of the instrument to resolve or to see as separate, the images of two close objects.
- Resolving power (R) $= \frac{1}{\text{Resolving limit}}$
  - Resolving power of microscope, $R = \frac{1}{\Delta d} = \frac{2\mu\sin\theta}{\lambda}$
    $\mu \rightarrow$ Refractive index of the medium between the object and objective lens of the microscope
    $\lambda \rightarrow$ wavelength of light
    $\theta \rightarrow$ angle subtended by radius of objective lens on the object.
  - Resolving power of telescope, $R = \frac{1}{\Delta\theta} = \frac{a}{1.22\lambda}$
    a = diameter of objective lens of the telescope.

☞ Polarisation of Light

- The phenomenon of restricting the vibrations of light (electric vector) in a particular direction, perpendicular to direction of wave motion is called polarisation of light.

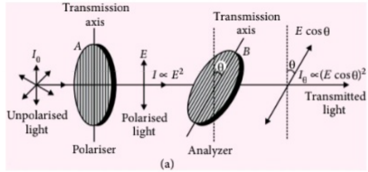
- Polarisation of light confirms the transverse nature of light. The plane in which vibrations of polarised light are confined is called plane of vibration. A plane which is perpendicular to the plane of vibration is called plane of polarisation.
- Plane polarised light can be produced by the following methods
  - By reflection
  - By scattering
  - By refraction
  - By dichroism
  - By double refraction
- **Polarising sheets (Polaroid) :** Unpolarised light is linearly polarised and reduced in intensity by half after passing through a single polarising sheet. The parallel lines, which are not actually visible on the sheet, suggest its polarisation direction.



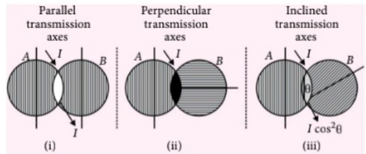
- **Malus's Law :** Intensity of transmitted light by analyser depends on the angle between the polariser and analyser.

  $I_\theta \propto \cos^2\theta \Rightarrow I_\theta = I\cos^2\theta$

  I = intensity of polarised light passing through the polariser.



(a)



(b)

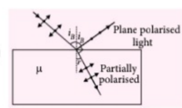
- **Polarisation by reflection (Brewster's law)**
  - When unpolarised light is reflected from a plane boundary between two transparent media, the reflected light is partially polarised. The degree of polarisation depends on the angle of incidence and the ratio of wave speeds in the two media.
  - At polarising angle (Brewster's angle), the reflected and refracted beams are mutually perpendicular to each other.

    $\sin i_B = \mu\sin(r)$

    $\sin i_B = \mu\sin(90 - i_B)$

    $\Rightarrow \mu = \tan i_B$

    This is the Brewster's law.



WRAP it up!

1. When a light of wavelength 4000 Å in vacuum travels through the same thickness in diamond and water separately, the difference in the number of waves is 200. Find the thickness, if refractive indices of diamond and water are $\frac{5}{2}$ and $\frac{4}{3}$ respectively.

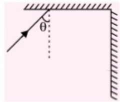
   (a) 0.685 mm (b) 0.0685 mm
   (c) 68.5 mm (d) 6.85 mm

2. A beam of natural light falls on a system of 5 polaroids, which are arranged in succession such that the pass axis of each polaroid is turned through 60° with respect to the preceding one. The fraction of the incident light intensity that passes through the system is

   (a) $\frac{1}{64}$ (b) $\frac{1}{32}$ (c) $\frac{1}{256}$ (d) $\frac{1}{512}$

3. Two plane mirrors are arranged at right angles to each other as shown in figure. A ray of light is incident on the horizontal mirror at an angle θ.

For what value of θ, the ray emerges parallel to the incoming ray after reflection from the vertical mirror?



(a) 60°   (b) 30°

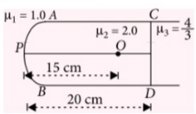
(c) 45°   (d) All of these

4. In a single slit diffraction pattern, the distance between the first minimum on the left and the first minimum on the right is 5 mm. The screen on which the diffraction pattern is displayed is at a distance of 80 cm from the slit. The wavelength is 6000 Å. The slit width (in mm) is about

(a) 0.576   (b) 0.348

(c) 0.192   (d) 0.096

5. A slab of a material of refractive index 2 shown in figure. has a curved surface *APB* of radius of curvature 10 cm and a plane surface *CD*. On the left of *APB* is air and on the right of *CD* is water with refractive indices as given in the figure. An object *O* is placed at a distance of 15 cm from the pole *P* as shown. The distance of the final image of *O* from *P*, as viewed from the left is



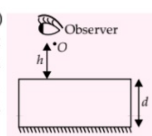
(a) 10 cm   (b) 20 cm

(c) –30 cm   (d) –20 cm

6. In Young's double slit experiment, the 10th maximum of wavelength λ_1 is at a distance of y_1 from the central maximum. When the wavelength of the source is changed to λ_2, 5th maximum is at a distance of y_2 from its central maximum. The ratio $\left(\frac{y_1}{y_2}\right)$ is

(a) $\frac{2\lambda_1}{\lambda_2}$ (b) $\frac{2\lambda_2}{\lambda_1}$ (c) $\frac{\lambda_1}{2\lambda_2}$ (d) $\frac{\lambda_2}{2\lambda_1}$

7. A point luminous object (*O*) is at a distance *h* from front face of a glass slab of width *d* and of refractive index μ. On the back face of slab is a reflecting plane mirror.



An observer sees the image of object in mirror as shown in figure. Distance of image from front face as seen by observer will be

(a) $h+\frac{2d}{\mu}$ (b) $2h + 2d$ (c) $h + d$ (d) $h+\frac{d}{\mu}$

8. A compound microscope has an eye piece of focal length 10 cm and an objective of focal length 4 cm. Calculate the magnification, if an object is kept at a distance of 5 cm from the objective, so that the final image is formed at the least distance of distinct vision 25 cm.

(a) 12   (b) 11   (c) 13   (d) 14

9. A beaker contains water up to a height h_1 and kerosene of height h_2 above water so that the total height of (water + kerosene) is $(h_1 + h_2)$. Refractive index of water is μ_1 and that of kerosene is μ_2. The apparent shift in the position of the bottom of the beaker when viewed from above is

(a) $\left(1-\frac{1}{\mu_1}\right)h_2+\left(1-\frac{1}{\mu_2}\right)h_1$

(b) $\left(1+\frac{1}{\mu_1}\right)h_1-\left(1+\frac{1}{\mu_2}\right)h_2$

(c) $\left(1-\frac{1}{\mu_1}\right)h_1+\left(1-\frac{1}{\mu_2}\right)h_2$

(d) $\left(1+\frac{1}{\mu_1}\right)h_2-\left(1+\frac{1}{\mu_2}\right)h_1$

10. The focal length of the lenses of an astronomical telescope are 50 cm and 5 cm. The length of the telescope when the image is formed at the least distance of distinct vision is

(a) 45 cm   (b) 55 cm

(c) $\frac{275}{6}$ cm   (d) $\frac{325}{6}$ cm

11. In Young's double slit experiment, first slit has width four times the width of the second slit. The ratio of the maximum intensity to the minimum intensity in the interference fringe system is

(a) 2 : 1   (b) 4 : 1   (c) 9 : 1   (d) 8 : 1

12. In a compound microscope, the focal lengths of two lenses are 1.5 cm and 6.25 cm. If an object is placed at 2 cm from objective and the final image is formed at 25 cm from eye lens, the distance between the two lenses is

(a) 6.00 cm   (b) 7.75 cm

(c) 9.25 cm   (d) 11.0 cm

13. Two identical glass ($\mu_g = 3/2$) equiconvex lenses of focal length f each are kept in contact. The space between the two lenses is filled with water ($\mu_w = 4/3$). The focal length of the combination is

(a) $f/3$ (b) f

(c) $4f/3$ (d) $3f/4$

14. In a diffraction pattern due to a single slit of width a, the first minimum is observed at an angle 30° when light of wavelength 5000 Å is incident on the slit. The first secondary maximum is observed at an angle of

(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{3}{4}\right)$

(c) $\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\sin^{-1}\left(\frac{2}{3}\right)$

15. The intensity at the maximum in a Young's double slit experiment is I_0. Distance between two slits is $d = 5\lambda$, where λ is the wavelength of light used in the experiment. What will be the intensity in front of one of the slits on the screen placed at a distance $D = 10d$?

(a) $\frac{3}{4}I_0$ (b) $\frac{I_0}{2}$

(c) I_0 (d) $\frac{I_0}{4}$

16. An astronomical telescope has objective and eyepiece of focal lengths 40 cm and 4 cm respectively. To view an object 200 cm away from the objective, the lenses must be separated by a distance

(a) 50.0 cm (b) 54.0 cm

(c) 37.3 cm (d) 46.0 cm

17. The angle of incidence for a ray of light at a refracting surface of a prism is 45°. The angle of prism is 60°. If the ray suffers minimum deviation through the prism, the angle of minimum deviation and refractive index of the material of the prism respectively, are

(a) $45°; \sqrt{2}$ (b) $30°; \frac{1}{\sqrt{2}}$

(c) $45°; \frac{1}{\sqrt{2}}$ (d) $30°; \sqrt{2}$

18. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ, the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when

(a) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(b) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(c) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

(d) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$

19. In Young's double slit experiment, the distance between slits and the screen is 1.0 m and monochromatic light of 600 nm is being used. A person standing near the slits is looking at the fringe pattern. When the separation between the slits is varied, the interference pattern disappears for a particular distance d_0 between the slits. If the angular resolution of the eye is $\frac{1°}{60}$, the value of d_0 is close to

(a) 1 mm (b) 3 mm

(c) 2 mm (d) 4 mm

20. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygen's principle leads us to conclude that as it travels, the light beam

(a) bends downwards

(b) bends upwards

(c) becomes narrower

(d) goes horizontally without any deflection.

SOLUTIONS

1. **(b):** Here, $\mu_{diamond} = \frac{5}{2}$, $\mu_{water} = \frac{4}{3}$

$\lambda_{vacuum} = 4000 \text{ Å} = 4000 \times 10^{-10}$ m

Refractive index of diamond

$$\mu_{diamond} = \frac{\text{Wavelength of light in vacuum}}{\text{Wavelength of light in diamond}} = \frac{\lambda_{vacuum}}{\lambda_{diamond}}$$

$$\lambda_{diamond} = \frac{4000 \text{ Å}}{(5/2)} = 1600 \text{ Å}$$

Refractive index of water

$$\mu_{water} = \frac{\text{Wavelength of light in vacuum}}{\text{Wavelength of light in water}} = \frac{\lambda_{vacuum}}{\lambda_{water}}$$

$$\lambda_{water} = \frac{4000\text{ Å}}{(4/3)} = 3000\text{ Å}$$

Number of waves in thickness t of diamond,

$$n_{diamond} = \frac{t}{\lambda_{diamond}}$$

Number of waves in same thickness t of water,

$$n_{water} = \frac{t}{\lambda_{water}}$$

According to question,

$$n_{diamond} - n_{water} = 200$$

$$\frac{t}{\lambda_{diamond}} - \frac{t}{\lambda_{water}} = 200$$

$$t\left(\frac{\lambda_{water} - \lambda_{diamond}}{\lambda_{diamond}\lambda_{water}}\right) = 200$$

$$t\left(\frac{3000\text{ Å} - 1600\text{ Å}}{(1600\text{ Å})(3000\text{ Å})}\right) = 200$$

On solving, we get

$$t = 6.85 \times 10^{-5}\text{ m} = 0.0685\text{ mm}$$

2. **(d):** Let I_0 be the intensity of incident light. Then the intensity of light from the 1st polaroid is

$$I_1 = \frac{I_0}{2}$$

Intensity of light from the 2nd polaroid is

$$I_2 = I_1\cos^2 60° = \frac{I_0}{2}\left(\frac{1}{2}\right)^2 = \frac{I_0}{8}$$

Intensity of light from the 3rd polaroid is

$$I_3 = I_2\cos^2 60° = \frac{I_0}{8}\left(\frac{1}{2}\right)^2 = \frac{I_0}{32}$$

Intensity of light from the 4th polaroid is

$$I_4 = I_3\cos^2 60° = \frac{I_0}{32}\left(\frac{1}{2}\right)^2 = \frac{I_0}{128}$$

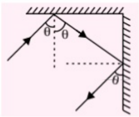
Intensity of light from 5th polaroid is

$$I_5 = I_4\cos^2 60° = \frac{I_0}{128}\left(\frac{1}{2}\right)^2 = \frac{I_0}{512}$$

Therefore, the fraction of the incident light that passes through the system is

$$\frac{I_5}{I_0} = \frac{1}{512}$$

3. **(d):**



The incident and the second reflected ray make the same angle θ with vertical. Therefore, they are parallel for any value of θ.

4. **(c) :** Distance between the first minimum on the left and the first minimum on the right is also the width of central maximum.

Width of central maximum, $W = \frac{2\lambda D}{a}$

where, λ = Wavelength of light

a = Width of the slit

D = Distance of the screen from the slit

$$\therefore \quad a = \frac{2\lambda D}{W}$$

Here, $\lambda = 6000\text{ Å} = 6000 \times 10^{-10}\text{ m}$,

$D = 80\text{ cm} = 80 \times 10^{-2}\text{ m}$

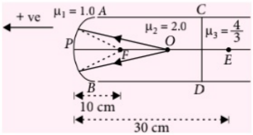
$W = 5\text{ mm} = 5 \times 10^{-3}\text{ m}$

$$\therefore \quad a = \frac{2 \times 6000 \times 10^{-10}\text{ m} \times 80 \times 10^{-2}\text{ m}}{5 \times 10^{-3}\text{ m}}$$

$$= 19.2 \times 10^{-5}\text{ m} = 0.192 \times 10^{-3}\text{ m} = 0.192\text{ mm}$$

5. **(c) :** When light travels from medium of refractive index μ_2 to medium of refractive index μ_1 at a single spherical surface, the formula used is

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$



Direction of light is in positive direction.

$$\therefore \quad \frac{1.0}{v} - \frac{2.0}{-15} = \frac{1.0 - 2.0}{-10}$$

F is centre of curvature of APB.

or $\frac{1}{v} = \frac{1}{10} - \frac{2}{15} = \frac{3-4}{30} = \frac{-1}{30}$ or $v = -30$ cm

$\therefore$ The distance of the final image of O from P, as viewed from the left, is 30 cm to right of P. The image formed will be virtual at E.

6. **(a) :** The distance of 10th maximum of wavelength λ_1 from the central maximum is

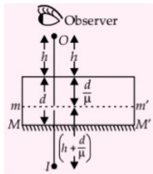
$$y_1 = 10\lambda_1\frac{D}{d}$$

where D is the distance of the slits from the screen and d is the distance between the slits.

The distance of 5[th] maximum of wavelength λ_2 from the central maximum is

$$y_2 = 5\lambda_2 \frac{D}{d} \qquad \therefore \quad \frac{y_1}{y_2} = \frac{2\lambda_1}{\lambda_2}$$

7. (a) : As shown in figure, glass slab will form the image of bottom *i.e.*, mirror MM' at a depth $\left(\frac{d}{\mu}\right)$ from its front face.



So the distance of object O from virtual mirror mm' will be $h + \left(\frac{d}{\mu}\right)$.

Now as a plane mirror forms image behind the mirror at the same distance as the object is in front of it, the distance of image I from mm' will be $h + \left(\frac{d}{\mu}\right)$ and as the distance of virtual mirror from the front face of slab is $\left(\frac{d}{\mu}\right)$, the distance of image I from front face as seen by observer will be

$$= \left[h + \frac{d}{\mu}\right] + \frac{d}{\mu} = h + \frac{2d}{\mu}$$

8. (d): Here, f_o = 4 cm, f_e = 10 cm, u_o = – 5 cm

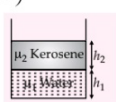
For objective

$$\frac{1}{v_o} - \frac{1}{-5} = \frac{1}{4}, \; \frac{1}{v_o} = \frac{1}{4} - \frac{1}{5} = \frac{1}{20} \text{ or } v_o = 20 \text{ cm}$$

Magnification when the final image is formed at the least distance of distinct vision D (= 25 cm) is

$$M = \frac{v_o}{|u_o|}\left(1 + \frac{D}{f_e}\right) = \frac{20}{5}\left(1 + \frac{25}{10}\right) = 14$$

9. (c) : Apparent shift of bottom position of beaker in water is



$$x_1 = h_1 - \frac{h_1}{\mu_1} = h_1\left(1 - \frac{1}{\mu_1}\right)$$

Apparent shift of bottom position in kerosene is

$$x_2 = h_2 - \frac{h_2}{\mu_2} = h_2\left(1 - \frac{1}{\mu_2}\right)$$

$$\text{Total shift} = x_1 + x_2 = h_1\left(1 - \frac{1}{\mu_1}\right) + h_2\left(1 - \frac{1}{\mu_2}\right)$$

10. (d): Here, f_o = 50 cm, f_e = 5 cm, D = 25 cm

The length of the telescope when the image is formed at the least distance of distinct vision is

$$L = f_o + \frac{f_e D}{f_e + D} = 50 + \frac{5 \times 25}{5 + 25} = 50 + \frac{25}{6} = \frac{325}{6} \text{ cm}$$

11. (c) : If W_1 and W_2 are widths of two slits, then

$$\frac{I_1}{I_2} = \frac{W_1}{W_2} = 4$$

$$\text{Also, } \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} \quad \therefore \quad \frac{A_1^2}{A_2^2} = 4 \text{ or } \frac{A_1}{A_2} = 2$$

$$\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \frac{\left(\frac{A_1}{A_2} + 1\right)^2}{\left(\frac{A_1}{A_2} - 1\right)^2} = \frac{(2+1)^2}{(2-1)^2} = \frac{9}{1}$$

12. (d): Here, f_o = 1.5 cm, f_e = 6.25 cm, u_o = –2 cm, v_e = –25 cm

For objective,

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o} \qquad \therefore \quad \frac{1}{v_o} - \frac{1}{-2} = \frac{1}{1.5}$$

$$\frac{1}{v_o} = \frac{1}{1.5} - \frac{1}{2} \text{ or } v_o = 6 \text{ cm}$$

For eye piece,

$$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$$

$$\frac{1}{-25} - \frac{1}{u_e} = \frac{1}{6.25}$$

$$-\frac{1}{u_e} = \frac{1}{6.25} + \frac{1}{25} \text{ or } u_e = -5 \text{ cm}$$

Distance between two lenses = $|v_o| + |u_e|$

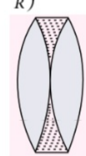
= 6 cm + 5 cm = 11 cm

13. (d): Let R be the radius of curvature of each surface.

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R} + \frac{1}{R}\right) \qquad \therefore \quad R = f$$

$$\text{For the water lens, } \frac{1}{f'} = \left(\frac{4}{3} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right)$$

$$= \frac{1}{3}\left(-\frac{2}{f}\right)$$



$$\text{or} \quad \frac{1}{f'} = -\frac{2}{3f}$$

$$\text{Using, } \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f}+\frac{1}{f'}=\frac{2}{f}-\frac{2}{3f}=\frac{4}{3f}$$

$\therefore\quad F=\frac{3f}{4}$

14. (b): For first minimum,
the path difference between extreme waves,
$a\sin\theta=\lambda$

Here $\theta=30^\circ \Rightarrow \sin 30^\circ=\frac{1}{2}$

$\therefore\quad a=2\lambda$ ...(i)

For first secondary maximum, the path difference between extreme waves

$a\sin\theta'=\frac{3}{2}\lambda$ or $(2\lambda)\sin\theta'=\frac{3}{2}\lambda$ [Using eqn (i)]

or $\sin\theta'=\frac{3}{4}\quad\therefore\quad \theta'=\sin^{-1}\left(\frac{3}{4}\right)$

15. (b): Here, $d=5\lambda$, $D=10d$, $y=\frac{d}{2}$.

Resultant Intensity at $y=\frac{d}{2}, I_y=?$

The path difference between two waves at $y=\frac{d}{2}$

$$\Delta x=d\tan\theta=d\times\frac{y}{D}=\frac{d\times\frac{d}{2}}{10d}=\frac{d}{20}=\frac{5\lambda}{20}=\frac{\lambda}{4}$$

Corresponding phase difference, $\phi=\frac{2\pi}{\lambda}\Delta x=\frac{\pi}{2}$.

Now, maximum intensity in Young's double slit experiment,

$I_{max}=I_1+I_2+2\sqrt{I_1I_2}$

$I_0=4I \Rightarrow I=\frac{I_0}{4}$. $(\because I_1=I_2=I)$

Required intensity $I_y=I_1+I_2+2I_1I_2\cos\frac{\pi}{2}$

$=2I=\frac{I_0}{2}$

16. (b): Here $f_o=40$ cm, $f_e=4$ cm
Tube length(l) = Distance between lenses = v_o+f_e
For objective lens,

$u_o=-200$ cm, $v_o=?$

$\frac{1}{v_o}-\frac{1}{u_o}=\frac{1}{f_o}$ or $\frac{1}{v_o}-\frac{1}{-200}=\frac{1}{40}$

or $\frac{1}{v_o}=\frac{1}{40}-\frac{1}{200}=\frac{4}{200}\quad\therefore\ v_o=50$ cm

$\therefore\quad l=50+4=54$ cm

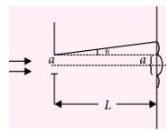
17. (d): Given, $i=45^\circ$, $A=60^\circ$
Since the ray undergoes minimum deviation, therefore, angle of emergence from second face, $e=i=45^\circ$

$\therefore\quad \delta_m=i+e-A=45^\circ+45^\circ-60^\circ=30^\circ$

$$\mu=\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}=\frac{\sin\left(\frac{60^\circ+30^\circ}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$$

$$=\frac{\sin 45^\circ}{\sin 30^\circ}=\frac{1}{\sqrt{2}}\times\frac{2}{1}=\sqrt{2}$$

18. (c): Size of spot, b
= Geometrical spread + diffraction spread



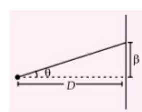
$\therefore\quad b=a+L\frac{\lambda}{a}$

Now, value of b would be minimum if $\frac{db}{da}=0$

$1+L\left(\frac{-\lambda}{a^2}\right)=0\Rightarrow a^2=\lambda L\Rightarrow a=\sqrt{\lambda L}$

$\therefore\quad b_{min}=\sqrt{\lambda L}+\frac{\lambda L}{\sqrt{\lambda L}}=2\sqrt{\lambda L}=\sqrt{4\lambda L}$

19. (c): For a particular distance d_0 between the slits, the eye is not able to resolve two consecutive bright fringes.



Now, $\theta=\frac{\beta}{D}$ but $\beta=\frac{\lambda D}{d_0}$

$\therefore\quad \theta=\frac{\lambda}{d_0}$

or $d_0=\frac{\lambda}{\theta}=\frac{600\times10^{-9}\text{ m}}{\frac{1}{60}\times\frac{\pi}{180}\text{rad}}=2.06\times10^{-3}\text{ m}\approx 2\text{ mm}$

20. (b): Consider a plane wavefront travelling horizontally.

As refractive index of air increases with height, so speed of wavefront decreases with height. Hence, the light beam bends upwards.

CBSE warm-up!

CLASS-XII

Practice questions for CBSE Exams as per the reduced syllabus, latest pattern and marking scheme issued by CBSE for the academic session 2020-21.

Series 4

CHAPTERWISE PRACTICE PAPER : Electromagnetic Induction | Alternating Current

Time Allowed : 3 hours
Maximum Marks : 70

GENERAL INSTRUCTIONS

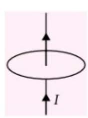
(1) All questions are compulsory. There are 33 questions in all.
(2) This question paper has five sections: Section A, Section B, Section C, Section D and Section E.
(3) Section A contains ten very short answer questions and four assertion reasoning MCQs of 1 mark each, Section B has two case based questions of 4 marks each, Section C contains nine short answer questions of 2 marks each, Section D contains five short answer questions of 3 marks each and Section E contains three long answer questions of 5 marks each.
(4) There is no overall choice. However internal choice is provided. You have to attempt only one of the choices in such questions.

Section-A

1. A light metal disc on the top of an electromagnet is thrown up as the current is switched on. Why? Give reason.

2. The frequency of a.c. is doubled. How do R, X_L and X_C get affected?

3. The current I in a wire passing normally through the centre of a conducting loop is increasing at a constant rate. Will any current be induced in the loop?



OR

On what factors does the magnitude of the emf induced in the circuit due to magnetic flux depend?

4. The peak value of emf in ac is E_0. Write its (i) rms (ii) average value over a complete cycle.

5. Explain the term 'sharpness of resonance' in ac circuit.

OR

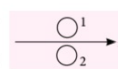
Define capacitive reactance. Write its S.I. units.

6. State Lenz's law.

7. Give one example of use of eddy currents.

OR

What is the direction of induced current in metal rings 1 and 2 when current I in the wire is increasing steadily ?



8. Why is the core of a transformer laminated ?

9. A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease? Explain.

10. What is the function of a step-up transformer ?

OR

Plot a graph showing variation of capacitive reactance with the change in the frequency of the ac source.

For question numbers 11, 12, 13 and 14, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false and R is also false

11. **Assertion (A) :** Only a change in magnetic flux will maintain an induced current in the coil.
Reason (R) : The presence of large magnetic flux through a coil maintains a current in the coil if the circuit is continuous.

12. **Assertion (A) :** The alternating current lags behind the e.m.f. by a phase angle of $\pi/2$, when AC flows through an inductor.
Reason (R) : The inductive reactance increases as the frequency of AC source decreases.

13. **Assertion (A) :** The possibility of an electric bulb fusing is higher at the time of switching on and off.
Reason (R) : Inductive effects produce a surge at the time of switch-off and switch-on.

14. **Assertion (A) :** The quantity L/R possesses dimension of time.
Reason (R) : To reduce the rate of increase of current through a solenoid, we should increase the time constant (L/R).

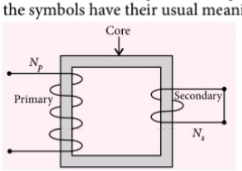
Section-B

Questions 15 and 16 are Case Study based questions and are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

15. A transformer is an electrical device which is used for changing the a.c. voltage. It is based on the principle of mutual induction, *i.e.*, whenever the amount of magnetic flux linked with a coil changes, an e.m.f. is induced in the neighbouring coil.

For an ideal transformer, $\frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$

where the symbols have their usual meaning.



(i) A transformer has 20 turns in primary and 200 turns in secondary. If the primary is connected to 220 V d.c. supply then the voltage across secondary coil is
(a) 10 V (b) 25 V (c) 8 V (d) 0 V

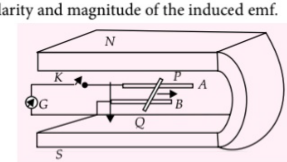
(ii) The output voltage of an ideal transformer connected to 240 V a.c. mains is 24 V. When this transformer is used to light a bulb with rating 24 V, 24 W, then the primary current is
(a) 2.0 A (b) 0.1 A (c) 2.5 A (d) 3.0 A

(iii) A transformer which increases the a.c. voltage is called
(a) step up transformer (b) step down transformer
(c) ideal transformer (d) none of these.

(iv) For a step up transformer
(a) $k > 1$ (b) $k < 1$ (c) $k = 1$ (d) $k \leq 1$

(v) The number of turns in secondary coil of a transformer is 100 times the number of turns in primary coil. The transformer ratio is
(a) 50 (b) 100 (c) 200 (d) 150

16. Figure shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0.50 T, resistance of the closed loop containing the rod = 9.0 mΩ. Assume the field to be uniform. Suppose K is open and the rod is moved with a speed of 12 cm s^{-1} in the direction shown. Give the polarity and magnitude of the induced emf.



(i) Suppose K is open and the rod is moved with a speed of 12 cm s^{-1} in the direction shown. The magnitude of the induced emf
(a) 3 mV (b) 9 mV (c) 12 mV (d) 13 mV

(ii) In the given figure, if K is open then
(a) upper end of the rod become positively charged
(b) upper end of the rod become negatively charged
(c) lower end of the rod become positively charged
(d) none of these.

(iii) When K is closed, the retarding force on the rod is
(a) 0.57 N (b) 0.053 N
(c) 0.075 N (d) 0.23 N

(iv) When K is closed, the power required (by an external agent) to keep the rod moving at the same speed (= 12 cm s^{-1}) is
(a) 5 mW (b) 7 mW (c) 9 mW (d) 3 mW

(v) The power dissipated as heat in the close circuit is
(a) 9 mW (b) 3 mW (c) 6 mW (d) 0 mW

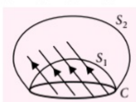
Section-C

17. Show that the current leads the voltage in phase by $\pi/2$ in an ac circuit containing an ideal capacitor.

18. A coil with an average diameter of 0.02 m is placed perpendicular to a magnetic field of 6000 T. If the induced emf is 11 V when the magnetic field is changed to 1000 T in 4 s, what is the number of turns in the coil?

OR

Consider a closed loop C in a magnetic field (figure shown). The flux passing through the loop is defined by choosing a surface whose edge coincides with the loop and using the formula $\phi = B_1 \cdot dA_1 + B_2 \cdot dA_2 + ...$ Now if we choose two different surfaces S_1 and S_2 having C as their edge, would we get the same answer for flux. Justify your answer.



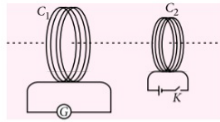
19. An a.c. source of frequency 50 Hz is connected to a 50 mH inductor and a bulb. The bulb glows with some brightness. Calculate the capacitance of the capacitor to be connected in series with the circuit so that the bulb glows with maximum brightness.

OR

A coil offers an inductive reactance of 18.0 Ω at a frequency of 50 Hz. What is the self-inductance of the coil? What reactance will the coil offer at a frequency of 20 kHz?

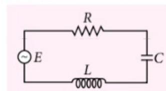
20. There are two coils A and B separated by some distance. If a current of 2 A flows through A, a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and a current of 1 A passes through B, what is the flux through A?

21. A current is induced in coil C_1 due to the motion of current carrying coil C_2. (a) Write any four ways by which a large deflection can be obtained in the galvanometer G. (b) Suggest an alternative device to demonstrate the induced current in place of a galvanometer.

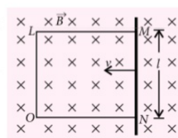


22. How does the mutual inductance of a pair of coils change when
(i) distance between the coils is increased and
(ii) number of turns in the coils is increased?

23. The figure shows a series LCR circuit connected to a variable frequency 200 V source with L = 50 mH, C = 80 μF and R = 40 Ω. Determine
(i) the source frequency which drives the circuit in resonance;
(ii) the quality factor (Q) of the circuit.



24. A rectangular conductor $LMNO$ is placed in a uniform magnetic field of 0.5 T. The field is directed perpendicular to the plane of the conductor. When the arm MN of length of 20 cm is moved towards left with a velocity of 10 m s^{-1}, calculate the emf induced in the arm. Given the resistance of the arm to be 5 Ω (assuming that other arms are of negligible resistance), find the value of the current in the arm.



OR

A wheel with 8 metallic spokes each 50 cm long is rotated with a speed of 120 rev min^{-1} in a plane normal to the horizontal component of the Earth's magnetic field. The Earth's magnetic field at the place is 0.4 G and the angle of dip is 60°. Calculate the emf induced between the axle and the rim of the wheel. How will the value of emf be affected if the number of spokes were increased?

25. State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers?

Section-D

26. Figure given below shows an arrangement by which current flows through the bulb (X) connected with coil B, when a.c. is passed through coil A.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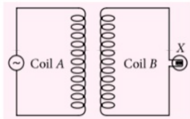
Explain the following observations.

(a) Bulb lights up.

(b) Bulb gets dimmer if the coil 'B' is moved upwards.

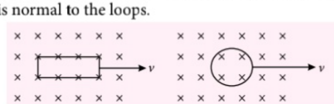
(c) If a copper sheet is inserted in the gap between the coils, how the brightness of the bulb would change?



27. Name the SI unit of magnetic flux and show that it equals volt-sec. Give three possible ways of producing an induced emf in a coil giving an example in each case.

OR

A rectangular loop and a circular loop are moving out of a uniform magnetic field region with a constant velocity $\vec{v}$ as shown in the figure. In which loop do you expect the induced emf to be constant during the passage out of the field region? The field is normal to the loops.



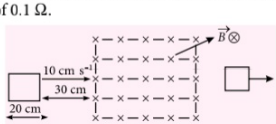
28. A square loop of side 20 cm is initially kept 30 cm away from a region of uniform magnetic field of 0.1 T as shown in the figure. It is then moved towards the right with a velocity of 10 cm s^{-1} till it goes out of the field.

Plot a graph showing the variation of

(i) magnetic flux (ϕ) through the loop with time (t).

(ii) induced emf (ε) in the loop with time t.

(iii) induced current in the loop if it has resistance of 0.1 Ω.



29. (a) An electric heater is connected, turn by turn, to DC and AC sources of equal voltages. Will the rate of heat production be same in the two cases? Explain.

(b) What are the disadvantages of AC as compared to DC?

OR

When a circuit element X is conneted across an a.c. source, a current of $\sqrt{2}$ A flows through it and this current is in phase with the applied voltage. When another element Y is connected across the same a.c. source, the same current flows in the circuit but it leads the voltage by $\pi/2$ radians.

(a) Name the circuit elements X and Y.

(b) Find the current that flows in the circuit when the series combination of X and Y is connected across the same a.c. voltage.

(c) Plot a graph showing variation of the net impedance of this series combination of X and Y as a function of the angular frequency of the applied voltage.

30. A 25.0 μF capacitor, a 0.10 H inductor and a 25.0 Ω resistor are connected in series with an a.c. source whose emf is given by $\varepsilon = 310 \sin 314\, t$ V.

(a) What is the frequency of the emf?

(b) What is the reactance of the circuit?

(c) What is the impedance of the circuit?

Section-E

31. (a) Draw a schematic diagram of an ac generator describing its basic elements. State briefly its working principle.

(b) A rectangular coil of area A, having number of turns N is rotated at 'f' revolutions per second in a uniform magnetic field B, the field being perpendicular to the coil. Prove that the maximum emf induced in the coil is $2\pi f NBA$.

OR

A jet plane is travelling towards west at a speed of 1800 km h^{-1}.

(a) Estimate voltage difference developed between the ends of the wing having a span of 25 m if the earth's magnetic field at the location has a magnitude of 5×10^{-4} T and dip angle is 30°.

(b) How will the voltage developed be affected if the jet changes its direction from west to north?

32. (a) A conductor of length 'l' is rotated about one of its ends at a constant angular speed 'ω' in a plane perpendicular to a uniform magnetic field B. Plot graphs to show variations of the emf induced across the ends of the conductor with (i) angular speed ω and (ii) length of the conductor l.

(b) A circular coil of cross-sectional area 200 cm^2 and 20 turns is rotated about the vertical diameter with angular speed of 50 rad s^{-1} in a uniform magnetic field of magnitude 3.0×10^{-2} T. Calculate the maximum value of the current in the coil.

OR

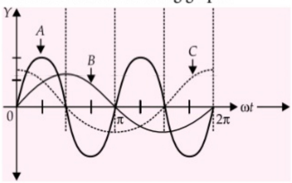
The current flowing through an inductor of self inductance L is continuously increasing. Plot a graph showing the variation of

(i) Magnetic flux versus the current

(ii) Induced emf versus dI/dt

(iii) Magnetic potential energy stored versus the current.

33. A device 'X' is connected to an ac source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph :



(a) Identify the device 'X'.

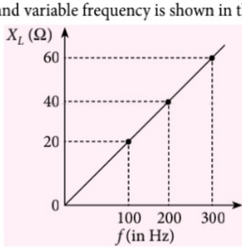
(b) Which of the curves A, B and C represent the voltage, current and the power consumed in the circuit? Justify your answer.

(c) How does its impedance vary with frequency of the ac source? Show graphically.

(d) Obtain an expression for the current in the circuit and its phase relation with ac voltage.

OR

The variation of inductive reactance (X_L) of an inductor with the frequency (f) of the ac source of 100 V and variable frequency is shown in the fig.



(a) Calculate the self-inductance of the inductor.

(b) When this inductor is used in series with a capacitor of unknown value and a resistor of 10 Ω at 300 s^{-1}, maximum power dissipation occurs in the circuit. Calculate the capacitance of the capacitor.

SOLUTIONS

1. As the electric current is switched on, the increasing magnetic flux sets up eddy currents in the disc and convert it into a small magnet. This disc has same polarity as that on the top of the electromagnet and hence it is thrown up due to repulsion.

2. (i) R remains unaffected.

(ii) X_L gets doubled, because $X_L \propto \upsilon$

(iii) X_C becomes one-half of the original value, because

$$X_C \propto \frac{1}{\upsilon}$$

3. The magnetic lines of force due to the current I are parallel to the plane of the loop. The flux linked with the loop is zero. Hence no current is induced in the loop.

OR

The magnitude of the emf induced in the circuit due to magnetic flux depends on the rate of change of magnetic flux with time through the circuit.

$$|\varepsilon| = \frac{\Delta\phi}{\Delta t}$$

4. E_0 = peak value of emf

(i) rms value $[E_{rms}] = \frac{E_0}{\sqrt{2}}$

(ii) rms value $[E_{rms}] = \frac{E_0}{\sqrt{2}}$

5. Sharpness of resonance : It is defined as the ratio of the voltage developed across the inductance (L) or capacitance (C) at resonance to the voltage developed across the resistance (R).

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

It may also be defined as the ratio of resonant angular frequency to the bandwidth of the circuit.

$$Q = \frac{\omega_r}{2\Delta\omega}$$

OR

Capacitive reactance is the resistance offered by a capacitor to the flow of ac through it. It is denoted by X_C. Mathematically,

$$X_C = \frac{1}{2\pi\upsilon C}$$

where υ = frequency of ac source

C = capacitance of the capacitor.

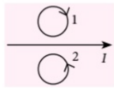
Ohm (Ω) is the SI unit of capacitive reactance.

6. Lenz's law states that the direction of the induced emf and the direction of induced current are such that they oppose the cause which produces them.

7. Eddy current is used for magnetic braking in trains. Strong electromagnets are situated in the train, just above the rails. When the electromagnets are activated, the eddy currents induced in the rails opposes the motion of the train.

OR

When current I in the wire is increasing steadily, the direction of induced current in metal ring 1 is clockwise and in metal ring 2 it is anticlockwise.



8. The core of a transformer is laminated to reduce the energy losses due to eddy currents, so that its efficiency may remain nearly 100%.

9. When an iron core is inserted into a solenoid, magnetic flux increases, induced emf is set up in the coil which opposes current according to Lenz's law and hence current in solenoid decreases.

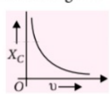
10. A step-up transformer is used to convert a low voltage at high current into a high voltage at low current.

OR

The variation of capacitive reactance with the change in the frequency of the AC source is shown in the figure.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C}$$

$$X_C \propto \frac{1}{\upsilon}$$



11. (c) : Induced current (emf) in a coil is directly proportional to the rate of change of magnetic flux linked with the coil. If there is no change in the flux, there is no induced current also.

12. (c) : Assertion is correct but reason is false because inductive reactance $X_L = 2\pi f L$. So, higher the frequency greater will be the inductive reactance.

13. (a) : The assertion is true. The reason is also true. It explains assertion. The bulb fuses when potential difference across it becomes more than 220 volt.
At on and off hour, inductive effect may produce a surge in voltage and the bulb may be fused under higher voltage.

14. (b): Induced e.m.f. $|\varepsilon| = \frac{Ldi}{dt}$

and current $(i) = \frac{\varepsilon}{R} = \frac{1}{R} \times \frac{Ldi}{dt}$

or $\frac{L}{R} = \frac{idt}{di} = \frac{\text{current} \times \text{time}}{\text{current}} = \text{time}$

In order to reduce the rate of increase of current through a solenoid, we increase the time constant (L/R).

15. (i) (d) : Here, $N_p = 20, N_s = 200, E_p = 220$ V d.c., $E_s = ?$
Voltage across secondary will be zero, as a transformer does not work on d.c. It is based on mutual induction.

(ii) (b) : Here, $E_p = 240$ V, $E_s = 24$ V

$$I_s = \frac{P_0}{E_s} = \frac{24}{24} = 1\text{ A}, I_p = ?$$

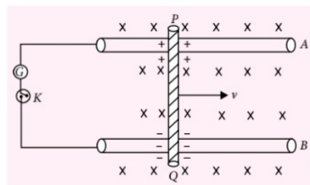
From $\frac{E_s}{E_p} = \frac{I_p}{I_s}, I_p = \frac{E_s \times I_s}{E_p} = \frac{24 \times 1}{240} = 0.1$ A

(iii) (a)

(iv) (a) : For step up transformer, the transformer ratio, $k > 1$.

(v) (b) : $k = \frac{N_s}{N_p} = 100$

16. (i) (b) : Here rails, rod and magnetic field are in three mutually perpendicular directions.

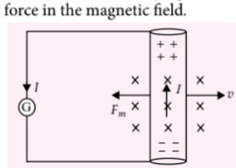


If switch K is open and rod moves with speed of 12 cm s^{-1},
Induced emf/motional emf, $\varepsilon = Bvl$

$\therefore \quad \varepsilon = 0.5 \times 12 \times 10^{-2} \times 15 \times 10^{-2} = 9$ mV

(ii) (a) : When K is open, upper end of the rod become positively charged, and lower end become negatively charged.
When the K is closed the charge flows in closed circuit but the excess charge is maintained by the flow of charge in the moving rod under magnetic force.

(iii) (c) : When the key is closed, the current flows in a loop and the current carrying wire experience a retarding force in the magnetic field.



$F_m = IBl$

where $I = \frac{Bvl}{R} = \frac{9 \times 10^{-3}}{9 \times 10^{-3}} = 1$ A

$\therefore \quad F_m = 1 \times 0.5 \times 15 \times 10^{-2} = 0.075$ N

(iv) (c) : To keep the rod moving in closed circuit at constant speed, the force required is $F = 0.075$ N.
So, power required

$P = \vec{F} \cdot \vec{v} = Fv \cos 0° = Fv$

$P = 0.075 \times 12 \times 10^{-2} = 9$ mW

When key K is open, no current flows and hence no retarding force, so no power is required to move at constant speed.

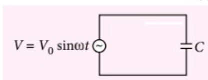
(v) (a) : Power lost in closed circuit due to flow of current

$P = I^2R = (1)^2 \times 9 \times 10^{-3} = 9$ mW

17. Let us consider a capacitor C connected to an ac source as shown in the figure.
Let the ac voltage applied be

$V = V_0 \sin\omega t$ ...(i)

$\therefore \quad V = \frac{q}{C}$ or $q = CV$

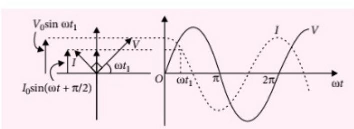


$I = \frac{dq}{dt}$

$I = \frac{d}{dt}(CV_0 \sin \omega t) = \omega CV_0 \cos\omega t = I_0 \cos \omega t$

$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ ...(ii)

where, $I_0 = \omega CV_0 = \frac{V_0}{\frac{1}{\omega C}}$ = current amplitude.



Hence, the current leads the voltage in phase by $\pi/2$.

18. Here, $r = \frac{0.02}{2} = 0.01$ m

$B_1 = 6000$ T, $B_2 = 1000$ T, $t = 4$ s, $\varepsilon = 11$ V

Now $\varepsilon = N\frac{|\phi_2 - \phi_1|}{t} = NA\frac{|B_2 - B_1|}{t}$

$= N \cdot \pi r^2 \cdot \frac{|B_2 - B_1|}{t}$

$\therefore \quad 11 = N \times \frac{22}{7} \times (0.01)^2 \times \frac{|1000 - 6000|}{4}$

Number of turns, $N = \frac{11 \times 7 \times 4}{22 \times (0.01)^2 \times 5000} = 28$

OR

We would get the same answer for magnetic flux because magnetic field lines form closed loops and as such cannot end or start in space. Thus, number of lines passing through S_1 must be the same as those passing through surface S_2.

19. As per question, frequency, $\upsilon = 50$ Hz,

$L = 50$ mH $= 0.05$ H

The bulb glows with maximum brightness when its impedance is minimum or equal to the resistance of the bulb filament and for this,

$X_L - X_C = 0$ or $X_L = X_C$

or $\quad \omega L = \frac{1}{\omega C}$

$\Rightarrow \quad C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 \upsilon^2 L} = \frac{1}{4 \times (3.14)^2 \times (50)^2 \times 0.05}$

$= 2.03 \times 10^{-4}$ F $= 203$ μF

OR

Here, initial frequency, $\upsilon = 50$ Hz and inductive reactance, $X_L = 18.0\ \Omega$

As per relation, $X_L = \omega L = 2\pi\upsilon L$, we have

$\therefore$ Inductance,

$L = \frac{X_L}{2\pi\upsilon} = \frac{18.0}{2 \times 3.14 \times 50} = 0.0573$ H $\simeq 57$ mH

If frequency is changed to $\upsilon' = 20$ kHz $= 20 \times 10^3$ Hz, the new value of reactance,

$X'_L = 2\pi\upsilon' L = 2 \times 3.14 \times 20 \times 10^3 \times 0.0573$

$= 7.2 \times 10^3\ \Omega$

20. Here, $I_A = 2$ A, $\phi_{BA} = 10^{-2}$ Wb

As $\phi_{BA} = M_{BA} I_A, M_{BA} = \frac{\phi_{BA}}{I_A} = \frac{10^{-2}\text{ Wb}}{2\text{ A}}$

$= 5 \times 10^{-3}$ H

When, $I_B = 1$ A, $\phi_{AB} = M_{AB} I_B = (5 \times 10^{-3}$ H) (1 A)

$= 5 \times 10^{-3}$ Wb

(As $M_{AB} = M_{BA} = 5 \times 10^{-3}$ H)

21. (a) To obtain large deflection in galvanometer we can take following steps :

(i) Connect the coil C_2 to a powerful battery for large current.

(ii) Switch on and off the key at a rapid rate.

(iii) Develop a relative shift/motion between the two coils.

(iv) Use a ferromagnetic material like iron inside the coil C_2 to increase the magnetic flux.

(b) Galvanometer can be replaced by a torch bulb. In this case, a relative motion between two coils or switch on and off of the key glows the bulb and shows presence of induced current.

22. (i) With the increase in distance between the coils the magnetic flux linked with the secondary coil decreases and hence, the mutual inductance of the two coils will decrease with the increase in separation between them.

(ii) Since, the mutual inductance of the two coils is given as $M = \mu_0 n_1 n_2 Al$. So, with the increase in number of turns mutual inductance increases.

23. (i) $L = 50 \times 10^{-3}$ H, $C = 80 \times 10^{-6}$ F, $R = 40\ \Omega$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50\times10^{-3}\times80\times10^{-6}}}$$

$$\omega = \frac{10^3}{2} = 500 \text{ rad s}^{-1} \Rightarrow \upsilon = \frac{500}{2\pi} = 80 \text{ Hz}$$

(ii) $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{40}\sqrt{\frac{50\times10^{-3}}{80\times10^{-6}}} = \frac{1}{40}\times\sqrt{625} = 0.625$

24. Induced emf $\varepsilon = Blv = 0.5 \times 0.2 \times 10 = 1$ V

Current $I = \frac{\varepsilon}{R}$

$I = \frac{1}{5} = 0.2$ A

OR

Horizontal component $B_H = B\cos\delta$
$= 0.4 \times \cos 60°$
$= 0.2$ G $= 0.2 \times 10^{-4}$ T

Induced emf $\varepsilon = \frac{1}{2}BL^2\omega$

$= \frac{1}{2}\times0.2\times10^{-4}\times(0.5)^2\times2\pi\times2 = 3.14 \times 10^{-5}$ V

The emf induced between the axle and the rim of the wheel is independent of number of spokes in the wheel.

25. A transformer is based on principle of mutual induction which states that due to continuous change in the current in the primary coil, an emf gets induced across the secondary coil.

Electric power generated at the power station, is stepped up to very high voltages by means of a step-up transformer and transmitted to a distant place. At receiving end, it is stepped down by a step down transformer.

26. (a) Bulb lights up due to the induced current set up in coil B because of alternating current in coil A.

(b) Bulb gets dimmer when the coil B is moved upwards because the flux linked with coil B decreases and induced current also decreases.

(c) When the copper sheet is inserted, eddy currents are set up in it which opposes the passage of magnetic flux. The induced emf in coil B decreases. This decreases the brightness of the bulb.

27. SI unit of magnetic flux is weber (Wb). By Faraday's law

$$|\varepsilon| = \frac{d\phi}{dt}$$

$\therefore$ $1 \text{ volt} = \frac{1 \text{ weber}}{1 \text{ second}}$ or 1 weber = 1 volt-sec

Magnetic flux linked with a closed coil is given by
$\phi = BA\cos\theta$

Whenever ϕ changes, emf is induced. Hence emf may be produced by three methods,

(i) by changing the magnetic field B, *e.g.*, by changing the relative separation between a closed coil and magnet.

(ii) by changing the area A of the closed coil, *e.g.*, by moving a closed loop into and out of a region of magnetic field.

(iii) by changing the relative orientation θ between a closed coil and a magnetic field, *e.g.*, by rotating a closed coil about an axis perpendicular to the magnetic field.

OR

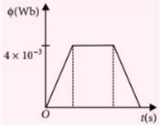
Magnitude of induced emf is directly proportional to the rate of area moving out of the field, for a constant magnetic field,

$$\varepsilon = -\frac{d\phi}{dt} = -B\frac{dA}{dt}$$

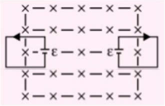
For the rectangular coil, the rate of area moving out of the field remains same while it is not so for the circular coil. Therefore, the induced emf for the rectangular coil remains constant.

28. Given $l = 20$ cm $= 0.2$ m,
$B = 0.1$ T, $v = 10$ cm s^{-1} $= 0.1$ m s^{-1}

(i) Magnetic flux through loop
$\phi = B \cdot A = Blx$
$\phi_{max} = 0.1 \times 0.2 \times 0.2 = 0.004$ Wb $= 4 \times 10^{-3}$ Wb

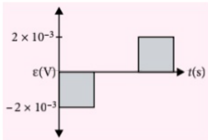


(ii) Induced emf,

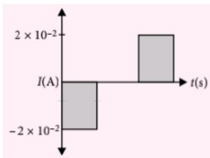


$$\varepsilon = \frac{-d\phi}{dt} = -Blv$$

$|\varepsilon|_{max} = 0.1 \times 0.2 \times 0.1$
$= 0.002$ V $= 2 \times 10^{-3}$ V



(iii) Induced current, $I = \frac{|\varepsilon|}{R} = \frac{2\times10^{-3}}{0.1} = 2\times10^{-2}$ A



29. (a) The heating element of the electric heater is a coil of nichrome wire which has a resistance R and an inductance L. When the heater is connected to DC source of voltage V, the current, $I_{DC} = \frac{V}{R}$

and rate of heat production $= I_{DC}^2 R = \frac{V^2}{R}$

When the same heater is connected to an AC supply of rms voltage V, the impedance of the circuit,

$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$

and hence, the current, $I_{AC} = \frac{V}{Z}$

Therefore, the rate of heat production

$= I_{AC}^2 R = \frac{V^2 R}{Z^2} = \frac{V^2 R}{(R^2 + \omega^2 L^2)}$

From the above explanation, it is clear that rate of heat production in AC circuit is less than that in DC circuit of same voltage.

(b) Main disadvantages of AC as compared to DC are as follows :

(i) AC is more dangerous than DC having same effective voltage.

(ii) For a given effective voltage, the peak voltage in AC is $\sqrt{2}$ times the effective voltage, hence better insulation is required for AC line.

(iii) AC supply cannot be used as such for chemical and magnetic effects of current and in electronic devices.

OR

(a) The circuit element X is a resistor and Y is a capacitor.

(b) Here, $R = X_C ; V_{eff} = \sqrt{2}R$

When X and Y are connected in series, the impedance becomes

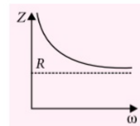
$Z = \sqrt{R^2 + X_C^2} = \sqrt{2R^2} = \sqrt{2}R$

Current, $I_{eff} = \frac{V_{eff}}{Z} = \frac{V_{eff}}{\sqrt{2}R} = \frac{\sqrt{2}R}{\sqrt{2}R} = 1$ A

(c) Impedance,

$Z = \sqrt{R^2 + X_C^2}$

$= \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$



30. (a) Given ε = 310 sin 314 t V

Comparing it with $\varepsilon = \varepsilon_0 \sin 2\pi \upsilon t$, we get

$2\pi\upsilon = 314$ or $\upsilon = \frac{314}{2\pi} = \frac{314}{2\times3.14} = 50$ Hz

(b) $X_C = \frac{1}{2\pi\upsilon C} = \frac{1\times7}{2\times22\times50\times25\times10^{-6}} = 127.3\ \Omega$

$X_L = 2\pi\upsilon L = 2\times\frac{22}{7}\times50\times0.1 = 31.4\ \Omega$

As X_L and X_C are out of phase by 180°, therefore,
Net reactance $= X_C - X_L = 127.3 - 31.4 = 95.9\ \Omega$
and it is capacitive.

(c) Impedance,

$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(25)^2 + (95.9)^2}$

$= \sqrt{625 + 9196.81} = \sqrt{9821.81} = 99.1\ \Omega$

31. (a) *Refer to answer 67, page no. 144* (MTG CBSE Champion Physics Class 12)

(b) *Refer to answer 72, page no. 144* (MTG CBSE Champion Physics Class 12)

OR

Refer to answer 35, page no. 138 (MTG CBSE Champion Physics Class 12)

32. (a) *Refer to answer 27, page no. 137* (MTG CBSE Champion Physics Class 12)

(b) *Refer to answer 70 (b), page no. 144* (MTG CBSE Champion Physics Class 12)

OR

Refer to answer 63, page no. 143 (MTG CBSE Champion Physics Class 12)

33. *Refer to Answer 17, page no. 159* (MTG CBSE Champion Physics Class 12)

OR

Refer to Answer 36, page no. 162 (MTG CBSE Champion Physics Class 12)

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 120

Electromagnetic Induction, Alternating Current and Electromagnetic Waves

Time Taken : 60 Min.

NEET

Only One Option Correct Type

1. Magnetic flux through a stationary loop with a resistance R varies during the time interval τ as $\phi = at(\tau - t)$ where a is a constant. The amount of heat generated in the loop during the time interval τ is

(a) $\frac{a^2\tau^3}{6R}$ (b) $\frac{a^2\tau^3}{4R}$ (c) $\frac{a^2\tau^3}{3R}$ (d) $\frac{a^2\tau^3}{2R}$

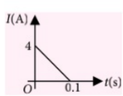
2. An alternating current is given by $I = I_0(\sin\omega t + \cos\omega t)$. The rms current is

(a) $\sqrt{2}I_0$ (b) $\frac{I_0}{\sqrt{2}}$ (c) I_0 (d) $2I_0$

3. A series LCR circuit is connected to an ac source of frequency υ and a voltage V. At this frequency, reactance of the capacitor is 350 Ω while the resistance of the circuit is 180 Ω. Current in the circuit leads the voltage by 54° and power dissipated in the circuit is 140 W. Then the voltage V is

(a) 250 V (b) 260 V (c) 270 V (d) 280 V

4. Some magnetic flux is changed from a coil of resistance 10 Ω. As a result, an induced current is developed in it, which varies with time as shown in figure.



The magnitude of change in magnetic flux through the coil in weber is

(a) 2 (b) 4 (c) 6 (d) 8

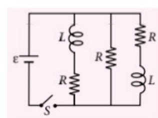
5. In a series LCR circuit, R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit, the current lags behind the voltage by 30°. On taking out the inductor from the circuit, the current leads the voltage by 30°. The power dissipated in the LCR circuit is

(a) zero (b) 210 W (c) 242 W (d) 305 W

6. A 16 μF capacitor is charged to 20 V. The battery is then disconnected and pure 40 mH coil is connected across the capacitor so that LC oscillations are set up. The maximum current in the coil is

(a) 0.2 A (b) 40 mA (c) 2 A (d) 0.4 A

7. Figure shows a circuit that contains three identical resistors with resistance R = 9 Ω, two identical inductors with inductance L = 2 mH, and an ideal battery with emf ε = 18 V.

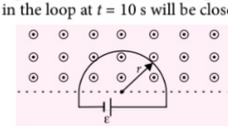


The current in the circuit long after the switch S is closed is

(a) 2 A (b) 4 A (c) 6 A (d) 8 A

8. Figure shows a conducting loop consisting of a half-circle of radius r = 0.2 m and three straight sections. The half-circle lies in a uniform magnetic field $\vec{B}$ that is directed out of the page, the field magnitude is given by $B = (4t^2 + 2t + 3)$ T, where t is in seconds.

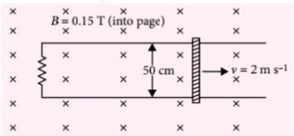
An ideal battery with emf ε = 2 V is connected to the loop. The resistance of the loop is 2 Ω. The current in the loop at t = 10 s will be close to



(a) 3.6 A (b) 1.6 A (c) 6.2 A (d) 4.2 A

9. The electric field of an electromagnetic wave in free space is given by $\vec{E} = 10\cos(10^7 t + kx)\hat{j}$ V m^{-1} where t and x are in seconds and metres, respectively. It can be inferred that
(i) the wavelength λ is 188.4 m.
(ii) the wave number k is 0.33 rad m^{-1}.
(iii) the wave amplitude is 10 V m^{-1}
(iv) the wave is propagating along $+x$ direction.
Which one of the following pairs of statements is correct?
(a) (iii) and (iv) (b) (i) and (ii)
(c) (ii) and (iii) (d) (i) and (iii)

10. As shown in the figure, a metal rod makes contact with a partial circuit and completes the circuit. The circuit area is perpendicular to a magnetic field with $B = 0.15$ T. If the resistance of the total circuit is 3 Ω, the force needed to move the rod as indicated with a constant speed of 2 m s^{-1} will be



(a) 3.75×10^{-3} N (b) 2.75×10^{-3} N
(c) 6.57×10^{-4} N (d) 4.36×10^{-4} N

11. A transformer with efficiency 80% works at 4 kW and 100 V. If the secondary voltage is 200 V, then the primary and secondary currents are respectively
(a) 40 A, 16 A (b) 16 A, 40 A
(c) 20 A, 40 A (d) 40 A, 20 A

12. A coaxial cable consists of two thin cylindrical conducting shells of radii a and b ($a < b$). The inductance per unit length of the cable is

(a) $\frac{\mu_0}{2\pi}\frac{(a+b)}{a}$ (b) $\frac{\mu_0}{4\pi}\ln\left(\frac{a}{b}\right)$

(c) $\frac{\mu_0}{4\pi}\ln\left(\frac{b}{a}\right)$ (d) $\frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$

Assertion & Reason Type

Directions : In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** No power loss is associated with a pure capacitor in an ac circuit.
Reason : No current is flowing in this circuit.

14. **Assertion :** In series LCR circuit, the resonance occurs at one frequency only.
Reason : At resonance, the inductive reactance is equal and opposite to the capacitive reactance.

15. **Assertion :** Dipole oscillations produce electromagnetic waves.
Reason : Accelerated charge produces electromagnetic waves.

JEE MAIN / JEE ADVANCED

Only One Option Correct Type

16. A magnetic field directed along z axis varies as $B = B_0 x/a$, where a is a positive constant. A square loop of side l and made of copper is placed with its edges parallel to x and y axes. If the loop is made to move with a constant velocity v_0 directed along x axis, the emf induced is

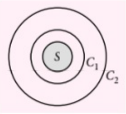
(a) $\frac{B_0 v_0 l^2}{a}$ (b) $B_0 v_0 l$ (c) $\frac{B_0 v_0 l^2}{2a}$ (d) $\frac{B_0 v_0 l^3}{a^2}$

17. In a series LCR circuit, impedance Z is the same at two frequencies υ_1 and υ_2. Then, the resonant frequency of the circuit is

(a) $\frac{\upsilon_1 + \upsilon_2}{2}$ (b) $\frac{2\upsilon_1\upsilon_2}{\upsilon_1 + \upsilon_2}$

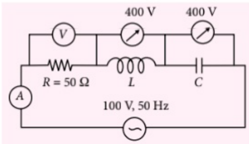
(c) $\frac{\sqrt{\upsilon_1^2 + \upsilon_2^2}}{2}$ (d) $\sqrt{\upsilon_1\upsilon_2}$

18. A spatially uniform magnetic field $\vec{B}$ exists in the circular region S and this field is decreasing in magnitude with time at a constant rate (see figure). The wooden ring C_1 and the conducting ring C_2 are concentric with a magnetic field. The magnetic field is perpendicular to the plane of the figure. Then,



(a) there is no induced electric field in C_1
(b) there is an induced electric field in C_1 and its magnitude is greater than the magnitude of the induced electric field in C_2
(c) there is an induced electric field in C_2 and its magnitude is greater than induced electric field in C_1
(d) there is no induced electric field in C_2.

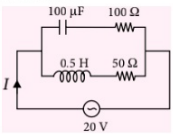
19. In the series LCR circuit, the voltmeter and ammeter readings are



(a) $V = 100$ V, $I = 2$ A (b) $V = 100$ V, $I = 5$ A
(c) $V = 800$ V, $I = 2$ A (d) $V = 300$ V, $I = 1$ A

More than One Options Correct Type

20. In the given circuit, the ac source has $\omega = 100$ rad s^{-1}. Considering the inductor and capacitor to be ideal, the correct choices are



(a) the current through the circuit I is 0.3 A
(b) the current through the circuit I is $0.3\sqrt{2}$ A
(c) the voltage across 100 Ω resistor is $10\sqrt{2}$ V
(d) the voltage across 50 Ω resistor is 10 V.

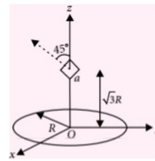
21. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statements are
(a) the emf induced in the loop is zero if the current is constant
(b) the emf induced in the loop is finite if the current is constant
(c) the emf induced in the loop is zero if the current decreases at a steady rate
(d) The emf induced in the loop is finite if the current decreases at a steady rate.

22. As the frequency of an ac circuit increases, the current first increases and then decreases. What combination of circuit elements is most likely to comprise the circuit?
(a) Inductor and capacitor.
(b) Resistor and inductor.
(c) Resistor and capacitor.
(d) Resistor, inductor and capacitor.

23. The mutual inductance M_{12} of coil 1 with respect to coil 2
(a) increases when they are brought nearer
(b) depends on the current passing through the coils
(c) increases when one of them is rotated about an axis
(d) is the same as M_{12} of coil 2 with respect to coil 1.

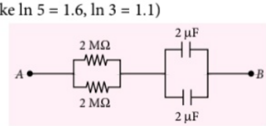
Numerical Value Type

24. A circular wire loop of radius R is placed in the x - y plane centered at the origin O. A square loop of side a ($a << R$) having two turns is placed with its center at $z = \sqrt{3}R$ along the axis of the circular wire loop as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then find the value of p.



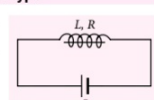
25. A step down transformer transforms a supply line voltage of 2200 V into 220 V. The primary coil has 5000 turns. The efficiency and power transmitted by the transformer are 90% and 8 kW respectively. If the number of turns in secondary coil is $k \times 10^2$, then what is the value of k?

26. At time $t = 0$, a battery of 10 V is connected across points A and B in the circuit shown in figure. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them becomes 4 V?
(Take ln 5 = 1.6, ln 3 = 1.1)



Comprehension Type

A solenoid of resistance R and inductance L has a piece of soft iron inside it. A battery of emf ε and of negligible internal resistance is connected across the solenoid as shown in figure. At any instant, the piece of soft iron is pulled out suddenly so that inductance of the solenoid decreases to ηL ($\eta < 1$) with battery remaining connected.



27. The work done to pull out the soft iron piece is

(a) $\frac{\eta L\varepsilon^2}{2R^2}$ (b) $\frac{(1-\eta)L\varepsilon^2}{2R^2}$

(c) $\frac{(1-\eta)L\varepsilon^2}{\eta R^2}$ (d) $\frac{(1-\eta)L\varepsilon^2}{2\eta R^2}$

28. Assume $t = 0$ is the instant when iron piece has been pulled out, the current as a function of time after this is

(a) $I=\frac{\varepsilon}{R}\left[1-\left(1-\frac{1}{\eta}\right)e^{-\frac{tR}{\eta L}}\right]$

(b) $I=\frac{\varepsilon}{R}\left[1+\left(1+\frac{1}{\eta}\right)e^{-\frac{tR}{\eta L}}\right]$

(c) $I=\frac{\varepsilon}{R}\left[1-\left(1+\frac{1}{\eta}\right)e^{-\frac{tR}{\eta L}}\right]$

(d) $I=\frac{\varepsilon}{R}\left[1+\left(1-\frac{1}{\eta}\right)e^{-\frac{tR}{\eta L}}\right]$

Matrix Match Type

29. You are given many resistors, capacitors and inductors. These are connected to variable dc voltage source (the first two circuits) or ac voltage source of 50 Hz frequency (the next two circuits) in different ways as shown in column II. When a current I (steady state for dc or rms for ac) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in column I. Match the entries of column I with those given in column II.

Column I

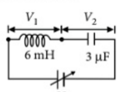
(A) $I \neq 0$, V_1 is proportional to I

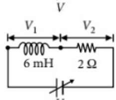
(B) $I \neq 0$, $V_2 > V_1$

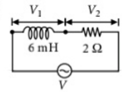
(C) $V_1 = 0$, $V_2 = V$

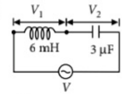
(D) $I \neq 0$, V_2 is proportional to I

Column II

(P)


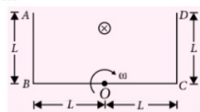
(Q)


(R)


(S)


	A	B	C	D
(a)	P, Q, R	Q, R, S	Q, S	R, S
(b)	R, S	Q, R, S	P, Q	Q, R, S
(c)	P, Q, R	R, S	Q, R, S	P, S
(d)	R, S	P, Q, S	P, S	Q, R, S

30. A frame $ABCD$ is rotating with an angular velocity ω about an axis passing through point O perpendicular to the plane of paper as shown in the figure. A uniform magnetic field $\vec{B}$ is applied into the plane of the paper in the region as in the figure. Match the entries of column I with those given in column II.



Column I	Column II
(A) Potential difference between A and O is	(P) zero
(B) Potential difference between O and D is	(Q) $\frac{B\omega L^2}{2}$
(C) Potential difference between C and D is	(R) $B\omega L^2$
(D) Potential difference between A and D is	(S) constant

	A	B	C	D
(a)	P, Q	Q, R	Q, S	R, S
(b)	Q, S	P, S	R, S	R, S
(c)	R, S	R, S	Q, S	P, S
(d)	P, S	Q, S	R, S	R, S

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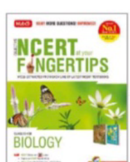
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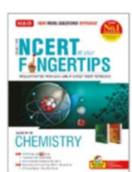
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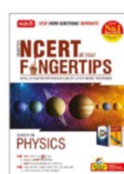
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