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# PHYSICS for you

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# NEET

# SOLVED PAPER 2020

Hurray!! We are happy to inform our readers that out of the 45 questions asked in NEET 2020, more than 90% questions were either exactly same or of similar type from the **MTG Books.** Hurray!!

Here, the references of few are given :

| Exam Q. No. | MTG Book | Q. No. | P. No. |
|---|---|---|---|
| 2 | NEET Guide (XII) | 66 | 360 |
| 4 | NEET Guide (XII) | 103 | 18 |
| 5 | Fingertips (XII) | 91 | 322 |
| 6 | Fingertips (XII) | 29 | 159 |
| 7 | NEET 32 Years (XI) | 13 | 115 |
| 8 | NEET Guide (XI) | 67 | 18 |
| 9 | NEET Guide (XI) | 4 | 331 |
| 10 | NEET Guide (XII) | 187 | 132 |
| 11 | NEET Guide (XII) | 69 | 16 |
| 12 | Fingertips (XI) | 47 | 175 |

| Exam Q. No. | MTG Book | Q. No. | P. No. |
|---|---|---|---|
| 14 | NEET Guide (XII) | 24 | 277 |
| 15 | NEET 32 Years (XII) | 23 | 231 |
| 16 | NEET Guide (XII) | 57 | 15 |
| 17 | NEET 32 Years (XI) | 81 | 72 |
| 18 | Fingertips (XII) | 16 | 58 |
| 19 | NEET 32 Years (XII) | 47 | 257 |
| 20 | NEET Guide (XI) | 139 | 267 |
| 21 | NEET 32 Years (XII) | 99 | 301 |
| 22 | Fingertips (XI) | 39 | 320 |
| 23 | Fingertips (XII) | 92 | 63 |

*and more such questions ……*

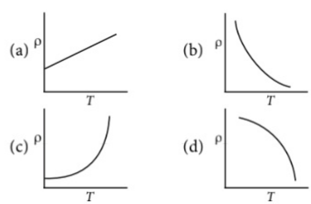
**1.** Light with an average flux of 20 W $cm^{-2}$ falls on a non-reflecting surface at normal incidence having surface area 20 $cm^2$. The energy received by the surface during time span of 1 minute is

(a) $10 \times 10^3$ J (b) $12 \times 10^3$ J
(c) $24 \times 10^3$ J (d) $48 \times 10^3$ J

**2.** For transistor action, which of the following statements is correct?

(a) Base, emitter and collector regions should have same doping concentrations.
(b) Base, emitter and collector regions should have same size.
(c) Both emitter junction as well as the collector junction are forward biased.
(d) The base region must be very thin and lightly doped.

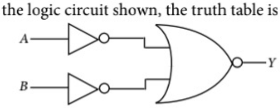
**3.** Which of the following graph represents the variation of resistivity ($\rho$) with temperature ($T$) for copper?




**4.** In a certain region of space with volume 0.2 $m^3$, the electric potential is found to be 5 V throughout. The magnitude of electric field in this region is

(a) zero (b) 0.5 N $C^{-1}$
(c) 1 N $C^{-1}$ (d) 5 N $C^{-1}$

**5.** For the logic circuit shown, the truth table is



(a)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(b)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

(c)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(d)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

**6.** A 40 μF capacitor is connected to a 200 V, 50 Hz ac supply. The r.m.s value of the current in the circuit is, nearly
(a) 1.7 A (b) 2.05 A (c) 2.5 A (d) 25.1 A

**7.** A cylinder contains hydrogen gas at pressure of 249 kPa and temperature 27°C. Its density is ($R = 8.3$ J mol$^{-1}$ K$^{-1}$)
(a) 0.5 kg m$^{-3}$ (b) 0.2 kg m$^{-3}$
(c) 0.1 kg m$^{-3}$ (d) 0.02 kg m$^{-3}$

**8.** Taking into account of the significant figures, what is the value of 9.99 m – 0.0099 m?
(a) 9.9801 m (b) 9.98 m
(c) 9.980 m (d) 9.9 m

**9.** The mean free path for a gas, with molecular diameter $d$ and number density $n$ can be expressed as
(a) $\frac{1}{\sqrt{2}n\pi d}$ (b) $\frac{1}{\sqrt{2}n\pi d^2}$
(c) $\frac{1}{\sqrt{2}n^2\pi d^2}$ (d) $\frac{1}{\sqrt{2}n^2\pi^2 d^2}$

**10.** An iron rod of susceptibility 599 is subjected to a magnetising field of 1200 A m$^{-1}$. The permeability of the material of the rod is ($\mu_0 = 4\pi \times 10^{-7}$ T m A$^{-1}$)
(a) $2.4\pi \times 10^{-4}$ T m A$^{-1}$ (b) $8.0 \times 10^{-5}$ T m A$^{-1}$
(c) $2.4\pi \times 10^{-5}$ T m A$^{-1}$ (d) $2.4\pi \times 10^{-7}$ T m A$^{-1}$

**11.** A short electric dipole has a dipole moment of $16 \times 10^{-9}$ C m. The electric potential due to the dipole at a point at a distance of 0.6 m from the centre of the dipole, situated on a line making an angle of 60° with the dipole axis is $\left(\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{ N m}^2\text{ C}^{-2}\right)$
(a) 50 V (b) 200 V (c) 400 V (d) zero

**12.** A body weighs 72 N on the surface of the earth. What is the gravitational force on it, at a height equal to half the radius of the earth?
(a) 48 N (b) 32 N (c) 30 N (d) 24 N

**13.** The solids which have the negative temperature coefficient of resistance are
(a) metals (b) insulators only
(c) semiconductors only
(d) insulators and semiconductors.

**14.** Light of frequency 1.5 times the threshold frequency is incident on a photosensitive material. What will be the photoelectric current if the frequency is halved and intensity is doubled?
(a) Doubled (b) Four times
(c) One-fourth (d) Zero

**15.** A series $LCR$ circuit is connected to an ac voltage source. When $L$ is removed from the circuit, the phase difference between current and voltage is $\pi/3$. If instead $C$ is removed from the circuit, the phase difference is again $\pi/3$ between current and voltage. The power factor of the circuit is
(a) zero (b) 0.5 (c) 1.0 (d) –1.0

**16.** A spherical conductor of radius 10 cm has a charge of $3.2 \times 10^{-7}$ C distributed uniformly. What is the magnitude of electric field at a point 15 cm from the centre of the sphere? $\left(\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{ N m}^2\text{ C}^{-2}\right)$
(a) $1.28 \times 10^4$ N C$^{-1}$ (b) $1.28 \times 10^5$ N C$^{-1}$
(c) $1.28 \times 10^6$ N C$^{-1}$ (d) $1.28 \times 10^7$ N C$^{-1}$

**17.** Find the torque about the origin when a force of $3\hat{j}$ N acts on a particle whose position vector is $2\hat{k}$ m.
(a) $6\hat{i}$ N m (b) $6\hat{j}$ N m
(c) $-6\hat{i}$ N m (d) $6\hat{k}$ N m

**18.** A charged particle having drift velocity of $7.5\times10^{-4}$ m s$^{-1}$ in an electric field of $3\times10^{-10}$ V m$^{-1}$, has a mobility in m$^2$ V$^{-1}$ s$^{-1}$ of
(a) $2.25 \times 10^{15}$ (b) $2.5 \times 10^6$
(c) $2.5 \times 10^{-6}$ (d) $2.25 \times 10^{-15}$

**19.** A ray is incident at an angle of incidence $i$ on one surface of a small angle prism (with angle of prism $A$) and emerges normally from the opposite surface. If the refractive index of the material of the prism is μ, then the angle of incidence is nearly equal to
(a) $A/2\mu$ (b) $2A/\mu$ (c) $\mu A$ (d) $\mu A/2$

**20.** The quantities of heat required to raise the temperature of two solid copper spheres of radii $r_1$ and $r_2$ ($r_1 = 1.5r_2$) through 1 K are in the ratio
(a) $\frac{27}{8}$ (b) $\frac{9}{4}$ (c) $\frac{3}{2}$ (d) $\frac{5}{3}$

21. When a uranium isotope $^{235}_{92}U$ is bombarded with a neutron, it generates $^{89}_{36}Kr$, three neutrons and
(a) $^{144}_{56}Ba$ (b) $^{91}_{40}Zr$ (c) $^{101}_{36}Kr$ (d) $^{103}_{36}Kr$

22. The phase difference between displacement and acceleration of a particle in a simple harmonic motion is
(a) $\pi$ rad (b) $3\pi/2$ rad
(c) $\pi/2$ rad (d) zero

23. A resistance wire connected in the left gap of a metre bridge balances a 10 Ω resistance in the right gap at a point which divides the bridge wire in the ratio 3 : 2. If the length of the resistance wire is 1.5 m, then the length of 1 Ω of the resistance wire is
(a) $1.0 \times 10^{-2}$ m (b) $1.0 \times 10^{-1}$ m
(c) $1.5 \times 10^{-1}$ m (d) $1.5 \times 10^{-2}$ m

24. A capillary tube of radius $r$ is immersed in water and water rises in it to a height $h$. The mass of the water in the capillary is 5 g. Another capillary tube of radius $2r$ is immersed in water. The mass of water that will rise in this tube is
(a) 2.5 g (b) 5.0 g (c) 10.0 g (d) 20.0 g

25. The ratio of contributions made by the electric field and magnetic field components to the intensity of an electromagnetic wave is ($c$ = speed of electromagnetic waves)
(a) $c : 1$ (b) $1 : 1$ (c) $1 : c$ (d) $1 : c^2$

26. In Young's double slit experiment, if the separation between coherent sources is halved and the distance of the screen from the coherent sources is doubled, then the fringe width becomes
(a) double (b) half
(c) four times (d) one-fourth

27. A long solenoid of 50 cm length having 100 turns carries a current of 2.5 A. The magnetic field at the centre of the solenoid is ($\mu_0 = 4\pi \times 10^{-7}$ T m $A^{-1}$)
(a) $6.28 \times 10^{-4}$ T (b) $3.14 \times 10^{-4}$ T
(c) $6.28 \times 10^{-5}$ T (d) $3.14 \times 10^{-5}$ T

28. A ball is thrown vertically downward with a velocity of 20 m $s^{-1}$ from the top of a tower. It hits the ground after some time with a velocity of 80 m $s^{-1}$. The height of the tower is ($g$ = 10 m $s^{-2}$)
(a) 360 m (b) 340 m (c) 320 m (d) 300 m

29. For which one of the following, Bohr model is not valid?
(a) Hydrogen atom
(b) Singly ionised helium atom ($He^+$)
(c) Deuteron atom
(d) Singly ionised neon atom ($Ne^+$)

30. The average thermal energy for a mono-atomic gas is ($k_B$ is Boltzmann constant and $T$, absolute temperature)
(a) $\frac{1}{2}k_BT$ (b) $\frac{3}{2}k_BT$ (c) $\frac{5}{2}k_BT$ (d) $\frac{7}{2}k_BT$

31. Two particles of mass 5 kg and 10 kg respectively are attached to the two ends of a rigid rod of length 1 m with negligible mass. The centre of mass of the system from the 5 kg particle is nearly at a distance of
(a) 33 cm (b) 50 cm (c) 67 cm (d) 80 cm

32. In a guitar, two strings $A$ and $B$ made of same material are slightly out of tune and produce beats of frequency 6 Hz. When tension in $B$ is slightly decreased, the beat frequency increases to 7 Hz. If the frequency of $A$ is 530 Hz, the original frequency of $B$ will be
(a) 523 Hz (b) 524 Hz (c) 536 Hz (d) 537 Hz

33. Two cylinders $A$ and $B$ of equal capacity are connected to each other via a stop cock. $A$ contains an ideal gas at standard temperature and pressure. $B$ is completely evacuated. The entire system is thermally insulated. The stop cock is suddenly opened. The process is
(a) isothermal (b) adiabatic
(c) isochoric (d) isobaric

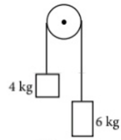
34. The capacitance of a parallel plate capacitor with air as medium is 6 μF. With the introduction of a dielectric medium, the capacitance becomes 30 μF. The permittivity of the medium is ($\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2 N^{-1} m^{-2}$)
(a) $0.44 \times 10^{-13}$ $C^2 N^{-1} m^{-2}$
(b) $1.77 \times 10^{-12}$ $C^2 N^{-1} m^{-2}$
(c) $0.44 \times 10^{-10}$ $C^2 N^{-1} m^{-2}$
(d) 5.00 $C^2 N^{-1} m^{-2}$

35. An electron is accelerated from rest through a potential difference of $V$ volt. If the de Broglie wavelength of the electron is $1.227 \times 10^{-2}$ nm, the potential difference is
(a) 10 V (b) $10^2$ V (c) $10^3$ V (d) $10^4$ V

36. A wire of length $L$, area of cross section $A$ is hanging from a fixed support. The length of the wire changes to $L_1$ when mass $M$ is suspended from its free end. The expression for Young's modulus is
(a) $\frac{MgL_1}{AL}$ (b) $\frac{Mg(L_1 - L)}{AL}$
(c) $\frac{MgL}{AL_1}$ (d) $\frac{MgL}{A(L_1 - L)}$

37. The Brewster's angle $i_b$ for an interface should be
(a) $0° < i_b < 30°$ (b) $30° < i_b < 45°$
(c) $45° < i_b < 90°$ (d) $i_b = 90°$

38. Two bodies of mass 4 kg and 6 kg are tied to the ends of a massless string. The string passes over a pulley which is frictionless (see figure). The acceleration of the system in terms of acceleration due to gravity ($g$) is



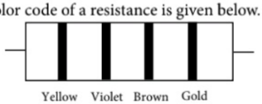

(a) $g$ (b) $g/2$ (c) $g/5$ (d) $g/10$

39. Dimensions of stress are
(a) $[MLT^{-2}]$ (b) $[ML^2T^{-2}]$
(c) $[ML^0T^{-2}]$ (d) $[ML^{-1}T^{-2}]$

40. A screw gauge has least count of 0.01 mm and there are 50 divisions in its circular scale. The pitch of the screw gauge is
(a) 0.01 mm (b) 0.25 mm
(c) 0.5 mm (d) 1.0 mm

41. The energy required to break one bond in DNA is $10^{-20}$ J. This value in eV is nearly
(a) 6 (b) 0.6 (c) 0.06 (d) 0.006

42. The color code of a resistance is given below.




The values of resistance and tolerance, respectively, are
(a) 470 kΩ, 5% (b) 47 kΩ, 10%
(c) 4.7 kΩ, 5% (d) 470 Ω, 5%

43. Assume that light of wavelength 600 nm is coming from a star. The limit of resolution of telescope whose objective has a diameter of 2 m is
(a) $3.66 \times 10^{-7}$ rad (b) $1.83 \times 10^{-7}$ rad
(c) $7.32 \times 10^{-7}$ rad (d) $6.00 \times 10^{-7}$ rad

44. The increase in the width of the depletion region in a *p-n* junction diode is due to
(a) forward bias only (b) reverse bias only
(c) both forward bias and reverse bias
(d) increase in forward current

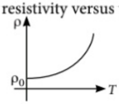
45. The energy equivalent of 0.5 g of a substance is
(a) $4.5 \times 10^{16}$ J (b) $4.5 \times 10^{13}$ J
(c) $1.5 \times 10^{13}$ J (d) $0.5 \times 10^{13}$ J

## SOLUTIONS

1. **(c) :** Energy received in 1 minute = Intensity × Area × Time
$E = (20\ W\ cm^{-2}) \times (20)\ cm^2 \times (1 \times 60\ s) = 24 \times 10^3$ J

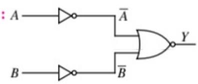
2. **(d):** For transistor action, emitter has greater doping concentration than collector or base. The base region is made thin and lightly doped so that only a few of electron recombine.
For the transistor to work in active region, emitter junction is forward biased whereas collector region is reverse biased.

3. **(c) :** For metals, resistivity versus time graph is




4. **(a) :** Electric field in a region, $E = -\frac{dV}{dr}$
But here electric potential is constant. Therefore electric field will be zero.

5. **(a) :**




Here, $Y = \overline{\bar{A} + \bar{B}} = \overline{\overline{A \cdot B}} = A \cdot B$
This is a AND Gate. So, the truth table is

| *A* | *B* | *Y* |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

6. **(c) :** Here, $C = 40$ mF $= 40 \times 10^{-6}$F; $\varepsilon_{rms} = 200$ V; $\upsilon = 50$ Hz
The value of the current, $I_{rms} = \frac{\varepsilon_{rms}}{\frac{1}{\omega C}} = \varepsilon_{rms}\omega C$
or $I_{rms} = 200\ (2\pi \times 50) \times (40 \times 10^{-6}) = 2.51$ A
$(\because \omega = 2\pi\upsilon)$

7. **(b):** Here, $P = 249$ kPa $= 249 \times 10^3$ Pa
$T = 273°C = 273 + 27 = 300$ K
$M = 2 \times 10^{-3}$ kg
Equation of state, $PV = nRT$
or $PM = \rho RT$ $\left[\because n = \frac{m}{M}\right]$
or $\rho = \frac{PM}{RT}$
Substituting the values,
$$\rho = \frac{(249 \times 10^3)(2 \times 10^{-3})}{8.3 \times 300} = 0.2\ kg\ m^{-3}$$

8. **(b):** $9.99 - 0.0099 = 9.9801$ m
Least number of significant figure are 3.
Hence, required answer will be 9.98 m.

9. **(b):** Mean free path for a gas, $\lambda = \frac{1}{\sqrt{2}\, n \pi d^2}$

10. **(a):** Given, $\chi_m = 599$
Relative permeability of the material, $\mu_r = 1 + \chi_m$
or $\mu_r = 1 + 599 = 600$
$\therefore \mu = \mu_r \mu_0 = 600 \times (4\pi \times 10^{-7}) = 24\pi \times 10^{-5}$ T m $A^{-1}$

11. **(b):** Potential due to dipole, $V = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2} = \frac{kp\cos\theta}{r^2}$

or $V = \frac{(9\times10^9)(16\times10^{-9})\times\cos 60^\circ}{(0.6)^2} = 200\text{ V}$

12. **(b):** Gravitational force at a height $h$,

$$mg_h = \frac{mg_0}{\left(1+\frac{h}{r}\right)^2} = \frac{72}{\left(1+\frac{R/2}{R}\right)^2} \text{ or } mg_h = 32\text{ N}$$

or $F_g = 32\text{ N}$

13. **(d)**

14. **(d):** Initially, $\upsilon = 1.5\,\upsilon_0$

If the frequency is halved, $\upsilon' = \frac{\upsilon}{2} = \frac{1.5\,\upsilon_0}{2} < \upsilon_0$

Hence, no photoelectric emission will take place.

15. **(c) :** When $L$ is removed

$$\tan\phi = \frac{|X_C|}{R} \Rightarrow \tan\frac{\pi}{3} = \frac{X_C}{R} \quad \text{...(i)}$$

When $C$ is removed,

$$\tan\phi = \frac{|X_L|}{R} \Rightarrow \tan\frac{\pi}{3} = \frac{X_L}{R} \quad \text{...(ii)}$$

From (i) and (ii), $X_C = X_L$.

Since, $X_L = X_C$, the circuit is in resonance.

$\therefore\ Z = R$

Power factor, $\cos\phi = \frac{Z}{R} = \frac{R}{R} = 1$

16. **(b):** Here, $r = 10$ cm, $q = 3.2\times10^{-7}$ C

$$E = \frac{kq}{r^2} = \frac{9\times10^9\times3.2\times10^{-7}}{225\times10^{-4}} = 1.28\times10^5\text{ N C}^{-1}$$

17. **(c) :** Here, $\vec{F} = 3\hat{j}$ N, $\vec{r} = 2\hat{k}$ m

Torque, $\vec{\tau} = \vec{r}\times\vec{F} = 2\hat{k}\times3\hat{j} = -6\hat{i}$ N m

18. **(b):** Here, $v_d = 7.5\times10^{-4}\text{ m s}^{-1}$, $E = 3\times10^{-10}\text{ V m}^{-1}$

Mobility, $\mu = \frac{v_d}{E} = \frac{7.5\times10^{-4}}{3\times10^{-10}}$

$\mu = 2.5\times10^6\text{ m}^2\text{ V}^{-1}\text{ s}^{-1}$

19. **(c) :** Light ray emerges normally from another surface so angle of emergence ($e$) = 0

$r_2 = 0$

$r_1 + r_2 = A \Rightarrow r_1 = A$

Using Snell's law on first surface,

$1\times\sin i = \mu\sin r_1$

$\sin i = \mu\sin A$

For small angles, $\sin A \approx A$ Hence, $i = \mu A$

20. **(a) :** Heat required, $\Delta Q = ms\Delta T$

$\Delta Q = (V\times\rho)\times s\,\Delta T$

$= \frac{4}{3}\pi r^3\rho\cdot s\,\Delta T$

$$\frac{\Delta Q_1}{\Delta Q_2} = \frac{r_1^3}{r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = (1.5)^3 = \frac{27}{8}$$

21. **(a) :** ${}_{92}U^{235} + {}_0n^1 \longrightarrow {}_{36}Kr^{89} + 3n_0^1 + {}_zX^A$

$92 + 0 = 36 + Z \Rightarrow Z = 56$

Now, $235 + 1 = 89 + 3 + A \Rightarrow A = 144$

So, ${}_{56}Ba^{144}$ is generated.

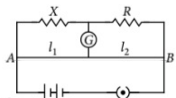
22 **(a) :** Displacement of the particle, $y = a\sin\omega t$,

$v = \frac{dy}{dt} = a\omega\cos\omega t$

Acceleration, $a = \frac{dv}{dt} = -a\omega^2\sin\omega t$

So, phase difference between displacement and acceleration is $\pi$.

23. **(b):** Unknown is $X$, $R = 10\ \Omega$.




Here, $\frac{l_1}{l_2} = \frac{3}{2}$

$\frac{X}{R} = \frac{l_1}{l_2}$

$\Rightarrow X = \frac{3}{2}\times10 \Rightarrow X = 15\ \Omega$

Thus, 1.5 m length has resistance 15 Ω.

Hence, length of 1 Ω of the resistance wire $= \frac{1.5}{15} = 0.1\text{ m} = 1.0\times10^{-1}\text{ m}$

24. **(c) :** Force of surface tension balances the weight of water in capillary tube.

$F_s = T\cos\theta\,(2\pi r) = mg$

Hence, $m \propto r$

$\frac{m'}{m} = \frac{r'}{r}$

$\frac{m'}{5\text{ g}} = \frac{2r}{r} \Rightarrow m' = 10\text{ g}$

25. **(b):** Energy intensity of electromagnetic wave is equally distributed in the form of electric and magnetic field, so ratio $\frac{U_E}{U_B} = \frac{1}{1}$.

26. **(c) :** Fringe width, $\beta = \frac{D\lambda}{d}$

$d$ becomes half $\Rightarrow d' = d/2$

$D$ doubles, so $\Rightarrow D' = 2D$

New fringe width, $\beta' = \frac{2D\lambda}{\left(\frac{d}{2}\right)} = 4\beta$

27. **(a) :** Here, $l = 50$ cm, $N = 100$, $i = 2.5$ A

Magnetic field inside the solenoid,

$B = \mu_0 ni = \frac{\mu_0 NI}{l}$

$$\Rightarrow B = \frac{4\pi\times10^{-7}\times100\times2.5}{0.5} = 6.28\times10^{-4}\text{ T}$$

28. **(d):** Here, $u = 20\text{ m s}^{-1}$, $v = 80\text{ m s}^{-1}$, $g = 10\text{ m s}^{-2}$, $h = ?$

$v^2 = u^2 + 2gh \Rightarrow 80^2 = 20^2 + 2\times10\times h$

Hence, $h = 300$ m

**29. (d):** Bohr's atomic model is valid for single electron species only. A singly ionised neon contains more than one electron. Hence option (d) is correct.

**30. (b):** Degree of freedom = 3

Energy associated with each degree of freedom $= \frac{1}{2}k_BT$

So, energy is $\frac{3}{2}k_BT$.

**31. (c) :** Given : $m_1 = 5$ kg, $m_2 = 10$ kg and $L = 1$ m

Here centre of mass, $X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$

$= \frac{5\times0+10\times1}{15} = \frac{10}{15} = \frac{2}{3}$

$= 0.666 \text{ m} \simeq 0.67 \text{ m} = 67 \text{ cm}$

**32. (b):** Given : frequency of $\upsilon_A = 530$ Hz and beat frequency $\Delta\upsilon_1 = 6$ Hz.

Since, $\upsilon_B = \upsilon_A \pm \Delta\upsilon_1$, we have $\upsilon_B = 536$ Hz or 524 Hz.

Now, when tension on the string is reduced, its frequency reduces.

Now, the beat frequency, $\Delta\upsilon_2 = 7$ Hz.

$\therefore$ The original frequency of $B$, $\upsilon_B = 524$ Hz.

**33. (b):** Since, the entire system is thermally insulated, no heat flows into the system. When the stop cock is removed, the gas expands adiabatically.

**34. (c) :** Given : capacitance without dielectric, $C = 6\,\mu$F and capacitance with dielectric, $C' = 30\,\mu$F.

$\therefore$ Dielectric constant, $K = \frac{C'}{C} = \frac{30}{6} = 5.$

Now, permittivity of the medium, $\varepsilon = K\varepsilon_0$

$= 5 \times 8.85 \times 10^{-12} = 0.44 \times 10^{-10}\ C^2\ N^{-1}\ m^{-2}$

**35. (d):** Given : de-Broglie wavelength of electron

$\lambda = 1.227 \times 10^{-2}$ nm $= 0.1227$ Å

$\because \quad \lambda = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}}$ Å

We have, $\sqrt{V} = \frac{12.27}{0.1227} = 100 \Rightarrow V = 10^4$ V.

**36. (d):** Given : initial length = $L$, area of cross section = $A$

New length after mass $M$ is suspended on the wire = $L$

$\therefore$ Change in length, $\Delta L = L_1 - L$.

Now Young's modulus, $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A} \times \frac{L}{\Delta L}$

$= \frac{mg}{A}\frac{L}{\Delta L}$ or $\frac{MgL}{A(L_1 - L)}$

**37. (c) :** We know, $\mu = \tan i_b$

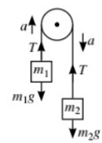
As $1 < \mu < \infty$

$\therefore \quad 1 < \tan i_b < \infty$

$\tan(45°) < \tan i_b < \tan(90°)$

or $\quad 45° < i_b < 90°$.

**38. (c) :** Given : $m_1 = 4$ kg, $m_2 = 6$ kg




From the diagram,

$T - m_1g = m_1a$ ...(i)

$m_2g - T = m_2a$ ...(ii)

Solving equation (i) and (ii)

$a = \frac{(m_2 - m_1)g}{m_2 + m_1} = \frac{(6-4)g}{10} = \frac{2}{10}g = \frac{g}{5}$

**39. (d):** We know, stress = $\frac{\text{Force}}{\text{Area}}$

Dimensions of force is $[M^1L^1T^{-2}]$ and that of area is $[L^2]$.

$\therefore$ Dimensions of stress = $\frac{[M^1L^1T^{-2}]}{[L^2]} = [M^1L^{-1}T^{-2}]$.

**40. (c) :** Given : least count = 0.01 mm and number of circular scale divisions = 50.

$\therefore$ Pitch = L.C × Number of circular scale division $= 0.01 \times 50 = 0.5$ mm.

**41. (c) :** Given : energy, $E = 10^{-20}$ J

Now, 1 J $= \frac{1}{1.6\times10^{-19}}$ eV

$\therefore \quad E = \frac{10^{-20}}{1.6\times10^{-19}}$ eV $= 0.0625$ eV $\simeq 0.06$ eV.

**42. (d):** The colour code of the given resistor is yellow, violet, brown and gold.

According to the colour code digits are

Yellow - 4

Violet - 7

Brown - 1

and Gold = 5%

$\therefore \quad R = 47 \times 10^1 \pm 5\% = 470\ \Omega \pm 5\%$

**43. (a) :** Given : $\lambda = 600$ nm $= 600 \times 10^{-9}$ m and $D = 2$ m

$\therefore$ Limit of resolution $= \frac{1.22\lambda}{D} = \frac{1.22\times600\times10^{-9}}{2}$

$= 366 \times 10^{-9} = 3.66 \times 10^{-7}$ rad.

**44. (b):** Width of the depletion layer increases in reverse biasing.

**45. (b):** Given : mass $m = 0.5$ g $= 0.5 \times 10^{-3}$ kg

According to Einstein mass-energy equivalence,

$E = mc^2 = 0.5 \times 10^{-3} \times (3 \times 10^8)^2 = 4.5 \times 10^{13}$ J

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# JEE Advanced

## SOLVED PAPER 2020

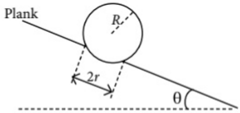
### PAPER - I

**SECTION 1 (Maximum Marks : 18)**

- *This section contains SIX (06) questions.*
- *Each question has FOUR options. ONLY ONE of these four options is the correct answer.*
- *For each question, choose the correct option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*

| | | |
|---|---|---|
| Full Marks : | +3 | *If ONLY the correct option is chosen;* |
| Zero Marks : | 0 | *If none of the options is chosen (i.e. the question is unanswered);* |
| Negative Marks : | −1 | *In all other cases.* |

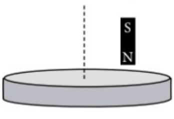
1. A football of radius $R$ is kept on a hole of radius $r$ ($r < R$) made on a plank kept horizontally. One end of the plank is now lifted so that it gets tilted making an angle $\theta$ from the horizontal as shown in the figure below. The maximum value of $\theta$ so that the football does not start rolling down the plank satisfies (Figure is schematic and not drawn to scale.)




(a) $\sin\theta = \frac{r}{R}$ (b) $\tan\theta = \frac{r}{R}$

(c) $\sin\theta = \frac{r}{2R}$ (d) $\cos\theta = \frac{r}{2R}$

2. A light disc made of aluminium (a non-magnetic material) is kept horizontally and is free to rotate about its axis as shown in the figure. A strong magnet is held vertically at a point above the disc away from its axis. On revolving the magnet about the axis of the disc, the disc will (Figure is schematic and not drawn to scale.)



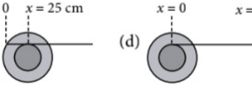

(a) rotate in the direction opposite to the direction of magnet's motion

(b) rotate in the same direction as the direction of magnet's motion

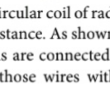
(c) not rotate and its temperature will remain unchanged

(d) not rotate but its temperature will slowly rise.

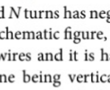
3. A small roller of diameter 20 cm has an axle of diameter 10 cm (see figure below on the left). It is on a horizontal floor and a meter scale is positioned horizontally on its axle with one edge of the scale on top of the axle (see figure on the right). The scale is now pushed slowly on the axle so that it moves without slipping on the axle, and the roller starts rolling without slipping. After the roller has moved 50 cm, the position of the scale will look like (Figures are schematic and not drawn to scale.)



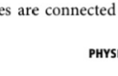
(a)



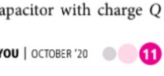

(b)




(c)

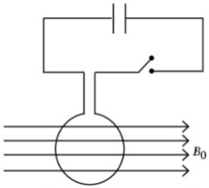



(d)



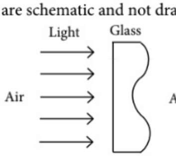

4. A circular coil of radius $R$ and $N$ turns has negligible resistance. As shown in the schematic figure, its two ends are connected to two wires and it is hanging by those wires with its plane being vertical. The wires are connected to a capacitor with charge $Q$

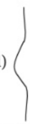
through a switch. The coil is in a horizontal uniform magnetic field $B_0$ parallel to the plane of the coil. When the switch is closed, the capacitor gets discharged through the coil in a very short time. By the time the capacitor is discharged fully, magnitude of the angular momentum gained by the coil will be (Assume that the discharge time is so short that the coil has hardly rotated during this time.)



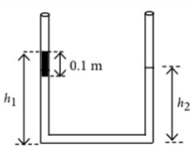

(a) $(\pi/2)NQB_0R^2$ (b) $\pi NQB_0R^2$
(c) $2\pi NQB_0R^2$ (d) $4\pi NQB_0R^2$

**5.** A parallel beam of light strikes a piece of transparent glass having cross-section as shown in the figure below. Correct shape of the emergent wavefront will be (Figures are schematic and not drawn to scale.)




(a)  (b)  (c)  (d) 

**6.** An open-ended U-tube of uniform cross-sectional area contains water (density $10^3$ kg m$^{-3}$). Initially the water level stands at 0.29 m from the bottom in each arm. Kerosene oil (a water-immiscible liquid) of density 800 kg m$^{-3}$ is added to the left arm until its length is 0.1 m, as shown in the schematic figure. The ratio $\left(\frac{h_1}{h_2}\right)$ of the heights of the liquid in the two arms is




(a) $\frac{15}{14}$ (b) $\frac{35}{33}$ (c) $\frac{7}{6}$ (d) $\frac{5}{4}$

## SECTION 2 (Maximum Marks : 24)

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***
- ***For each question, choose the option(s) corresponding to (all) the correct answer(s).***
- ***Answer to each question will be evaluated according to the following marking scheme:***

| | |
|---|---|
| Full Marks : | +4 If only (all) the correct option(s) is (are) chosen; |
| Partial Marks : | +3 If all the four options are correct but ONLY three options are chosen; |
| Partial Marks : | +2 If three or more options are correct but ONLY two options are chosen and both of which are correct; |
| Partial Marks : | +1 If two or more options are correct but ONLY one option is chosen and it is a correct option; |
| Zero Marks : | 0 If none of the options is chosen (i.e. the question is unanswered); |
| Negative Marks : | –2 In all other cases. |

**7.** A particle of mass $m$ moves in circular orbits with potential energy $V(r) = Fr$, where $F$ is a positive constant and $r$ is its distance from the origin. Its energies are calculated using the Bohr model. If the radius of the particle's orbit is denoted by $R$ and its speed and energy are denoted by $v$ and $E$, respectively, then for the $n^{\text{th}}$ orbit (Here $h$ is the Planck's constant.)

(a) $R \propto n^{1/3}$ and $v \propto n^{2/3}$

(b) $R \propto n^{2/3}$ and $v \propto n^{1/3}$

(c) $E = \frac{3}{2}\left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}$ (d) $E = 2\left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}$

**8.** The filament of a light bulb has surface area 64 mm$^2$. The filament can be considered as a black body at temperature 2500 K emitting radiation like a point source when viewed from far. At night the light bulb is observed from a distance of 100 m. Assume the pupil of the eyes of the observer to be circular with radius 3 mm. Then

(Stefan-Boltzmann constant = $5.67 \times 10^{-8}$ Wm$^{-2}$K$^{-4}$, Wien's displacement constant = $2.90 \times 10^{-3}$ m-K, Planck's constant = $6.63 \times 10^{-34}$ Js, speed of light in vacuum = $3.00 \times 10^8$ ms$^{-1}$)

(a) power radiated by the filament is in the range 642 W to 645 W

(b) radiated power entering into one eye of the observer is in the range $3.15 \times 10^{-8}$ W to $3.25 \times 10^{-8}$ W

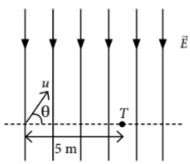
(c) the wavelength corresponding to the maximum intensity of light is 1160 nm

(d) taking the average wavelength of emitted radiation to be 1740 nm, the total number of photons entering per second into one eye of the observer is in the range $2.75 \times 10^{11}$ to $2.85 \times 10^{11}$.

9. Sometimes it is convenient to construct a system of units so that all quantities can be expressed in terms of only one physical quantity. In one such system, dimensions of different quantities are given in terms of a quantity $X$ as follows: [position] = $[X^{\alpha}]$; [speed] = $[X^{\beta}]$; [acceleration] = $[X^{p}]$; [linear momentum] = $[X^{q}]$; [force] = $[X^{r}]$. Then

(a) $\alpha + p = 2\beta$
(b) $p + q - r = \beta$
(c) $p - q + r = \alpha$
(d) $p + q + r = \beta$

10. A uniform electric field, $\vec{E} = -400\sqrt{3}\,\hat{y}\ \text{NC}^{-1}$ is applied in a region. A charged particle of mass $m$ carrying positive charge $q$ is projected in this region with an initial speed of $2\sqrt{10} \times 10^{6}\ \text{ms}^{-1}$. This particle is aimed to hit a target $T$, which is 5 m away from its entry point into the field as shown schematically in the figure. Take $\frac{q}{m} = 10^{10}\ \text{C kg}^{-1}$. Then



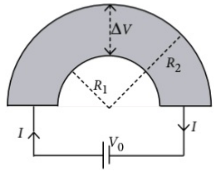

(a) the particle will hit $T$ if projected at an angle 45° from the horizontal

(b) the particle will hit $T$ if projected either at an angle 30° or 60° from the horizontal

(c) time taken by the particle to hit $T$ could be $\sqrt{\frac{5}{6}}\ \mu\text{s}$ and $\sqrt{\frac{5}{2}}\ \mu\text{s}$

(d) time taken by the particle to hit $T$ is $\sqrt{\frac{5}{3}}\ \mu\text{s}$.

11. Shown in the figure is a semicircular metallic strip that has thickness $t$ and resistivity $\rho$. Its inner radius is $R_1$ and outer radius is $R_2$. If a voltage $V_0$ is applied between its two ends, a current $I$ flows in it. In addition, it is observed that a transverse voltage $\Delta V$ develops between its inner and outer surfaces due to purely kinetic effects of moving electrons (ignore any role of the magnetic field due to the current). Then (figure is schematic and not drawn to scale)



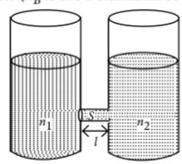

(a) $I = \frac{V_0 t}{\pi\rho} \ln\left(\frac{R_2}{R_1}\right)$

(b) the outer surface is at a higher voltage than the inner surface

(c) the outer surface is at a lower voltage than the inner surface

(d) $\Delta V \propto I^2$.

12. As shown schematically in the figure, two vessels contain water solutions (at temperature $T$) of potassium permanganate ($KMnO_4$) of different concentrations $n_1$ and $n_2$ ($n_1 > n_2$) molecules per unit volume with $\Delta n = (n_1 - n_2) << n_1$. When they are connected by a tube of small length $l$ and cross-sectional area $S$, $KMnO_4$ starts to diffuse from the left to the right vessel through the tube. Consider the collection of molecules to behave as dilute ideal gases and the difference in their partial pressure in the two vessels causing the diffusion. The speed $v$ of the molecules is limited by the viscous force $-\beta v$ on each molecule, where $\beta$ is a constant. Neglecting all terms of the order $(\Delta n)^2$, which of the following is/are correct? ($k_B$ is the Boltzmann constant)




(a) The force causing the molecules to move across the tube is $\Delta n k_B T S$.

(b) Force balance implies $n_1 \beta v l = \Delta n k_B T$.

(c) Total number of molecules going across the tube per sec is $\left(\frac{\Delta n}{l}\right)\left(\frac{k_B T}{\beta}\right)S$.

(d) Rate of molecules getting transferred through the tube does not change with time.

## SECTION 3 (Maximum Marks : 24)

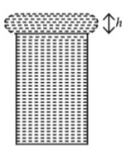
- ***This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.***
- ***For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

**Full Marks :** ***+4 If ONLY the correct numerical value is entered.***

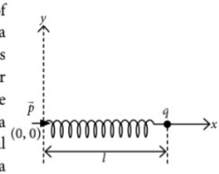
**Zero Marks :** ***0 In all other cases.***

**13.** Put a uniform meter scale horizontally on your extended index fingers with the left one at 0.00 cm and the right one at 90.00 cm. When you attempt to move both the fingers slowly towards the center, initially only the left finger slips with respect to the scale and the right finger does not. After some distance, the left finger stops and the right one starts slipping. Then the right finger stops at a distance $x_R$ from the center (50.00 cm) of the scale and the left one starts slipping again. This happens because of the difference in the frictional forces on the two fingers. If the coefficients of static and dynamic friction between the fingers and the scale are 0.40 and 0.32, respectively, the value of $x_R$ (in cm) is ______.

**14.** When water is filled carefully in a glass, one can fill it to a height $h$ above the rim of the glass due to the surface tension of water. To calculate $h$ just before water starts flowing, model the shape of the water above the rim as a disc of thickness $h$ having semicircular edges, as shown schematically in the figure. When the pressure of water at the bottom of this disc exceeds what can be withstood due to the surface tension, the water surface breaks near the rim and water starts flowing from there. If the density of water, its surface tension and the acceleration due to gravity are $10^3$ kg m$^{-3}$, 0.07 Nm$^{-1}$ and 10 ms$^{-2}$, respectively, the value of $h$ (in mm) is ________.




**15.** One end of a spring of negligible unstretched length and spring constant $k$ is fixed at the origin (0, 0). A point particle of mass $m$ carrying a positive charge $q$ is attached at its other end. The entire system is kept on a smooth horizontal surface. When a point dipole $\vec{p}$ pointing towards the charge $q$ is fixed at the origin, the spring gets stretched to a length $l$ and attains a new equilibrium position (see figure). If the point mass is now displaced slightly by $\Delta l << l$ from its equilibrium position and released, it is found to oscillate at frequency $\frac{1}{\delta}\sqrt{\frac{k}{m}}$. The value of $\delta$ is ______.




**16.** Consider one mole of helium gas enclosed in a container at initial pressure $P_1$ and volume $V_1$. It expands isothermally to volume $4V_1$. After this, the gas expands adiabatically and its volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are $W_{iso}$ and $W_{adia}$, respectively. If the ratio $\frac{W_{iso}}{W_{adia}} = f \ln 2$, then $f$ is ________.

**17.** A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of 2 ms$^{-1}$ in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is 320 ms$^{-1}$, the smallest value of the percentage change required in the length of the pipe is ____________.

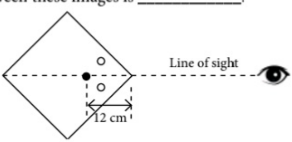
**18.** A circular disc of radius $R$ carries surface charge density $\sigma(r) = \sigma_0\left(1 - \frac{r}{R}\right)$, where $\sigma_0$ is a constant and $r$ is the distance from the center of the disc. Electric flux through a large spherical surface that encloses the charged disc completely is $\phi_0$. Electric flux through another spherical surface of radius $R/4$ and concentric with the disc is $\phi$. Then the ratio $\frac{\phi_0}{\phi}$ is_________.

## SECTION 1 (Maximum Marks : 18)

- ***This section contains SIX (06) questions.***
- ***The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.***
- ***For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numerical keypad in the place designated to enter the answer.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

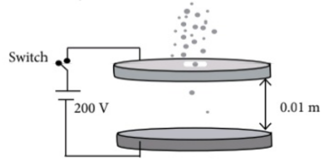
  **Full Marks :** +3 *If ONLY the correct integer is chosen;*
  **Zero Marks :** 0 *If the question is unanswered;*
  **Negative Marks :** –1 *In all other cases.*

**1.** A large square container with thin transparent vertical walls and filled with water (refractive index $\frac{4}{3}$) is kept on a horizontal table. A student holds a thin straight wire vertically inside the water 12 cm from one of its corners, as shown schematically in the figure. Looking at the wire from this corner, another student sees two images of the wire, located symmetrically on each side of the line of sight as shown. The separation (in cm) between these images is ____________.



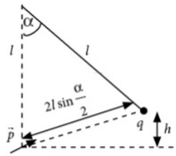

**2.** A train with cross-sectional area $S_t$ is moving with speed $v_t$ inside a long tunnel of cross-sectional area $S_0$ ($S_0=4S_t$). Assume that almost all the air (density $\rho$) in front of the train flows back between its sides and the walls of the tunnel. Also, the air flow with respect to the train is steady and laminar. Take the ambient pressure and that inside the train to be $p_0$. If the pressure in the region between the sides of the train and the tunnel walls is $p$, then $p_0 - p = \frac{7}{2N}\rho v_t^2$. The value of $N$ is ________.

**3.** Two large circular discs separated by a distance of 0.01 m are connected to a battery via a switch as shown in the figure. Charged oil drops of density 900 kg m$^{-3}$ are released through a tiny hole at the center of the top disc. Once some oil drops achieve terminal velocity, the switch is closed to apply a voltage of 200 V across the discs. As a result, an oil drop of radius $8\times10^{-7}$ m stops moving vertically and floats between the discs. The number of electrons present in this oil drop is ________. (neglect the buoyancy force, take acceleration due to gravity =10 ms$^{-2}$ and charge on an electron ($e$) = $1.6\times10^{-19}$ C)

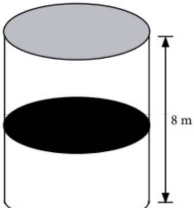



**4.** A hot air balloon is carrying some passengers, and a few sandbags of mass 1 kg each so that its total mass is 480 kg. Its effective volume giving the balloon its buoyancy is $V$. The balloon is floating at an equilibrium height of 100 m. When $N$ number of sandbags are thrown out, the balloon rises to a new equilibrium height close to 150 m with its volume $V$ remaining unchanged. If the variation of the density of air with height $h$ from the ground is $\rho(h) = \rho_0 e^{-\frac{h}{h_0}}$, where $\rho_0$=1.25 kg m$^{-3}$ and $h_0$ = 6000 m, the value of $N$ is __________.

**5.** A point charge $q$ of mass $m$ is suspended vertically by a string of length $l$. A point dipole of dipole moment $\vec{p}$ is now brought towards $q$ from infinity so that the charge moves away. The final equilibrium position of the system including the direction of the dipole, the angles and distances is shown in the figure. If the work done in bringing the dipole to this position is $N\times(mgh)$, where $g$ is the acceleration due to gravity, then the value of $N$ is __________ . (Note that for three coplanar forces keeping a point mass in equilibrium, $\frac{F}{\sin\theta}$ is the same for all forces, where $F$ is any one of the forces and $\theta$ is the angle between the other two forces.)




6. A thermally isolated cylindrical closed vessel of height 8 m is kept vertically. It is divided into two equal parts by a diathermic (perfect thermal conductor) frictionless partition of mass 8.3 kg. Thus the partition is held initially at a distance of 4 m from the top as shown in the schematic figure. Each of the two parts of the vessel contains 0.1 mole of an ideal gas at temperature 300 K. The partition is now released and moves without any gas leaking from one part of the vessel to the other. When equilibrium is reached, the distance of the partition from the top (in m) will be ________ (take the acceleration due to gravity $=10\ \text{ms}^{-2}$ and the universal gas constant $=8.3\ \text{J mol}^{-1}\text{K}^{-1}$).




## SECTION 2 (Maximum Marks : 24)

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***
- ***For each question, choose the option(s) corresponding to (all) the correct answer(s).***
- ***Answer to each question will be evaluated according to the following marking scheme:***

**Full Marks :** +4 *If only (all) the correct option(s) is (are) chosen;*

**Partial Marks :** + 3 *If all the four options are correct but ONLY three options are chosen;*

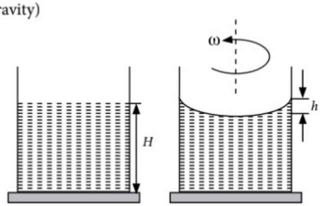
**Partial Marks :** + 2 *If three or more options are correct but ONLY two options are chosen and both of which are correct;*

**Partial Marks :** + 1 *If two or more options are correct but ONLY one option is chosen and it is a correct option;*

**Zero Marks :** 0 *If none of the options is chosen (i.e. the question is unanswered);*

**Negative Marks :** –2 *In all other cases.*

7. A beaker of radius $r$ is filled with water (refractive index $\frac{4}{3}$) up to a height $H$ as shown in the figure on the left. The beaker is kept on a horizontal table rotating with angular speed ω. This makes the water surface curved so that the difference in the height of water level at the center and at the circumference of the beaker is $h$ ($h << H, h << r$), as shown in the figure on the right. Take this surface to be approximately spherical with a radius of curvature $R$. Which of the following is/are correct? ($g$ is the acceleration due to gravity)

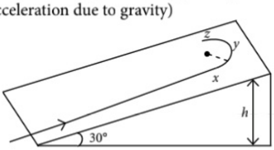



(a) $R=\dfrac{h^2+r^2}{2h}$ (b) $R=\dfrac{3r^2}{2h}$

(c) Apparent depth of the bottom of the beaker is close to $\dfrac{3H}{2}\left(1+\dfrac{\omega^2 H}{2g}\right)^{-1}$.

(d) Apparent depth of the bottom of the beaker is close to $\dfrac{3H}{4}\left(1+\dfrac{\omega^2 H}{4g}\right)^{-1}$.

8. A student skates up a ramp that makes an angle 30° with the horizontal. He/she starts (as shown in the figure) at the bottom of the ramp with speed $v_0$ and wants to turn around over a semicircular path $xyz$ of radius $R$ during which he/she reaches a maximum height $h$ (at point $y$) from the ground as shown in the figure. Assume that the energy loss is negligible and the force required for this turn at the highest point is provided by his/her weight only. Then ($g$ is the acceleration due to gravity)

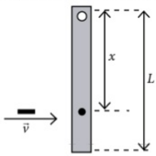


(a) $v_0^2 - 2gh = \frac{1}{2}gR$ (b) $v_0^2 - 2gh = \frac{\sqrt{3}}{2}gR$

(c) the centripetal force required at points $x$ and $z$ is zero

(d) the centripetal force required is maximum at points $x$ and $z$.

**9.** A rod of mass $m$ and length $L$, pivoted at one of its ends, is hanging vertically. A bullet of the same mass moving at speed $v$ strikes the rod horizontally at a distance $x$ from its pivoted end and gets embedded in it. The combined system now rotates with angular speed $\omega$ about the pivot. The maximum angular speed $\omega_M$ is achieved for $x = x_M$. Then




(a) $\omega = \frac{3vx}{L^2 + 3x^2}$ (b) $\omega = \frac{12vx}{L^2 + 12x^2}$

(c) $x_M = \frac{L}{\sqrt{3}}$ (d) $\omega_M = \frac{v}{2L}\sqrt{3}$

**10.** In an X-ray tube, electrons emitted from a filament (cathode) carrying current $I$ hit a target (anode) at a distance $d$ from the cathode. The target is kept at a potential $V$ higher than the cathode resulting in emission of continuous and characteristic X-rays. If the filament current $I$ is decreased to $\frac{I}{2}$, the potential difference $V$ is increased to $2V$, and the separation distance $d$ is reduced to $\frac{d}{2}$, then

(a) the cut-off wavelength will reduce to half, and the wavelengths of the characteristic X-rays will remain the same

(b) the cut-off wavelength as well as the wavelengths of the characteristic X-rays will remain the same

(c) the cut-off wavelength will reduce to half, and the intensities of all the X-rays will decrease

(d) the cut-off wavelength will become two times larger, and the intensity of all the X-rays will decrease.

**11.** Two identical non-conducting solid spheres of same mass and charge are suspended in air from a common point by two non-conducting, massless strings of same length. At equilibrium, the angle between the strings is $\alpha$. The spheres are now immersed in a dielectric liquid of density 800 kg m$^{-3}$ and dielectric constant 21. If the angle between the strings remains the same after the immersion, then

(a) electric force between the spheres remains unchanged

(b) electric force between the spheres reduces

(c) mass density of the spheres is 840 kg m$^{-3}$

(d) the tension in the strings holding the spheres remains unchanged.

**12.** Starting at time $t = 0$ from the origin with speed 1 ms$^{-1}$, a particle follows a two-dimensional trajectory in the $x$-$y$ plane so that its coordinates are related by the equation $y = \frac{x^2}{2}$. The $x$ and $y$ components of its acceleration are denoted by $a_x$ and $a_y$, respectively. Then

(a) $a_x = 1$ ms$^{-2}$ implies that when the particle is at the origin, $a_y = 1$ ms$^{-2}$

(b) $a_x = 0$ implies $a_y = 1$ ms$^{-2}$ at all times

(c) at $t = 0$, the particle's velocity points in the $x$-direction

(d) $a_x = 0$ implies that at $t = 1$ s, the angle between the particle's velocity and the $x$ axis is 45°.

## SECTION 3 (Maximum Marks : 24)

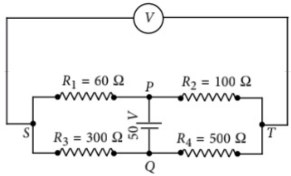
- ***This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.***
- ***For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

***Full Marks :*** ***+4 If ONLY the correct numerical value is entered;***

***Zero Marks :*** ***0 In all other cases.***

**13.** A spherical bubble inside water has radius $R$. Take the pressure inside the bubble and the water pressure to be $p_0$. The bubble now gets compressed radially in an adiabatic manner so that its radius becomes $(R - a)$. For $a << R$ the magnitude of the work done in the process is given by $(4\pi p_0 R a^2)X$, where $X$ is a constant and $\gamma = \frac{C_p}{C_v} = \frac{41}{30}$. The value of $X$ is________.

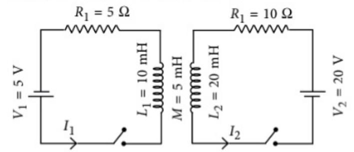
**14.** In the balanced condition, the values of the resistances of the four arms of a Wheatstone bridge are shown in the figure. The resistance $R_3$ has temperature coefficient 0.0004°C$^{-1}$. If the temperature of $R_3$ is increased by 100°C, the voltage developed between $S$ and $T$ will be ________ volt.




**15.** Two capacitors with capacitance values $C_1 = 2000 \pm 10$ pF and $C_2 = 3000 \pm 15$ pF are connected in series. The voltage applied across this combination is $V = 5.00 \pm 0.02$ V. The percentage error in the calculation of the energy stored in this combination of capacitors is _______.

**16.** A cubical solid aluminium block $\left(\text{bulk modulus} = -V\dfrac{dP}{dV} = 70\,\text{GPa}\right)$ has an edge length of 1 m on the surface of the earth. It is kept on the floor of a 5 km deep ocean. Taking the average density of water and the acceleration due to gravity to be $10^3$ kg m$^{-3}$ and 10 ms$^{-2}$, respectively, the change in the edge length of the block in mm is _____.

**17.** The inductors of two *LR* circuits are placed next to each other, as shown in the figure. The values of the self-inductance of the inductors, resistances, mutual-inductance and applied voltages are specified in the given circuit. After both the switches are closed simultaneously, the total work done by the batteries against the induced *EMF* in the inductors by the time the currents reach their steady state values is________ mJ.

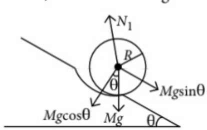



**18.** A container with 1 kg of water in it is kept in sunlight, which causes the water to get warmer than the surroundings. The average energy per unit time per unit area received due to the sunlight is 700 Wm$^{-2}$ and it is absorbed by the water over an effective area of 0.05 m$^2$. Assuming that the heat loss from the water to the surroundings is governed by Newton's law of cooling, the difference (in °C) in the temperature of water and the surroundings after a long time will be ______. (Ignore effect of the container, and take constant for Newton's law of cooling = 0.001 s$^{-1}$, Heat capacity of water = 4200 J kg$^{-1}$ K$^{-1}$)

## SOLUTIONS

### PAPER-1

**1.** **(a) :** When the football is about to roll, but not yet started to roll, the situation is given in the figure.




In equilibrium, the net torque acting on the body is zero.

*i.e.*, $(Mg\cos\theta)r = (Mg\sin\theta)\sqrt{R^2 - r^2}$

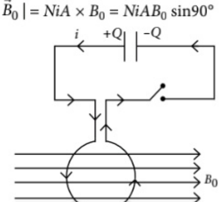
or $\tan\theta = \dfrac{r}{\sqrt{R^2 - r^2}}$ or $\sin\theta = \dfrac{r}{R}$

**2.** **(b) :** Due to the motion of magnet, an induced e.m.f is produced on the disc, which according to the Lenz law, opposes the motion of the magnet. Then according to Newton's third law of motion, an equal and opposite force is applied on the disc by the magnet, which rotates the disc on the same direction as that of the magnet.

**3.** **(b)**

**4.** **(b) :** Here, the net torque acting on the coil due to the external magnetic field is $B_0$

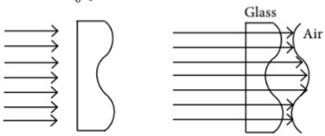
$\vec{\tau} = |\vec{m} \times \vec{B}_0| = NiA \times B_0 = NiAB_0 \sin 90°$




We know that the change in angular momentum,

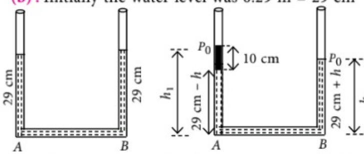
$$\Delta L = \tau\Delta t = \int \tau dt = \int NiAB_0 dt$$
$$= NAB_0 \int idt = NAB_0 Q \qquad [\because \int idt = Q]$$
or $\Delta L = N\pi R^2 B_0 Q$

**5. (a) :**




Since the upper and lower part of the wavefront has to travel a larger distance through glass, the emerging wavefront will be similar to that given in option (a).

**6. (b) :** Initially the water level was 0.29 m = 29 cm




Now the pressure at point $A$ = pressure at point $B$

*i.e.*, $P_0 + \rho_k g h_k + \rho_w g(29 - h) = P_0 + \rho_w g\,(29 + h)$

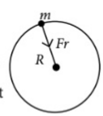
$\rho_k h_k + \rho_w(29 - h) = \rho_w(29 + h)$

$800 \times 10 + 1000\,(29 - h) = 1000(29 + h)$

$\Rightarrow \quad h = 4$ cm

$$\therefore \quad \frac{h_1}{h_2} = \frac{10\text{ cm} + 29\text{ cm} - 4\text{ cm}}{29\text{ cm} + 4\text{ cm}} = \frac{35}{33}$$

**7. (b, c) :** Given potential energy




$V(r) = Fr$ ...(i)

$\therefore$ The force acting on the particle,

$f = -\dfrac{dV(r)}{dr} = -F$, which is a constant

Now, this force provides the necessary centripetal force to the particle to be in its circular orbit. Then

$$f = \frac{mv^2}{r} \qquad \text{...(ii)}$$

where $v$ is the velocity of the particle.

Now according to Bohr's postulate, the angular momentum of the particle.

$$mvr = \frac{nh}{2\pi} \qquad \text{...(iii)}$$

From (i), (ii) and (iii), we have

$$v = \left(\frac{nhF}{2\pi m^2}\right)^{1/3}$$

and $r = \left(\dfrac{n^2h^2}{4\pi^2 Fm}\right)^{1/3}$ ...(iv)

$\Rightarrow \quad v \propto n^{\frac{1}{3}}$ and $r \propto n^{\frac{2}{3}}$

Now, the kinetic energy of the particle

$$K = \frac{1}{2}mv^2 = \frac{Fr}{2} \qquad \left[\because \frac{mv^2}{r} = F\right]$$

and potential energy is given by, $V(r) = Fr$

$\therefore$ The total energy $E = V(r) + K = \dfrac{3}{2}Fr$.

Substituting the value of $r$ from equation (iv)

$$F = \frac{3}{2}\left(\frac{n^2F^2h^2}{4\pi^2 m}\right)^{1/3}$$

**8. (b, c, d) :** Given: surface area of filament,

$A = 64\text{ mm}^2$, $T = 2500$ K

Then from Stefan-Boltzmann law, power radiated

$P = \sigma AeT^4 = 5.67 \times 10^{-8} \times 64 \times 10^{-6} \times 1 \times (2500)^4$

$= 1.42 \times 10^2 \simeq 142$ W

Now the radiated power entering the observer's eye,

$$P' = \frac{P}{4\pi d^2}\pi r^2$$

where $d$ is the distance of the observer and $r$ is the radius of the eye.

$$\therefore \quad P' = \frac{142\text{ W}}{4 \times 3.14 \times (100)^2} \times 3.14 \times (3 \times 10^{-3})^2$$

$= 3.20 \times 10^{-8}$ W

From Wien's displacement law, the wavelength corresponding to maximum intensity,

$$\lambda_m = \frac{b}{T} = \frac{2.90 \times 10^{-3}}{2500} = 1.16 \times 10^{-6}\text{ m} \simeq 1160\text{ nm}$$

The total number of photons entering the eye

$$N = \frac{3.20 \times 10^{-8}}{hc}\lambda = \frac{3.20 \times 10^{-8} \times 1740 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^8}$$

$= 2.80 \times 10^{11}$ photons

**9. (a, b) :** Given [position] = $[X^\alpha]$ ; [speed] = $[X^\beta]$

[acceleration] = $[X^p]$ ; [linear momentum] = $[X^q]$

[force] = $[X^r]$

$$\text{Now, } \frac{\text{linear momentum}}{\text{force}} = \frac{mv}{ma} = \frac{v}{a} = [X^{q-r}]$$

$$\text{But } \frac{\text{speed}}{\text{acceleration}} = \frac{v}{a} = [X^{\beta - p}]$$

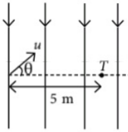
$\therefore \quad q - r = \beta - p$ or $\beta = q + p - r$

Now from equation of motion,

$(\text{speed})^2 = 2(\text{acceleration}) \times (\text{position})$

$X^{2\beta} = X^{p+\alpha}$ or $\alpha + p = 2\beta$

**10. (b, c) :** Here the force due to electric field is acting downwards, so the charge will be accelerated in the $-y$ direction.




$$i.e., a_y = \frac{F}{m} = \frac{qE}{m}$$

$$= 400\sqrt{3}\times 10^{10}\ \text{m/s}^2$$

The maximum range of the particle,

$$R = \frac{u^2 \sin 2\theta}{a_y} \Rightarrow \frac{(2\sqrt{10}\times 10^6)^2 \sin 2\theta}{400\sqrt{3}\times 10^{10}} = 5$$

$$\Rightarrow \sin 2\theta = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30° \text{ or } 60°$$

($\because$ For angle $\theta$ or $90° - \theta$, horizontal range remains the same.)

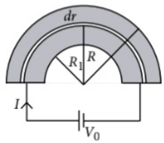
Total time taken by the projectile,

$$\text{Time period, } T = \frac{2u\sin\theta}{a_y}$$

$$= \frac{2(2\sqrt{10}\times 10^6)\sin 30°}{400\sqrt{3}\times 10^{10}} \text{ or } \frac{2(2\sqrt{10}\times 10^6)}{400\sqrt{3}\times 10^{10}}\sin 60°$$

$$= \sqrt{\frac{5}{6}}\ \mu\text{s or } \sqrt{\frac{5}{2}}\ \mu\text{s}$$

**11. (a, c, d) :** Consider a small portion of metallic strip with a negligible breadth $dr$. Then the small current through this strip,




$di = \frac{V_0}{R'}$, where $R'$ is the resistance offered by the portion.

$$di = \frac{V_0}{\rho\frac{l}{A}} = \frac{V_0}{\rho\pi R}(tdR) \quad \text{...(i)}$$

$\therefore$ The total current passing through the strip,

$$I = \int di = \int_{R_1}^{R_2} \frac{V_0}{\rho\pi R}tdR = \frac{V_0 t}{\rho\pi}\ln\frac{R_2}{R_1} \quad \text{...(ii)}$$

Since, the current is in clockwise direction, the electrons are moving in anti-clockwise direction. Thus, the electric field which applies force on the electrons, should be directed radially outwards, *i.e.*, the outer. Surface of the strip is at a lower potential compared to inner surface.

Now, the current through the strip,

$i = neAv_d$, where $v_d$ is the drift velocity.

$\therefore \quad di = nev_d\, dA = nev_d tdR$

$$\text{From (i), } \frac{V_0}{\rho\pi R}tdR = nev_d tdR$$

$$\text{or } \frac{V_0}{\rho\pi R} = nev_d \text{ or } v_d = \frac{V_0}{\rho\pi ne}\frac{1}{R} \quad \text{...(iii)}$$

$$\text{Now force on electron } eE = \frac{mv_d^2}{R} = \frac{m}{R}\frac{V_0^2}{\rho^2\pi^2 n^2 e^2}\frac{1}{R^2}$$

$$\Rightarrow \quad E = \frac{k}{R^3} \text{ or } E \propto \frac{1}{R^3}$$

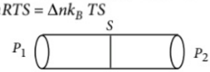
$$\text{Now, } dV = -EdR = \frac{-k}{R^3}dR = \frac{k'}{R^2} \propto v_d^2$$

From (ii) and (iii), $\Delta V \propto I^2$

**12. (a, b, c) :** Since $n_1 > n_2$, the fluid will flow towards first vessel. Then force, $F = \Delta P \cdot S = (P_1 - P_2)S$

$= (n_1RT - n_2RT)S = (n_1 - n_2)RTS$

$= \Delta nRTS = \Delta nk_B TS \quad \text{...(i)}$




Given: Viscous force $f = \beta v$ for each molecule

$\therefore$ Total force, $n_1 \times S \times l \times \beta v = n_1 Sl\beta v \quad \text{...(ii)}$

From (i) and (ii)

$n_1 Sl\beta v = \Delta nk_B TS$ or $n_1 l\beta v = \Delta nk_B T \quad \text{...(iii)}$

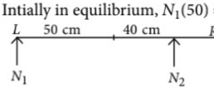
$$\frac{dN}{dt} = \frac{n_1 Sl}{t} = \frac{n_1 Sl}{\frac{l}{v}} = n_1 Sv$$

$$\text{From equation (iii), } \frac{dN}{dt} = n_1 Sv$$

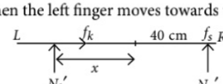
$$= n_1 S\frac{\Delta nk_B T}{n_1 l\beta} = \frac{\Delta nk_B TS}{\beta l}$$

Since $\Delta n$ changes with time, $\frac{dN}{dt}$ also changes with time.

**13. (25.60) :** Intially in equilibrium, $N_1(50) = N_2(40)$




Now when the left finger moves towards the centre:



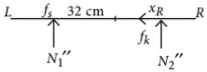

For equilibrium $N'_1 x = N_2'(40) \quad \text{...(i)}$

and $f_k N_1' = f_s N_2'$ or $0.32\ N_1' = 0.40\ N_2' \quad \text{...(ii)}$

From (i) and (ii), $x = \frac{40}{0.40} \times 0.32 = 32$ cm

When the right finger moves towards the centre and reaches a distance $x_R$ from the centre, for equilibrium

$N_1''(32) = N_2'' x_R$ ...(iii)




and $f_s N_1' = f_k N_2''$ or $0.40\, N_1'' = 0.32\, N_2''$ ...(iv)

From (iii) and (iv)

$$x_R = \frac{32}{0.40} \times 0.32 = 25.6 \text{ cm}$$

**14. (3.65)** **15. (3.14)**

**16. (1.78) :** Intially the gas is expanding isothermally.

$\therefore$ Work done, $W_1 = nRT \ln \frac{4V_1}{V_1} = 2nRT \ln 2$

Now for an adiabatic process,

$TV^{\gamma - 1}$= constant

$\Rightarrow \quad T_1(4V_1)^{\gamma - 1} = T_2(32V_1)^{\gamma - 1}$

$$T_1 = T_2\left(\frac{32V_1}{4V_1}\right)^{\gamma-1} = T_2(8)^{\gamma-1}$$

For helium gas, $\gamma = \frac{5}{3}$

$$\therefore \quad T_1 = T_2(8)^{\frac{5}{3}-1} = T_2(8)^{\frac{2}{3}}$$

or $T_1 = 4T_2$ or $T_2 = \frac{T}{4}$ and $T_1 = T$

Now work done during adiabatic process,

$$W_2 = \frac{nR}{(\gamma-1)}[T_1 - T_2] = \frac{3nR}{2} \times \frac{3T}{4} = \frac{9}{8}nRT$$

$$\therefore \quad \frac{W_1}{W_2} = \frac{2nRT \ln 2}{\frac{9}{8}nRT} = \frac{16}{9}\ln 2 \quad \therefore \quad f = \frac{16}{9} \simeq 1.78$$

**17. (0.63) :** For resonance to occur, the frequency should be,

$f = \frac{nV}{4L}$, where $n$ is a positive odd integer.

Now let the initial resonant frequency be

$$f_0 = \frac{V}{4L_1}$$

where $L_1$ is the initial length of the tube.

Now, since the fork is moving towards or away from the tube, the new frequency

$$f' = \left(\frac{V}{V \pm V_s}\right) f_0 = \left(\frac{320}{320 \mp 2}\right) f_0$$

$$= \left(\frac{320}{318}\right) f_0 \text{ or } \left(\frac{320}{322}\right) f_0$$

$$= \frac{320}{318}\frac{V}{4L_1} \text{ or } \frac{320}{322}\frac{V}{4L_1}$$

$\therefore$ The new length, $L_2 = \frac{V}{4f'}$

$$= \frac{V \times 318 \times 4L_1}{4 \times 320 \times V} \text{ or } \frac{322 \times 4L_1 V}{320 \times 4V}$$

$$L_2 = \frac{318}{320}L_1 \text{ or } \frac{322}{320}L_1$$

$\therefore$ Fraction change in length, $\frac{\Delta L}{L_1} = 6.25 \times 10^{-3}$

Percentage change in length = $\frac{\Delta L}{L} \times 100 = 0.625\%$

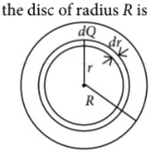
**18. (6.40) :** Consider an element of thickness $dr$ of radius $r$ in the given disc.

The charge on this element,

$dQ = \sigma dA = \sigma(2\pi r)\, dr$

$\therefore$ Total charge enclosed by the disc of radius $R$ is

$$Q_R = \int dQ = \int_0^R \sigma 2\pi r dr$$




$$= \int_0^R 2\pi\sigma_0\left(1 - \frac{r}{R}\right) r dr$$

$$= \pi\sigma_0 \frac{R^2}{3}$$

$$\therefore \; \phi_0 = \frac{Q_R}{\varepsilon} = \frac{\pi\sigma_0 R^2}{3\varepsilon} \quad \text{... (i)}$$

For a sphere of radius $R/4$, $Q_{R/4} = \int_0^{R/4} \sigma(2\pi r)\, dr$

or $Q_{R/4} = \frac{5}{12}\frac{2\pi\sigma_0 R^2}{16}$

$$\therefore \; \phi = \frac{Q_{R/4}}{\varepsilon} = \frac{5}{12}\frac{2\pi\sigma_0 R^2}{16} \quad \text{... (ii)}$$

From equation (i) and (ii), $\frac{\phi_0}{\phi} = 6.40$

## PAPER-2 ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (3) | 2. | (9) | 3. | (6) | 4. | (4) | 5. | (2) |
| 6. | (6) | 7. | (a, d) | 8. | (a, d) | 9. | (a,c,d) | 10. | (a,c) |
| 11. | (b,c) | 12. | (a,b,c,d) | 13. | (2.05) | 14. | (0.27) | 15. | (1.30) |
| 16. | (0.24) | 17. | (55.00) | 18. | (8.33) | | | | |

# SOLVED PAPER

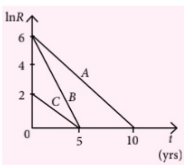
Held on
5th Sep, Morning Shift

# JEE MAIN 2020

**1.** Assume that the displacement($s$) of air is proportional to the pressure difference ($\Delta P$) created by a sound wave. Displacement($s$) further depends on the speed of sound ($v$), density of air ($\rho$) and the frequency ($f$). If $\Delta P \sim 10$ Pa, $v \sim 300$ m/s, $\rho \sim 1$ kg/m$^3$ and $f \sim 1000$ Hz, then $s$ will be of the order of (take the multiplicative constant to be 1)

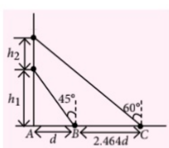
(a) $\frac{3}{100}$ mm (b) 10 mm
(c) $\frac{1}{10}$ mm (d) 1 mm

**2.** Activities of three radioactive substances $A$, $B$ and $C$ are represented by the curves $A$, $B$ and $C$ in the figure. Then their half lives $T_{1/2}(A) : T_{1/2}(B) : T_{1/2}(C)$ are in the ratio




(a) 2 : 1 : 1 (b) 3 : 2 : 1
(c) 2 : 1 : 3 (d) 4 : 3 : 1

**3.** A balloon is moving up in air vertically above a point $A$ on the ground. When it is at a height $h_1$, a girl standing at a distance $d$ (point $B$) from $A$ (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height $h_2$, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 $d$ (point $C$). Then the height $h_2$ is (given tan 30° = 0.5774)




(a) 1.464 $d$ (b) 0.732 $d$
(c) 0.464 $d$ (d) $d$

**4.** An electron is constructed to move along the $y$-axis with a speed of 0.1 $c$ ($c$ is the speed of light) in the presence of electromagnetic wave, whose electric field is $\vec{E} = 30\hat{j}\sin(1.5\times10^{7}t - 5\times10^{-2}x)$ V m$^{-1}$. The maximum magnetic force experienced by the electron will be (given $c = 3 \times 10^8$ m s$^{-1}$ and electron charge $= 1.6 \times 10^{-19}$ C)

(a) $3.2 \times 10^{-18}$ N (b) $2.4 \times 10^{-18}$ N
(c) $4.8 \times 10^{-19}$ N (d) $1.6 \times 10^{-19}$ N

**5.** A helicopter rises from rest on the ground vertically upwards with a constant acceleration $g$. A food packet is dropped from the helicopter when it is at a height $h$. The time taken by the packet to reach the ground is close to [$g$ is the acceleration due to gravity]

(a) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$ (b) $t = 1.8\sqrt{\left(\frac{h}{g}\right)}$
(c) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$ (d) $t = \sqrt{\left(\frac{2h}{3g}\right)}$

**6.** A galvanometer of resistance $G$ is converted into a voltmeter of range 0-1 V by connecting a resistance $R_1$ in series with it. The additional resistance that should be connected in series with $R_1$ to increase the range of the voltmeter to 0-2 V will be

(a) $G$ (b) $R_1$ (c) $R_1 - G$ (d) $R_1 + G$

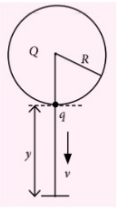
**7.** Two capacitors of capacitances $C$ and $2C$ are charged to potential differences $V$ and $2V$ respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is

(a) $\frac{25}{6}CV^2$ (b) $\frac{3}{2}CV^2$ (c) zero (d) $\frac{9}{2}CV^2$

**8.** With increasing biasing voltage of a photodiode, the photocurrent magnitude
(a) remains constant
(b) increases initially and after attaining certain value, it decreases
(c) increases linearly
(d) increases initially and saturates finally.

**9.** The value of the acceleration due to gravity is $g_1$ at a height $h = \frac{R}{2}$ ($R$ = radius of the earth) from the surface of the earth. It is again equal to $g_1$ at a depth $d$ below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals
(a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{1}{3}$ (d) $\frac{7}{9}$

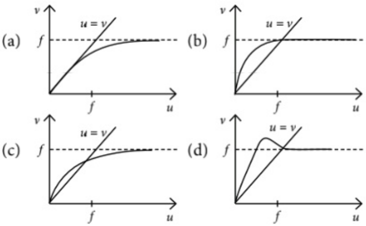
**10.** A solid sphere of radius $R$ carries a charge $Q + q$ distributed uniformly over its volume. A very small point like a piece of it of mass $m$ gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge $q$. If it acquires a speed $v$ when it has fallen through a vertical height $y$ (see figure), then (assume the remaining portion to be spherical)



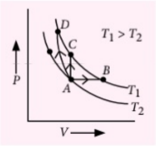

(a) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R^2 ym} + g\right]$
(b) $v^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)\, m} + g\right]$
(c) $v^2 = 2y\left[\frac{qQR}{4\pi\varepsilon_0 (R+y)^3\, m} + g\right]$
(d) $v^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)\, m} + g\right]$

**11.** A wheel is rotating freely with an angular speed $\omega$ on a shaft. The moment of inertia of the wheel is $I$ and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia $3I$ initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is
(a) $\frac{5}{6}$ (b) $\frac{1}{4}$ (c) 0 (d) $\frac{3}{4}$

**12.** For a concave lens of focal length $f$, the relation between object and image distances $u$ and $v$, respectively, from its pole can best be represented by ($u = v$ is the reference line)




**13.** Three different process that can occur in an ideal monoatomic gas are shown in the $P$ versus $V$ diagram. The paths are labelled as $A \to B$, $A \to C$ and $A \to D$. The change in internal energies during these processes are taken as $E_{AB}$, $E_{AC}$ and $E_{AD}$ and the work done as $W_{AB}$, $W_{AC}$ and $W_{AD}$. The correct relation between these parameters are




(a) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$
(b) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} > 0$
(c) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$
(d) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$.

**14.** Number of molecules in a volume of 4 $cm^3$ of a perfect monoatomic gas at some temperature $T$ and at a pressure of 2 cm of mercury is close to ? (Given, mean kinetic energy of a molecule (at $T$) is $4 \times 10^{-14}$ erg, $g = 980$ cm $s^{-2}$, density of mercury = 13.6 g $cm^{-3}$)
(a) $4.0 \times 10^{18}$ (b) $4.0 \times 10^{16}$
(c) $5.8 \times 10^{16}$ (d) $5.8 \times 10^{18}$

**15.** A square loop of side $2a$, and carrying current $I$, is kept in $XZ$ plane with its centre at origin. A long wire carrying the same current $I$ is placed parallel to the $Z$-axis and passing through the point $(0, b, 0)$, $(b >> a)$. The magnitude of the torque on the loop about $Z$-axis is given by
(a) $\frac{\mu_0 I^2 a^2}{2\pi b}$ (b) $\frac{\mu_0 I^2 a^3}{2\pi b^2}$
(c) $\frac{2\mu_0 I^2 a^2}{\pi b}$ (d) $\frac{2\mu_0 I^2 a^3}{\pi b^2}$

**16.** A physical quantity $z$ depends on four observables $a$, $b$, $c$ and $d$, as $z = \frac{a^2 b^{2/3}}{\sqrt{c} d^3}$. The percentages of error in the measurement of $a$, $b$, $c$ and $d$ are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in $z$ is
(a) 12.25% (b) 16.5% (c) 13.5% (d) 14.5%

**17.** A hollow spherical shell at outer radius $R$ floats just submerged under the water surface. The inner radius of the shell is $r$. If the specific gravity of the shell material is $\frac{27}{8}$ with respect to water, the value of $r$ is

(a) $\frac{8}{9}R$ (b) $\frac{4}{9}R$ (c) $\frac{2}{3}R$ (d) $\frac{1}{3}R$

**18.** In a resonance tube experiment the tube is filled with water upto a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m $s^{-1}$, the tuning fork frequency is

(a) 2200 Hz (b) 550 Hz
(c) 1100 Hz (d) 3300 Hz

**19.** A electrical power line, having a total resistance of 2 Ω, delivers 1 kW at 220 V. The efficiency of the transmission line is approximately

(a) 72% (b) 91% (c) 85% (d) 96%

**20.** A bullet of mass 5 g, travelling with a speed of 210 m $s^{-1}$, strikes a fixed wooden target. One half of its kinetic energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal g°$C^{-1}$ (1 cal = $4.2 \times 10^7$ ergs) close to

(a) 87.5°C (b) 83.3°C (c) 119.2°C (d) 38.4°C

**21.** A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})$ m. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ m will be $\sqrt{x}$ N m. The value of $x$ is ______.

**22.** A beam of electrons of energy $E$ scatters from a target having atomic spacing of 1 Å. The first maximum intensity occurs at $\theta = 60°$. Then $E$ (in eV) is ______.
(Planck constant, $h = 6.64 \times 10^{-34}$ J s, 1 eV = $1.6 \times 10^{-19}$ J, electron mass $m = 9.1 \times 10^{-31}$ kg)

**23.** A particle of mass 200 MeV/$c^2$ collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of $N$ is ______.
(Given the mass of the hydrogen atom to be 1 GeV/$c^2$)

**24.** A compound microscope consists of an objective lens of focal length 1 cm and an eye piece of focal length 5 cm with a separation of 10 cm. The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of $n$ is ______.

**25.** Two concentric circular coils, $C_1$ and $C_2$, are placed in the $XY$ plane. $C_1$ has 500 turns and a radius of 1 cm. $C_2$ has 200 turns and radius of 20 cm. $C_2$ carries a time dependent current $I(t) = (5t^2 - 2t + 3)$ A where $t$ is in s. The emf induced in $C_1$ (in mV), at the instant $t$ = 1 s is $\frac{4}{x}$. The value of $x$ is ______.

## SOLUTIONS

**1. (a)**

**2. (c)** : Rate of disintegration, $R = R_0 e^{-\lambda t}$

$\ln R = \ln R_0 - \lambda t$

$$\lambda_A = \frac{6}{10} \Rightarrow T_A = \frac{10}{6}\ln 2 \quad \ldots\text{(i)}$$

$$\lambda_B = \frac{6}{5} \Rightarrow T_B = \frac{5\ln 2}{6} \quad \ldots\text{(ii)}$$

$$\lambda_C = \frac{2}{5} \Rightarrow T_C = \frac{5\ln 2}{2} \quad \ldots\text{(iii)}$$

From eq. (i), (ii) and (iii), we get

$$\frac{10}{6}:\frac{5}{6}:\frac{15}{6}::2:1:3$$

$$\frac{1}{\lambda_A}:\frac{1}{\lambda_B}:\frac{1}{\lambda_C}::2:1:3$$

**3. (d)** : $\frac{h_1}{d} = \tan 45° \Rightarrow h_1 = d$ ...(i)

$$\frac{h_1 + h_2}{d + 2.464\,d} = \tan 30°$$

$\Rightarrow (h_1 + h_2) = 0.5774 \times 3.464\,d\,;\,(h_1 + h_2) = 2.0\,d$

$\Rightarrow d + h_2 = 2\,d$

$\Rightarrow h_2 = d$ (Using eq. (i))

**4. (c)** : We know that, $\vec{E} = E_0 \sin(kt - \omega x)$

Given, $\vec{E} = 30\hat{j}\,\sin(1.5 \times 10^7\,t - 5 \times 10^{-2}\,x)$ V $m^{-1}$

In electromagnetic wave, $\frac{E_0}{B_0} = c$

So, maximum value of magnetic field, $B_0 = \frac{E_0}{c}$

The maximum magnetic force experienced by the electron will be,

$F_{max} = qvB_{max} \sin 90°$

$$= qv\frac{E_0}{c} = 1.6 \times 10^{-19} \times 0.1 \times 30 = 4.8 \times 10^{-19}\text{ N}$$

**5. (c)** : For upward motion of helicopter.

From equation of motion, $v^2 = u^2 + 2as$

$\Rightarrow \quad u^2 = 0 + 2gh \Rightarrow u = \sqrt{2gh}$ ...(i)

This is initial velocity of packet in upward direction.
Now food packet will start moving under gravity,

$v^2 = u^2 + 2as$

$v^2 = 2gh + 2gh$ (Using eq. (i))

$v = \sqrt{4gh}$

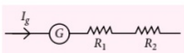
This is the velocity of packet just before hitting the ground.

We know, $v = u + at$

$$\sqrt{4gh} = -\sqrt{2gh} + gt \Rightarrow t = \sqrt{\frac{4h}{g}} + \sqrt{\frac{2h}{g}} = 3.4\sqrt{\frac{h}{g}}$$

**6. (d) :** 


$1 = I_g(G + R_1)$ ...(i)



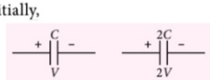

$2 = I_g(G + R_1 + R_2)$ ...(ii)

Divide eq. (i) by (ii),

$\Rightarrow \quad \frac{1}{2} = \frac{G + R_1}{G + R_1 + R_2} \Rightarrow G + R_1 + R_2 = 2G + 2R_1$

$\Rightarrow \quad G = R_2 - R_1 \,;\, R_2 = G + R_1$

**7. (b) :** Initially,

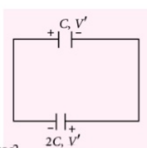



$Q_1 = CV \,;\, Q_2 = 2C \times 2V = 4CV$

$\Rightarrow$ By conservation of charge

$q_i = q_f$

$4CV - CV = (C + 2C)V'$

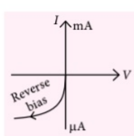



Common potential, $V' = V$

$\text{Energy} = \frac{1}{2}\, CV'^2$

$= \frac{1}{2} \times (3C) \times V^2 = \frac{3}{2}\, CV^2$

**8. (d) :** $I$-$V$ characteristic of a photodiode is shown in the given figure.




On increasing the potential difference, the current first increases and then attains saturation.

**9. (b) :** Acceleration due to gravity, $g = \frac{GM}{R^2}$

At height, $h = \frac{R}{2}$ the acceleration due to gravity is,

$$g_1 = \frac{GM}{\left(R + \frac{R}{2}\right)^2} \quad \text{...(i)}$$

At depth $d$, $g_2 = \frac{GM(R-d)}{R^3}$ ...(ii)

Given that, $g_1 = g_2$

$$\therefore \quad \frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GM(R-d)}{R^3} \Rightarrow \frac{4}{9} = \frac{R-d}{R}$$

$\Rightarrow \quad 4R = 9R - 9d \Rightarrow \frac{d}{R} = \frac{5}{9}$

**10. (d)**

**11. (d) :** By angular momentum conservation

$\omega I + 3I \times 0 = 4I\omega'$

$\Rightarrow \quad \omega' = \frac{\omega}{4}$

$$(\text{K.E.})_i = \frac{1}{2} I\omega^2 \,;\, (\text{K.E.})_f = \frac{1}{2} \times (4I) \times \left(\frac{\omega}{4}\right)^2 = \frac{I\omega^2}{8}$$

$\Delta\text{K.E.} = (\text{K.E.})_f - (\text{K.E.})_i = \frac{3}{8} I\omega^2$

$$\text{Fractional loss, } \frac{\Delta\,\text{K.E.}}{\text{K.E}_i} = \frac{\frac{3}{8} I\omega^2}{\frac{1}{2} I\omega^2} = \frac{3}{4}$$

**12. (a) :** By using relation, $v = \frac{uf}{u + f}$ ...(i)

Case-I : If $v = u$

$\Rightarrow \quad f + u = f$ (Using eq. (i))

$\Rightarrow \quad u = f - f = 0$

Case-II : If $u = \infty$

then $v = f$ (Using eq. (i))

So, option (a) is correct answer.

**13. (*) :** At constant pressure, $\Delta U = nC_v\Delta T$ is same.

In process $A \to B$ there is change in volume and pressure is constant.

Work done = area under $P - V$ curve

As area under the $P - V$ curve is increases for $A \to B$ process.

$\therefore \quad W_{AB} > 0$

In process $A \to C$, there is no change in volume in isochoric process.

$\therefore \quad W_{AC} = 0$

In process $A \to D$, volume is decreasing

$\therefore \quad W_{AD} < 0.$

*None of the given options are correct.

**14. (a) :** Given, K.E. = $4 \times 10^{-14}$ erg

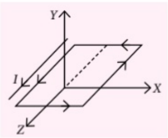
$\text{K.E.} = \frac{3}{2} PV = \frac{3}{2} kT$ $(\because PV = kT)$

K.E. = $\frac{3}{2}kT = 4 \times 10^{-14}$ erg ; K.E. = $RT = \frac{8}{3} \times 10^{-14}$ ...(i)

From ideal gas equation, $PV = nRT$

$$n = \frac{PV}{RT} = \frac{\rho \times g \times h \times V}{\frac{8}{3} \times 10^{-14}} = \frac{2 \times 980 \times 13.6 \times 4 \times 3}{8 \times 10^{-14}} = 4 \times 10^{18}$$

**15. (c) :** The magnitude of torque on the loop is given by,




$\tau = MB \sin\theta$ ...(i)

Here, $M = IA = I(2a)^2$

and $B = \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I}{2\pi b}$ $(\because d = b)$

From equation (i), $\tau = MB \sin 90°$

$$\tau = 4a^2 I \times \frac{\mu_0 I}{2\pi b} = \frac{2\mu_0 a^2 I^2}{\pi b}$$

**16. (d) :** Here, $z = \frac{a^2 b^{2/3}}{\sqrt{c}\, d^3}$

$$\frac{\Delta z}{z} = \frac{2\Delta a}{a} + \frac{2}{3}\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{3\Delta d}{d}$$

$$\left(\frac{\Delta z}{z} \times 100\right)\% = \left(\frac{2\Delta a}{a} + \frac{2}{3}\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{3\Delta d}{d}\right) \times 100\%$$

$\therefore$ The percentage error in $z$,

$$= \left(2 \times 2 + \frac{2}{3} \times 1.5 + \frac{1}{2} \times 4 + 3 \times 2.5\right)\% = 14.5\%$$

**17. (a) :** In equilibrium Buoyant force is $F_B = mg$

$$\frac{4}{3}\pi(R^3 - r^3)\rho_0 g = \frac{4}{3}\pi R^3 \rho_w g \; ; \left[1 - \left(\frac{r}{R}\right)^3\right] = \frac{\rho_w}{\rho_0}$$

$$1 - \frac{r^3}{R^3} = \frac{8}{27} \Rightarrow \frac{r}{R} = \left(\frac{19}{27}\right)^{1/3} = \frac{(19)^{1/3}}{3} \; ; r = 0.88\, R = \frac{8}{9}R$$

**18. (a) :** A resonance tube is an organ pipe closed at one end. If $L_1$ and $L_2$ be the first and second resonance lengths with a tuning fork of frequency $\upsilon$, then the speed of sound in air is given by, $v = 2\upsilon(L_2 - L_1)$

$\Rightarrow \upsilon\lambda = 2\upsilon(L_2 - L_1)$ $(\because v = \upsilon\lambda)$

$\Rightarrow \lambda = 2(L_2 - L_1) = 2 \times (24.5 - 17) \Rightarrow \lambda = 15$ cm

and $v = \upsilon\lambda \Rightarrow 330 = \upsilon \times 15 \times 10^{-2}$

$\therefore$ $\upsilon = \frac{330}{15} \times 100 = 2200$ Hz

**19. (d) :** Here, $R = 2\ \Omega$, $P = 1$ kW, $V = 220$ V

Power dissipated, $P = VI = I^2R = \left(\frac{1000}{220}\right)^2 \times 2 = 41$ W

Efficiency, $\eta = \frac{\text{Total potential difference}}{\text{emf}} \times 100\%$

$$= \frac{1000}{1000 + I^2R} \times 100 = \frac{1000}{1000 + 41} \times 100 = 96\%$$

**20. (a) :** Given, $m = 5$ g; $v = 210$ m s$^{-1}$

As $W = JQ \Rightarrow \left(\frac{1}{2}mv^2\right) \times \frac{1}{2} = J \times ms\Delta T$

or $\Delta T = \frac{v^2}{4Js} = \frac{(210)^2}{4 \times 4.200 \times 30} = 87.5°C$

**21. (195) :** Here, $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{r}_1 = \hat{i} + 2\hat{j} + \hat{k}$ ; $\vec{r}_2 = 4\hat{i} + 3\hat{j} - \hat{k}$

$\vec{\tau} = (\vec{r}_2 - \vec{r}_1) \times \vec{F}$ ; $\vec{\tau} = [(4\hat{i} + 3\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k}) \times \vec{F}$

$= (3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

$$|\vec{\tau}| = \sqrt{(7)^2 + (-11)^2 + (5)^2} = \sqrt{195} \Rightarrow x = 195$$

**22. (50.47) :** Using Bragg's formula,

$2d\sin\theta = n\lambda$

$2d\sin 60° = \lambda = \frac{h}{\sqrt{2mE}}$ ; $2 \times 10^{-10} \times \frac{\sqrt{3}}{2} = \frac{6.6 \times 10^{-34}}{\sqrt{2mE}}$

$$E = \frac{1}{2} \times \frac{(6.64)^2 \times 10^{-48}}{9.1 \times 10^{-31} \times 3 \times 1.6 \times 10^{-19}} = 50.47 \text{ eV}$$

**23. (51)** **24. (50)**

**25. (5) :** Given,

$I = (5t^2 - 2t + 3) \Rightarrow \frac{dI}{dt} = 10t - 2$

Now, magnetic field produced by $C_2$ at its centre,

$$B_2 = \frac{\mu_0 I N_2}{2r_2}$$

Now magnetic flux linked with coil $C_1$,

$$\phi_{12} = B_2 \times A_1 = \frac{N_2 \mu_0 I}{2r_2} \times \pi r_1^2$$

$\therefore$ Induced emf across coil $C_1$,

$$\varepsilon = -N_1 \frac{d\phi}{dt} = \frac{-\mu_0 N_2 N_1 \pi r_1^2}{2r_2}\frac{dI}{dt}$$

At $t = 1$ s,

$$= \frac{4\pi \times 10^{-7} \times 500 \times 200 \times \pi \times (1 \times 10^{-2})^2 \times 8}{2 \times (20 \times 10^{-2})}$$

$$= \frac{4\pi^2 \times 800000 \times 10^{-11}}{40 \times 10^{-2}} = 4 \times 0.19 \times 10^{-3} \simeq 4 \times 0.2 \text{ mV}$$

or $\frac{4}{5}$ mV $\Rightarrow x = 5$

**CLASS-XII**

# BRUSH UP for NEET/JEE 2021

Brush up your concepts to get high rank in NEET/JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

## Unit 4 Electromagnetic Induction, Alternating Current and Electromagnetic Waves

### Faraday's Law of Induction

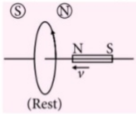
- First law : Whenever there is change in magnetic flux with respect to time for a coil or circuit, an emf induced in it and remains in it till change in flux takes place.
- Second law : The magnitude of the induced emf is directly proportional to the rate of change of flux through the coil.
- Mathematically, the induced emf is given by

  $$\varepsilon = -\frac{d\phi_B}{dt}$$

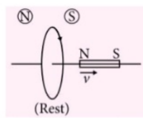
  The negative sign indicates the direction of $\varepsilon$ and hence the direction of current in the closed loop.
- Induced emf does not depend on nature of the coil *i.e.* resistance.
- Magnitude of induced emf is directly proportional to the relative speed of coil and magnet system.

### Lenz's Law

- This law states that the direction of induced current in the coil is in such a way that it always opposes the cause by which it is produced.



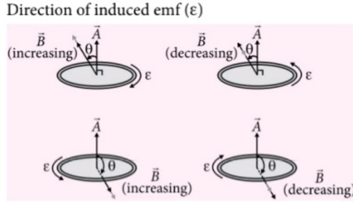

(Coil face behaves as north pole to oppose the motion of magnet)




(Coil face behaves as south pole to oppose the motion of magnet)

$\varepsilon = (-)\frac{d\phi}{dt}$ where negative sign indicates the Lenz's law.

- Direction of induced emf ($\varepsilon$)



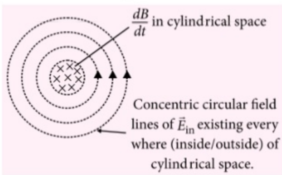

- Some basic induced parameters in a circuit
  - Average induced emf $\varepsilon_{av} = \frac{-\Delta\phi}{\Delta t}$
  - Instantaneous induced emf

    $$\varepsilon = \lim_{\Delta t \to 0}\left(\frac{-\Delta\phi}{\Delta t}\right) = -\frac{d\phi}{dt}$$
  - Induced current flow at this instant in the closed circuit $I = \frac{\varepsilon}{R} = \frac{1}{R}\left(\frac{d\phi}{dt}\right)$

- In time interval $dt$, induced charge $dq = Idt = \frac{d\phi}{R}$
- Induced heat $H = \int_0^t I^2 R dt = \int_0^t \frac{\varepsilon^2}{R} dt$
- Induced electric field : A time varying magnetic field ($dB/dt$) always produces induced electric field in all space surrounding it. Induced emf $\varepsilon = \oint \vec{E}_{\text{in}} \cdot d\vec{l}$

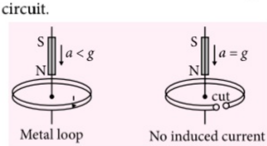
  As $\varepsilon = -\frac{d\phi}{dt}$

  so $\varepsilon = \oint \vec{E}_{\text{in}} \cdot d\vec{l} = -\frac{d\phi}{dt}$




  – It is non conservative and non electrostatic in nature.
  – Its field lines are concentric circular closed curves.

- Induced emf may exist in an open circuit, but there is no induced current and induced charge in the open circuit.




## ☞ Motional Electromotive Force

- When a conducting rod moves in an external magnetic field in such a way it cuts the field. So all the free electrons in the rod transfer from one end to another end ($C$ to $D$) and emf is induced in the rod. This emf is known as motional emf.
- emf induced within $d\vec{l}$

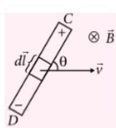
  $d\varepsilon = (\vec{v} \times \vec{B}) \cdot d\vec{l}$

  Net emf across the rod,

  $\varepsilon = \int (\vec{v} \times \vec{B}) \cdot d\vec{l} = vBl \sin\theta$

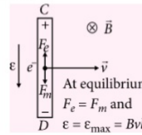
  Induced electric field in the rod,

  $\vec{E}_{\text{in}} = -(\vec{v} \times \vec{B}) = (\vec{B} \times \vec{v})$




- If any two out of $\vec{v}, \vec{B}$ and $d\vec{l}$ become parallel or antiparallel, ε will become zero.
- Some special cases for motional emf :
  - If θ = 90° *i.e.* $\vec{v}, \vec{B}$ and $\vec{l}$ are perpendicular to each other.

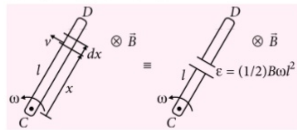
    Induced emf $\varepsilon = Blv$




  - Rotating straight conductor :

    emf induced in small element $dx$

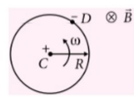
    $d\varepsilon = (\vec{v} \times \vec{B}) \cdot d\vec{x} = -vBdx$




    Net emf $\varepsilon = -\int_0^l vBdx = -B\omega \int_0^l xdx = -\frac{1}{2} B\omega l^2$

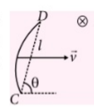
    Negative sign indicates that end $C$ will be at higher potential.

  - A rotating conducting disc : Induced emf between centre $C$ and circumference $D$ is



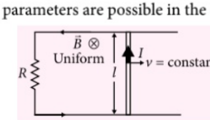

    $\varepsilon = \frac{1}{2} B\omega R^2$

  - A conductor of arbitrary shape : Induced emf in this conductor = Induced emf in straight conductor connected between $C$ and $D$ = $Blv \sin\theta$




## ☞ Effect of Motional emf Developed in a Circuit

- For a given circuit, if the metal rod moves with uniform velocity $v$ by an external agent then all the induced parameters are possible in the circuit.



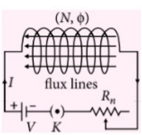

- Induced emf in the circuit $\varepsilon = Bvl$
- Current flows through the circuit $I = \frac{\varepsilon}{R} = \frac{Bvl}{R}$

- Retarding opposing force exerted on metal rod by action of induced current
  $\vec{F}_m = I(\vec{l} \times \vec{B})$ ; $F_m = BIl$ where $\theta = 90°$
  $$F_m = \frac{B^2 l^2 v}{R}$$
- External mechanical force required for uniform velocity of metal rod
  - For constant velocity resultant force on metal rod must zero and for that $F_{ext} = F_m$
  - $F_{ext} = F_m = \frac{B^2 l^2 v}{R}$
  - If $(B, l, R) \rightarrow$ constant $\Rightarrow F_{ext} \propto v$
- For uniform motion of metal rod, mechanical power delivered by external source is given as
  $P_{mech} = P_{ext} = \vec{F}_{ext} \cdot \vec{v} = F_{ext} v$
  $$P_{ext} = P_{mech} = \frac{B^2 l^2 v^2}{R}$$
  - If $(B, l, R) \rightarrow$ constant $\Rightarrow P_{mech} \propto v^2$
- Thermal power developed across resistor
  $$P_{th} = I^2 R = \left(\frac{Bvl}{R}\right)^2 R = \frac{B^2 l^2 v^2}{R}$$
  - It is clear that $P_{th} = P_{mech}$ which is consistent with the principle of conservation of energy.

## Self Induction

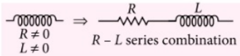
- When current through a coil changes with respect to the time then magnetic flux linked with the coil also changes with respect to time. Due to this an emf and a current induced in the coil. According to Lenz's law induced current opposes the change in magnetic flux. This phenomenon is called self induction and a factor by virtue the coil shows opposition for change in magnetic flux called self inductance of coil.
- When current through a coil is constant,
  - $I \rightarrow B \rightarrow \phi \rightarrow$ constant
  
  $\Rightarrow$ No electromagnetic induction
  - Total flux of coil $(N\phi) \propto$ current through the coil

  $N\phi \propto I$
  $N\phi = LI$
  $$L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{\phi_{total}}{I},$$
  where $L$ is self inductance of the coil.



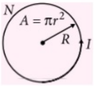

- When current through a coil changes with respect to time
  - $\frac{dI}{dt} \rightarrow \frac{dB}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow$ electromagnetic induction link
  - $N\phi = LI$ or, $-N\frac{d\phi}{dt} = -L\frac{dI}{dt}$,

  where $-N\frac{d\phi}{dt}$ called self induced emf of coil $\varepsilon_s$.
  $$\varepsilon_s = -L\frac{dI}{dt}$$
- Self inductance always opposes the change of current in a electric circuit so it is also called inertia of the electric circuit.
- For a real inductor




Role of $L$: To oppose the change in current. If current becomes constant then there is no role of $L$.

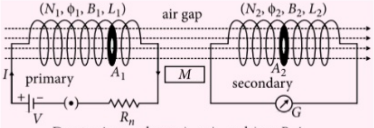
- Coefficient of self inductance of planar circular coil
  $$L_c = \frac{\mu_0 N^2 \pi R}{2}$$




- Coefficient of self inductance of solenoid
  $$L_s = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 Al = \mu_0 n^2 V$$
  Here, $V$ = volume of solenoid = $Al$
  $A$ = area of cross section of frame of solenoid

## Mutual Induction

- Whenever current passing through a primary coil or circuit changes with respect to time then magnetic flux in a neighbouring secondary coil or circuit will also changes with respect to time. According to Lenz's law for opposition of flux change an emf and a current induced in the neighbouring coil or circuit. This phenomenon is called as mutual induction.

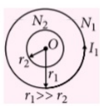



Due to air gap always $\phi_2 < \phi_1$ and $\phi_2 = B_1 A_2$

- When current through primary coil is constant,
  - Total flux linked into secondary coil is directly proportional to the current flowing through the primary coil.
  - $N_2\phi_2 \propto I_1$, $N_2\phi_2 = MI_1$
  - $M = \dfrac{N_2\phi_2}{I_1} = \dfrac{N_2B_1A_2}{I_1} = \dfrac{(\phi_T)_s}{I_p}$,
    where $M$ is mutual inductance of circuits.
- When current through primary coil changes with respect to time,
  - $\dfrac{dI_1}{dt} \to \dfrac{dB_1}{dt} \to \dfrac{d\phi_1}{dt} \to \dfrac{d\phi_2}{dt}$
    $\Rightarrow$ Electromagnetic induction link
  - $N_2\phi_2 = MI_1$ or, $-N_2\dfrac{d\phi_2}{dt} = -M\dfrac{dI_1}{dt}$,

  where $\left(-N_2\dfrac{d\phi_2}{dt}\right)$ called total mutual induced emf of secondary coil $\varepsilon_m$.

  $\varepsilon_m = -M\left(\dfrac{dI_1}{dt}\right)$
- Mutual inductance of two concentric and coplanar coils




  $M_{c_1c_2} = \dfrac{\mu_0 N_1 N_2 \pi r_2^2}{2r_1}$
- Mutual inductance of two co-axial solenoids

  $M_{s_1s_2} = \dfrac{\mu_0 N_1 N_2 A}{l}$

## ☞ Combination of Inductors

- Two inductors of inductances $L_1$ and $L_2$ are connected in series and are kept apart so that their mutual inductance is negligible, then the equivalent inductance of the combination is given by

  $L_S = L_1 + L_2$
- Two inductors of self-inductance $L_1$ and $L_2$ are connected in series and they have mutual inductance $M$, then the equivalent inductance of the combination is given by

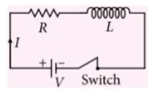
  $L = L_1 + L_2 \pm 2M$

  The plus sign occurs if windings in the two coils are in the same sense, while minus sign occurs if windings are in opposite sense.
- Two inductors of inductances $L_1$ and $L_2$ are connected in parallel and are kept far apart so that mutual inductance between them is negligible, then their equivalent inductance is given by

  $\dfrac{1}{L} = \dfrac{1}{L_1} + \dfrac{1}{L_2}$ or $L = \dfrac{L_1L_2}{L_1 + L_2}$

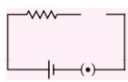
## ☞ Current growth in an *LR* circuit

- Emf equation : $V = IR + L\dfrac{dI}{dt}$



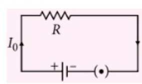

- Current at any instant : When key is closed the current in circuit increases exponentially with respect to time. The current in circuit at any instant $t$ is given by

  $I = I_0(1 - e^{-t/\tau})$
- Just after the closing of key, inductance behaves like open circuit and current in circuit is zero.



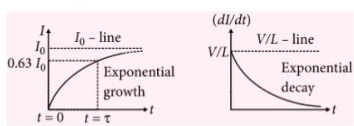
(Open circuit, $t = 0$, $I = 0$)
(Inductor provide infinite resistance)

- Some time after closing of the key inductance behaves like short circuit and current in circuit is constant.



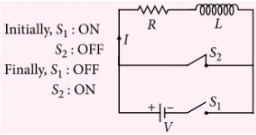

(Short circuit, $t \to \infty$, $I \to I_0$)
(Inductor provide zero resistance)

  - $I_0 = \dfrac{V}{R}$ (maximum or peak value of current)
  - Peak value of current in circuit does not depend on self inductance of coil.
- Time constant : It is a time in which current increases up to 63% or 0.63 times of peak current value. $\tau = \dfrac{L}{R}$



## ☞ Current decay in an *LR* circuit

- Emf equation : $IR + L\dfrac{dI}{dt} = 0$

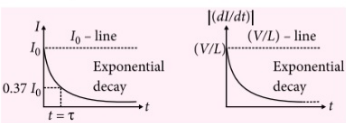



- Current at any instant : Once current acquires its final maximum steady value, if suddenly required switching positions ($S_1$ and $S_2$) are interchanged then current starts decreasing exponentially with respect to time. The current in the circuit at any instant $t$ is given by

  $I = I_0\, e^{-t/\tau}$

  - Just after opening of key $t = 0 \Rightarrow I = I_0 = \dfrac{V}{R}$
  - Some time after opening of key $t \to \infty \Rightarrow I_0 \to 0$

- Time constant ($\tau$) : It is a time in which current decreases up to 37% or 0.37 times of peak current value $\tau = \dfrac{L}{R}$




## ☞ Fundamental of Alternating Current

- Voltage or current is said to be alternating if it changes continuously magnitude and periodic in direction with the time. It can be represented by a sine curve or cosine curve.

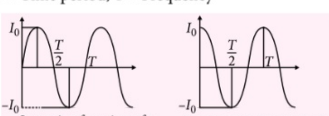
  $I = I_0 \sin \omega t$ or $I = I_0 \cos \omega t$

  $I$ = Instantaneous value of current at time $t$

  $I_0$ = Amplitude or peak value

  $\omega$ = Angular frequency (rad s$^{-1}$) $= \dfrac{2\pi}{T} = 2\pi\upsilon$

  $T$ = Time period, $\upsilon$ = Frequency




*I* as a sine function of *t*     *I* as a cosine function of *t*

- Average value or mean value
  - The mean value of *ac* over any half cycle is that value of *dc* which would send same amount of charge through a circuit as is sent by the *ac* through same circuit in the same time.
  - Average value of current for half cycle

$$\langle I \rangle = \frac{\int_0^{T/2} I\,dt}{\int_0^{T/2} dt}$$

  - Average value of $I = I_0 \sin \omega t$ over the positive half cycle :

$$I_{av} = \frac{\int_0^{T/2} I_0 \sin \omega t\, dt}{\int_0^{T/2} dt} = \frac{2I_0}{\omega T}[-\cos \omega t]_0^{T/2} = \frac{2I_0}{\pi}$$

- Root mean square (rms) value
  - It is that value of *dc* which would produce same heat in a given resistance in a given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}} = \sqrt{\frac{\int_0^T (I_0 \sin \omega t)^2 dt}{\int_0^T dt}}$$

$$= I_0 \sqrt{\frac{1}{T}\int_0^T \left[\frac{1-\cos 2\omega t}{2}\right] dt} = \frac{I_0}{\sqrt{2}}$$

  - rms value = virtual value = apparent value

- Phase : $I = I_0 \sin(\omega t \pm \phi)$
  - Initial phase = $\phi$ (it does not change with time)
  - Instantaneous phase = $\omega t$ (it changes with time)

- Phase difference :

  $V = V_0 \sin(\omega t + \phi_1)$, $I = I_0 \sin(\omega t + \phi_2)$
  - Phase difference of $I$ with respect to $V$

    $\phi = \phi_2 - \phi_1$
  - Phase difference of $V$ with respect to $I$

    $\phi = \phi_1 - \phi_2$

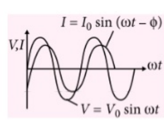
- $V$ leads $I$ or $I$ lags $V \to$ It means, $V$ reaches maximum before $I$.

  Let if $V = V_0 \sin \omega t$ then

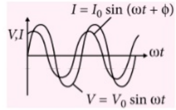
  $I = I_0 \sin(\omega t - \phi)$

  and if $V = V_0 \sin(\omega t + \phi)$

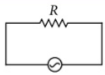
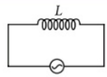
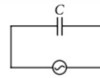
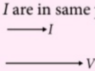
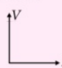
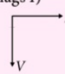
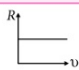
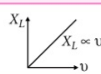
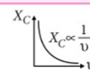
  then $I = I_0 \sin \omega t$



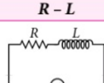
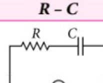
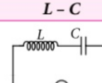
- $V$ lags $I$ or $I$ leads $V \rightarrow$ It means, $V$ reaches maximum after $I$
  Let if $V = V_0 \sin \omega t$ then
  $I = I_0 \sin (\omega t + \phi)$
  and if $V = V_0 \sin (\omega t - \phi)$
  then $I = I_0 \sin \omega t$

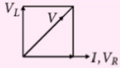
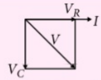
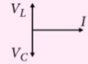



- Components $L$, $C$ and $R$ in $ac$ circuit separately

| Term | R | L | C |
|---|---|---|---|
| Circuit |   |   |   |
| Supply voltage | $V = V_0 \sin \omega t$ | $V = V_0 \sin \omega t$ | $V = V_0 \sin \omega t$ |
| Current | $I = I_0 \sin \omega t$ | $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ | $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ |
| Peak current | $I_0 = \frac{V_0}{R}$ | $I_0 = \frac{V_0}{\omega L}$ | $I_0 = \frac{V_0}{1/\omega C}$ |
| Impedance (Ω) $Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$ | $\frac{V_0}{I_0} = R$<br>$R$ = Resistance | $\frac{V_0}{I_0} = \omega L = X_L$<br>$X_L$ = Inductive reactance | $\frac{V_0}{I_0} = \frac{1}{\omega C} = X_L$<br>$X_C$ = Capacitive reactance |
| Phase difference and Phasor diagram | zero<br>($V$ and $I$ are in same phase)<br>  | $+\frac{\pi}{2}$ ($V$ leads $I$)<br>  | $-\frac{\pi}{2}$ ($V$ lags $I$)<br>  |
| Variation of $Z$ with υ |   $R$ does not depend on υ |   |   |
| $G$, $S_L$, $S_C$<br>(mho, seiman) | $G = \frac{1}{R}$ = conductance | $S_L = \frac{1}{X_L}$<br>Inductive susceptance | $S_C = \frac{1}{X_C}$<br>Capacitive susceptance |
| Behaviour of device in $dc$ and $ac$ | Same in $ac$ and $dc$ | $L$ passes $dc$ easily (because $X_L = 0$) while gives a high impedance for the $ac$ of high frequency ($X_L \propto \upsilon$) | $C$ blocks $dc$ (because $X_C = \infty$) while provides an easy path for the $ac$ of high frequency $\left(X_C \propto \frac{1}{\upsilon}\right)$ |
| Ohm's law | $V_R = IR$ | $V_L = IX_L$ | $V_C = IX_C$ |

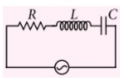
### ☞ Combination of $R - L$, $R - C$ and $L - C$ in an $ac$ circuit

| Term | R – L | R – C | L – C |
|---|---|---|---|
| Circuit |  <br>$I$ is same in $R$ and $L$ |  <br>$I$ is same in $R$ and $C$ |  <br>$I$ is same in $L$ and $C$ |

| | | | |
|---|---|---|---|
| Phasor diagram |  $V^2 = V_R^2 + V_L^2$ |  $V^2 = V_R^2 + V_C^2$ |  $V = V_L - V_C \quad (V_L > V_C)$<br>$V = V_C - V_L \quad (V_C > V_L)$ |
| Phase difference between $V$ and $I$ | $V$ leads $I$ $\left(0 \text{ to } \frac{\pi}{2}\right)$ | $V$ lags $I$ $\left(-\frac{\pi}{2} \text{ to } 0\right)$ | $V$ lags $I$ $\left(-\frac{\pi}{2}, \text{ if } X_C > X_L\right)$<br>$V$ leads $I$ $\left(+\frac{\pi}{2}, \text{ if } X_L > X_C\right)$ |
| Impedance | $Z = \sqrt{R^2 + X_L^2}$ | $Z = \sqrt{R^2 + X_C^2}$ | $Z = \lvert X_L - X_C \rvert$ |
| At very low υ | $Z \simeq R (X_L \to 0)$ | $Z \simeq X_C$ | $Z \simeq X_C$ |
| At very high υ | $Z \simeq X_L$ | $Z \simeq R \quad (X_C \to 0)$ | $Z \simeq X_L$ |

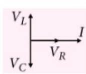
## ☞ Series *LCR* Circuit

- Circuit diagram

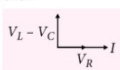



$I$ is same for $R$, $L$ and $C$.

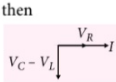
- Phasor diagram




  - If $V_L > V_C$ then



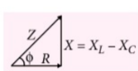

  - If $V_C > V_L$ then




$$V = \sqrt{V_R^2 + (V_L - V_C)^2}, \; Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- Impedance triangle

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{V_L - V_C}{V_R}$$




  - $V = IZ$, $V_R = IR$, $V_L = IX_L$, $V_C = IX_C$

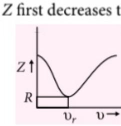
## ☞ Resonance in Series *LCR* Circuit

- A circuit is said to be resonant when the natural frequency of the circuit is equal to frequency of the applied voltage. For resonance both $L$ and $C$ must be present in circuit.
- At resonance,
  - $X_L = X_C$, $V_L = V_C$
  - $\phi = 0$ ($V$ and $I$ are in same phase)
  - $Z_{min} = R$, $I_{max} = \frac{V}{R}$
- Resonance frequency : $\because X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$

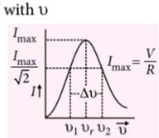
$$\omega = \frac{1}{\sqrt{LC}} \text{ or, } \upsilon_r = \frac{1}{2\pi\sqrt{LC}}$$

- Variation of $Z$ with υ
  - If $\upsilon < \upsilon_r$ then $X_L < X_C$, circuit is capacitive, $\phi$ (negative).
  - At $\upsilon = \upsilon_r$, $X_L = X_C$, circuit is resistive, $\phi$ = zero.
  - If $\upsilon > \upsilon_r$ then $X_L > X_C$ circuit is inductive, $\phi$ (positive).

As υ increases, $Z$ first decreases then increases,




- Variation of $I$ with υ




as υ increases, $I$ first increases then decreases.

- At resonance impedance of the series resonant circuit is minimum so it is called acceptor circuit as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.
- Band width $\Delta\upsilon = \upsilon_2 - \upsilon_1$
- Quality factor ($Q$) : $Q$-factor of ac circuit basically gives an idea about stored energy and lost energy.

$$Q = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{maximum energy lost per cycle}}$$

  - It represents the sharpness of resonance.
  - It is unitless and dimensionless quantity.
  - $Q = \frac{(X_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{2\pi\upsilon_r L}{R}$

    $= \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\upsilon_r}{\Delta\upsilon} = \frac{\upsilon_r}{\text{band width}}$
  - Sharpness $\propto$ quality factor
    $R$ decrease $\Rightarrow$ $Q$ increases $\Rightarrow$ sharpness increases

## Power in *ac* circuit

- Let $V = V_0 \sin \omega t$ and $I = I_0 \sin(\omega t - \phi)$
  Instantaneous power $P = V_0 \sin \omega t \cdot I_0 \sin(\omega t - \phi)$
  $= V_0 I_0 \sin \omega t (\sin \omega t \cos \phi - \sin\phi \cos \omega t)$
- Average power $\langle P \rangle$

$$= \frac{1}{T}\int_0^T (V_0 I_0 \sin^2 \omega t \cos\phi - V_0 I_0 \sin\omega t \cos\omega t \sin\phi)dt$$

$$\langle P \rangle = \frac{V_0 I_0 \cos\phi}{2} \Rightarrow \langle P \rangle = V_{rms} I_{rms} \cos\phi$$

- rms power $P_{rms} = V_{rms} I_{rms}$
- Power factor $(\cos \phi) = \frac{\text{Average power}}{\text{rms power}}$
  $\cos\phi = \frac{R}{Z}$
- Power dissipation is maximum in resistive circuit or at resonance in a $LCR$ series circuit.
- No power is dissipated in purely inductive or capacitive circuit even a current is flowing in the circuit $\left(\because \phi = \frac{\pi}{2}\right)$. This current is refered as wattless current.
- Power is dissipated only in resistor even circuit has $RL$, $RC$ or $LCR$ combination.

## Choke Coil

- Circuit with a choke coil is a series $L - R$ circuit. If resistance of choke coil = $r$ (very small). Current in the circuit, $I = \frac{V}{Z}$ with $Z = \sqrt{(R+r)^2 + (\omega L)^2}$
- It has a high inductance and negligible resistance coil.
- It is used to control current in ac circuit at negligible power loss.
  $\therefore \quad \cos\phi = \frac{V}{Z} = \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \frac{r}{\omega L} \to 0$
- Resistance of an ideal coil is zero.

## *LC* Oscillation

- The oscillation of energy between capacitor (electric field energy) and inductor (magnetic field energy) is called $LC$-oscillation.
- Frequency of oscillation, $\upsilon = \frac{1}{2\pi\sqrt{LC}}$
- If charge varies sinusoidally with time $t$ as $q = q_0 \cos \omega t$ then current varies periodically with $t$ as $I = \frac{dq}{dt} = q_0 \omega \cos\left(\omega t + \frac{\pi}{2}\right)$
- If initial charge on the capacitor is $q_0$ then electrical energy stored in capacitor is $U_E = \frac{1}{2}\frac{q_0^2}{C}$
- If the capacitor is fully discharged, then total electrical energy is stored in the inductor in the form of magnetic energy.
  $U_B = \frac{1}{2}LI_0^2$, where $I_0$ = Maximum current

## Transformer

- A transformer is an electrical device which is used for changing alternating voltages. It is based on the phenomenon of mutual induction.
- If it is assumed that there is no loss of energy in the transformer then the power input = the power output, and since $P = I \times V$ then
  $I_P V_P = I_S V_S$
- Although some energy is always lost, this is a good approximation, since a well designed transformer may have an efficiency of more than 95%.
  $\frac{I_P}{I_S} = \frac{V_S}{V_P} = \frac{N_S}{N_P}$
- A transformer affects the voltage and current. We have

$$V_S = \left(\frac{N_S}{N_P}\right)V_P \text{ and } I_S = \left(\frac{N_P}{N_S}\right)I_P$$

- Now, if $N_S > N_P$, the voltage is stepped up ($V_S > V_P$). This type of arrangement is called a step-up transformer.
- If $N_S < N_P$, we have a step-down transformer. In this case $V_S < V_P$ and $I_S > I_P$. The voltage is stepped down and the current is increased.

## Displacement Current

- Maxwell assumed that a current also flows in the gap between the two plates of a capacitor, during the process of charging, known as displacement current $I_D$. This displacement current originates due to time varying electric field between the plates of capacitor and is given by

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

where $\phi_E$ is the electric flux linked with the space between the two plates of the capacitor.

- Using the concept of displacement current $I_D$, Ampere's circuital law can be modified as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(I_C + I_D)$$

- Maxwell's equations:
  - $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$ (Gauss's law for electricity)
  - $\oint \vec{B} \cdot d\vec{A} = 0$ (Gauss's law for magnetism)
  - $\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$ (Faraday's law)
  - $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_C + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$ (Ampere-Maxwell law)

## Electromagnetic Waves

- An electromagnetic wave is a wave radiated by an accelerated charge which propagates through space as coupled electric and magnetic fields, oscillating perpendicular to each other and to the direction of propagation of the wave.
- Velocity of electromagnetic waves in free space is given by,

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m s}^{-1}$$

- The instantaneous magnitude of the electric and magnetic field vectors in electromagnetic wave are related as

$$\frac{|\vec{E}|}{|\vec{B}|} = c \text{ or } E = Bc$$

- In a medium of refractive index $n$, the velocity $v$ of an electromagnetic wave is given by

$$v = \frac{c}{n} = \frac{1}{n} \cdot \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Also $v = \frac{1}{\sqrt{\mu\varepsilon}}$, so that $n = \sqrt{\frac{\mu\varepsilon}{\mu_0 \varepsilon_0}}$

- The energy is equally shared between electric field and magnetic field vectors of electromagnetic wave. Therefore the energy density of the electric field, $u_E = \frac{1}{2}\varepsilon_0 E^2$ and

the energy density of magnetic field, $u_B = \frac{1}{2}\frac{B^2}{\mu_0}$.

- Average energy density of the electric field, $\langle u_E \rangle = \frac{1}{4}\varepsilon_0 E_0^2$ and average energy density of the magnetic field $\langle u_B \rangle = \frac{B_0^2}{4\mu_0} = \frac{1}{4}\varepsilon_0 E_0^2$.
- Average energy density of electromagnetic wave is

$$\langle u \rangle = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}.$$

- Intensity of electromagnetic wave is defined as energy crossing per unit area per unit time perpendicular to the directions of propagation of electromagnetic wave. The intensity $I$ is given by the relation

$$I = \langle u \rangle c = \frac{1}{2}\varepsilon_0 E_0^2 c$$

- The electromagnetic wave also carries linear momentum with it. The linear momentum carried by the portion of wave having energy $U$ is given by $p = U/c$.
- If the electromagnetic wave incident on a material surface is completely absorbed, it delivers energy $U$ and momentum $p = U/c$ to the surface.
- If the incident wave is totally reflected from the surface, the momentum delivered to the surface is $U/c - (-U/c) = 2U/c$. It follows that the electromagnetic wave incident on a surface exerts a force on the surface.

# WRAP it up!

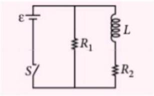
1. Two coils have a mutual inductance 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10$ A and $\omega = 100\,\pi$ rad s$^{-1}$. The maximum value of emf in the second coil is (in V)

   (a) $2\pi$ (b) $5\pi$ (c) $6\pi$ (d) $12\pi$

2. An express train takes 16 hours to cover the distance of 960 km between Patna and Gaziabad. The rails are separated by 130 cm and the vertical component of the earth's magnetic field is $4.0 \times 10^{-5}$ T. If the leakage resistance between the rails is 100 Ω, the retarding force on the train due to the magnetic field will be

   (a) $5 \times 10^{-10}$ N (b) $8 \times 10^{-10}$ N
   (c) $15 \times 10^{-5}$ N (d) $5 \times 10^{-5}$ N

3. An inductor of inductance $L = 400$ H and resistors of resistances $R_1 = 2\ \Omega$ and $R_2 = 2\ \Omega$ are connected to a battery of emf 12 V as shown in figure. The internal resistance of the battery is negligible. The switch $S$ is closed at $t = 0$. The potential drop across $L$ as a function of time is




   (a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V
   (c) $6(1 - e^{-t/0.2})$ V (d) $12e^{-5t}$ V

4. A uniform magnetic field $B$ exists in a direction perpendicular to the plane of a square frame made of copper wire. The wire has a diameter of 2 mm and a total length of 40 cm. The magnetic field changes with time at a steady rate $dB/dt = 0.02$ T s$^{-1}$. What will be the current induced in the frame? (Resistivity of copper = $1.7 \times 10^{-8}\ \Omega$ m)

   (a) 0.1 A (b) 0.2 A (c) 0.3 A (d) 0.4 A

5. In a uniform magnetic field of induction $B$, a wire in the form of semicircle of radius $r$ rotates about the diameter of the circle with angular frequency $\omega$. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is $R$, then mean power generated per period of rotation is

   (a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$
   (c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$

6. If a resistance of 100 Ω, an inductance of 0.5 H and a capacitance of $10 \times 10^{-6}$ F are connected in series through 50 Hz *ac* supply, the impedance will be

   (a) 1.87 Ω (b) 101.3 Ω
   (c) 18.7 Ω (d) 189.7 Ω

7. Two circular loops of equal radii are placed coaxially at some separation. The first loop is cut and a battery is inserted in between to drive a current in it. The current changes slightly because of the variation in resistance with temperature. During this period, the two loops

   (a) attract each other
   (b) repel each other
   (c) do not exert any force on each other
   (d) attract or repel each other depending on the sense of the current.

8. The instantaneous values of alternating current and voltage in a circuit are given as

   $I = \frac{1}{\sqrt{2}}\sin(100\pi t)$ A and $\varepsilon = \frac{1}{\sqrt{2}}\sin(100\pi t + \pi/3)$ V

   The average power in watts consumed in the circuit is

   (a) $\frac{1}{4}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

9. An electromagnetic wave of frequency 3 MHz passes from vacuum into a dielectric medium with permittivity $\varepsilon_r = 4$. Then

   (a) the wavelength and frequency both remain unchanged
   (b) the wavelength is doubled and the frequency remains unchanged
   (c) the wavelength is doubled and the frequency becomes half
   (d) the wavelength is halved and the frequency remains unchanged.

10. The r.m.s. value of the electric field of the light coming from the sun is 720 N $C^{-1}$. The average total energy density of the electromagnetic wave is
(a) $3.3 \times 10^{-3}$ J $m^{-3}$ (b) $4.58 \times 10^{-6}$ J $m^{-3}$
(c) $6.37 \times 10^{-9}$ J $m^{-3}$ (d) $81.35 \times 10^{-12}$ J $m^{-3}$

11. The amplitude of the electric field in a parallel beam of light of intensity 2.0 W $m^{-2}$ is
(a) 38.8 N $C^{-1}$ (b) 49.5 N $C^{-1}$
(c) 32.7 N $C^{-1}$ (d) 35.5 N $C^{-1}$

12. A free electron is placed in the path of a plane electromagnetic wave. The electron will start moving
(a) along the electric field
(b) along the magnetic field
(c) along the direction of propagation of the wave
(d) in a plane containing the magnetic field and the direction of propagation.

13. An inductor 20 mH, a capacitor 50 μF and a resistor 40 Ω are connected in series across a source of emf $V = 10 \sin 340t$. The power loss in *ac* circuit is
(a) 0.76 W (b) 0.89 W
(c) 0.46 W (d) 0.67 W

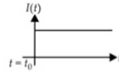
14. A small signal voltage $V(t) = V_0 \sin\omega t$ is applied across an ideal capacitor $C$
(a) Current $I(t)$ is in phase with voltage $V(t)$
(b) Current $I(t)$ leads voltage $V(t)$ by 180°
(c) Current $I(t)$, lags voltage $V(t)$ by 90°
(d) Over a full cycle the capacitor $C$ does not consume any energy from the voltage source.

15. A long solenoid has 1000 turns. When a current of 4 A flows through it, the magnetic flux linked with each turn of the solenoid is $4 \times 10^{-3}$ Wb. The self-inductance of the solenoid is
(a) 2 H (b) 1 H (c) 4 H (d) 3 H

16. Out of the following options which one can be used to produce a propagating electromagnetic wave?
(a) A chargeless particle
(b) An accelerating charge
(c) A charge moving at constant velocity
(d) A stationary charge

17. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (r.m.s.), 50 Hz *ac* supply, the series inductor needed for it to work is close to
(a) 80 H (b) 0.08 H (c) 0.044 H (d) 0.065 H

18. A series $LR$ circuit is connected to a voltage source with $V(t) = V_0 \sin\omega t$. After very large time, current $I(t)$ behaves as $\left(t_0 >> \frac{L}{R}\right)$

(a)


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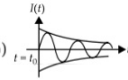

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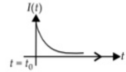
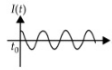


(d)



19. A conducting metal circular-wire-loop of radius $r$ is placed perpendicular to a magnetic field which varies with time as $B = B_0 e^{-\frac{t}{\tau}}$, where $B_0$ and $\tau$ are constants, at time $t = 0$. If the resistance of the loop is $R$ then the heat generated in the loop after a long time ($t \to \infty$) is
(a) $\frac{\pi^2 r^4 B_0^4}{2\tau R}$ (b) $\frac{\pi^2 r^4 B_0^2}{2\tau R}$
(c) $\frac{\pi^2 r^4 B_0^2 R}{\tau}$ (d) $\frac{\pi^2 r^4 B_0^2}{\tau R}$

20. Microwave oven acts on the principle of
(a) giving rotational energy to water molecules
(b) giving translational energy to water molecules
(c) giving vibrational energy to water molecules
(d) transferring electrons from lower to higher energy levels in water molecule.

## SOLUTIONS

1. **(b):** As $|\varepsilon| = M\frac{dI}{dt} = M\frac{d}{dt}(I_0 \sin \omega t) = MI_0\,\omega \cos \omega t$,

$\varepsilon_{max} = MI_0\omega = (0.005)(10)(100\pi)$ V $= 5\pi$ V

2. **(a):** As the train moves in a magnetic field, a motional emf $\varepsilon = vBl$ is produced across its width. Here $B$ is the component of the magnetic field in a direction perpendicular to the plane of the motion, *i.e.*, the vertical component.

The speed of the train is $v = \frac{960 \text{ km}}{16 \text{ h}} = 16.67 \text{ m s}^{-1}$

Thus, $\varepsilon = (16.67 \text{ m s}^{-1})(4.0 \times 10^{-5} \text{ T})(1.3 \text{ m})$
$= 8.6 \times 10^{-4}$ V

The leakage current is $I = \varepsilon/R$ and the retarding force is

$$F = IlB = \frac{8.6\times10^{-4}\text{ V}}{100\ \Omega}\times1.3 \text{ m}\times4.0\times10^{-5}\text{ T}$$

$$= 4.47 \times 10^{-10} \text{ N} \simeq 5\times10^{-10} \text{ N}$$

3. **(d):** If $I_1$ is the current through $R_1$ and $I_2$ is the current through $L$ and $R_2$, then $I_1 = \frac{\varepsilon}{R_1}$ and $I_2 = I_0(1 - e^{-t/\tau})$,

where $\tau = \frac{L}{R_2} = \frac{400\times10^{-3}}{2} = 0.2$ s

and $I_0 = \frac{\varepsilon}{R_2} = \frac{12}{2} = 6$ A

Thus, $I_2 = 6(1 - e^{-t/0.2})$

Potential drop across $L$, *i.e.*,

$\varepsilon - R_2I_2 = 12\text{ V} - 2 \times 6(1 - e^{-t/0.2})\text{ V} = (12e^{-5t})\text{ V}$

4. **(a):** Here, total length $l = 40$ cm $= 40 \times 10^{-2}$ m,

Resistivity $= 1.7 \times 10^{-8}\ \Omega$ m

The area $A$ of the loop

$$=\left(\frac{40\text{ cm}}{4}\right)\left(\frac{40\text{ cm}}{4}\right) = 0.01\text{ m}^2$$

If the magnetic field at an instant is $B$, the flux through the frame at that instant will be $\phi = BA$. As the area remains constant, the magnitude of the emf induced will be

$$\varepsilon = \frac{d\phi}{dt} = A\frac{dB}{dt}$$

$= (0.01\text{ m})(0.02\text{ T s}^{-1}) = 2 \times 10^{-4}$ V

The resistance of the loop is

$$\frac{(1.7\times10^{-8}\,\Omega\text{ m})(40\times10^{-2}\text{ m})}{3.14\times1\times10^{-6}\text{ m}^2} = 2.16\times10^{-3}\,\Omega$$

Hence, the current induced in the loop will be

$$I = \frac{2\times10^{-4}\text{ V}}{2.16\times10^{-3}\,\Omega} = 9.3\times10^{-2}\text{ A} \approx 0.1\text{ A}$$

5. **(b):** As $\phi_B = BA\cos\omega t = B\left(\frac{1}{2}\pi r^2\right)\cos\omega t$,

$$\varepsilon = -\frac{d\phi_B}{dt} = \frac{1}{2}B\pi r^2\omega\sin\omega t$$

Instantaneous power, $P = \frac{\varepsilon^2}{R} = \frac{(B\pi r^2\omega)^2}{4R}\sin^2\omega t$

$$P_{av} = \frac{\int_0^T P\,dt}{T} = \frac{(B\pi r^2\omega)^2}{4R}\frac{(T/2)}{T}$$

$$\left(as \int_0^T \sin^2\omega t = T/2\right)$$

$$= \frac{(B\pi r^2\omega)^2}{8R}$$

6. **(d):** As $X_L = 2\pi\upsilon L = 2\pi(50)(0.5) = 157.1\ \Omega$,

$$X_C = \frac{1}{2\pi\upsilon C} = \frac{1}{2\pi(50)(10^{-5})}\ \Omega = 318.4\ \Omega$$

$$|X_L - X_C| = 161.3\ \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100)^2 + (161.3)^2}\ \Omega$$

$$= 189.7\ \Omega$$

7. **(a)**

8. **(d):** As $\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}} = \frac{(1/\sqrt{2})}{\sqrt{2}} = \frac{1}{2}$ V,

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{(1/\sqrt{2})}{\sqrt{2}} = \frac{1}{2}\text{ A},$$

and $\cos\phi = \cos\pi/3 = \frac{1}{2}$

$$P_{av} = \varepsilon_{rms}I_{rms}\cos\phi = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}\text{ W}$$

9. **(d):** Frequency remains unchanged with change of medium.

Refractive index, $n = \frac{c}{v} = \frac{1/\sqrt{\varepsilon_0\mu_0}}{1/\sqrt{\varepsilon\mu}} = \sqrt{\varepsilon_r\mu_r}$

Since $\mu_r$ is very close to 1, $n = \sqrt{\varepsilon_r} = \sqrt{4} = 2$

Thus, $\lambda_{medium} = \frac{\lambda}{n} = \frac{\lambda}{2}$

10. **(b):** $u_{av} = \frac{1}{2}\varepsilon_0E_0^2 = \frac{1}{2}\varepsilon_0(\sqrt{2}E_{rms})^2 = \varepsilon_0E_{rms}^2$

$$\left(\text{as } E_{rms} = \frac{E_0}{\sqrt{2}}\right)$$

$= (8.85 \times 10^{-12})(720)^2 = 4.58 \times 10^{-6}\text{ J m}^{-3}$

11. **(a):** The intensity of a plane electromagnetic wave is

$$I = u_{av}c = \frac{1}{2}\varepsilon_0E_0^2c$$

or $E_0 = \sqrt{\frac{2I}{\varepsilon_0c}} = \sqrt{\frac{2\times2.0}{8.85\times10^{-12}\times3\times10^8}}$

$= 38.8\text{ N C}^{-1}$

12. **(a)**

13. **(c):** Here, $L = 20$ mH $= 20 \times 10^{-3}$ H,

$C = 50\ \mu\text{F} = 50 \times 10^{-6}$ F

$R = 40\ \Omega$, $V = 10\sin 340t = V_0\sin\omega t$

$\omega = 340\text{ rad s}^{-1}$, $V_0 = 10$ V

$X_L = \omega L = 340 \times 20 \times 10^{-3} = 6.8\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{340 \times 50 \times 10^{-6}} = \frac{10^4}{34 \times 5} = 58.82\,\Omega$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(40)^2 + (58.82 - 6.8)^2}$$

$$= \sqrt{(40)^2 + (52.02)^2} = 65.62\,\Omega$$

The peak current in the circuit is

$$I_0 = \frac{V_0}{Z} = \frac{10}{65.62}\text{ A}, \cos\phi = \frac{R}{Z} = \left(\frac{40}{65.62}\right)$$

Power loss in ac circuit,

$$= V_{rms} I_{rms} \cos\phi = \frac{1}{2} V_0 I_0 \cos\phi$$

$$= \frac{1}{2} \times 10 \times \frac{10}{65.62} \times \frac{40}{65.62} = 0.46\text{ W}$$

**14. (d):** When an ideal capacitor is connected with an ac voltage source, current leads voltage by 90°. Since, energy stored in capacitor during charging is spent in maintaining charge on the capacitor during discharging. Hence over a full cycle, the capacitor does not consume any energy from the voltage source.

**15. (b):** Here, $N = 1000$, $I = 4$ A, $\phi_0 = 4 \times 10^{-3}$ Wb

Total flux linked with the solenoid, $\phi = N\phi_0$

$= 1000 \times 4 \times 10^{-3}$ Wb $= 4$ Wb

Since, $\phi = LI$

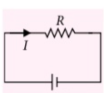
$\therefore$ Self-inductance of solenoid,

$$\text{L} = \frac{\phi}{I} = \frac{4\text{ Wb}}{4\text{ A}} = 1\text{ H}$$

**16. (b):** An accelerating charge is used to produce oscillating electric and magnetic fields, hence the electromagnetic wave.

**17. (d):** For a dc source

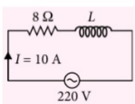
$I = 10$ A, $V = 80$ V




Resistance of the arc lamp,

$$R = \frac{V}{I} = \frac{80}{10} = 8\,\Omega$$

For an ac source,




$\varepsilon_{rms} = 220$ V

$\upsilon = 50$ Hz

$\omega = 2\pi \times 50 = 100\,\pi$ rad s$^{-1}$

Arc lamp will glow if $I = 10$ A,

$$\therefore \quad I = \frac{\varepsilon_{rms}}{\sqrt{R^2 + \omega^2 L^2}} \text{ or } R^2 + \omega^2 L^2 = \left(\frac{\varepsilon_{rms}}{I}\right)^2$$

$$\text{or } 8^2 + (100\,\pi)^2 L^2 = \left(\frac{220}{10}\right)^2$$

$$\text{or } L^2 = \frac{22^2 - 8^2}{(100\,\pi)^2}$$

$$\therefore \quad L = \frac{\sqrt{30 \times 14}}{100\,\pi} = 0.065\text{ H}$$

**18. (d):** Current in *LR* circuit is

$$I = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t - \frac{\pi}{2}\right)$$

*i.e.*, it is sinusoidal in nature.

**19. (b):** Here, $B = B_0 e^{-\frac{t}{\tau}}$

Area of the circular loop, $A = \pi r^2$

Flux linked with the loop at any time $t$,

$$\phi = BA = \pi r^2 B_0 e^{-\frac{t}{\tau}}$$

Emf induced in the loop, $\varepsilon = -\frac{d\phi}{dt} = \pi r^2 B_0 \frac{1}{\tau} e^{-\frac{t}{\tau}}$

Net heat generated in the loop

$$= \int_0^\infty \frac{\varepsilon^2}{R} dt = \frac{\pi^2 r^4 B_0^2}{\tau^2 R} \int_0^\infty e^{-\frac{2t}{\tau}} dt$$

$$= \frac{\pi^2 r^4 B_0^2}{\tau^2 R} \times \frac{1}{\left(-\frac{2}{\tau}\right)} \times \left[e^{-\frac{2t}{\tau}}\right]_0^\infty$$

$$= \frac{-\pi^2 r^4 B_0^2}{2\tau^2 R} \times \tau(0-1) = \frac{\pi^2 r^4 B_0^2}{2\tau R}$$

**20. (a)**

## Monthly Test Drive CLASS XI ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (d) | **2.** | (a) | **3.** | (c) | **4.** | (d) | **5.** | (a) |
| **6.** | (c) | **7.** | (c) | **8.** | (c) | **9.** | (d) | **10.** | (a) |
| **11.** | (b) | **12.** | (a) | **13.** | (a) | **14.** | (c) | **15.** | (b) |
| **16.** | (a) | **17.** | (d) | **18.** | (c) | **19.** | (a) | **20.** | (a,c) |
| **21.** | (b,c) | **22.** | (a,b,c) | **23.** | (b,d) | **24.** | (4) | **25.** | (2) |
| **26.** | (4) | **27.** | (a) | **28.** | (d) | **29.** | (b) | **30.** | (c) |

# CBSE warm-up!

CLASS-XII

Practice questions for CBSE Exams as per the reduced syllabus, latest pattern and marking scheme issued by CBSE for the academic session 2020-21.

**Series 3**

## CHAPTERWISE PRACTICE PAPER : Magnetic Effect of Current and Magnetism

Time Allowed : 3 hours
Maximum Marks : 70

### GENERAL INSTRUCTIONS

Read the following instructions very carefully and strictly follow them.

(i) This question paper comprises four section – A, B, C and D.

(ii) There are 37 questions in the question paper. All questions are compulsory.

(iii) Section A – Question no. 1 to 20 are very short answer type questions carrying 1 mark each.

(iv) Section B – Question no. 21 to 27 are short answer type questions carrying 2 marks each.

(v) Section C – Question no. 28 to 34 are long answer type questions carrying 3 marks each.

(vi) Section D – Question no. 35 to 37 are also long answer type questions, carrying 5 marks each.

(vii) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of 1 mark, 2 questions of 2 marks, 1 question of 3 marks and all the 3 questions of 5 marks. You have to attempt only one of the choices in such questions.

(viii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(ix) Use of calculators and log tables is not permitted.

(x) You may use the values of physical constants wherever necessary.

### SECTION-A

**Directions (Q. No. 1-10) : Select the most appropriate option from those given below each question.**

1. A strong magnetic field is applied on a stationary electron. Then the electron
   (a) moves in the direction of the field
   (b) remains stationary
   (c) moves perpendicular to the direction of the field
   (d) moves opposite to the direction of the field.

2. A charged particle would continue to move with a constant velocity in a region wherein, which of the following conditions is not correct?
   (a) $E = 0, B \neq 0$ (b) $E \neq 0, B \neq 0$
   (c) $E \neq 0, B = 0$ (d) $E = 0, B = 0$

3. The magnitude of the magnetic field at the centre of the tightly wound 150 turn coil of radius 12 cm carrying a current of 2 A is
   (a) 18 G (b) 19.7 G (c) 15.7 G (d) 17.7 G

4. The nature of parallel and anti-parallel currents are
   (a) parallel currents repel and antiparallel currents attract
   (b) parallel currents attract and antiparallel currents repel
   (c) both currents attract
   (d) both currents repel.

5. A current carrying loop is placed in a uniform magnetic field. The torque acting on it does not depend upon
(a) shape of loop (b) area of loop
(c) value of current (d) magnetic field.

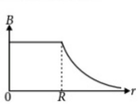
6. The angles of dip at the poles and the equator respectively are
(a) 30°, 60° (b) 0°, 90° (c) 45°, 90° (d) 90°, 0°

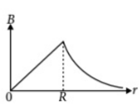
7. Which one of the following is correct statement about magnetic forces?
(a) Magnetic forces always obey Newton's third law.
(b) Magnetic forces do not obey Newton's third law.
(c) For very high current, magnetic forces obey Newton's third law.
(d) Inside low magnetic field, magnetic forces obey Newton's third law.

8. If electron moving with velocity $\vec{v}$ produces a magnetic field $\vec{B}$, then
(a) the direction of field $\vec{B}$ will be same in the direction of velocity $\vec{v}$
(b) the direction of field $\vec{B}$ will be opposite to the direction of velocity $\vec{v}$
(c) the direction of field $\vec{B}$ will be perpendicular to the direction of velocity $\vec{v}$
(d) the direction of field $\vec{B}$ does not depend upon the direction of velocity $\vec{v}$.

9. The magnetic force per unit length on a wire carrying a current of 10 A and making an angle of 45° with the direction of a uniform magnetic field of 0.20 T is
(a) $2\sqrt{2}$ N m$^{-1}$ (b) $\frac{2}{\sqrt{2}}$ N m$^{-1}$
(c) $\frac{\sqrt{2}}{2}$ N m$^{-1}$ (d) $4\sqrt{2}$ N m$^{-1}$

10. The correct plot of the magnitude of magnetic field $\vec{B}$ vs distance $r$ from centre of the wire is, if the radius of wire is $R$

(a)



(b)
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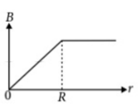
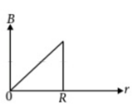



(d)



**Directions (Q. No. 11-15) : Fill in the blanks with appropriate answer.**

11. A solenoid of length 50 cm, having 100 turns carries a current of 2.5 A. The magnetic field at one end of the solenoid is______.

12. If the current sensitivity of a galvanometer is doubled, then its voltage sensitivity will be ______.

13. A proton and an α-particle enter in a uniform magnetic field perpendicularly with same speed. The ratio of time periods of both particle $\left(\frac{T_p}{T_\alpha}\right)$ will be ______.

14. Two α-particles have the ratio of their velocities as 3 : 2 on entering the field. If they move in different circular paths, then the ratio of the radii of their paths is ______.

15. A 200 turn closely wound circular coil of radius 15 cm carries a current of 4A. The magnetic moment of this coil is ______.

**OR**

A straight wire having mass of 1.2 kg and length of 1 m carries a current of 5A. If the wire is suspended in mid-air by a uniform horizontal magnetic field, then the magnitude of field is ______.

**Directions (Q. No. 16-20) : Answer the following.**

16. At a certain place the horizontal component of earth's magnetic field is $B_0$ and the angle of dip is 45°. Find the total intensity of the field at that place.

17. What is the function of radial magnetic field in a moving coil galvanometer?

18. In a hydrogen atom, the electron moves in an orbit of radius 0.5 Å, making $10^{16}$ rps. Calculate the magnetic moment associated with the relative motion of electron.

**OR**

State Gauss's law for magnetism.

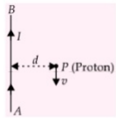
19. Define one tesla using the expression for the magnetic force acting on a particle of charge '$q$' moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$.

20. Why do magnetic lines of force form continuous closed loops?

## SECTION-B

21. A charge $Q$ is spread uniformly over an insulated ring of radius $R$. What is the magnetic moment of the ring if it is rotated with an angular velocity ω with respect to normal axis?

22. A long straight wire $AB$ carries a current $I$. A proton $P$ travels with a speed $v$, parallel to the wire, at a distance $d$ from it in a direction opposite to the current as shown in figure. What is the force experienced by the proton and what is its direction?




23. A long wire is bent into a circular coil of one turn A similar wire is bent into a circular coil of smaller radius having $n$ turns. If the same current passes in both the cases, find the ratio of the magnetic fields produced at the centres in the two cases.

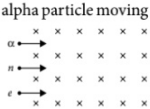
24. A magnetising field of 1500 $Am^{-1}$ produces a magnetic flux of $2.4 \times 10^{-5}$ Wb in an iron bar of cross section 0.5 $cm^2$. Calculate permeability and susceptibility of the iron bar used.

**OR**

Out of the two magnetic materials, $A$ has relative permeability slightly greater than unity while $B$ has less than unity. Identify the nature of the materials $A$ and $B$. Will their susceptibilities be positive or negative?

25. A charged particle is moving on a circular path of radius $R$ in a uniform magnetic field under the Lorentz force $F$. How much work is done by the force in one round? Is the momentum of the particle changing?

26. A neutron, an electron and an alpha particle moving with equal velocities, enter a uniform magnetic field going into the plane of the paper as shown. Trace their paths in the field.



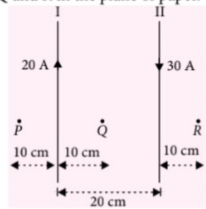

**OR**

The magnetic force depends on $\vec{v}$ which depends on the inertial frame of reference. Does then the magnetic force differ from inertial frame to frame? Is it reasonable that the net acceleration has a different value in different frames of reference?

27. A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment?

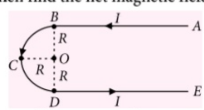
## SECTION-C

28. In the figure two long parallel current carrying wires I and II are shown. Find the magnitudes and directions of the magnetic field induction at the points $P$, $Q$ and $R$ in the plane of paper.




29. A galvanometer having 30 divisions has a current sensitivity of 20 μA per division. It has a resistance of 20 Ω. How will you convert it into an ammeter measuring upto 1 ampere? How will you convert this ammeter into voltmeter reading upto 1 V?

30. A combination of two long straight wires $AB$ and $DE$ and a semicircular arc $BCD$ of radius $R$ is shown in the figure. If a current $I$ flows as shown in the figure, then find the net magnetic field at point $O$.




31. An electron of mass $m_e$ revolves around a nucleus of charge $+Ze$. Show that it behaves like a tiny magnetic dipole. Hence prove that the magnetic moment associated with it is expressed as $\vec{\mu} = -\frac{e}{2m_e}\vec{L}$, where $\vec{L}$ is the orbital angular momentum of the electron. Give the significance of negative sign.

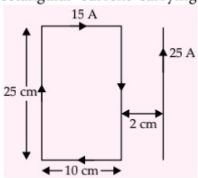
32. Two identical coils, each of radius '$R$' and number of turns '$N$' are lying in perpendicular planes such that their centres coincide. Find the magnitude and direction of the resultant magnetic field at the centre of the coils, if they are carrying currents $I$ and $\sqrt{3}I$ respectively.

**OR**

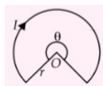
An electron beam passes through a magnetic field of 2.0 mT and an electric field of $3.4 \times 10^4$ V $m^{-1}$ both acting simultaneously in mutually perpendicular directions. If the path of electrons remains undeflected, calculate the speed of the electrons.

If the electric field is switched off, what will be the radius of the electron path?
Given that $m_e = 9.1 \times 10^{-31}$ kg and $e = 1.60 \times 10^{-19}$ C.

**33.** The figure shows a rectangular current carrying loop, placed 2 cm away from a long straight, current carrying conductor. What is the direction and magnitude of the net force acting on the loop?




**34.** A circular segment of radius $r$ and angle $\theta$ carries current $I$ as shown in figure. What is the value of magnetic field at the centre $O$ of segment?




## SECTION-D

**35. (a)** Derive the expression for the torque on a rectangular current carrying loop suspended in a uniform magnetic field.

**(b)** A proton and a deuteron having equal momenta enter in a region of a uniform magnetic field at right angle to the direction of a the field. Depict their trajectories in the field.

**OR**

Write any two important points of similarities and differences each between Coulomb's law for the electrostatic field and Biot-Savart's law of the magnetic field. Use Biot-Savart's law to find the expression for the magnetic field due to a circular loop of radius '$r$' carrying current '$I$', at its centre.

**36.** State Biot-Savart's law. Using this law derive an expression for the magnetic field at a point situated at a distance of $x$ metre from the centre of a circular coil of $N$ turns and radius $r$ carrying a current of $I$ A.

**OR**

**(a)** Draw a labelled diagram of a moving coil galvanometer. Describe briefly its principle and working.

**(b)** A galvanometer has resistance G and shows full scale deflection for current $I_g$. How can it be converted into an ammeter to measure current upto $I_0 (I_0 > I_g)$?

**37. (a)** A long straight wire of uniform cross section of radius $a$ is carrying a steady current $I$. Use Ampere's circuital law to obtain a relation showing the variation of the magnetic field ($B$) inside and outside the wire with distance $r$, ($r < a$) and ($r > a$) at the field point from the centre of its cross section.

**(b)** Explain how Biot-Savart's law enables one to express the Ampere's circuital law in the integral form, viz.

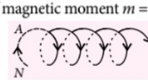
$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

where $I$ is the total current passing through the surface.

**OR**

**(a)** State Ampere's circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius '$r$', having '$n$' turns per unit length and carrying steady current $I$.

**(b)** An observer to the left of a solenoid of $N$ turns each of cross section area '$A$' observes that a steady current $I$ in it flows in the clockwise direction. Depict the magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic moment $m = NIA$.




## SOLUTIONS

**1. (b) :** Magnetic force, $\vec{F} = \vec{q}(\vec{v} \times \vec{B})$
The electron is stationary, *i.e.*, velocity $\vec{v} = 0$.
$\therefore$ $\vec{F} = 0$. So, electron will remain stationary.

**2. (c)**

**3. (c) :** Here, $N = 150$ ; $r = 12$ cm $= 12 \times 10^{-2}$ m ; $I = 2$A

$$B = \frac{\mu_0 NI}{2R} = \frac{2\pi \times 10^{-7} \times 150 \times 2}{12 \times 10^{-2}} = 1.57 \times 10^{-3} \text{ T}$$

$= 15.7 \times 10^{-4}$ T $= 15.7$ G

**4. (b)** **5. (a)** **6. (d)**

**7. (b) :** As the action and reaction are instantaneous in III$^{rd}$ law. But magnetic forces transmit with finite speed $c$.

**8. (c) :** According to Biot Savart's law, the magnetic field

$$\vec{B} = \frac{\mu_o}{4\pi} \cdot \frac{q(\vec{v} \times \vec{r})}{r^3}$$

The direction of $\vec{B}$ will be along $\vec{v} \times \vec{r}$, *i.e.* perpendicular to the plane containing $\vec{v}$ and $\vec{r}$.

**9. (b) :** $I = 10$ A, $\theta = 45°$, $B = 0.2$ T

$\therefore$ $\vec{F} = I\vec{l} \times \vec{B} \Rightarrow F = IlB \sin\theta$

$$\therefore \quad \frac{F}{l} = IB \sin 45° = 10 \times 0.2 \times \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} \text{ N m}^{-1}$$

**10.** **(b)**

**11.** Here, $I = 2.5$ A, $l = 50$ cm $= 0.50$ m

and $n = \frac{100}{0.50} = 200\ \text{m}^{-1}$

$\therefore\ B = \frac{\mu_0 nI}{2} = \frac{4\pi \times 10^{-7} \times 200 \times 2.5}{2} = 3.14 \times 10^{-4}\ \text{T}$

**12.** Current sensitivity of galvanometer is deflection per unit current *i.e.*

$$\frac{\phi}{I} = \frac{NAB}{k} \quad \text{...(i)}$$

Similarly voltage sensitivity is deflection per unit voltage *i.e.*

$$\frac{\phi}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V} = \left(\frac{NAB}{k}\right)\frac{1}{R} \quad \text{... (ii)}$$

From (i) and (ii),

Voltage sensitivity = current sensitivity $\times \frac{1}{\text{Resistance}}$

Now if current sensitivity is doubled, then the resistance in the circuit will also be doubled since it is proportional to the length of the wire, then voltage sensitivity

$= (2 \times \text{current sensitivity}) \times \frac{1}{(2 \times \text{Resistance})}$

$= (\text{current sensitivity}) \times \frac{1}{\text{Resistance}}$

Hence, voltage sensitivity will remain unchanged.

**13.** The time period of revolution of a charged particle in a magnetic field is

$T = \frac{2\pi m}{Bq}$

For proton, $m_P = m;\ q_P = q$

$\therefore\ T_p = \frac{2\pi m}{Bq}$

Now, for $\alpha$-particle,

$m_\alpha = 4\ m,\ q_\alpha = 2q$

$\therefore\ T_\alpha = \frac{2\pi(4m)}{B(2q)} = 2\left(\frac{2\pi m}{Bq}\right)$ or $\frac{T_p}{T_\alpha} = \frac{1}{2}$

**14.** As $qvB = \frac{mv^2}{r}$, $r = \frac{mv}{qB} \Rightarrow r \propto v$ or $\frac{r_A}{r_B} = \frac{v_A}{v_B} = \frac{3}{2}$

**15.** The magnetic moment is given by

$|\vec{m}| = NIA = NI\pi r^2$

$= 200 \times 4 \times 3.14 \times (15 \times 10^{-2})^2$

$= 200 \times 4 \times 3.14 \times 15 \times 15 \times 10^{-4} = 56.5\ \text{Am}^2$

**OR**

For mid-air suspension the upward force $F$ on wire due to magnetic field $B$ must be balanced by the force due to gravity, then

$IlB = mg$

$B = \frac{mg}{Il}$

Here, $m = 1.2$ kg, $g = 10$ m s$^{-2}$, $I = 5$ A, $l = 1$ m

$B = \frac{1.2 \times 10}{5 \times 1} = 2.4$ T

**16.** Here, $B_H = B_0$, $\delta = 45°$

As, $B_H = B\cos\delta$

$B = \frac{B_H}{\cos\delta} = \frac{B_0}{\cos 45°} = \frac{B_0}{1/\sqrt{2}}$, $B = \sqrt{2}B_0$

**17.** Radial magnetic field provides constant magnetic field and hence the torque on the coil becomes constant in all positions of the coil for the given current. This provides a linear current scale.

**18.** Here, $r = 0.5$ Å $= 0.5 \times 10^{-10}$ m, $e = 1.6 \times 10^{-19}$C

As $m = IA = \left(\frac{e}{T}\right) \times \pi r^2 = e\upsilon\pi r^2$

$= 1.6 \times 10^{-19} \times 10^{16} \times 3.14 \times (0.5 \times 10^{-10})^2\ \text{A m}^2$

$= 1.256 \times 10^{-23}\ \text{A m}^2$

**OR**

Gauss's law for magnetism : Gauss's law for magnetism states that the net magnetic flux through any closed surface is zero.

$$\phi = \sum_{\substack{\text{all area}\\ \text{elements}}} \vec{B} \cdot \Delta\vec{S} = 0$$

**19.** One tesla is defined as the magnitude of magnetic field which produces a force of 1 newton when a charge of 1 coulomb moves perpendicularly in the region of the magnetic field at a velocity of 1 m/s.

$F = qvB \Rightarrow B = \frac{F}{qv}$ or $1\ \text{T} = \frac{1\ \text{N}}{(1\ \text{C})(1\ \text{m s}^{-1})}$

**20.** Magnetic lines of force form continuous closed loops because a magnet is always a dipole and as a result, the net magnetic flux of a magnet is always zero.

**21.** Charge on the element of length $dl$ of the ring is,

$dq = \frac{Q}{2\pi R}dl$

Current due to circular motion of this charge is,

$dI = dq \times \upsilon = \frac{Q}{2\pi R}dl \times \frac{\omega}{2\pi}$ (As, $\omega = 2\pi\upsilon$)

## Monthly Test Drive CLASS XII ANSWER KEY

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | (a) | 2. | (c) | 3. | (c) | 4. | (d) | 5. | (c) |
| 6. | (b) | 7. | (d) | 8. | (a) | 9. | (a) | 10. | (c) |
| 11. | (b) | 12. | (a) | 13. | (c) | 14. | (d) | 15. | (c) |
| 16. | (d) | 17. | (a) | 18. | (b) | 19. | (d) | 20. | (a,b,c) |
| 21. | (b,c) | 22. | (a,b,d) | 23. | (a,d ) | 24. | (1) | 25. | (6) |
| 26. | (5) | 27. | (a) | 28. | (a) | 29. | (c) | 30. | (b) |

Magnetic moment due to current $dI$,

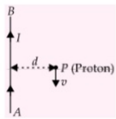
$$dM = dI \times \pi R^2 = \frac{Q}{2\pi R} dl \times \frac{\omega}{2\pi} \times \pi R^2$$

or $\quad M = \frac{Q\omega R}{4\pi}\int dl = \frac{Q\omega R}{4\pi} 2\pi R \quad (\because \int dl = 2\pi R)$

$$= \frac{1}{2} Q\omega R^2$$

22. Magnetic field induction at $P$ due to current $I$ in long straight wire $AB$ is $B = \frac{\mu_0}{4\pi}\frac{2I}{d}$

It acts perpendicular to the plane of paper inwards. Since the proton is moving in opposite direction to the current carrying straight wire, hence the proton is moving perpendicular to the direction of magnetic field due to current through straight wire.




$\therefore$ The force acting on the proton is

$$F = qvB\sin 90^\circ = ev\frac{\mu_0 2I}{4\pi d} = \frac{\mu_0 Iev}{2\pi d}$$

It acts in the plane of paper away from the wire.

23. Let $l$ be the length of the wire. When the wire is bent in the form of one turn circular coil,

$$l = 2\pi r_1, \text{ or } r_1 = \frac{l}{2\pi}, N = 1$$

$$\therefore \ B_1 = \frac{\mu_0 NI}{2r_1} = \frac{\mu_0 \times 1 \times I}{2\times(l/2\pi)} = \frac{\mu_0 \pi I}{l}$$

When the wire is bent in the form of $n$-turns coil,

$$l = n \times 2\pi r_2 \text{ or } r_2 = \frac{l}{2n\pi}$$

$$\therefore \ B_2 = \frac{\mu_0 NI}{2r_2} = \frac{\mu_0 nI}{2(l/2n\pi)} = \frac{\mu_0 \pi n^2 I}{l}$$

So, $\frac{B_1}{B_2} = \frac{1}{n^2}$ or, $B_1 : B_2 = 1 : n^2$

24. Here, $H = 1500$ A m$^{-1}$, $\phi = 2.4 \times 10^{-5}$ Wb,

$A = 0.5$ cm$^2$ $= 0.5 \times 10^{-4}$ m$^2$

Magnetic induction,

$$B = \frac{\phi}{A} = \frac{2.4\times 10^{-5}}{0.5\times 10^{-4}} = 0.48 \text{ Wb m}^{-2}$$

Permeability, $\mu = \frac{B}{H} = \frac{0.48}{1500} = 3.2\times 10^{-4}$ T m A$^{-1}$

As $\mu = \mu_0 (1 + \chi_m)$

$\therefore$ Susceptibility, $\chi_m = \frac{\mu}{\mu_0} - 1$

$$= \frac{3.2\times 10^{-4}}{4\times 3.14\times 10^{-7}} - 1 = 254.77 - 1 = 253.77$$

**OR**

Relative permeability, $\mu_r = 1 + \chi_m$

$\therefore \ \chi_m = \mu_r - 1$

As the relative permeability of $A$ is slightly greater than 1, so its susceptibility $\chi$ is small and positive. Hence $A$ is a paramagnetic substance.

As the relative permeability of $B$ is slightly less than 1, so its susceptibility $\chi$ is small and negative. Hence $B$ is a diamagnetic substance.

25. When a charged particle moving on a circular path of radius $R$ in a uniform magnetic field, the Lorentz magnetic force $F(= qvB)$ acting on the particle, provides the required centripetal force for its circular motion. It means the Lorentz force acts along the radius towards the centre of circular path. While moving on a circular path, the small displacement $d\vec{r}$ of the charged particle is always perpendicular to Lorentz force, *i.e.*, $\theta = 90^\circ$, therefore work done

$$dW = \vec{F}\cdot d\vec{r} = F\,dr\,\cos 90^\circ = 0.$$

Since the velocity of the charged particle, moving on a circular path is tangential to the path whose direction is changing continuously in circular motion of the particle, hence the momentum of the particle is changing.

26. Magnetic force on $\alpha$-particle

$\vec{F}_\alpha = q\vec{v}\times\vec{B} = 2evB$ upward

So, curve will bend upwards as force is perpendicular to the velocity.

Magnetic force on neutron, $F = 0$ (as $q = 0$)

So, neutron will move along straight line.

Magnetic force on electron

$\vec{F}_e = q\,\vec{v}\times\vec{B} = |-evB|$ downwards

So, curve will bend downwards as force is perpendicular to the velocity.

**OR**

Magnetic force is given by,

$$\vec{F}_m = q(\vec{v}\times\vec{B})$$

Here, $\vec{v}$ depends on the inertial frame of reference. Hence magnetic force is frame dependent. Net acceleration arising from this force is however frame independent for inertial frames.

27. A magnetic force can only change the direction of charged particle but never changes magnitude of speed as force act normal to direction of speed. So, charged particle may follow a complicated trajectory, but its speed remains the same.

BRAIN MAP

# CURRENT ELECTRICITY

Our society relies heavily on the convenience and versatility of electricity. It powers our microwave, helps light our house, lets us watch TV and so much more. Even it plays a role in the way our heart beats. Muscle cells in the heart are contracted by electricity going through the heart. An electrocardiogram translates the heart's electrical activity into line tracings on paper.

## Ohm's Law

- $V = IR$
- Vector form, $\vec{J} = \sigma\vec{E}$, conductivity $\sigma = \frac{1}{\rho}$
- Resistance of a uniform conductor, $R = \frac{\rho l}{A}$
- Conductance, $G = \frac{1}{R}$
- Resistivity or specific resistance, $\rho = \frac{RA}{l}$

Ohm's law is not valid for semiconductor.

## Drift Speed

- Drift speed, $v_d = \frac{eE}{m}\tau$
- Mobility, $\mu_e = \frac{v_d}{E} = \frac{e\tau}{m}$
- Mobility of electron is more than that of hole.
- Current in terms of drift velocity,

$$I = neAv_d = \frac{ne^2 A\tau E}{m} = neA\mu_e E = neA\mu_e \frac{V}{l}$$

- $\sigma = ne\mu_e$
- In terms of relaxation time $\tau$,

$$R = \frac{ml}{ne^2\tau A} \text{ and } \rho = \frac{m}{ne^2\tau}$$

## Resistors

- Resistance of a resistor with colour code
$R$ = [(digit 1)(digit 2) Decimal Multiplier ± Tolerance]
- Variation of resistance with temperature,
$R_T = R_0[1 + \alpha(T - T_0)]$
Temperature coefficient of resistance, $\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$
If $T_1 = 0\,°C$ and $T_2 = T\,°C$ then
$$\alpha = \frac{R_T - R_0}{R_0 \times T} \text{ or } R_T = R_0(1 + \alpha T)$$
- Resistivity of conductor increases with increase in temperature.
- Resistivity of semiconductor decreases with increase in temperature.

## Combination of Resistors

- In series, equivalent resistance
$R_S = R_1 + R_2 + R_3 + ....$
- In parallel, equivalent resistance
$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ....$$
- For two resistances in parallel current through the two resistances, $I_1 = \frac{R_2 I}{R_1 + R_2}$, $I_2 = \frac{R_1 I}{R_1 + R_2}$
- When resistances are connected in series, the current through each resistance is same. In parallel combination voltage is same.

## Potentiometer

- Principle : Fall of potential across any portion of the wire, $V \propto l$, $V = Kl$
Here, $K = \frac{V}{l}$ = potential gradient.
- Comparision of emfs of two cells,
$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$$
- Internal resistance of a cell
$$r = R\left(\frac{\varepsilon}{V} - 1\right) = R\left(\frac{l_1}{l_2} - 1\right)$$
Here, $l_1$ = balancing length of potentiometer wire corresponding to emf of the cell.
$l_2$ = balancing length of potentiometer wire corresponding to potential difference of the cell when a resistance $R$ is connected in series with the cell whose internal resistance to be determined.

## Electric Current

- Rate of flow of electric charge (positive)
$$I = \frac{dQ}{dt} = \oint \vec{J} \cdot d\vec{S}$$
- $J$ is the current density which is the current passing through a cross-section of the wire.
- Dot product of $\vec{J}$ and $d\vec{S}$ shows current is scalar inspite of this we represent current with an arrow.
- In case of an electron revolving in a circle of radius $r$ with speed $v$, period of revolution of electron is $T = \frac{2\pi r}{v}$.
- Frequency of revolution $\upsilon = \frac{v}{2\pi r}$
- Current in an orbit of a moving electron with velocity $v$, $I = \frac{e}{T} = \frac{ev}{2\pi r}$

## Flow of Charge

- $\Delta Q = I\Delta t$ (Current is constant)
$= \int I dt$ (Current is a function of time)
= Area under $I$-$t$ graph

## Kirchhoff's Rules

- **Junction Rule :** At any junction of circuit elements, the sum of currents entering the junction must be equal to the sum of currents leaving it.
$\Sigma I = 0.$
It is based on conservation of charge.
So bending or reorienting the wire does not change the validity of Kirchhoff's junction rule.
- **Loop Rule :** The algebraic sum of changes in potential around any closed loop must be zero.
$\Sigma\varepsilon = \Sigma IR$
It is based on conservation of energy.

## Sign Convention for Kirchhoff's Rules

- While traversing a closed loop (clockwise or anti clockwise) if negative pole of the cell is encountered first, then its emf is negative, otherwise positive.
- The product of resistance and current in an arm of the circuit is taken positive if the direction of current in that arm is in the same sense as one moves in a closed path and is taken negative if the direction of current in that arm is opposite to the sense as one moves in a closed path.
- This convention depends on your choice. You can go to reverse of this assumption and will get same answer.

## Wheatstone Bridge

- In balanced condition,
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$
- It provides a parallel method to determine the value of an unknown resistance.

## Meter Bridge

- Resistance of unknown resistor,
$$R = \frac{Sl_1}{(100 - l_1)}$$
- It is more sensitive if $l_1$(balance point) is close to 50 cm. This requires a suitable choice of $S$.

## Electrical Energy and Power

- Heat energy developed across a resistor
$H = I^2Rt$
- Power $P = I^2R = \frac{V^2}{R}$
- For transmission cable, power loss
$$P_C = I^2R_C = \frac{P^2R_C}{V_C^2}, P = \text{constant}$$

## Principle of Bulb

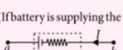
- Resistance of bulb
$$R = \frac{V^2}{P} \text{ or } R \propto \frac{1}{P}$$
- In parallel $P = P_1 + P_2$
- In series $\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$ or $P = \frac{P_1P_2}{P_1 + P_2}$
- In $R = \frac{V^2}{P}$, $V$ and $P$ are rated values for that bulb.
- In parallel a bulb having more rated power glows more brightly. In series a bulb having less rated power glows more brightly.

## Grouping of Batteries

- Series grouping : $I = \frac{\text{net emf}}{\text{net resistance}}$
If there are $n$ identical batteries of emf $\varepsilon$ and internal resistance $r$, then current through external resistance $R$ (if all batteries are additive in nature) is given by
$$I = \frac{n\varepsilon}{nr + R}$$
If polarity of $m$ batteries is reversed, then
$$I = \frac{(n - 2m)\varepsilon}{nr + R}$$
- Parallel grouping with identical batteries :
$$I = \frac{\varepsilon_{net}}{R_{net}}$$
Here, $\varepsilon_{net} = \varepsilon$, $R_{net}$ = Total internal resistance + Total external resistance $= \frac{r}{n} + R$
- Parallel grouping with unidentical batteries :
$$I = \frac{\varepsilon_{net}}{R_{net}}$$
Here, $\varepsilon_{net} = \frac{\Sigma(\varepsilon / r)}{\Sigma(1/r)}$ ...(i)
If any of the battery is oppositely connected, then place negative sign in numerator of eqn. (i) but, no change in denominator.
- Mixed grouping :
If there are $n$ rows of identical batteries ($\varepsilon$, $r$) with $m$ cells in each row. Then, $I = \frac{\varepsilon_{net}}{R_{net}}$
Here, $\varepsilon_{net} = m\varepsilon$, $R_{net} = \frac{mr}{n} + R$
In this case current through the external resistance is maximum when, $R = \frac{mr}{n}$

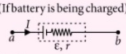
## Potential Difference across the Terminals of a Battery

- $V = \varepsilon$ (If $I = 0$)
- $V = 0$ (If battery is short circuited)
- $V = \varepsilon - Ir$ (If battery is supplying the energy)

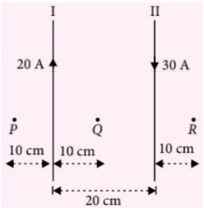



$V = V_a - V_b = V_{ab} = \varepsilon - Ir$

- $V = \varepsilon + Ir$ (If battery is being charged)




$V = V_a - V_b = V_{ab} = \varepsilon + Ir$

**28.** 


Resultant magnetic field induction at $P$ is

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi}\frac{2I_1}{r_1} - \frac{\mu_0}{4\pi}\frac{2I_2}{r_2} = 2\times10^{-5}\ \text{T}$$

$$= \frac{\mu_0}{4\pi}\times 2\left[\frac{I_1}{r_1} - \frac{I_2}{r_2}\right]$$

Here, $I_1 = 20$ A, $r_1 = 0.1$ m;

$I_2 = 30$ A ; $r_2 = 0.30$ m

$$B = 10^{-7}\times 2\left[\frac{20}{0.1} - \frac{30}{0.3}\right] = 2\times10^{-5}\ \text{T}$$

It will be acting perpendicular to the plane of paper upwards.

Resultant magnetic field induction at $Q$ is

$$B = \frac{\mu_0}{4\pi}\times 2\left[\frac{20}{0.1} + \frac{30}{0.1}\right] = 10^{-7}\times 2\times 500 = 10^{-4}\ \text{T}$$

It will be acting perpendicular to the plane of the paper downwards.

Resultant magnetic field induction at $R$ is

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi}\frac{2I_1}{r_1} - \frac{\mu_0}{4\pi}\frac{2I_2}{r_2} = 2\times10^{-5}\ \text{T}$$

It will be acting perpendicular to the plane of the paper upwards.

**29.** Here $n = 30$, $R_g = 20\ \Omega$

Current sensitivity, $k = 20\ \mu$A per division

$\therefore$ Current required for full-scale deflection is

$I_g = nk = 30\times 20 = 600\ \mu\text{A} = 6\times10^{-4}\ \text{A} = 0.0006$ A

(i) For conversion into ammeter, $I = 1$ A

$$\therefore\ R_s = \frac{I_g}{I - I_g}\times R_g = \frac{0.0006\times 20}{1 - 0.0006} = 0.012\ \Omega$$

*i.e.*, a shunt of 0.012 Ω should be connected across the galvanometer.

(ii) For conversion of resulting ammeter into voltmeter. The resistance of the ammeter formed is

$$R'_g = \frac{R_g R_s}{R_g + R_s} = \frac{20\times0.012}{20+0.012} = 0.012\ \Omega$$

Current for full scale deflection, $I'_g = 1$ A

Voltage range, $V = 1$ V

$\therefore$ Required series resistance,

$$R = \frac{V}{I'_g} - R'_g = \frac{1}{1} - 0.012 = 0.988\ \Omega$$

**30.** Let $\vec{B}_1$, $\vec{B}_2$ and $\vec{B}_3$ be the magnetic fields at point $O$ due to current carrying straight wire $AB$, semicircular wire $BCD$ and straight wire $DE$ respectively.

Then, $\vec{B}_1 = \dfrac{\mu_0 I}{4\pi R}\odot$

[Since point $O$ lies near one end of long wire at a normal distance $R$.]

$\vec{B}_2 = \dfrac{\mu_0 I}{4R}\odot$ and $\vec{B}_3 = \dfrac{\mu_0 I}{4\pi R}\odot$

$\therefore$ Net magnetic field at $O$ is given by

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \left[\frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\pi R}\right]\odot$$

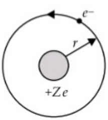
$$\Rightarrow |\vec{B}| = \frac{\mu_0 I}{4\pi R}[2+\pi]$$

and $\vec{B}$ is directed perpendicular to plane of paper directed out of it.

**31.** The electron of charge $(-e)$ performs uniform circular motion around a stationary heavy nucleus of charge $+Ze$. This constitutes a current $I$ and forms a loop which behaves like a tiny magnetic dipole.

$I = \dfrac{e}{T}$ ($T$ = time period) ...(i)

Let $r$ be the orbital radius of the electron and $v$ the orbital speed. Then,




$T = \dfrac{2\pi r}{v}$ ...(ii)

From eqn (i) and (ii),

$I = ev/2\pi r$

Magnetic moment $\mu = I\pi r^2 = evr/2$

The direction of magnetic moment is into the plane of paper.

Now, $\mu = \dfrac{e}{2m_e}(m_e vr) = \dfrac{e}{2m_e}L$

where $L$ is the magnitude of angular momentum of the electron.

Vectorially, $\vec{\mu} = \dfrac{-e}{2m_e}\vec{L}$

The negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment.

**32.** Magnetic field at the centre of the coils due to coil $P$, having current $I$ is

$B_P = \frac{\mu_0 I}{2R}$

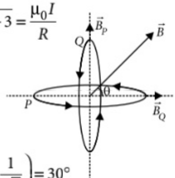
And magnetic field due to coil $Q$ having current $\sqrt{3}I$ is

$B_Q = \frac{\mu_0 \sqrt{3} I}{2R}$

Since both coils are inclined to each other at an angle of 90°, the magnitude of their resultant magnetic field at the common centre will be

$$B = \sqrt{B_P^2 + B_Q^2} = \frac{\mu_0 I}{2R}\sqrt{1+3} = \frac{\mu_0 I}{R}$$

The directions of $B_P$ and $B_Q$ are as indicated in the figure. The direction of the resultant field is at an angle θ given by




$$\theta = \tan^{-1}\left(\frac{B_P}{B_Q}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Hence, the direction of the magnetic field will be at an angle 30° to the plane of loop Q.

**OR**

Here, $m_e = 9.1 \times 10^{-31}$ kg,
$e = 1.6 \times 10^{-19}$ C, $B = 2.0$ mT $= 2.0 \times 10^{-3}$ T
and $E = 3.4 \times 10^4$ V m$^{-1}$

As electron beam goes undeflected,

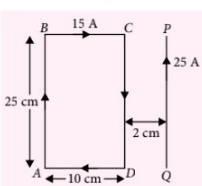
$$\therefore \quad v = \frac{E}{B} = \frac{3.4\times10^4}{2.0\times10^{-3}} = 1.7 \times 10^7 \text{ m s}^{-1}$$

If the electric field is switched off, then under the magnetic field, electron beam will describe a circular path of radius.

$$r = \frac{m_e v}{eB} = \frac{9.1\times10^{-31}\times1.7\times10^7}{1.6\times10^{-19}\times2.0\times10^{-3}}$$

$= 4.8 \times 10^{-2}$ m $= 4.8$ cm

33.




Force between wires $PQ$ and $CD$

$$F_1 = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

$$= \frac{2\times10^{-7}\times15\times25\times0.25}{2\times10^{-2}} = 93.75 \times 10^{-5}\text{N}$$

$= 9.375 \times 10^{-4}$ N (Repulsive)

Force between wires $PQ$ and $AB$

$$F_2 = \frac{2\times10^{-7}\times15\times25\times0.25}{0.12}$$

$= 1.56 \times 10^{-4}$ N (Attractive)

Net force on the rectangular loop

$F = F_1 - F_2 = (9.375 - 1.56) \times 10^{-4}$
$= 7.815 \times 10^{-4}$ N (Repulsive *i.e.*, towards left).

34. According to Biot-Savart's law, the magnetic field due to a small circular current element of length $dl$ at centre $O$ is

$$dB = \frac{\mu_0 I\, dl \sin 90°}{4\pi r^2}$$

while the magnetic field due to straight portions will be zero (since sin θ = 0).

∴ Total magnetic field at centre $O$ is

$$B = \frac{\mu_0 I}{4\pi r^2}\int dl = \frac{\mu_0 I}{4\pi r^2} r\theta \qquad \left(\because \int \Delta l = r\theta\right)$$

$$= \frac{\mu_0 I\theta}{4\pi r}$$

35. (a) *Refer to answer 81, Page no. 110* (MTG CBSE Champion Physics Class 12)
(b) *Refer to answer 19, Page no. 98* (MTG CBSE Champion Physics Class 12)

**OR**

*Refer to answer 52, Page no. 104* (MTG CBSE Champion Physics Class 12)

36. *Refer to answer 40, Page no. 101 and 102* (MTG CBSE Champion Physics Class 12)

**OR**

(a) *Refer to answer 94, Page no. 112* (MTG CBSE Champion Physics Class 12)
(b) *Refer to answer 87(b), Page no. 111* (MTG CBSE Champion Physics Class 12)

37. (a) *Refer to answer 59, Page no. 105* (MTG CBSE Champion Physics Class 12)
(b) *Refer to answer 58, Page no. 105* (MTG CBSE Champion Physics Class 12)

**OR**

*Refer to answer 64, Page no. 106* (MTG CBSE Champion Physics Class 12)

# MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

**Total Marks : 120**

## Magnetic Effect of Current and Magnetism

**Time Taken : 60 Min.**

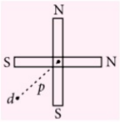
### NEET

#### Only One Option Correct Type

**1.** Consider a long, straight wire of cross-sectional area $A$ carrying a current $I$. Let there be $n$ free electrons per unit volume. An observer places himself on a trolley moving in the direction opposite to the current with a speed $v = \frac{I}{\pi A e}$ and separated from the wire by a distance $r$. The magnetic field seen by the observer is very nearly

(a) $\frac{\mu_0 I}{2\pi r}$ (b) zero (c) $\frac{\mu_0 I}{\pi r}$ (d) $\frac{2\mu_0 I}{\pi r}$

**2.** Two short magnets of equal dipole moments $M$ are fastened perpendicularly at their centres as shown in the figure. The magnitude of the magnetic field at a distance $d$ from the centre on the bisector of the right angle is




(a) $\frac{\mu_0}{4\pi}\frac{M}{d^3}$ (b) $\frac{\mu_0}{4\pi}\frac{\sqrt{2}M}{d^3}$

(c) $\frac{\mu_0}{4\pi}\frac{2\sqrt{2}M}{d^3}$ (d) $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$

**3.** An electron accelerated through a potential difference $V$ passes through a uniform transverse magnetic field and experiences a force $F$. If the accelerating potential is increased to $2V$, the electron in the same magnetic field will experience a force

(a) $F$ (b) $\frac{F}{2}$ (c) $\sqrt{2}F$ (d) $2F$

**4.** An electric current $I$ enters and leaves a uniform circular wire of radius $a$ through diametrically opposite points. A charged particle $q$ moving along the axis of the circular wire passes through its centre at speed $v$. The magnetic force acting on the particle when it passes through the centre has a magnitude

(a) $qv\frac{\mu_0 I}{2a}$ (b) $qv\frac{\mu_0 I}{2\pi a}$

(c) $qv\frac{\mu_0 I}{a}$ (d) zero

**5.** A beam consisting of protons and electrons moving at the same speed goes through a thin region in which there is a magnetic field perpendicular to the beam. The protons and the electrons

(a) will go undeviated

(b) will be deviated by the same angle and will not separate

(c) will be deviated by different angles and hence separate

(d) will be deviated by the same angle but will separate.

**6.** A charged particle of mass $10^{-3}$ kg and charge $10^{-5}$ C enters a magnetic field of induction 1 T. If $g = 10$ m s$^{-2}$, for what value of velocity will it pass straight through the field without deflection?

(a) $10^{-3}$ m s$^{-1}$ (b) $10^{3}$ m s$^{-1}$

(c) $10^{6}$ m s$^{-1}$ (d) 1 m s$^{-1}$

**7.** Mark out the correct options.

(a) Diamagnetism does not occur in all materials.

(b) Diamagnetism results from the partial alignment of permanent magnetic moment.

(c) The magnetising field intensity is always zero in free space.

(d) The magnetic field of induced magnetic moment is opposite to the applied field.

**8.** A particle is moving with velocity $\vec{v} = \hat{i} + 3\hat{j}$ and it produces an electric field at a point given by $\vec{E} = 2\hat{k}$. It will produce magnetic field at that point equal to (all quantities are in SI units)

(a) $(6\hat{i} - 2\hat{j})\mu_0 \varepsilon_0$

(b) $(6\hat{i} + 2\hat{j})\mu_0 \varepsilon_0$

(c) zero

(d) cannot be determined from the given data.

**9.** A point charge is moving in clockwise direction in a circle with constant speed. Consider the magnetic field produced by the charge at a fixed point *P* (not at the center of circle) on the axis of the circle. Then,

(a) it is constant in magnitude only

(b) it is constant in direction only

(c) it is constant both in direction and magnitude

(d) it is constant neither in magnitude nor in direction.

**10.** A dip circle is so that its needle moves freely in the magnetic meridian. In this position, the angle of dip is 40°. Now the dip circle is rotated so that the plane in which the needle moves makes an angle of 30° with the magnetic meridian. In this position, the needle will dip by an angle

(a) 40° (b) 30°

(c) more than 40° (d) less than 40°

**11.** What should be the current in a circular coil of radius 5 cm to annul $B_H = 5 \times 10^{-5}$ T?

(a) 0.4 A (b) 4 A

(c) 40 A (d) 1 A

**12.** A particle of mass $2 \times 10^{-5}$ kg moves horizontally between two horizontal plates of a charged parallel plate capacitor between which there is an electric field of 200 N C$^{-1}$ acing upward. A magnetic induction of 2.0 T is applied at right angles to the electric field in a direction normal to both $\vec{B}$ and $\vec{v}$. If $g$ is 9.8 m s$^{-2}$ and the charge on the particle is $10^{-6}$ C, then find the velocity of charge particle so that it continues to move horizontally.

(a) 2 m s$^{-1}$ (b) 20 m s$^{-1}$

(c) 0.2 m s$^{-1}$ (d) 100 m s$^{-1}$

## Assertion & Reason Type

**Directions :** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false.

(d) If both assertion and reason are false.

**13. Assertion :** The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.

**Reason :** Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.

**14. Assertion :** The true geographic north direction is found by using a compass needle.

**Reason :** The magnetic meridian of the earth is along the axis of rotation of the earth.

**15. Assertion :** The net force on a closed circular current carrying loop placed in a uniform magnetic field is zero.

**Reason :** The torque produced in a conducting circular ring is zero when it is placed in a uniform magnetic field such that the magnetic field is perpendicular to the plane of loop.

## JEE MAIN / JEE ADVANCED

## Only One Option Correct Type

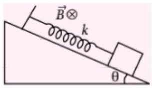
**16.** A charged particle of specific charge (charge/mass) $\alpha$ is released from origin at time $t = 0$ with velocity $\vec{v} = v_0(\hat{i} + \hat{j})$ in a uniform magnetic field $\vec{B} = B_0\hat{i}$. Coordinates of the particle at time $t = \frac{\pi}{(B_0\alpha)}$ are

(a) $\left(0, \frac{\sqrt{2}v_0}{\alpha B_0}, \frac{-v_0}{B_0\alpha}\right)$ (b) $\left(\frac{-v_0}{2B_0\alpha}, 0, 0\right)$

(c) $\left(0, \frac{2v_0}{B_0\alpha}, \frac{v_0\pi}{2B_0\alpha}\right)$ (d) $\left(\frac{v_0\pi}{B_0\alpha}, 0, \frac{-2v_0}{B_0\alpha}\right)$

**17.** A small block of mass *m*, having charge *q*, is placed on a frictionless inclined plane making an angle θ with the horizontal as shown in figure. There exists a uniform magnetic field *B* parallel to the inclined plane but perpendicular to the length of a spring.

If $m$ is slightly pulled on the incline in downward direction, the time period of oscillation will be (assume that the block does not leave contact with the plane)

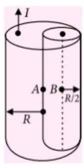



(a) $2\pi\sqrt{m/k}$ (b) $2\pi\sqrt{2m/k}$
(c) $2\pi\sqrt{qB/K}$ (d) $2\pi\sqrt{qB/2K}$

**18.** A thin, plastic disk of radius $R$ has a charge $q$ uniformly distributed over its surface. If the disk rotates at an angular frequency $\omega$ about its axis, then magnetic dipole moment of the disk is

(a) $\frac{\omega qR^2}{2}$ (b) $\frac{\omega qR^2}{4}$ (c) $\omega qR^2$ (d) $2\omega qR^2$

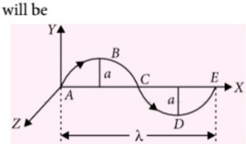
**19.** From a cylinder of radius $R$, a cylinder of radius $R/2$ is removed, as shown in the figure. Current flowing in the remaining cylinder is $I$. Then, magnetic field strength is




(a) zero at point $A$
(b) zero at point $B$
(c) $\frac{\mu_0 I}{2\pi R}$ at point $A$ (d) $\frac{\mu_0 I}{3\pi R}$ at point $B$

## More than One Options Correct Type

**20.** A conductor $ABCDEF$, shaded as shown, carries a current $I$. It is placed in the $xy$ plane with the ends $A$ and $E$ on the $x$-axis. A uniform magnetic field of magnitude $B$ exists in the region. The force acting on it will be

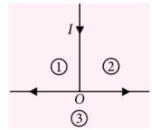



(a) zero, if $B$ is in the $x$-direction
(b) $\lambda BI$ in the $z$-direction, if $B$ is in the $y$-direction
(c) $\lambda BI$ in the negative $y$-direction, if $B$ is in the $z$-direction
(d) $2aBI$, if $B$ is in the $x$-direction.

**21.** A microammeter has a resistance of 100 Ω and a full scale range of 50 μA. It can be used as a voltmeter or as a higher range ammeter provided a resistance is added to it. Pick the correct range and resistance combination(s).

(a) 50 V range with 10 kΩ resistance in series
(b) 10 V range with 200 kΩ resistance in series
(c) 5 mA range with 1 Ω resistance in parallel
(d) 10 mA range with 1 Ω resistance in parallel

**22.** A direct current $I$ flows along a long straight wire as shown in the figure. From point $O$ the current spreads radially all over on infinite conducting plane perpendicular to the wire. Then




(a) magnetic field in region ① is non-uniform
(b) magnetic field in region ② in non-uniform
(c) magnetic field in region ③ is non-uniform
(d) magnetic field in region ③ is zero.

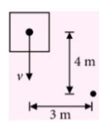
**23.** A current $I$ flows in a long round uniform cylindrical wire made of paramagnetic material with susceptibility $\chi$. Which of following statements are correct regarding the surface molecular current ($I_s$) and the volume molecular current ($I_v$) ?

(a) Both the currents $I_s$ and $I_v$ have same magnitude.
(b) Both the currents $I_s$ and $I_v$ have different magnitude.
(c) Both the currents $I_s$ and $I_v$ flow in the same direction.
(d) Both the currents $I_s$ and $I_v$ flow in the opposite directions.

## Numerical Value Type

**24.** A current $I = 10$ A flows in a ring of radius $r_0 = 15$ cm made of a very thin wire. The tensile strength of the wire is equal to $T = 1.5$ N. The ring is placed in a magnetic field, which is perpendicular to the plane of the ring so that the forces tend to break the ring. Find $B$ (in T) at which the ring is broken.

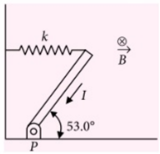
**25.** An elevator carrying a charge of 0.5 C is moving down with a velocity of $5 \times 10^3$ m s$^{-1}$. The elevator is 4 m from the bottom and 3 m horizontally from $P$ as shown in figure. What magnetic field (in μT) does it produce at point $P$?




**26.** An iron of volume $10^{-4}$ m$^3$ and relative permeability 1000 is placed inside a long solenoid wound with 5 turns per cm. If a current of 0.1 A is passed through the solenoid, find the magnetic moment of the rod.

## Comprehension Type

A thin, uniform rod with negligible mass and length 0.200 m is attached to the floor by a frictionless hinge at point $P$ as shown in the figure. A horizontal spring with force constant $k = 4.80\ \text{N m}^{-1}$ connects the other end of the rod to a vertical wall.



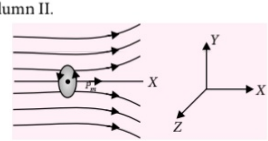

The rod is in a uniform magnetic field $B = 0.340$ T directed into the plane of the proper. There is current $I = 6.50$ A in the rod, in the direction shown.

**27.** Calculate the torque due to the magnetic force on the rod, for an axis at $P$.
(a) $0.0442\ \text{N m}^{-1}$, clockwise
(b) $0.0442\ \text{N m}^{-1}$, anticlockwise
(c) $0.022\ \text{N m}^{-1}$, clockwise
(d) $0.022\ \text{N m}^{-1}$, anticlockwise

**28.** When the rod is in equilibrium and makes an angle of 53.0° with the floor, is the spring stretched or compressed?
(a) 0.05765 m, stretched
(b) 0.05765 m, compressed
(c) 0.0242 m, stretched
(d) 0.0242 m, compressed

## Matrix Match Type

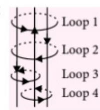
**29.** An elementary current loop is placed in a non-uniform magnetic field as shown in the figure, where, $P_m$ is magnetic moment of loop. In column I different orientations of loop are described and in column II, the corresponding forces experienced by the loop. Match the entries of column I with entries of column II.




| Column I | Column II |
|---|---|
| (A) In the given situation, | (P) Resultant force is acting along $\vec{P}_m$ |
| (B) If loop is rotated such that $\vec{P}_m$ is along positive $z$-direction | (Q) Resultant force is acting opposite to $\vec{P}_m$ |
| (C) If loop is rotated such that $\vec{P}_m$ is along negative $z$-direction | (R) $F_x = 0, F_y = 0$ |
| (D) If loop is rotated such that $\vec{P}_m$ is along positive $y$-direction | (S) $F_x = 0, F_z = 0$ |

| | A | B | C | D |
|---|---|---|---|---|
| (a) | P | P, R | S | Q |
| (b) | S | P, R | P, R | Q, S |
| (c) | Q | P, R | P, R | P, S |
| (d) | R | R, S | Q, R | P, Q |

**30.** Three wires are carrying same constant current $I$ in different directions. Four loops enclosing the wires in different manners as shown in the figure. The direction of $d\vec{l}$ is shown in each loop. Match the entries of column I with entries of column II.




| Column I | Column II |
|---|---|
| (A) Along closed loop 1 | (P) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$ |
| (B) Along closed loop 2 | (Q) $\oint \vec{B}\cdot d\vec{l} = -\mu_0 I$ |
| (C) Along closed loop 3 | (R) $\oint \vec{B}\cdot dl = 0$ |
| (D Along closed loop 4 | (S) Net work done by the magnetic force to move a unit charge along the loop is zero |

| | A | B | C | D |
|---|---|---|---|---|
| (a) | P, Q | Q, R | Q, S | R, S |
| (b) | Q, S | P, S | R, S | R, S |
| (c) | P, Q | P, R | Q, S | P, S |
| (d) | P, S | Q, S | R, S | R, S |

◆◆

*Keys are published in this issue. Search now!* ☺

**SELF CHECK**

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

**Check your score! If your score is**

| | | |
|---|---|---|
| > 90% | EXCELLENT WORK ! | You are well prepared to take the challenge of final exam. |
| 90-75% | GOOD WORK ! | You can score good in the final exam. |
| 74-60% | SATISFACTORY ! | You need to score more next time |
| < 60% | NOT SATISFACTORY! | Revise thoroughly and strengthen your concepts. |

CLASS-XI

BRUSH UP

for

# NEET/JEE 2021

Brush up your concepts to get high rank in NEET/JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

Unit 4

## System of Particles and Rotational Motion

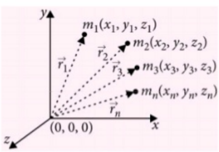
### CENTRE OF MASS OF A RIGID BODY

- Ideally a rigid body is a body with a perfectly definite and unchanged shape. The distances between all pairs of particles of such a body do not change. *e.g.*, fan, pen, stone etc.
- For a system of particles, centre of mass is an imaginary point at which its total mass is supposed to be concentrated.
- If co-ordinates of particles of masses $m_1, m_2, ......$ are $(x_1, y_1, z_1), (x_2, y_2, z_2), ......$ then position vector of their center of mass is $\vec{R}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{j} + z_{CM}\hat{k}$

$$= \frac{m_1(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})+m_2(x_2\hat{i}+y_2\hat{j}+z_2\hat{k}) + m_3(x_3\hat{i}+y_3\hat{j}+z_3\hat{k})+......}{m_1+m_2+m_3+......}$$

$$= \frac{(m_1x_1+m_2x_2+......)\hat{i}+(m_1y_1+m_2y_2.....)\hat{j} + (m_1z_1+m_2z_2+.....)\hat{k}}{m_1+m_2+m_3+......}$$

- $x_{CM} = \left(\frac{m_1x_1+m_2x_2+............}{m_1+m_2+..........}\right) = \frac{1}{M}\sum m_i x_i$
- $y_{CM} = \left(\frac{m_1y_1+m_2y_2+............}{m_1+m_2+....................}\right) = \frac{1}{M}\sum m_i y_i$
- $z_{CM} = \left(\frac{m_1z_1+m_2z_2+............}{m_1+m_2+....................}\right) = \frac{1}{M}\sum m_i z_i$




- If the system has continuous distribution of mass, treating the mass element $dm$ at position $\vec{r}$ as a point mass and replacing summation by integration.

$$\vec{R}_{CM} = \frac{1}{M}\int \vec{r}\, dm$$

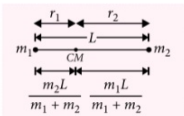
and $x_{CM} = \frac{1}{M}\int x dm,\ y_{CM} = \frac{1}{M}\int y dm,\ z_{CM} = \frac{1}{M}\int z dm$

- It may be inside or outside of the body.
- Its position depends on the shape of the body.
- For a given shape it depends on the distribution of mass within the body and is closer to massive part.
- For symmetrical bodies having homogeneous distribution of mass it coincides with centre of symmetry of geometrical centre.
- If we know the centre of mass of parts of the system and their masses, we can get the combined centre of mass by treating the parts as point particles placed at their respective centre of masses.

- It is independent of the co-ordinate system, *e.g.*, the centre of mass of a ring is at its centre whatever be the co-ordinate system.
- If the origin of co-ordinate system is at centre of mass, *i.e.*, $\vec{R}_{CM} = 0$

  Then by definition $\frac{1}{M}\sum m_i \vec{r}_i = 0 \Rightarrow \sum m_i \vec{r}_i = 0$

  - The sum of the moments of the masses of a system about its centre of mass is always zero.
  - For a system of two point masses $m_1 r_1 = m_2 r_2$

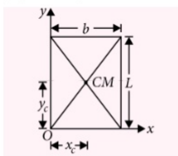



  The centre of mass lies closer to the heavier mass.

- Centre of mass of symmetric body
  - Rectangular plate (By symmetry)

    $x_c = \frac{b}{2}$,

    $y_c = \frac{L}{2}$

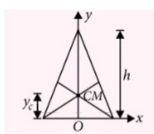



  - Triangular plate (By qualitative argument)

    At the centroid,

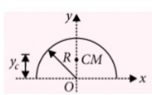
    $y_c = \frac{h}{3}$

    $x_c = 0$



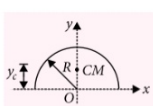

  - Semi-circular ring

    $y_c = \frac{2R}{\pi}, \quad x_c = 0$



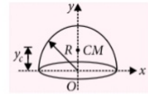

  - Semi-circular disc

    $y_c = \frac{4R}{3\pi}, \quad x_c = 0$



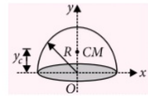

  - Hemispherical shell

    $y_c = \frac{R}{2}, \; x_c = 0$



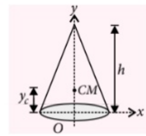

  - Solid hemisphere

    $y_c = \frac{3R}{8}, \; x_c = 0$



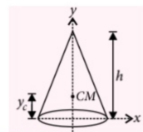

  - Circular cone (solid)

    $y_c = \frac{h}{4}, \; x_c = 0$




  - Circular cone (hollow)

    $y_c = \frac{h}{3}, \; x_c = 0$




## ☞ MOTION OF CENTRE OF MASS

- For a system of particles, position of centre of mass is $\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + .......}{m_1 + m_2 + m_3 + ......}$
- Velocity of *CM*

  $$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ......}{m_1 + m_2 + ......} \quad \left(\because \; \frac{d\vec{r}}{dt} = \vec{v}\right)$$

- Acceleration of *CM*

  $$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ....}{m_1 + m_2 + ....} \quad \left(\because \; \vec{a} = \frac{d\vec{v}}{dt}\right)$$

- Linear momentum of a system of particles is equal to the product of mass of the system with the velocity of its centre of mass.
- If no external force acts on a system the velocity of its centre of mass remains constant, *i.e.*, velocity of centre of mass is unaffected by internal forces.

  From Newton's second law $\vec{F}_{ext} = \frac{d(M\vec{v}_{CM})}{dt}$

  If $\vec{F}_{ext} = 0$ then $\vec{v}_{CM} = \text{constant}$.

## ☞ PROBLEM SOLVING TRICKS

(For centre of mass problems)

- Make full use of the symmetry of the object, be it point, line, or plane.

- If the object can be divided into several parts, treat each of these parts as a particle, located at its own centre of mass.
- Choose the axis wisely : If given system is a group of particles, choose one of the particles as origin. If system is a body with a line of symmetry, consider it as $x$-axis. The choice of origin is completely arbitrary ; the location of the centre of mass is same regardless of the origin from which it is measured.

## PURE ROTATIONAL MOTION

- A body is said to be in pure rotational motion if the perpendicular distance of each particle remains constant from a fixed line or point and do not move parallel to the line, and that line is known as axis of rotation.
- Angular displacement $\theta = \frac{s}{r}$

  Where $s$ = length of arc traced by the particle.

  $r$ = distance of particle from the axis of rotation.
- Angular velocity $\omega = \frac{d\theta}{dt}$
- Angular acceleration $\alpha = \frac{d\omega}{dt}$
- All the parameters $\theta$, $\omega$ and $\alpha$ are same for all the particles. Axis of rotation is perpendicular to the plane of rotation of particles.
- If $\alpha$ = constant, then

  $\omega = \omega_0 + \alpha t$ where $\omega_0$ = initial angular speed

  $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $t$ = time interval

  $\omega^2 = \omega_0^2 + 2\alpha\theta$

  These equations are known as equations of rotational motion.
- Total kinetic energy $= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + ..........$

  $= \frac{1}{2}(m_1 r_1^2 + m_2 r_2^2 + ........)\omega^2$

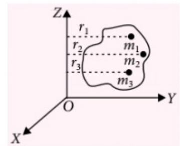
  $= \frac{1}{2} I\omega^2$; where $I$ = Moment of inertia $= m_1 r_1^2 + m_2 r_2^2 + ....$

  $\omega$ = angular speed of body.

## MOMENT OF INERTIA

- The property of a body by virtue of which it opposes any change in its state of rotation about an axis is known as moment of inertia.
- The moment of inertia of a rigid body about a given axis is the sum of the product of the masses of its constituent particles and the square of their respective distances from the axis of rotation.

  $$I = \sum_{i=1}^{n} m_i r_i^2$$
- Moment of inertia of a body about an axis not only depend upon the mass of the body but also upon the distribution of its mass about the axis of rotation.
- Greater is the part of the mass of the body away from the axis of rotation, greater is the moment of inertia of the body about that axis.
- The radius of gyration $K$ of a body about an axis of rotation is defined as the root mean square distances of the particles from the axis of rotation and its square when multiplied with the mass of the body gives moment of inertia of the body about the axis.




  $$K = \sqrt{\frac{r_1^2 + r_2^2 + ..... + r_n^2}{n}}$$

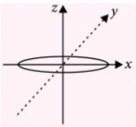
  = root mean square distance of a particles from axis $OZ$.
- **Two Important Theorems on Moment of Inertia :**
  - Perpendicular axes theorem (Only applicable to plane lamina (that means for 2-$D$ objects only)):

    $I_z = I_x + I_y$ (Object is in $x$-$y$ plane)

    Where $I_z$ = moment of inertia of the body about $z$-axis.

    $I_x$ = moment of inertia of the body about $x$-axis.

    $I_y$ = moment of inertia of the body about $y$-axis.



Body is in $x$-$y$ plane

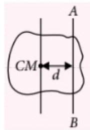
    $I_y = I_x + I_z$ (Object is in $x$-$z$ plane)

    $I_x = I_y + I_z$ (Object is in $y$-$z$ plane)
  - Parallel axes theorem (Applicable to any type of object) :

$I_{AB} = I_{cm} + Md^2$

Where

$I_{cm}$ = Moment of inertia of the object about an axis passing through centre of mass and parallel to axis $AB$




$I_{AB}$ = Moment of inertia of the object about axis $AB$

$M$ = Total mass of object

$d$ = Perpendicular distance between axis about which moment of inertia is to be calculated and the one passing through the centre of mass.

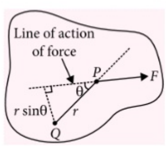
- **Moment of inertia and radius of gyration of some regular bodies about specific axis**

| Body | Axis of rotation | Moment of inertia ($I$) | Radius of gyration ($K$) |
|---|---|---|---|
| Uniform circular ring of mass $M$ and radius $R$ | about an axis passing through centre and perpendicular to its plane | $MR^2$ | $R$ |
| | about a diameter | $\frac{1}{2}MR^2$ | $\frac{R}{\sqrt{2}}$ |
| | about a tangent in its own plane | $\frac{3}{2}MR^2$ | $\sqrt{\frac{3}{2}}R$ |
| | about a tangent perpendicular to its plane | $2MR^2$ | $R\sqrt{2}$ |
| Uniform circular disc of mass $M$ and radius $R$ | about an axis passing through centre and perpendicular to its plane | $\frac{1}{2}MR^2$ | $\frac{R}{\sqrt{2}}$ |
| | about a diameter | $\frac{1}{4}MR^2$ | $R/2$ |
| | about a tangent in its own plane | $\frac{5}{4}MR^2$ | $\sqrt{5}\frac{R}{2}$ |
| | about a tangent perpendicular to its own plane | $\frac{3}{2}MR^2$ | $\sqrt{\frac{3}{2}}R$ |
| Solid sphere of radius $R$ and mass $M$ | about its diameter | $\frac{2}{5}MR^2$ | $\sqrt{\frac{2}{5}}R$ |
| | about a tangential axis | $\frac{7}{5}MR^2$ | $\sqrt{\frac{7}{5}}R$ |
| Hollow sphere of radius $R$ and mass $M$ | about its diameter | $\frac{2}{3}MR^2$ | $\sqrt{\frac{2}{3}}R$ |
| | about a tangential axis | $\frac{5}{3}MR^2$ | $\sqrt{\frac{5}{3}}R$ |
| Solid cylinder of length $l$, radius $R$ and mass $M$ | about its own axis | $\frac{1}{2}MR^2$ | $\frac{R}{\sqrt{2}}$ |
| | about an axis passing through centre of mass and perpendicular to its own axis | $M\left[\frac{l^2}{12}+\frac{R^2}{4}\right]$ | $\sqrt{\frac{l^2}{12}+\frac{R^2}{4}}$ |
| | about the diameter of one of the faces of cylinder | $M\left[\frac{l^2}{3}+\frac{R^2}{4}\right]$ | $\sqrt{\frac{l^2}{3}+\frac{R^2}{4}}$ |

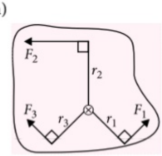
| | | | |
|---|---|---|---|
| Hollow cylinder of mass $M$ and radius $R$ | about its own axis | $MR^2$ | $R$ |
| | about an axis passing through its centre of mass and perpendicular to its own axis | $M\left(\frac{R^2}{2}+\frac{l^2}{12}\right)$ | $\sqrt{\frac{R^2}{2}+\frac{l^2}{12}}$ |
| Thin rod of length $L$ | about an axis through centre of mass and perpendicular to the rod | $\frac{ML^2}{12}$ | $\frac{L}{\sqrt{12}}$ |
| | about an axis through one end and perpendicular to rod | $\frac{ML^2}{3}$ | $\frac{L}{\sqrt{3}}$ |
| Rectangular lamina of length $l$ and breadth $b$ | about an axis passing through its centre of mass and perpendicular to plane | $M\left[\frac{l^2+b^2}{12}\right]$ | $\sqrt{\frac{l^2+b^2}{12}}$ |
| Uniform cone of radius $R$ and height $h$ | about an axis passing through its centre of mass and joining its vertex to centre of base | $\frac{3}{10}MR^2$ | $R\sqrt{\frac{3}{10}}$ |
| Parallelopiped of length $l$, breadth $b$, height $h$ and mass $M$ | about its central axis | $M\left(\frac{l^2+b^2}{12}\right)$ | $\sqrt{\frac{l^2+b^2}{12}}$ |

## ☞ TORQUE

- It represents the capability of a force to produce change in the rotational motion of the body.
- Torque of force $\vec{F}$ about a point $\vec{\tau}=\vec{r}\times\vec{F}$,
  where $\vec{F}$ = force applied
  $P$ = point of application of force
  $Q$ = point about which we want to calculate the torque.




  $\vec{r}$ = position vector of the point of application of force from the point about which we want to determine the torque.
  - $|\vec{\tau}| = rF\sin\theta = r_{\perp}F = rF_{\perp}$
    Where θ = angle between the direction of force and the position vector of $P$ with respect to $Q$.
    $r_{\perp}$ = perpendicular distance of line of action of force from point $Q$.
    $F_{\perp}$ = force arm
- Torque acting on the body about the axis of rotation, $\vec{\tau}=\vec{r}\times\vec{F}$
  $\vec{r}$ = position vector of the point of application of force about the axis of rotation
  $\vec{F}$ = force applied on the body
  - If forces $F_1$ and $F_2$ are applied on the body to rotate it in anti-clockwise direction and $F_3$ makes body to rotate in clockwise direction. Then,
    $\tau_{resultant} = F_1r_1 + F_2r_2 - F_3r_3$ (in anti-clockwise direction)




  - $\vec{\tau}_{net} = \vec{\tau}_1+\vec{\tau}_2+\vec{\tau}_3+\ldots\ldots$
  - Torque produced by a force about an axis can be zero if force vector
    (i) is parallel to the axis of rotation.
    (ii) passes through the axis of rotation.
- If net external torque acting on the body is zero, then the body is said to be in rotational equilibrium. *i.e.*, $\sum\vec{\tau}_{ext}=0$.
- Relation between $\vec{\tau}$ and $\vec{\alpha}$ (angular acceleration):
  $\vec{\tau} = I\,\vec{\alpha}$

- Torque is also change in angular momentum

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

- Work done by a torque and power of a torque : If a torque $\tau$ applied on a body rotates it through an angle $\Delta\theta$, the work done by the torque is

$$\Delta W = \tau\Delta\theta$$

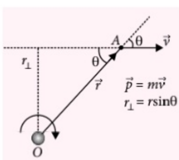
or work done = torque × angular displacement
Power of a torque is given as

$$P = \frac{\Delta W}{\Delta t} = \frac{\tau\Delta\theta}{\Delta t} = \tau\omega$$

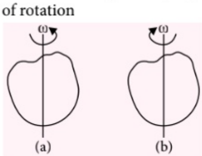
*i.e.*, Power of a torque = torque × angular velocity

## ANGULAR MOMENTUM

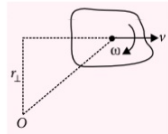
- Angular momentum of a particle in motion about a fixed point ($O$)
  - Suppose a particle $A$ has a linear momentum $\vec{p} = m\vec{v}$ as shown in the figure.




  - Angular momentum of particle $A$ about point $O$ will be, $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times (m\vec{v}) = m(\vec{r} \times \vec{v})$
  - Magnitude of $\vec{L}$ is $L = mvr\sin\theta = mvr_\perp$, where $\theta$ is the angle between $\vec{r}$ and $\vec{p}$.
  - Direction of $\vec{L}$ will be given by right hand screw rule. As shown in figure, direction of $\vec{L}$ is perpendicular to paper inwards.
  - If the particle passes through point $O$, $r_\perp = 0$. Therefore, angular momentum is zero.
- Angular momentum of a rigid body in pure rotation about axis of rotation



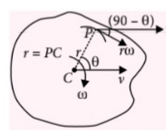

  - If a rigid body is in pure rotation about a fixed axis, then angular momentum of rigid body about this axis will be given by $L = I\omega$
- Angular momentum of a rigid body in rotation plus translation about a general axis




  - There will be two terms
    (a) $I_c\omega$  (b) $mv_c r_\perp = mvr_\perp$
  - From right hand screw rule, we can see that $I_c\omega$ and $mvr_\perp$ both terms are perpendicular to the paper in inward direction. Hence, they are added.
    or $L_{Total} = I_c\omega + mvr_\perp$
  - In a problem, if these two terms are in opposite directions then they will be subtracted.
  - Law of conservation of angular momentum : If no external torque acts on a system, total angular momentum of the system remains unchanged. In the absence of any external torque,
    $L = I\omega$ = constant
    or $I_1\omega_1 = I_2\omega_2$ or $I_1 \cdot \frac{2\pi}{T_1} = I_2 \cdot \frac{2\pi}{T_2}$

## ROTATIONAL PLUS TRANSLATIONAL MOTION OF A RIGID BODY

- A complex motion of rotation plus translation can be simplified by considering
  - The translational motion of centre of mass of the rigid body.
  - Rotational motion about the centre of mass.
- Now, velocity of point $P$ is the vector sum of two terms $v$ and $r\omega$. Here $v$ is common for all points, while $r\omega$ is different for different points, as $r$ is different.



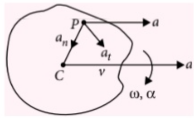

$$v_p = \sqrt{v^2 + (r\omega)^2 + 2(v)(r\omega)\cos(90° - \theta)}$$

$$= \sqrt{v^2 + r^2\omega^2 + 2vr\omega\sin\theta}$$

- Acceleration of point $P$ is the vector sum of three terms.
  - $a$ (acceleration of $CM$)
  - $a_n = r\omega^2$ (acting towards centre $O$)

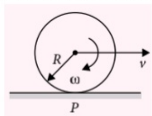
- $a_t = r\alpha = r\dfrac{d\omega}{dt}$ (acting tangentially)

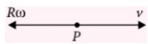
  Again, here $a$ is common for all points, while $a_n$ and $a_t$ are different.



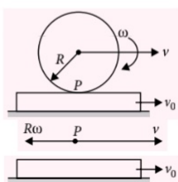

- Uniform pure rolling : In which $v$ and $\omega$ remain constant.

  Condition of pure rolling is $v = R\omega$. In this case bottommost point of the spherical body is at rest. It has no slipping with its contact point on ground. Because ground point is also at rest.



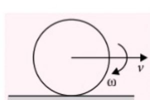

  - If $v > R\omega$, then net velocity of point $P$ is in the direction of $v$. This is called forward slipping.
  - If $v < R\omega$, then net velocity of point $P$ is in opposite direction of $v$. This is called backward slipping.




  - If a spherical body is rolling over a plank, condition for no slipping between spherical body and plank is, $v - R\omega = v_0$

- Accelerated pure rolling : If $v$ and $\omega$ are not constant then, $a = R\alpha$ is an additional condition for pure rolling on horizontal ground, which takes place in the presence of some external forces.

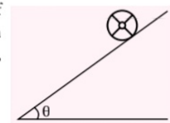
  Here friction plays very important role. Magnitude and direction of friction is so adjusted that equation $a = R\alpha$ is satisfied. If friction is insufficient for satisfying the equation $a = R\alpha$, slipping (either forward or backward) will occur and kinetic friction will act.




## MOTION OF A SPHERICAL BODY ON ROUGH INCLINED SURFACE

- Minimum value of coefficient of friction required for pure rolling,

$$\mu_{min} = \frac{\tan\theta}{1+\dfrac{mR^2}{I}}$$




- If $\mu = 0$, body will slip downwards (only translational motion) with an acceleration,

  $a_1 = g\sin\theta$

- If $\mu > \mu_{min}$, body will roll down without slipping with an acceleration, (rotation + translation both)

$$a_2 = \frac{g\sin\theta}{1+\dfrac{I}{mR^2}}$$

  - When $\mu > \mu_{min}$ force of friction will act upwards. Magnitude of this force is,

$$f = \frac{mg\sin\theta}{1+\dfrac{mR^2}{I}}$$

  - If $\mu < \mu_{min}$, body will roll downwards with forward slipping. Maximum friction will act in this case. The acceleration of body is

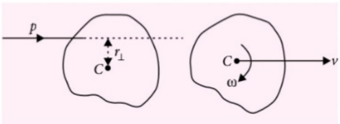
    $a_3 = g\sin\theta - \mu g\cos\theta$

- Comparison of pure rolling and pure sliding on a inclined plane

| Physical quantities | Pure rolling | Pure sliding |
|---|---|---|
| Acceleration | $\dfrac{g\sin\theta}{1+\dfrac{K^2}{r^2}}$ | $g\sin\theta$ |
| Velocity | $\dfrac{2gh}{1+\dfrac{K^2}{r^2}}$ | $\sqrt{2gh}$ |
| Time taken to cover the distance $s$, where $K$ is the radius of gyration. | $\dfrac{2s\left(1+\dfrac{K^2}{r^2}\right)}{g\sin\theta}$ | $\sqrt{\dfrac{2s}{g\sin\theta}}$ |

## ☞ ANGULAR IMPULSE

- Linear impulse when multiplied by perpendicular distance gives angular impulse. Angular impulse is also equal to change in angular momentum.




- A rigid body is kept over a smooth table. It is hit at a point by a linear impulse $p$ at a perpendicular distance $r_\perp$ from $C$ as shown. Since it is hit at some perpendicular distance from $C$ its motion is rotational plus translational. Velocity of centre of mass will be given by, $v = \dfrac{p}{m}$

  Angular velocity of rigid body is,

  $$\omega = \frac{p \times r_\perp}{I}$$

  ➤ $(p \times r_\perp)$ = angular impulse = change in angular momentum = $I\omega$.

## ☞ ANALOGY BETWEEN TRANSLATIONAL MOTION AND ROTATIONAL MOTION

| Translational motion | Rotational motion about a fixed axis |
|---|---|
| Displacement $x$ | Angular displacement $\theta$ |
| Velocity $v = dx/dt$ | Angular velocity $\omega = d\theta/dt$ |
| Acceleration $a = dv/dt$ | Angular acceleration $\alpha = d\omega/dt$ |
| Mass $M$ | Moment of inertia $I$ |
| Force $F = Ma$ | Torque $\tau = I\alpha$ |
| Work $dW = Fds$ | Work $dW = \tau d\theta$ |
| Kinetic energy of a translational motion $K_T = Mv^2/2$ | Kinetic energy of a rotational motion $K_R = I\omega^2/2$ |
| Power $P = Fv$ | Power $P = \tau\omega$ |
| Linear momentum $p = Mv$ | Angular momentum $L = I\omega$ |
| Equations of translational motion<br>(i) $v = u + at$<br>(ii) $s = ut + \frac{1}{2}at^2$<br>(iii) $v^2 - u^2 = 2as$<br>(iv) $s_{nth} = u + \frac{a}{2}(2n-1)$ | Equations of rotational motion<br>$\omega = \omega_0 + \alpha t$<br>$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$<br>$\omega^2 - \omega_0^2 = 2\alpha\theta$<br>$\theta_{nth} = \omega_0 + \frac{\alpha}{2}(2n-1)$ |

# WRAP it up!

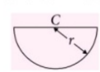
1. The centre of mass of a non-uniform rod of length $L$ whose mass per unit length $\lambda = \dfrac{Kx^2}{L}$, where $K$ is a constant and $x$ is the distance from one end is

   (a) $\dfrac{3L}{4}$ (b) $\dfrac{L}{8}$ (c) $\dfrac{K}{L}$ (d) $\dfrac{3K}{L}$

2. A uniform rod $AB$ of mass $m$ and length $2a$ is falling freely without rotation under gravity with $AB$ horizontal. Suddenly the end $A$ is fixed when the speed of the rod is $v$. The angular speed with which the rod begins to rotate is

   (a) $\dfrac{v}{2a}$ (b) $\dfrac{4v}{3a}$ (c) $\dfrac{v}{3a}$ (d) $\dfrac{3v}{4a}$

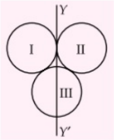
3. A semicircular lamina of mass $m$, radius $r$ and centre at $C$ is shown in the figure. Its centre of mass is at a distance $x$ from $C$. Its moment of inertia about an axis through its centre of mass and perpendicular to its plane is




   (a) $\dfrac{1}{2}mr^2$ (b) $\dfrac{1}{4}mr^2$

(c) $\frac{1}{2}mr^2+mx^2$ (d) $\frac{1}{2}mr^2-mx^2$

**4.** Three identical rings, each of mass $M$ and radius $R$ are arranged as shown in figure. The moment of inertia of the arrangement about $YY'$ is



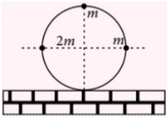

(a) $\frac{1}{2}MR^2$

(b) $MR^2$

(c) $\frac{5}{2}MR^2$

(d) $\frac{7}{2}MR^2$

**5.** A ring of mass $m$ and radius $R$ has three particles attached to the ring as shown in the figure. The centre of the ring has a speed $v_0$. If the system is in pure rolling then the kinetic energy of the system is



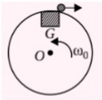

(a) $6\,mv_0^2$ (b) $12\,mv_0^2$

(c) $4\,mv_0^2$ (d) $8\,mv_0^2$

**6.** A solid sphere and a solid cylinder of same mass are rolled down on two inclined planes of heights $h_1$ and $h_2$ respectively. If at the bottom of the plane the two objects have same linear velocities, then the ratio of $h_1 : h_2$ is

(a) 2 : 3 (b) 7 : 5 (c) 14 : 15 (d) 15 : 14

**7.** A horizontal turn table in the form of a disc of radius $r$ carries a gun at $G$ and rotates with angular velocity $\omega_0$ about a vertical axis passing through the centre $O$. The increase in angular velocity of the system if the gun fires a bullet of mass $m$ with a tangential velocity $v$ with respect to the gun is




(moment of inertia of gun + table about $O$ is $I_0$)

(a) $\frac{mvr}{I_0+mr^2}$ (b) $\frac{2mvr}{I_0}$

(c) $\frac{v}{2r}$ (d) $\frac{mvr}{2I_0}$

**8.** Three identical rods, each of length $l$, are joined to form an equilateral triangle. Its radius of gyration about an axis passing through a corner and perpendicular to the plane of the triangle is

(a) $\frac{l}{2}$ (b) $\sqrt{\frac{3}{2}}\,l$ (c) $\frac{l}{\sqrt{2}}$ (d) $\frac{l}{\sqrt{3}}$

**9.** A uniform disc of radius $R$ lies in $x$-$y$ plane with its centre at origin. Its moment of inertia about the axis $x = 2R$ and $y = 0$ is equal to the moment of inertia about the axis $y = d$ and $z = 0$. Where $d$ is equal to

(a) $\frac{4}{3}R$ (b) $\frac{\sqrt{17}}{2}R$ (c) $3R$ (d) $\frac{\sqrt{15}}{2}R$

**10.** A constant power is supplied to a rotating disc. Angular velocity ($\omega$) of disc varies with number of rotations ($n$) made by the disc as

(a) $\omega \propto n^{1/3}$ (b) $\omega \propto n^{3/2}$

(c) $\omega \propto n^{2/3}$ (d) $\omega \propto n^2$

**11.** A disc is rolling on the inclined plane. What is the ratio of its rotational KE to the total KE?

(a) 1 : 3 (b) 3 : 1 (c) 1 : 2 (d) 2 : 1

**12.** A boat of 90 kg is floating in still water. A boy of mass 30 kg walks from the stern to the bow. The length of the boat is 3 m. Find the distance through which the boat will move.

(a) 0.75 m (b) 0.90 m (c) 1.0 m (d) 1.5 m

**13.** A disk and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first?

(a) Both reach at the same time

(b) Depends on their masses

(c) Disk

(d) Sphere

**14.** From a disc of radius $R$ and mass $M$, a circular hole of diameter $R$, whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre?

(a) $11\,MR^2/32$ (b) $9\,MR^2/32$

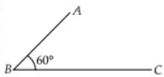
(c) $15\,MR^2/32$ (d) $13\,MR^2/32$

**15.** A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad $s^{-2}$. Its net acceleration in m $s^{-2}$ at the end of 2.0 s is approximately

(a) 6.0 (b) 3.0 (c) 8.0 (d) 7.0

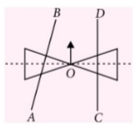
16. An automobile moves on a road with a speed of 54 km $h^{-1}$. The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is 3 kg $m^2$. If the vehicle is brought to rest in 15 s, the magnitude of average torque transmitted by its brakes to the wheel is
(a) 10.86 kg $m^2$ $s^{-2}$ (b) 2.86 kg $m^2$ $s^{-2}$
(c) 6.66 kg $m^2$ $s^{-2}$ (d) 8.58 kg $m^2$ $s^{-2}$

17. In the figure shown $ABC$ is a uniform wire. If centre of mass of wire lies vertically below point $A$, then $\frac{BC}{AB}$ is close to



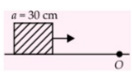

(a) 1.85 (b) 1.5 (c) 1.37 (d) 3

18. A roller is made by joining together two cones at their vertices $O$. It is kept on two rails $AB$ and $CD$ which are placed asymmetrically (see figure), with its axis perpendicular to $CD$ and its centre $O$ at the centre of line joining $AB$ and $CD$ (see figure). It is given a light push so that it starts rolling with its centre $O$ moving parallel to $CD$ in the direction shown. As it moves, the roller will tend to




(a) turn left (b) turn right
(c) go straight
(d) turn left and right alternately

19. A cubical block of side 30 cm is moving with velocity 2 m $s^{-1}$ on a smooth horizontal surface.

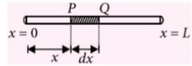



The surface has a bump at a point $O$ as shown in figure. The angular velocity (in rad/s) of the block immediately after it hits the bump, is
(a) 13.3 (b) 5.0 (c) 9.4 (d) 6.7

20. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute (rpm) to ensure proper mixing is close to
(Take the radius of the drum to be 1.25 m and its axle to be horizontal)
(a) 27.0 (b) 0.4 (c) 1.3 (d) 8.0

## SOLUTIONS

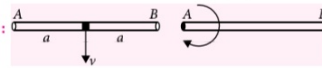
1. **(a) :**




Mass of the element $PQ$ is $dm = \lambda dx = \frac{Kx^2}{L} \cdot dx$

$$\therefore \quad x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L \frac{Kx^3}{L}dx}{\int_0^L \frac{Kx^2}{L}dx} = \frac{\left(\frac{L^4}{4}\right)}{\left(\frac{L^3}{3}\right)} = \frac{3L}{4}$$

2. **(d) :**

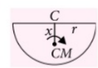



Angular momentum about $A$ will be conserved, *i.e.*, $L_i = L_f$

or $mva = I\omega$ or $mva = \frac{m(2a)^2}{3}\omega$

$\therefore \quad \omega = \frac{3v}{4a}$

3. **(d) :** We know, $I_C = mr^2/2$




Using parallel axes theorem,

$I_C = I_{CM} + mx^2$

$\therefore \quad I_{CM} = I_C - mx^2 = mr^2/2 - mx^2$

4. **(d) :** Moment of inertia of ring I about $YY' = \frac{3}{2}MR^2$

Moment of inertia of ring II about $YY' = \frac{3}{2}MR^2$

Moment of inertia of ring III about $YY' = \frac{1}{2}MR^2$

Moment of inertia of the system about $YY'$

$$= \frac{3}{2}MR^2 + \frac{3}{2}MR^2 + \frac{1}{2}MR^2 = \frac{7}{2}MR^2$$

5. **(a) :** As we know, $KE = KE_{\text{tran.}} + KE_{\text{rot.}}$

$$\therefore \quad KE = \frac{1}{2}m(v_0^2) + \frac{1}{2}(mR^2)\left(\frac{v_0}{R}\right)^2 + \frac{1}{2}(3m)(\sqrt{2}v_0)^2 + \frac{1}{2}(m)(2v_0)^2 = 6mv_0^2$$

6. **(c)**

7. **(a) :** Given, $I_0$ is the moment of inertia of table and gun and $m$ is the mass of bullet.

Initial angular momentum of system about centre

$L_i = (I_0 + mr^2)\,\omega_0$ ...(i)

Let $\omega$ be the angular velocity of table after the bullet is fired. Final angular momentum

$$L_f = I_0\omega + mr^2\omega - mvr$$

or $L_f = I_0\omega - m(v - r\omega)r$ ...(ii)

where $(v - r\omega)$ is absolute velocity of bullet to the right.

From eqs. (i) and (ii), we get

$$(\omega - \omega_0) = \frac{mvr}{I_0 + mr^2}$$

This is the increase in angular velocity.

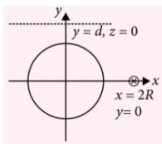
**8. (c) :** Moment of inertia of all the three rods is

$$I = \frac{Ml^2}{3} + \frac{Ml^2}{3} + \left(\frac{Ml^2}{12} + M\left(\frac{\sqrt{3}l}{2}\right)^2\right) = \frac{3}{2}Ml^2$$

As $I = MK^2$

So, $3MK^2 = \frac{3}{2}Ml^2 \Rightarrow K = \frac{l}{\sqrt{2}}$

**9. (b):** An axis passing through $x = 2R$, $y = 0$ is in ⊗ direction as shown in figure. Moment of inertia about this axis will be




$$I_1 = \frac{1}{2}mR^2 + m(2R)^2 = \frac{9}{2}mR^2 \quad ...(i)$$

Axis passing through $y = d$, $z = 0$ is shown by dotted line in figure. Moment of inertia about this axis will be

$$I_2 = \frac{1}{4}mR^2 + md^2 \quad ...(ii)$$

By equations (i) and (ii), we get

$$\frac{1}{4}mR^2 + md^2 = \frac{9}{2}mR^2 \text{ or } d = \frac{\sqrt{17}}{2}R$$

**10. (a) :** We have $P = \tau.\omega$

or $I\left(\omega\frac{d\omega}{d\theta}\right)\omega = P$ or $\omega^2 d\omega = \frac{P}{I}d\theta$

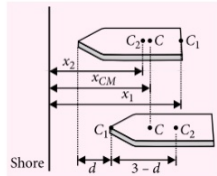
On integration, we find that $\omega \propto (\theta)^{1/3}$

or $\omega \propto (n)^{1/3}$

**11. (a)**

**12. (a) :** As shown in figure, let $C_1$, $C_2$ and $C$ be the centres of mass of the boy, boat and the system (boy and boat) respectively. Let $x_1$ and $x_2$ be the distances of $C_1$ and $C_2$ from the shore. Then the centre of mass will be at a distance,

$$x_{CM} = \frac{30x_1 + 90x_2}{30 + 90}$$




As the boy moves from the stern to the bow, the boat moves backward through a distance $d$ so that position of the centre of mass of the system remains unchanged.

$$x'_{CM} = \frac{30[x_1 - (3-d)] + 90(x_2 + d)}{30 + 90}$$

As $x'_{CM} = x_{CM}$

$$\frac{30(x_1 - 3 + d) + 90(x_2 + d)}{120} = \frac{30x_1 + 90x_2}{120}$$

or $-90 + 30\,d + 90\,d = 0$

or $d = 0.75$ m

**13. (d):** Time taken by the body to reach the bottom when it rolls down on an inclined plane without slipping is given by

$$t = \sqrt{\frac{2l\left(1 + \frac{k^2}{R^2}\right)}{g\sin\theta}}$$

Since $g$ is constant and $l$, $R$ and $\sin\theta$ are same for both

$$\therefore \frac{t_d}{t_s} = \frac{\sqrt{1 + \frac{k_d^2}{R^2}}}{\sqrt{1 + \frac{k_s^2}{R^2}}} = \sqrt{\frac{1 + \frac{R^2}{2R^2}}{1 + \frac{2R^2}{5R^2}}} \quad \left(\because k_d = \frac{R}{\sqrt{2}}, k_s = \sqrt{\frac{2}{5}}R\right)$$

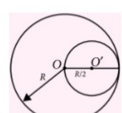
$$= \sqrt{\frac{3}{2} \times \frac{5}{7}} = \sqrt{\frac{15}{14}} \Rightarrow t_d > t_s$$

Hence, the sphere gets to the bottom first.

**14. (d):** Mass per unit area of disc = $\frac{M}{\pi R^2}$

Mass of removed portion of disc,

$$M' = \frac{M}{\pi R^2} \times \pi\left(\frac{R}{2}\right)^2 = \frac{M}{4}$$




Moment of inertia of removed portion about an axis passing through centre of disc $O$ and perpendicular to the plane of disc,

$I'_O = I_{O'} + M'd^2$

$= \frac{1}{2}\times\frac{M}{4}\times\left(\frac{R}{2}\right)^2 + \frac{M}{4}\times\left(\frac{R}{2}\right)^2$

$= \frac{MR^2}{32} + \frac{MR^2}{16} = \frac{3MR^2}{32}$

When portion of disc would not have been removed, the moment of inertia of complete disc about centre $O$ is

$I_O = \frac{1}{2}MR^2$

So, moment of inertia of the disc with removed portion is

$$I = I_O - I'_O = \frac{1}{2}MR^2 - \frac{3MR^2}{32} = \frac{13MR^2}{32}$$

**15. (c) :** Given, $r = 50$ cm $= 0.5$ m, $\alpha = 2.0$ rad s$^{-2}$, $\omega_0 = 0$

At the end of 2 s,

Tangential acceleration, $a_t = r\alpha = 0.5 \times 2 = 1$ m s$^{-2}$

Radial acceleration, $a_r = \omega^2 r = (\omega_0 + \alpha t)^2 r$

$= (0 + 2 \times 2)^2 \times 0.5 = 8$ m s$^{-2}$

$\therefore$ Net acceleration,

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{1^2 + 8^2} = \sqrt{65} \approx 8 \text{ m s}^{-2}$$

**16. (c) :** Here,

Speed of the automobile,

$$v = 54 \text{ km h}^{-1} = 54\times\frac{5}{18} \text{ m s}^{-1} = 15 \text{ m s}^{-1}$$

Radius of the wheel of the automobile, $R = 0.45$ m

Moment of inertia of the wheel about its axis of rotation, $I = 3$ kg m$^2$

Time in which the vehicle brought to rest, $t = 15$ s

The initial angular speed of the wheel is

$$\omega_i = \frac{v}{R} = \frac{15 \text{ m s}^{-1}}{0.45 \text{ m}} = \frac{1500}{45} \text{rad s}^{-1} = \frac{100}{3} \text{ rad s}^{-1}$$

and its final angular speed is

$\omega_f = 0$ (as the vehicle comes to rest)

$\therefore$ The angular retardation of the wheel is

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - \frac{100}{3}}{15} = -\frac{100}{45} \text{ rad s}^{-2}$$

The magnitude of required torque is

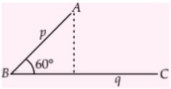
$$\tau = I|\alpha| = (3 \text{ kg m}^2)\left(\frac{100}{45}\text{rad s}^{-2}\right)$$

$$= \frac{20}{3} \text{ kg m}^2\text{s}^{-2} = 6.66 \text{ kg m}^2\text{s}^{-2}$$

**17. (c) :** Let $AB = p$

$BC = q$

$\lambda$ = linear mass density of the rod




According to question, centre of mass of the rod lies vertically below point $A$.

$$\therefore \quad X_{CM} = p\cos 60° = \frac{(\lambda q)\left(\frac{q}{2}\right) + (\lambda p)\left(\frac{p}{2}\right)\cos 60°}{\lambda(p+q)}$$

$$\Rightarrow \quad \frac{p}{2} = \frac{\frac{q^2}{2} + \frac{p^2}{4}}{(p+q)} \Rightarrow p^2 + pq = q^2 + \frac{p^2}{2}$$

$$\Rightarrow \quad 1 + \frac{q}{p} = \frac{q^2}{p^2} + \frac{1}{2} \Rightarrow \left(\frac{q}{p}\right)^2 - \frac{q}{p} - \frac{1}{2} = 0$$

$$\frac{q}{p} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)\left(-\frac{1}{2}\right)}}{2\times 1} = \frac{1 \pm \sqrt{3}}{2}$$

$$\therefore \quad \text{Possible value of } \frac{q}{p} = \frac{1+\sqrt{3}}{2} = 1.366 \approx 1.37$$

**18. (a)**

**19. (b):** Since no external torque acts on the system, therefore total angular momentum of the system about point $O$ remains constant.

Before hitting, $L_i = mv\frac{a}{2}$

After hitting , $L_f = I\omega$

$\therefore \; mv\frac{a}{2} = I\omega$ or $\omega = \frac{mva}{2I}$

Here, $I$ = moment of inertia of cube about its edge

$$= m\frac{a^2}{6} + m\left(\frac{\sqrt{2}a}{2}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2ma^2}{3}$$

$$\therefore \; \omega = \frac{mva\times 3}{2\times 2ma^2} = \frac{3v}{4a} = \frac{3\times 2}{4\times 0.3} = 5 \text{ rad s}^{-1}$$

**20. (a):** Radius of the drum, $R = 1.25$ m

For just one complete rotation, speed of the drum at top position,

$v = \sqrt{Rg}$

Angular velocity of the drum, $\omega = \frac{v}{R} = \sqrt{\frac{g}{R}}$

$$\omega = \sqrt{\frac{10}{1.25}} \text{ rad s}^{-1} = \frac{60}{2\pi}\sqrt{\frac{10}{1.25}} \text{ rpm} = 27 \text{ rpm}$$

◆◆

# CBSE warm-up!

CLASS-XI

**Practice questions for CBSE Exams as per the reduced syllabus, latest pattern and marking scheme issued by CBSE for the academic session 2020-21.**

**Series 2**

## CHAPTERWISE PRACTICE PAPER : Laws of Motion | Work, Energy and Power

Time Allowed : 3 hours
Maximum Marks : 70

### GENERAL INSTRUCTIONS

Read the following instructions very carefully and strictly follow them.

(i) This question paper comprises four section – A, B, C and D.
(ii) There are 37 questions in the question paper. All questions are compulsory.
(iii) Section A – Questions no. 1 to 20 are very short answer type questions carrying 1 mark each.
(iv) Section B – Question no. 21 to 27 are short answer type questions carrying 2 marks each.
(v) Section C – Questions no. 28 to 34 are long answer type questions carrying 3 marks each.
(vi) Section D – Questions no. 35 to 37 are also long answer type questions, carrying 5 marks each.
(vii) There is no overall choice in the question paper. However, an internal choice has been provided in 2 questions of 1 mark, 2 questions of 2 marks, 1 question of three marks and all the 3 questions of five marks. You have to attempt only one of the choices in such questions.
(viii) In addition to this, separate instructions are given with each section and question, wherever necessary.
(ix) Use of calculators and log tables is not permitted.
(x) You may use the values of physical constants wherever necessary.

## SECTION-A

**Directions (Q. No. 1-10) : Select the most appropriate option from those given below each question.**

1. If the normal force is doubled, the coefficient of friction, is
(a) halved (b) doubled
(c) not changed (d) one fourth

2. A block of mass 10 kg is placed on rough horizontal surface whose coefficient of friction is 0.5. If a horizontal force of 100 N is applied on it, then acceleration of the block will be
[Take $g = 10\ m\ s^{-2}$]
(a) $10\ m\ s^{-2}$ (b) $5\ m\ s^{-2}$
(c) $15\ m\ s^{-2}$ (d) $0.5\ m\ s^{-2}$

3. A cork of mass 10 g is floating on water. The net force acting on the cork is
(a) 10 N (b) $10^{-3}$ N (c) $10^{-2}$ N (d) zero

4. A shell of mass 200 g is fired by a gun of mass 100 kg. If the muzzle speed of the shell is $80\ m\ s^{-1}$, then the recoil speed of the gun is
(a) $16\ cm\ s^{-1}$ (b) $8\ cm\ s^{-1}$ (c) $8\ m\ s^{-1}$ (d) $16\ m\ s^{-1}$

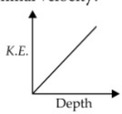
5. Which of the following statements is correct about friction?
(a) The coefficient of friction between a given pair of substances is largely independent of the area of contact between them.
(b) The frictional force can never exceed the reaction force on the body from the support surface.
(c) Rolling friction is only slightly smaller than sliding friction.
(d) The main source of friction is the irregularity of the surfaces in contact.

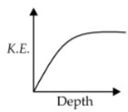
6. A body with mass 5 kg is acted upon by a force $\vec{F} = (-3\hat{i} + 4\hat{j})$ N. If its initial velocity at $t = 0$ is $\vec{u} = (6\hat{i} - 12\hat{j})$ m s$^{-1}$, the time at which it will just have a velocity along the $y$-axis is
(a) 20 s (b) 10 s (c) 2 s (d) 15 s

7. A body is being raised to a height $h$ from the surface of earth. What is the sign of work done by
(i) applied force (ii) gravitational force?
(a) Positive, Positive (b) Positive, Negative
(c) Negative, Positive (d) Negative, Negative

8. A force $(4\hat{i} + \hat{j} - 2\hat{k})$ N acting on a body maintains its velocity at $(2\hat{i} + 2\hat{j} + 3\hat{k})$ m s$^{-1}$. The power exerted is
(a) 4 W (b) 5 W (c) 2 W (d) 8 W

9. Which of the following figures correctly shows the change in kinetic energy of an iron sphere falling freely in a lake having sufficient depth to impart it a terminal velocity?

(a)



(b)
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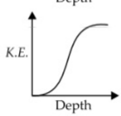
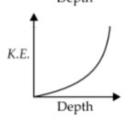



(d)



10. A body is initially at rest. It undergoes one dimensional motion with constant acceleration. The power delivered to it at time $t$ is proportional to
(a) $t^{1/2}$ (b) $t$ (c) $t^{3/2}$ (d) $t^2$

**Directions (Q. No. 11-15) : Fill in the blanks with appropriate answer.**

11. The motion of particle of mass $m$ is given by $y = ut + \frac{1}{2}gt^2$. The force acting on the particle is ______.

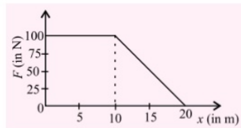
12. A girl press her physics text book against a rough vertical wall with her hand. The direction of the frictional force on the book exerted by the wall is ______.

13. A cyclist bends while taking turn to ______.

**OR**

The coefficient of friction between the tyres and the road is 0.1. The maximum speed with which a cyclist can take a circular turn of radius 3 m without skidding is ______. (Take $g = 10$ m s$^{-2}$)

14. A force $F$ acting on an object varies with distance $x$ as shown in the figure. The work done by the force in moving the object from $x = 0$ to $x = 20$ m is ______.




15. A ball of mass $m$ collides with a wall with speed $v$ and rebounds on the same line with the same speed. If the mass of the wall is taken as infinite, then the work done by the ball on the wall is ______.

**Directions (Q. No. 16-20) : Answer the following.**

16. About $4 \times 10^{10}$ kg of matter is converted into energy in the Sun each second. What is the power output of the Sun?

17. What is a spring force?

18. A light body and a heavy body have the same kinetic energy. Which one will have the greater momentum?

19. State the principle of conservation of energy.

20. Write any two conditions under which work done by a force is zero.

**OR**

How much height can a 60 kg man climb by using the energy from a slice of bread which produces 100,000 cal? Assume that the efficiency of human body is 28%?

## SECTION-B

21. What is the tension in rod of length $L$ and mass $M$ at a distance $y$ from $F_1$ when the rod is acted on by two unequal forces $F_1$ and $F_2$ ($F_2 < F_1$) at its ends?

22. A sphere of mass $m$ moving with a velocity $u$ hits another stationary sphere of same mass. If $e$ is the coefficient of restitution, what is the ratio of the velocities of two spheres after the collision?

**OR**

A block of mass $m$ moving at a speed $v_0$ compresses a spring through a distance $x$ before its speed is halved. Find the spring constant of the spring.

23. A particle is projected making an angle of 45° with horizontal having kinetic energy $K$. What is the kinetic energy at highest point ?

24. What is kinetic energy? Does it depend on frame of reference?

**OR**

The displacement $x$ and time $t$ for a particle are related to each other as $t = \sqrt{x} + 3$. What is work done in first six seconds of its motion?

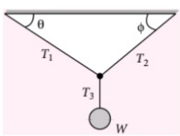
25. Why does a pilot looping a vertical loop not fall down even at the highest point?

26. Explain why a horse cannot pull a cart and run in empty space.

27. The kinetic energy of a body decreases by 19%. What is the percentage decrease in the magnitude of linear momentum of the body?

## SECTION-C

28. An object of weight $W$ hangs from a rope that is tied to other ropes that are fastened to the ceiling as shown in figure. The upper ropes make angles $\theta$ and $\phi$ with the horizontal.



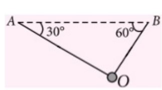

Find the tensions $T_1$, $T_2$ and $T_3$ in the three ropes.

29. Prove that the gravitational force is a conservative force.

30. A locomotive of mass $m$ starts moving so that its velocity varies according to the law $v = k\sqrt{s}$, where $k$ is constant, and $s$ is the distance covered. Find the total work performed by all the forces which are acting on the locomotive during the first $t$ second after the beginning of motion.

**OR**

A ball of mass 1 kg hangs in equilibrium from two strings $OA$ and $OB$ as shown in figure. What are the tensions in strings $OA$ and $OB$?
(Take $g = 10\ \text{m s}^{-2}$)




31. A truck can move up a road rising 1 in 25 with a speed of 24 km $h^{-1}$ with frictional force $(1/50)^{th}$ of the weight of truck. What will be the speed of the truck moving down the same road with the same power?

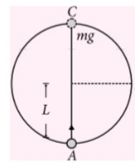
32. Using component of force, show that it is easier to pull a lawn roller than to push it.

33. Show that two identical particles move at right angles to each other after elastic collision in two dimensions.

34. If the elongation in a spring of force constant $k$ is tripled, calculate
(a) ratio of final to initial force in the spring
(b) ratio of elastic energies stored in the two cases
(c) work done in changing to the state of elongation.

## SECTION-D

35. A bob of mass $m$ is suspended by a light string of length $L$. It is imparted a horizontal velocity $v_0$ at the lowest point $A$ such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, $C$ as shown in figure. Obtain an expression for (a) $v_0$ (b) the speeds at points $B$ and $C$ (c) the ratio of the kinetic energies at $B$ and $C$ $(K_B/K_C)$.




**OR**

State and explain the Newton's second law of motion. Hence deduce first and third law of motion from second law of motion. A piece of cork is floating on water. What is the net force acting on it?

36. A body of mass $m_1$ moving with speed $u_1$, collides with another body of mass $m_2$ at rest and sticks to it. After collision they move together with speed $v$. Find the loss in kinetic energy in the collision.

**OR**

(a) Define angle of friction. Deduce its relation with coefficient of static friction.
(b) Define angle of repose. Deduce its relation with coefficient of static friction.

37. Why does a cyclist lean inward when moving along a curved path? Determine the angle through which a cyclist bends from the vertical while negotiating a curve.

**OR**

A body of mass $m_1$ moving with speed $u_1$, collides with another body of mass $m_2$ at rest and sticks to it. After collision they move together with speed $v$. Find the loss in kinetic energy in the collision.

## SOLUTIONS

1. (c)  2. (b)

3. (d): When the cork is floating, its weight is balanced by the upthrust. Therefore, net force on the cork is zero.

4. (a)

5. (a): The coefficient of friction between a given pair of substances is independent of the area of contact between them.
$\mu > 1$ for copper on cast iron.
Rolling friction is much smaller than sliding friction.
Irregularity of the surfaces in contact is not a main contributor of friction.

6. (b)  7. (b)

8. (a): Force, $\vec{F} = (4\hat{i} + \hat{j} - 2\hat{k})$ N
Velocity, $\vec{v} = (2\hat{i} + 2\hat{j} + 3\hat{k})$ m s$^{-1}$
Power, $P = \vec{F} \cdot \vec{v} = (4\hat{i} + \hat{j} - 2\hat{k}) \cdot (2\hat{i} + 2\hat{j} + 3\hat{k})$
$= (8 + 2 - 6)$ W $= 4$ W

9. (b): When an iron sphere falls freely in a lake, its motion is accelerated due to gravity and retarded due to viscous force. The overall effect is increase in velocity and hence increase in kinetic energy till the sphere acquires terminal velocity which is constant. Hence, kinetic energy of the sphere beyond this depth of lake becomes constant. Hence, option (b) represents the correct diagram.

10. (b): Using, $v = u + at$
$\therefore \quad v = at \qquad (\because u = 0)$
As power, $P = F \times v \quad \therefore P = (ma) \times at = ma^2 t$
As $m$ and $a$ are constants, $\quad \therefore P \propto t$

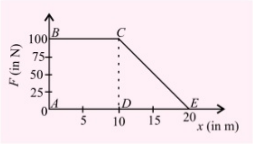
11. Here, $y = ut + \frac{1}{2}gt^2$
$\therefore$ Velocity, $v = \frac{dy}{dt} = u + gt$
Acceleration, $a = \frac{dv}{dt} = g$
The force acting on the particle is
$F = ma = mg$

12. Friction forces are always parallel to the surfaces in contact, which in this case, are the wall and the cover of the book. This tells us that the friction force is either upwards or downwards. Because the tendency of the book is to fall due to gravity, the friction force must be in the upwards direction.

13. A cyclist bends while taking turn to generate required centripetal force.

**OR**

Here, $r = 3$ m, $\mu = 0.1$, $g = 10$ m s$^{-2}$
The maximum speed with which a cyclist can take a turn without skidding is
$$v_{max} = \sqrt{\mu r g} = \sqrt{0.1 \times (3\text{ m})(10\text{ m s}^{-2})} = \sqrt{3}\text{ m s}^{-1}$$

14. 


Work done = Area under $F$-$x$ graph
$W$ = Area of rectangle $ABCD$ + Area of $\Delta CDE$
$= 100 \times 10 + \frac{1}{2} \times 10 \times 100 = 1000 + 500 = 1500$ J

15. Since the wall has infinite mass therefore the displacement is zero.

16. Energy liberated per second by the conversion of $4 \times 10^{10}$ kg of matter, *i.e.*,
$E = mc^2 = [(4 \times 10^{10})(3 \times 10^8)^2]$ J $= 3.6 \times 10^{27}$ J
Power output of the Sun = energy (work) liberated per second
$= 3.6 \times 10^{27}$ J s$^{-1}$ $= 3.6 \times 10^{27}$ W

17. Spring force is the restoring force arising because of compression or extension of spring.
Spring force, $F \propto (-x)$
$F = -kx$
where $x$ is the change in length and $k$ is a constant called spring constant or force constant. The negative sign indicates that the spring force is opposite to the direction of displacement of the spring from its normal position.

18. Kinetic energy, $K = \frac{p^2}{2m}$, $\therefore p = \sqrt{2mK}$.
Since $K$ is same for both bodies, $p \propto \sqrt{m}$, *i.e.*, the heavier body has more momentum than the lighter body.

19. Law of Conservation of Energy: It states that energy may be transformed from one form to another but it can neither be created nor be destroyed. The total energy of an isolated system remains constant.

20. Condition for work done by a force is zero:
(i) If force and displacement are perpendicular to each other.
(ii) If displacement of the particle is zero.

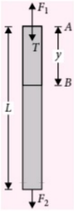
**OR**

Effective work = $mgh$

$$\frac{28}{100}\times 100{,}000\times 4.2 = 60\times 9.8\times h$$

or $h = \dfrac{28\times 100{,}000\times 4.2}{100\times 60\times 9.8}$ m = 200 m

**21.** Refer to figure, acceleration of the rod along $F_1$,




$$a = \frac{(F_1 - F_2)}{M}$$

Mass of the part (say $AB$) of length $y$,

$$m = \left(\frac{M}{L}\right)y$$

If $T$ is the tension in $AB$, then

$$F_1 - T = ma = \left(\frac{M}{L}y\right)\frac{(F_1 - F_2)}{M} = (F_1 - F_2)\frac{y}{L}$$

or $T = F_1 - (F_1 - F_2)\dfrac{y}{L} = F_1\left(1 - \dfrac{y}{L}\right) + F_2\left(\dfrac{y}{L}\right)$

**22.** Here, $u_1 = u$, $u_2 = 0$

$\therefore\quad e = \dfrac{v_2 - v_1}{u_1 - u_2} = \dfrac{v_2 - v_1}{u - 0}$

or $v_2 - v_1 = e\,u$ ...(i)

By the law of conservation of momentum,

$mu + m\times 0 = mv_1 + mv_2$

or $v_1 + v_2 = u$ ...(ii)

Adding (i) and (ii),

$2v_2 = u + eu = u\,(1 + e)$

or $v_2 = \dfrac{u(1+e)}{2}$ ...(iii)

Again, from (ii),

$v_1 = u - v_2 = u - \dfrac{u(1+e)}{2} = \dfrac{u(1-e)}{2}$ ...(iv)

Divide eqn (iii) by eqn (iv)

$$\frac{v_2}{v_1} = \frac{1+e}{1-e}.$$

**OR**

Loss in kinetic energy of the block

$$K = \frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{v_0}{2}\right)^2 = \frac{3}{8}mv_0^2$$

Gain in elastic potential energy of the spring,

$$U = \frac{1}{2}kx^2$$

As $U = K$, $\therefore\quad \dfrac{1}{2}kx^2 = \dfrac{3}{8}mv_0^2$

or $k = \dfrac{3mv_0^2}{4x^2}$

**23.** Initial kinetic energy, $K = \dfrac{1}{2}mu^2$

Velocity at the highest point, $u'$

= horizontal component of $u$ $= u\cos 45° = \dfrac{u}{\sqrt{2}}$

Hence KE at the highest point, $K' = \dfrac{1}{2}mu'^2$

$$K' = \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}\left(\frac{1}{2}mu^2\right) = \frac{K}{2}.$$

KE at highest point will be half of the initial kinetic energy.

**24.** Kinetic energy is the energy possessed by an objects due to its motion. Yes, it depends on frame of reference because $v$ is not an absolute value. It always given relative to a reference frame.

**OR**

Given; $t = \sqrt{x} + 3 \Rightarrow x = (t - 3)^2$

Now, $v = \dfrac{dx}{dt} = 2\,(t - 3)$

At $t = 0$, $v_1 = 2\,(-3) = -6$

At $t = 6$, $v_2 = 2\,(6 - 3) = 6$

Work done = Change in kinetic energy

$$= \frac{1}{2}m(v_2^2 - v_1^2) = \text{zero}$$

**25.** The forces acting on the pilot looping a vertical loop are due to acceleration due to gravity, normal reaction by the seat on the pilot and centripetal force.

At highest point, the normal force and weight act in vertically downward direction.

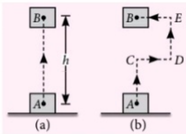
$\therefore$ For the pilot not to fall down,

$$N + mg < \frac{mv^2}{r} \Rightarrow N < \frac{mv^2}{r} - mg \qquad \text{...(i)}$$

$\therefore$ As long as (i) is satisfied, the pilot does not fall down.

**29.** As shown in figure (a), suppose a body of mass $m$ moves from point $A$ to $B$. The work done against gravity is

$W = mg\times AB = mgh$




As shown in figure (b), now suppose the body is taken from point $A$ to $B$ along the path $ACDEB$. During the horizontal path $CD$ and $EB$, the force of gravity is perpendicular to the displacement, so work done is zero. Work is done only along vertical paths $AC$ and $DE$. The total work done is

$$W = W_{AC} + W_{CD} + W_{DE} + W_{EB}$$
$$= mg \times AC + 0 + mg \times DE + 0$$
$$= mg\,(AC + DE) = mg \times h \quad \text{or} \quad W = mgh$$

Thus the work done in moving a body against gravity is independent of the path taken and depends only on the initial and final positions of the body. Hence gravitational force is a conservative force.

**27.** Initial kinetic energy, $K = \frac{p^2}{2m}$

Initial momentum, $p = \sqrt{2mK}$

Decreases in K.E. = 19% of $K = 0.19K$

Final K.E., $K' = K - 0.19\,K = 0.81K$

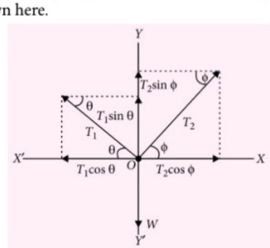
Final momentum,

$$p' = \sqrt{2mK'} = \sqrt{2m \times 0.81K} = 0.9\sqrt{2mK} = 0.9p$$

% decrease in momentum

$$= \frac{p - p'}{p} \times 100 = \frac{p - 0.9p}{p} \times 100 = 10\%$$

**28.** The free body diagram for the forces acting at $O$, is shown here.




From $\Sigma F_x = 0$,

$$T_2 \cos\phi - T_1 \cos\theta = 0 \quad \text{...(i)}$$

From $\Sigma F_y = 0$,

$$T_1 \sin\theta + T_2 \sin\phi - W = 0 \quad \text{...(ii)}$$

From eqns. (i) and (ii),

$$T_1 \sin\theta + \left(\frac{T_1 \cos\theta}{\cos\phi}\right)\sin\phi = W$$

or $\quad T_1\,(\sin\theta + \cos\theta\tan\phi) = W$

or $\quad T_1 = \frac{W}{(\sin\theta + \cos\theta\tan\phi)}$

Similarly, $T_2 = \frac{W}{(\sin\phi + \cos\phi\tan\theta)}$ and $T_3 = W$

**26.** While trying to pull a cart, a horse pushes the ground backwards with a certain force at an angle. The ground offers an equal reaction in the opposite direction, on the feet of the horse. The forward component of this reaction is responsible for motion of the cart. In empty space, there is no reaction and hence a horse cannot pull the cart and run.

**30.** Given: $v = k\sqrt{s}$

$$\frac{dv}{dt} = \frac{k}{2\sqrt{s}}\frac{ds}{dt} = \frac{k}{2\sqrt{s}}v = \frac{k}{2\sqrt{s}} \times k\sqrt{s} = \frac{1}{2}k^2$$

Force on the locomotive, $F = m\frac{dv}{dt} = \frac{1}{2}mk^2$

Again, $\frac{dv}{dt} = \frac{k^2}{2}$ or $dv = \frac{k^2}{2}dt$

Integrating, $v = \frac{k^2}{2}t + c$

where $c$ is the constant of integration.

Suppose $v = 0$ at $t = 0$. Then, $c = 0$

$\therefore \quad v = \frac{k^2}{2}t$ or $\frac{ds}{dt} = \frac{k^2 t}{2}$ or $ds = \frac{k^2 t}{2}dt$

Integrating, $s = \frac{k^2 t^2}{4} + c'$

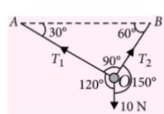
Suppose $s = 0$ at $t = 0$. Then, $c' = 0 \quad \therefore \quad s = \frac{k^2 t^2}{4}$

Work done = $Fs$

$$= \frac{1}{2}mk^2 \times \frac{k^2 t^2}{4} = \frac{mk^4 t^2}{8}$$

**OR**

Various forces acting at the point $O$ are as shown in figure. The three forces are in equilibrium. Using Lami's theorem,




$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{10}{\sin 90^\circ}$$

or $\dfrac{T_1}{\sin 30^\circ} = \dfrac{T_2}{\sin 60^\circ} = \dfrac{10}{1}$

$\therefore$ $T_1 = 10 \sin 30^\circ = 10 \times 0.5 = 5$ N

and $T_2 = 10 \sin 60^\circ = 10 \times \dfrac{\sqrt{3}}{2} = 5\sqrt{3}$ N

**31.** Here, $\sin\theta = \dfrac{1}{25}$

Total upward force required, $F = mg \sin\theta + f$

or $F = mg\left(\dfrac{1}{25}\right) + \left(\dfrac{1}{50}\right)mg = \left(\dfrac{3}{50}\right)mg$

Power, $P = Fv = \left(\dfrac{3}{50}\right)mgv$

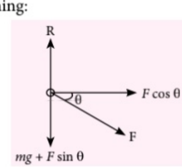
Net downward force, $F' = mg\sin\theta - f$

or $F' = mg\left(\dfrac{1}{25}\right) - \left(\dfrac{1}{50}\right)mg = \left(\dfrac{1}{50}\right)mg$

As $P = F'v'$, $\therefore$ $\left(\dfrac{3}{50}\right)mgv = \left(\dfrac{1}{50}\right)mgv'$

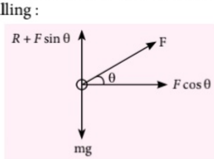
or $v' = 3v = (3 \times 24)$ km h$^{-1}$ = 72 km h$^{-1}$

**32.** While pushing:




Normal reaction force, $R = mg + F \sin\theta$

While pulling :

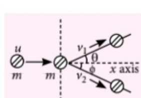



Normal reaction, $R = mg - F \sin\theta$

So, the normal reaction in case of pulling is less than the normal reaction while pushing the lawn roller. Therefore, it is easier to pull lawn roller than pushing it.

**33.** Let $m_1\vec{u}_1 = \vec{p}_1, m_1\vec{v}_1 = \vec{p}_1{}', m_2\vec{v}_2 = \vec{p}_2{}'$

Here, $\vec{p}_1, \vec{p}_1{}'$ initial and final momenta of $A$ and $\vec{p}_2{}'$ is the final momentum of $B$. The initial momentum of $B$ which is at rest is zero.




From law of conservation of momentum, $\vec{p}_1 = \vec{p}_1{}' + \vec{p}_2{}'$ ...(i)

$\because$ Collision is elastic, kinetic energy is also conserved thus, $\dfrac{p_1^2}{2m_1} = \dfrac{p_1'^2}{2m_1} + \dfrac{p_2'^2}{2m_2}$

As the two particles are identical, $m_1 = m_2$ so

$p_1^2 = p_1'^2 + p_2'^2$ ...(ii)

From eqn. (i) using parallelogram law of vectors,

$p_1^2 = p_1'^2 + p_2'^2 + 2p_1'p_2' \cos(\theta + \phi)$ ...(iii)

From eqn. (ii) and (iii)

$2p_1'p_2' \cos(\theta + \phi) = 0 \Rightarrow \theta + \phi = 90^\circ$

Thus, the two identical particles move at right angles after collision in two dimensions.

**34.** Force constant is given $k$, let the displacement be $x$, then force, $|F| = kx$

(a) If elongation in the spring is tripled, then final force

$F = k\,(3x)$

$\therefore$ Their ratio, $\dfrac{F'}{F} = \dfrac{k(3x)}{kx} = \dfrac{3}{1}$

(b) Initial elastic energy, $E = \dfrac{1}{2}kx^2$

Final elastic energy $E' = \dfrac{1}{2}k(3x)^2$

Their ratio, $\dfrac{E'}{E} = \dfrac{9}{1}$

(c) Work done in changing state = $E' - E$

$= \dfrac{1}{2} \times (9kx^2) - \dfrac{1}{2}kx^2 = 4kx^2$

**35.** *Refer to answer 115, Page no. 177* (MTG CBSE Champion Physics Class 11)

**OR**

*Refer to answer 43, Page no. 131* (MTG CBSE Champion Physics Class 11)

**36.** *Refer to answer 185, Page no. 186* (MTG CBSE Champion Physics Class 11)

**OR**

*Refer to answer 125, Page no. 141* (MTG CBSE Champion Physics Class 11)

**37.** *Refer to answer 144, Page no. 143, 144* (MTG CBSE Champion Physics Class 11)

**OR**

*Refer to answer 184, Page no. 186* (MTG CBSE Champion Physics Class 11)

# MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

**Total Marks : 120**

## Rotational Motion | Gravitation

**Time Taken : 60 Min.**

### NEET

#### Only One Option Correct Type

1. If $R$ is the average radius of earth, $\omega$ is its angular velocity about its axis and $g$ is the gravitational acceleration on the surface of earth then the cube of the radius of orbit of a geostationary satellite will be equal to
   (a) $\frac{R^2 g}{\omega}$ (b) $\frac{R^2\omega^2}{g}$ (c) $\frac{Rg}{\omega^2}$ (d) $\frac{R^2 g}{\omega^2}$

2. A man stands at one end of a boat which is stationary in water. Neglect water resistance. The man now moves to the other end of the boat and again becomes stationary. The centre of mass of the man plus boat system will remain stationary with respect to water
   (a) in all cases
   (b) only when the man is stationary initially and finally
   (c) only if the man moves without acceleration on the boat
   (d) only if the man and the boat have equal masses.

3. Weights of 1 g, 2 g, ...., 100 g are suspended from the 1 cm, 2 cm, ...., 100 cm marks respectively of a light metre scale. Where should it be supported for the system to be in equilibrium?
   (a) 55 cm mark (b) 60 cm mark
   (c) 66 cm mark (d) 72 cm mark

4. The gravitational force between two bodies is directly proportional to $\frac{1}{R}\left(\text{not } \frac{1}{R^2}\right)$, where $R$ is the distance between the bodies. Then the orbital speed for this force in circular orbit is proportional to
   (a) $\frac{1}{R^2}$ (b) $R^2$ (c) $R$ (d) $\frac{1}{\sqrt{R^0}}$

5. The imaginary angular velocity of the earth for which the effective acceleration due to gravity at the equator shall be zero is equal to
   (Take $g = 10$ m s$^{-2}$ and $R_e = 6400$ km)
   (a) $1.25 \times 10^{-3}$ rad s$^{-1}$ (b) $2.50 \times 10^{-3}$ rad s$^{-1}$
   (c) $3.75 \times 10^{-3}$ rad s$^{-1}$ (d) $5.0 \times 10^{-3}$ rad s$^{-1}$

6. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is $K$. If radius of the ball be $R$, then the fraction of total energy associated with its rotation will be
   (a) $\frac{K^2+R^2}{R^2}$ (b) $\frac{K^2}{R^2}$
   (c) $\frac{K^2}{K^2+R^2}$ (d) $\frac{R^2}{K^2+R^2}$

7. A uniform rod of mass $m$ and length $l$ makes a constant angle $\theta$ with an axis of rotation which passes through one end of the rod. Its moment of inertia about this axis is
   (a) $\frac{ml^2}{3}$ (b) $\frac{ml^2}{3}\sin\theta$
   (c) $\frac{ml^2}{3}\sin^2\theta$ (d) $\frac{ml^2}{3}\cos^2\theta$

8. A particle of mass $m$ is placed at the centre of a uniform spherical shell of mass $3m$ and radius $R$. The gravitational potential on the surface of the shell is

(a) $-\frac{Gm}{R}$ (b) $-\frac{3Gm}{R}$

(c) $-\frac{4Gm}{R}$ (d) $-\frac{2Gm}{R}$

9. A rope is wound round a hollow cylinder of mass 3 kg and radius 40 cm. If the rope is pulled with a force of 30 N, angular acceleration of the cylinder will be

(a) 10 rad $s^{-2}$ (b) 15 rad $s^{-2}$

(c) 20 rad $s^{-2}$ (d) 25 rad $s^{-2}$

10. Two point masses $A$ and $B$ having masses in the ratio 4 : 3 are separated by a distance of 1 m. When another point mass $C$ of mass $M$ is placed in between $A$ and $B$, the force between $A$ and $C$ is $\left(\frac{1}{3}\right)^{\text{rd}}$ of the force between $B$ and $C$. Then the distance of $C$ from $A$ is

(a) $\frac{2}{3}$ m (b) $\frac{1}{3}$ m (c) $\frac{1}{4}$ m (d) $\frac{2}{7}$ m

11. The diameter of a flywheel is 1 m. It has a mass of 20 kg. It is rotating about its axis with a speed of 120 rotations per minute. Its angular momentum in kg $m^2$ $s^{-1}$ is

(a) 13.4 (b) 31.4 (c) 41.4 (d) 43.4

12. Two bodies of masses $m_1$ and $m_2$ are initially at rest and infinite distance apart from each other. Now, they are allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a distance $r$ between them is

(a) $\sqrt{\frac{2G(m_1+m_2)}{r}}$ (b) $\sqrt{\frac{2Gm_1m_2}{(m_1+m_2)r}}$

(c) $\sqrt{\frac{G(m_1+m_2)}{r}}$ (d) $\sqrt{\frac{Gm_1m_2}{(m_1+m_2)r}}$

### Assertion & Reason Type

**Directions :** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion

(b) If both assertion and reason are true but reason is not the correct explanation of assertion

(c) If assertion is true but reason is false

(d) If both assertion and reason are false.

13. **Assertion :** The time period of revolution of a satellite close to surface of earth is smaller than that revolving far from surface of earth.

**Reason :** The square of time period of revolution of a satellite is directly proportional to cube of its orbital radius.

14. **Assertion :** Position of centre of mass is independent of the reference frame.

**Reason :** Centre of mass is same as centre of gravity.

15. **Assertion :** A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling motion).

**Reason :** For perfect rolling motion, work done against friction is zero.

## JEE MAIN / JEE ADVANCED

### Only One Option Correct Type

16. A uniform hollow sphere has internal radius $a$ and external radius $b$. Taking the potential at infinity be zero, the ratio of the gravitational potential at a point on the outer surface to that on the inner surface is

(a) $\frac{2(b^3-a^3)}{3b(b^2-a^2)}$ (b) $\frac{(b^3-a^3)}{3b(b^2-a^2)}$

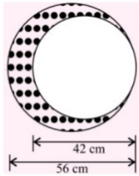
(c) $\frac{3b(b^2-a^2)}{2(b^3-a^3)}$ (d) $\frac{a(b^2-a^2)}{(b^3-a^3)}$

17. A carpet of mass $M$, made of inextensible, material is rolled along its length in the form of a cylinder of radius $R$ and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to $R/2$.

(a) $\sqrt{\frac{3}{2}Rg}$ (b) $\sqrt{3}Rg$ (c) $\sqrt{\frac{21}{2}Rg}$ (d) $\sqrt{\frac{14}{3}Rg}$

18. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in the figure. Find the position of the centre of mass of the remaining portion.

(a) 9 cm to right of centre of bigger circle
(b) 2.5 cm to left of centre of bigger circle
(c) 9 cm to left of centre of bigger circle
(d) 4.2 cm to right of centre of bigger circle



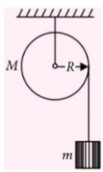

**19.** A cockroach, mass $m$, runs counterclockwise around the rim of a lazy Susan (a circular dish mounted on a vertical axle) of radius $R$ and rotational inertia $I$ with frictionless bearings. The cockroach's speed (relative to the earth) is $v$, whereas the lazy Susan turns clockwise with angular speed $\omega$. The cockroach finds a bread crumb on the rim and, of course, stops. What will be the angular speed of the lazy Susan after the cockroach stops?

(a) $\dfrac{I\omega - mvR}{I + mR^2}$ (b) $\dfrac{mvR - I\omega}{mR^2}$

(c) $\omega$ (d) $\dfrac{\omega(2m+1)}{m}$

## More than One Options Correct Type

**20.** A light thread with a body of mass $m$ tied to its end is wound on a uniform solid cylinder of mass $M$ and radius $R$ as shown in the figure. At a moment $t = 0$ the system is set in motion. Assuming the friction in the axle of the cylinder to be negligible, at any time $t$




(a) the angular velocity of the cylinder is $\dfrac{gt}{R}\left(1+\dfrac{M}{2m}\right)$

(b) the angular velocity of the cylinder is $\dfrac{gt}{R}\left(1+\dfrac{m}{2M}\right)$

(c) the kinetic energy of the whole system is $\dfrac{mg^2t^2}{2}\left(1+\dfrac{M}{2m}\right)$

(d) the kinetic energy of the whole system is $\dfrac{m^2g^2t^2}{2}\left(1+\dfrac{2m}{M}\right)$

**21.** A rocket is accelerated to speed of $2\sqrt{gR}$ near earth's surface and then coast upward. ($R$ is radius of the earth and $g$ is acceleration due to gravity.)

(a) Rocket will orbit the earth.
(b) Rocket will escape from the earth.
(c) Speed of the rocket at very far from the earth is $\sqrt{2gR}$.
(d) Speed of the rocket in the orbit is $\sqrt{gR/2}$.

**22.** A thin uniform rod of mass $m$ and length $l$ is free to rotate about its upper end. When it is at rest, it receives an impulse $J$ at its lowest point, normal to its length. Immediately after impact,

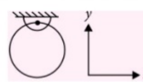
(a) the angular momentum of the rod is $Jl$
(b) the angular velocity of the rod is $3J/ml$
(c) the kinetic energy of the rod is $3J^2/2m$
(d) the linear velocity of the midpoint of the rod is $3J/m$.

**23.** A sphere of uniform density $\rho$ has within it a spherical cavity whose centre is at distance $a$ from the centre of the sphere. Then

(a) The gravitational field within the cavity is non-uniform.
(b) The gravitational field within the cavity is uniform.
(c) The magnitude of gravitational field within the cavity is $\dfrac{4\pi G\rho a^2}{3r}$.
(d) The magnitude of gravitational field within the cavity is $\dfrac{4\pi G\rho a}{3}$.

## Numerical Value Type

**24.** A bullet of mass $m$ moving with velocity $v_0(-\hat{k})$ strikes the bottom of a stationary vertical uniform ring of same mass $m$ and radius $R$ = 1 m. The ring lies in $xy$ plane with its topmost point hinged on the ceiling. The ring can rotate about $x$-axis. There is no friction between the hinge and the ring. The bullet gets embedded in the ring immediately after



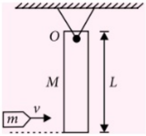
collision. Find the angular velocity of the system (in rad $s^{-1}$) just after collision. (Take $v_0 = 11$ m $s^{-1}$)

**25.** The gravitational potential energy of a satellite revolving around the earth in circular orbit is –4 MJ. Find the additional energy (in MJ) that should be given to the satellite so that it escapes from the gravitational field of earth. Assume earth's gravitational force to be the only gravitational force on the satellite and no atmospheric resistance.

**26.** A solid disc is rolling with slipping on a level surface at constant speed of 5 m $s^{-1}$. If the disc rolls up a 28° ramp, how far (in m) along the ramp will it move before it stops? (Given sin 28° = 0.4695)

## Comprehension Type

A rod of mass $M$ and length $L$ is suspended by a frictionless hinge at the point $O$ as shown in figure. A bullet of mass $m$ moving with velocity $v$ in a horizontal direction strikes the end of the rod and gets embedded in it.




**27.** The angular momentum of the system, about $O$ before collision is

(a) $mvL$ (b) $MvL$

(c) $\frac{1}{2}mvL$ (d) $\frac{1}{2}MvL$

**28.** The angular velocity acquired by the rod just after the collision is

(a) $\frac{mv}{(3m+M)L}$ (b) $\frac{2mv}{ML}$

(c) $\frac{3mv}{L}$ (d) $\frac{3mv}{(3m+M)L}$

## Matrix Match Type

**29.** A satellite is in a circular equatorial orbit of radius 7000 km around the earth. If it is transferred to a circular orbit of double the radius then match the entries of column I with those given in column II.

| | Column I | | Column II |
|---|---|---|---|
| (A) | Angular momentum | (P) | Increases |
| (B) | Area of earth covered by satellite signal | (Q) | Decreases |
| (C) | Potential energy | (R) | Becomes double |
| (D) | Kinetic energy | (S) | Becomes half |

| | A | B | C | D |
|---|---|---|---|---|
| (a) | S | Q, S | Q | S |
| (b) | P | P | P, S | Q, S |
| (c) | P | S | Q, S | P |
| (d) | Q | P | P, S | R |

**30.** In case of pure rolling of a rigid body of radius $R$ on a stationary horizontal surface with an angular velocity ω and with $v_0$ as the velocity of its centre of mass, match the entries in column I with those given in column II.

| | Column I | | Column II |
|---|---|---|---|
| (A) | Distance moved by the CM of the body in one full rotation while slipping forward | (P) | $\sqrt{2}\,v_0$ |
| (B) | The speed of the bottom-most point on its circumference | (Q) | $2v_0$ |
| (C) | The speed of the topmost point on its circumference | (R) | $> 2\pi R$ |
| (D) | The speed of a point on the circumference at angle 90° with line joining the bottom-most point and the centre of the body | (S) | zero |

| | A | B | C | D |
|---|---|---|---|---|
| (a) | S | Q | R | P |
| (b) | Q | P | Q | R |
| (c) | R | S | Q | P |
| (d) | P | P | R | S |

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*Keys are published in this issue. Search now!* ☺

**SELF CHECK**

| | |
|---|---|
| No. of questions attempted | ...... |
| No. of questions correct | ...... |
| Marks scored in percentage | ...... |

**Check your score! If your score is**

| | | |
|---|---|---|
| > 90% | EXCELLENT WORK ! | You are well prepared to take the challenge of final exam. |
| 90-75% | GOOD WORK ! | You can score good in the final exam. |
| 74-60% | SATISFACTORY ! | You need to score more next time |
| < 60% | NOT SATISFACTORY! | Revise thoroughly and strengthen your concepts. |

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