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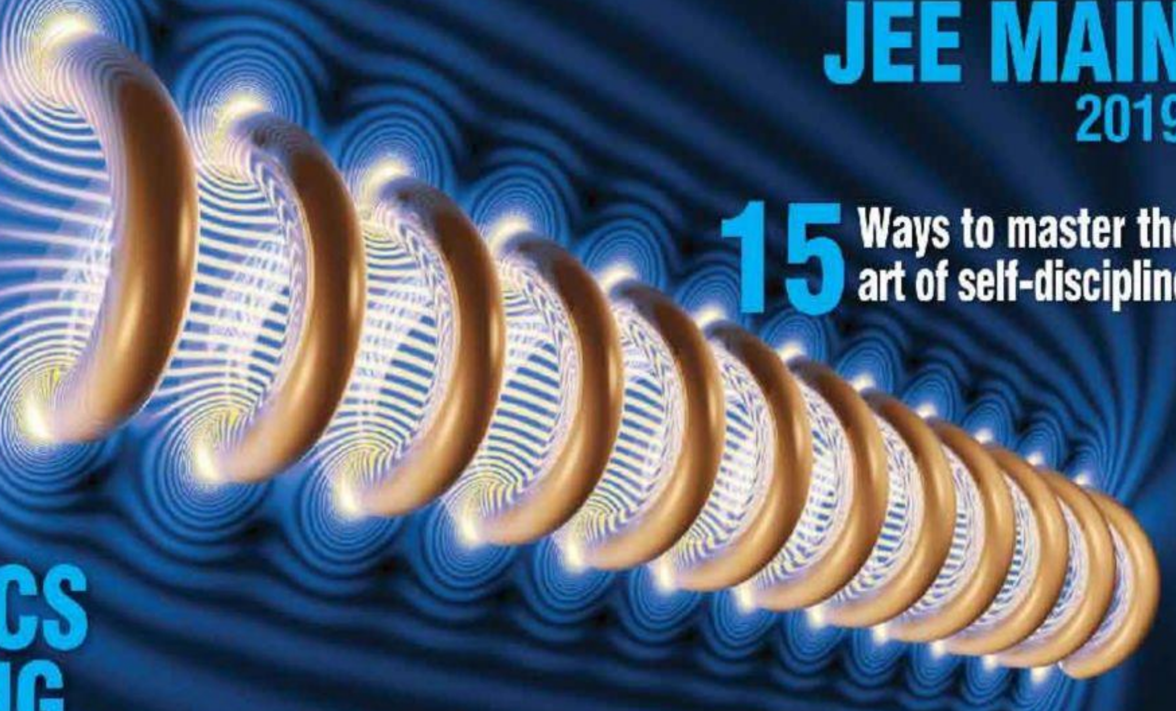

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FOCUS NEET/JEE 2019

Class XI

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UNIT - 4 : SYSTEM OF PARTICLES AND ROTATIONAL MOTION

Centre of Mass

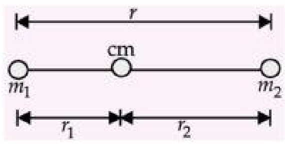
The centre of mass of a rigid body is an imaginary point at a fixed position with respect to the body as a whole. The position of centre of mass of a rigid body depends on two factors.

(i) The geometrical shape of the body.

(ii) The distribution of mass in the body.

Position of Centre of Mass (COM) of Two Particles

Centre of mass of two particles of mass m_1 and m_2 separated by a distance r lies in between the two particles. The distance of centre of mass from any of the particle (r) is inversely proportional to the mass of the particle (m)



i.e. $r \propto \frac{1}{m}$

or $\frac{r_1}{r_2} = \frac{m_2}{m_1}$

or $m_1 r_1 = m_2 r_2$

or $r_1 = \left(\frac{m_2}{m_2 + m_1}\right) r$ and $r_2 = \left(\frac{m_1}{m_1 + m_2}\right) r$

Here, r_1 = distance of COM from m_1

and r_2 = distance of COM from m_2

From the above discussion, we see that

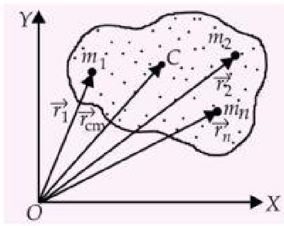
$r_1 = r_2 = 1/2\, r$ if $m_1 = m_2$, *i.e.*, COM lies midway between the two particles of equal masses.

Centre of Mass of a System of 'N' Discrete Particles

Consider a system of N point masses m_1, m_2, m_3, ... m_n whose position vectors from origin O are given by $\vec{r}_1, \vec{r}_2, \vec{r}_3, ...\ \vec{r}_n$ respectively. Then the position vector of the centre of mass C of the system is given by

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n}$$

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i}$$



$$\vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

where $M \left(= \sum_{i=1}^{n} m_i\right)$ is the total mass of the system.

Centre of Mass of a Continuous Mass Distribution

For continuous mass distribution the centre of mass can be located by replacing summation sign with an integral sign. Proper limits for the integral are chosen according to the situation

$$x_{cm} = \frac{\int x\,dm}{\int dm}, y_{cm} = \frac{\int y\,dm}{\int dm}, z_{cm} = \frac{\int z\,dm}{\int dm}$$

$\int dm = M$(mass of the body)

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm.$$

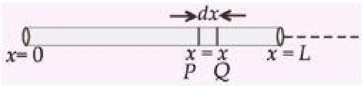
Note: If an object has symmetric uniform mass distribution about x axis then y coordinate of COM is zero and vice-versa

Centre of Mass of a Uniform Rod

Suppose a rod of mass M and length L is lying along the x-axis with its one end at $x = 0$ and the other at $x = L$.

Mass per unit length of the rod = M/L

Hence, dm, (the mass of the element dx situated at $x = x$ is) $= \frac{M}{L}dx$



The coordinates of the element PQ are $(x, 0, 0)$. Therefore, x-coordinate of COM of the rod will be

$$x_{cm} = \frac{\int_0^L x\,dm}{\int dm} = \frac{\int_0^L (x)\left(\frac{M}{L}dx\right)}{M} = \frac{1}{L}\int_0^L x\,dx = \frac{L}{2}$$

The y-coordinate of COM is $y_{cm} = \frac{\int y\,dm}{\int dm} = 0$

Similarly, $z_{cm} = 0$

i.e., the coordinates of COM of the rod are $\left(\frac{L}{2}, 0, 0\right)$.

Or it lies at the centre of the rod.

Rigid Body

A rigid body is one whose geometric shape and size remains unchanged under the action of any external force. When a rigid body performs rotational motion, the particles of the body move in circles. The centres of these circles lie on a straight line called the axis of rotation, which is fixed and perpendicular to the planes of circle.

Torque and Angular Momentum

Torque is the turning effect of a force. If a force acting on a object has a tendency to rotate the body about an axis, the force is said to exert a torque on the body. It is a vector quantity. In vector form,

Torque, $\tau = \vec{r} \times \vec{F}$

In magnitude, $\tau = r\,F \sin\theta$.

Here θ is the angle between $\vec{r}$ and $\vec{F}$.

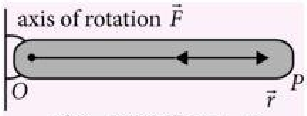
Torque has the same dimensions as that of work *i.e.* $[ML^2T^{-2}]$. But work is a scalar quantity whereas torque is a vector quantity.

By convention, anticlockwise moments are taken as positive and clockwise moments are taken as negative.

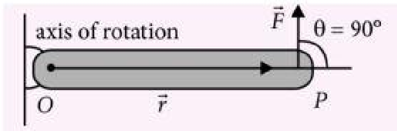
Special Cases

- 

If $\theta = 0°$, then $\tau = 0$ (the rod will not rotate)

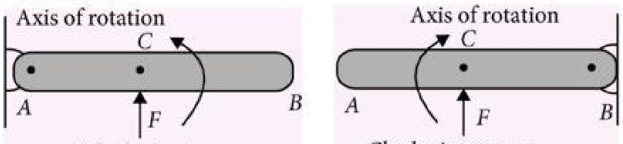
- 

If $\theta = 180°$, then $\tau = 0$ (the rod will not rotate)

- 

If $\theta = 90°$, then $\tau = rF$ (maximum torque)

Note : Same force acting at the same point can produce either anticlockwise or clockwise torque depending upon the location of the axis of rotation as shown in the figure.



Anticlockwise torque   Clockwise torque

Work Done by Torque

Work done, W = torque × angular displacement

$= \tau \times \Delta\theta$

Power, $P = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$

Angular momentum of a particle about a given axis is the moment of linear momentum of the particle about that axis. It is denoted by symbol $\vec{L}$.

Angular momentum $\vec{L} = \vec{r} \times \vec{p}$

In magnitude, $L = rp \sin\theta$

where θ is the angle between $\vec{r}$ and $\vec{p}$.

Angular momentum is a vector quantity. Its SI unit is kg $m^2 s^{-1}$. Its dimensional formula is $[ML^2T^{-1}]$.

Relationship between Torque and Angular Momentum

Rate of change of angular momentum of a body is equal to the external torque acting upon the body.

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

Law of Conservation of Angular Momentum

If no external torque acts on a system then the total angular momentum is conserved.

$$i.e., \ \vec{\tau}_{ext} = 0 \ \text{ then } \ \frac{d\vec{L}}{dt} = 0 \ \text{ or } \ \vec{L} = \text{constant}$$

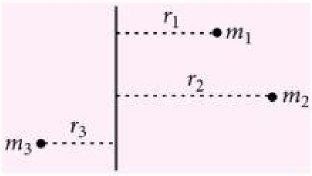
Equilibrium of Rigid Body

A rigid body is in mechanical equilibrium, if it is in translational equilibrium *i.e.* the total external force on it is zero. *i.e.* $\Sigma F_i = 0$.

Abody is it is in rotational equilibrium, if the total external torque on it is zero, *i.e.* $\Sigma \tau_i = 0$.

Moment of Inertia

Moment of inertia of a rigid body about a given axis of rotation is defined as the sum of the products of masses of the various particles and square of their respective perpendicular distances from the axis of rotation.



It is denoted by symbol I and is given by $I = \sum_{i=1}^{N} m_i r_i^2$

Moment of inertia is a quantity. Its S.I. unit is kg m^2.

Factors on which the moment of inertia depends :

(i) mass of the body, its shape and distribution of mass about the axis of rotation

(ii) position and orientation from the axis of rotation.

Radius of Gyration

It is defined as the distance from the axis of rotation at which, if whole mass of the body were concentrated, the moment of inertia of the body would be same as with the actual distribution of the mass of body. It is denoted by symbol K.

The SI unit of radius of gyration is metre.

Radius of gyration of a body about an axis of rotation may also be defined as the root mean square distance of the particles from the axis of rotation.

$$i.e., \ K = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}}$$

The moment of inertia of a body about a given axis is equal to the product of mass of the body and square of its radius of gyration about that axis.

i.e., $I = MK^2$.

Theorems of Perpendicular and Parallel Axes

The moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body is called perpendicular axes theorem.

$I_z = I_x + I_y$

where x and y are two perpendicular axes in the plane and z axis is perpendicular to its plane.

The moment of inertia of a body about an axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes is called parallel axis theorem.

$I = I_c + Md^2$

where I_c is the moment of inertia of the body about an axis passing through the centre of mass and d is the perpendicular distance between two parallel axes.

Moment of Inertia and Radius of Gyration of Some Regular Bodies About Specific Axis is Given Below

S.No.	Body	Axis of rotation	Moment of inertia (I)	Radius of gyration (K)
1.	Uniform circular ring of mass M and radius R	(i) about an axis passing through centre and perpendicular to its plane	MR^2	R
		(ii) about a diameter	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(iii) about a tangent in its own plane	$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
		(iv) about a tangent perpendicular to its plane	$2MR^2$	$R\sqrt{2}$

2.	Uniform circular disc of mass M and radius R	(i) about an axis passing through centre and perpendicular to its plane	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(ii) about a diameter	$\frac{1}{4}MR^2$	$R/2$
		(iii) about a tangent in its own plane	$\frac{5}{4}MR^2$	$\sqrt{5}\frac{R}{2}$
		(iv) about a tangent perpendicular to its own plane	$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
3.	Solid sphere of radius R and mass M	(i) about its diameter	$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$
		(ii) about a tangential axis	$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$
4.	Hollow sphere of radius R and mass M	(i) about its diameter	$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$
		(ii) about a tangential axis	$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$
5.	Solid cylinder of length l, radius R and mass M	(i) about its own axis	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(ii) about an axis passing through centre of mass and perpendicular to its own axis	$M\left[\frac{l^2}{12}+\frac{R^2}{4}\right]$	$\sqrt{\frac{l^2}{12}+\frac{R^2}{4}}$
		(iii) about the diameter of one of the faces of cylinder	$M\left[\frac{l^2}{3}+\frac{R^2}{4}\right]$	$\sqrt{\frac{l^2}{3}+\frac{R^2}{4}}$
6.	Hollow cylinder of mass M and radius R	(i) about its own axis	MR^2	R
		(ii) about an axis passing through its centre of mass and perpendicular to its own axis	$M\left(\frac{R^2}{2}+\frac{l^2}{12}\right)$	$\sqrt{\frac{R^2}{2}+\frac{l^2}{12}}$
7.	Thin rod of length L	(i) about an axis through centre of mass and perpendicular to the rod	$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$
		(ii) about an axis through one end and perpendicular to rod	$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$
8.	Rectangular lamina of length l and breadth b	about an axis passing through its centre of mass and perpendicular to plane	$M\left[\frac{l^2+b^2}{12}\right]$	$\sqrt{\frac{l^2+b^2}{12}}$
9.	Uniform cone of radius R and height h	about an axis through its centre of mass and joining its vertex to centre of base	$\frac{3}{10}MR^2$	$R\sqrt{\frac{3}{10}}$
10.	Parallelopiped of length l, breadth b and height h, mass M	about its central axis	$M\left(\frac{l^2+b^2}{12}\right)$	$\sqrt{\frac{l^2+b^2}{12}}$

Relation between Angular Momentum and Moment of Inertia

Angular momentum, $L = I\omega$

The angular acceleration α of a rigid body rotating about a fixed axis is given by $I\alpha = \tau$. If the external torque τ is zero, the component of angular momentum about the fixed axis $I\omega$ of such a rotating body is constant.

A Rolling Rigid Body

Kinetic energy of rotational motion, $K_R = \frac{1}{2}I\omega^2$.

Kinetic energy of a rolling body = translational kinetic energy (K_T) + rotational kinetic energy (K_R).

$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2\left[1 + \frac{K^2}{R^2}\right]$$

When a body rolls down an inclined plane of inclination θ without slipping its velocity at the bottom of incline is given by $v = \sqrt{\dfrac{2gh}{1 + \dfrac{K^2}{R^2}}}$

where h is the height of the incline.

When a body rolls down on an inclined plane without slipping, its acceleration down the inclined plane is given by $a = \dfrac{g\sin\theta}{1 + \dfrac{K^2}{R^2}}$.

When a body rolls down on an inclined plane without slipping, time taken by the body to reach the bottom is given by $t = \sqrt{\dfrac{2l\left(1 + \dfrac{K^2}{R^2}\right)}{g\sin\theta}}$

where l is the length of the inclined plane.

Analogy between Translational Motion and Rotational Motion

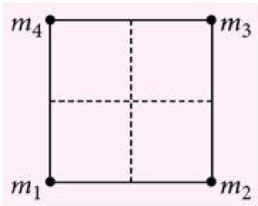
	Translational motion	Rotational motion about a fixed axis
1.	Displacement x	Angular displacement θ
2.	Velocity $v = dx/dt$	Angular velocity $\omega = d\theta/dt$
3.	Acceleration $a = dv/dt$	Angular acceleration $\alpha = d\omega/dt$
4.	Mass M	Moment of inertia I
5.	Force $F = Ma$	Torque $\tau = I\alpha$
6.	Work $dW = Fds$	Work $dW = \tau d\theta$
7.	Kinetic energy of a translational motion $K_T = Mv^2/2$	Kinetic energy of a rotational motion $K_R = I\omega^2/2$
8.	Power $P = Fv$	Power $P = \tau\omega$
9.	Linear momentum $p = Mv$	Angular momentum $L = I\omega$
10.	Equations of translational motion (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 - u^2 = 2as$ (iv) $s_{nth} = u + \frac{a}{2}(2n-1)$	Equations of rotational motion $\omega = \omega_0 + \alpha t$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 - \omega_0^2 = 2\alpha\theta$ $\theta_{nth} = \omega_0 + \frac{\alpha}{2}(2n-1)$

Thought Provoking Points

- Moment of inertia of a body has different values in different directions and as such it is not a scalar. Futher, it is not a vector either as its value about a given axis remains the same whether the direction of rotation is clockwise or anticlockwise, i.e., direction of rotation need not be specified. In fact, it is tensor quanity.
- Rolling motion is possible only if a frictional force is present between the rolling body and incline to produce a net torque about the centre of mass.
- The angular velocity of rotating body is the same relative to any point on it.
- If sense of angular velocity and angular acceleration is same, the rigid body is speeding up and if they have opposite sence, the body is slowing down.
- If an object is uncostrained (*i.e.*, not an pivot)a couple causes the object to rotate about its centre of mass.
- The gravitational torque on any extended object is equivalent to the torque of a single force, gravitational force, acting at the objects centre of mass. We can treat the object as if all its mass were concentrated at the centre of mass.
- Rotational kinetic energy is maximum for ring and minimum for solid and moving with same speed.

SPEED PRACTICE

1. Four particles of mass $m_1 = 2m$, $m_2 = 4m$, $m_3 = m$ and m_4 are placed at four corners of a square. What should be the value of m_4 so that the centre of mass of all the four particles are exactly at the centre of the square?



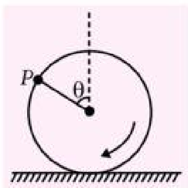
(a) $2\,m$ (b) $8\,m$
(c) $6\,m$ (d) None of these

2. For the same total mass, which of the following will have the largest moment of inertia about an axis passing through the centre of gravity and perpendicular to the plane of body?
(a) A ring of radius l
(b) A disc of radius l
(c) A square lamina of side $2l$
(d) Four rods forming square of side $2l$

3. A ring rolls down, starting from rest, down an inclined plane of length l and inclination θ. The velocity of the centre of mass of the ring at the mid point of the inclined plane is
(a) $\sqrt{gl\sin\theta}$ (b) $\sqrt{\frac{gl\sin\theta}{2}}$
(c) $\sqrt{2gl\sin\theta}$ (d) $\sqrt{\frac{4}{7}gl\sin\theta}$

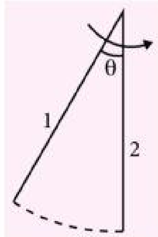
4. If a disc slides from top to bottom of an inclined plane, it takes time t_1. If it rolls, it takes time t_2. Now, $\frac{t_2^2}{t_1^2}$ is
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{2}{5}$

5. A wheel is rolling straight on ground without slipping. If the axis of the wheel has speed v, the instantaneous velocity of a point P on the rim, defined by angle θ, relative to the ground will be



(a) $v\cos\theta/2$ (b) $2v\cos\theta/2$
(c) $v(1 + \sin\theta)$ (d) $v(1 + \cos\theta)$.

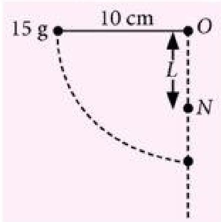
6. A uniform rod of length l is free to rotate in a vertical plane about a fixed horizontal axis through its centre. The rod begins rotation from rest from its unstable equilibrium position. When it has turned through an angle θ, its angular velocity ω is given as



(a) $\sqrt{\frac{6g}{l}}\sin\theta$ (b) $\sqrt{\frac{6g}{l}}\sin\frac{\theta}{2}$
(c) $\sqrt{\frac{6g}{l}}\cos\frac{\theta}{2}$ (d) $\sqrt{\frac{6g}{l}}\cos\theta$

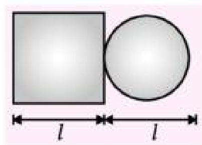
7. A wheel of radius r and mass m stands in front of a step of height h. The least horizontal force which should be applied to the axle of the wheel to allow it to raise on to the step is
(a) $\frac{mgh(2r-h)}{r-h}$ (b) $mgh\,(r-h)$
(c) $\frac{mg\sqrt{h(2r-h)}}{r-h}$ (d) none of these

8. A ball weighing 15 g is tied to a string 10 cm long. Initially the ball is held in position such that the string is horizontal. The ball is now released. A nail N is situated vertically below the support at a distance L. The minimum value of L such that the string will be wound round the nail is



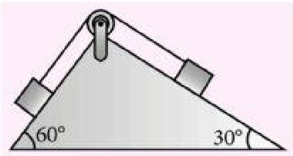
(a) 2 cm (b) 4 cm (c) 6 cm (d) 8 cm

9. If the density of the material of a square plate and a circular plate shown in the figure is the same, the center of mass of the composite system will be



(a) inside the square plate
(b) inside the circular plate
(c) at the point of contact
(d) outside the system

10. Two blocks of equal mass are tied with a light string which passes over a massless pulley as shown in the figure.

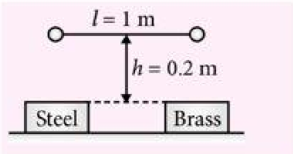


The magnitude of acceleration of centre of mass of both blocks is (neglect friction everywhere)

(a) $\left(\frac{\sqrt{3}-1}{4\sqrt{2}}\right)g$ (b) $\left(\frac{\sqrt{3}+1}{4\sqrt{2}}\right)g$

(c) $\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)g$ (d) $\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)g$

11. Two steel balls of equal diameter are connected by a rigid bar of negligible weight as shown and are dropped in the horizontal position from height h above the heavy steel and brass base plates. If the coefficient of restitution between the ball and steel base is 0.6 and that between the other ball and the brass base is 0.4. The angular velocity of the bar immediately after rebound is (Assume the two impacts are simultaneous).



(a) $\frac{2}{5}$ rad s^{-1} (b) $\frac{1}{5}$ rad s^{-1}

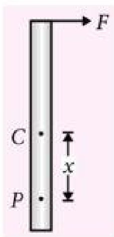
(c) $\frac{3}{5}$ rad s^{-1} (d) $\frac{1}{4}$ rad s^{-1}

12. A uniform rod of mass m and length l_0 is rotating with a constant angular speed ω about a vertical axis passing through its point of suspension. The moment of inertia of the rod about the axis of rotation if it makes an angle θ to the vertical (axis of rotation) is

(a) $\frac{ml_0^2\sin^2\theta}{12}$ (b) $\frac{ml_0^2\sin^2\theta}{6}$

(c) $\frac{2ml_0^2\sin^2\theta}{3}$ (d) $\frac{ml_0^2\sin^2\theta}{3}$

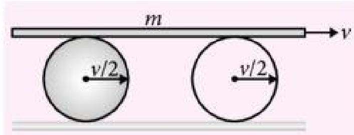
13. A rod of mass m is placed on smooth table. For what value of x, the point P on the rod of length l = 6 m has zero acceleration if a force F is applied at the end of rod as shown in figure.



(a) 1 m (b) 2 m

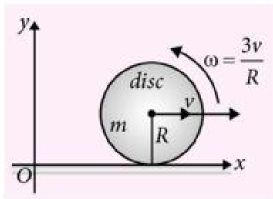
(c) 2.5 m (d) 1.5 m

14. A rod of mass m is placed on a solid and hollow spheres each of mass m. If the speed of the plate is v, assuming pure rolling of the spheres with all contacting surfaces, the kinetic energy of the system (spheres + plate) will be



(a) $\frac{43}{50}mv^2$ (b) $\frac{33}{50}mv^2$ (c) $\frac{43}{60}mv^2$ (d) $\frac{53}{60}mv^2$

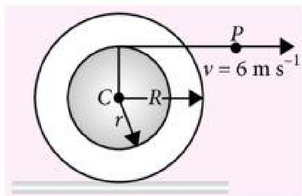
15. The angular momentum of the disc which spins with $\vec{\omega} = \frac{3v}{R}\hat{k}$ and its CM moves with a velocity $\vec{v} = v\hat{i}$ about O will be



(a) $\frac{3mvR}{2}\hat{k}$ (b) $\frac{mvR}{2}\hat{k}$

(c) $-\frac{3mvR}{2}\hat{k}$ (d) $-\frac{mvR}{2}\hat{k}$

16. A cotton reel rolls without sliding such that the point P of the string has velocity v = 6 m s^{-1}. If r = 10 cm and R = 20 cm, then the velocity of its centre C is



(a) 2.5 m s^{-1} (b) 5 m s^{-1}

(c) 4 m s^{-1} (d) 2 m s^{-1}

17. A homogenous rod of length $l = \eta x$ and mass M is lying on a smooth horizontal floor. A bullet of mass m hits the rod at a distance x from the middle of the rod at a velocity v_0 perpendicular to the rod and comes to rest after collision. If the velocity of the farther end of the rod just after the impact is in the opposite direction of v_0, then

(a) $\eta > 3$ (b) $\eta < 3$ (c) $\eta > 6$ (d) $\eta < 6$

18. A uniform rod of length L and mass M is held vertical, with its bottom end pivoted to the floor. The rod falls under gravity, freely turning about the pivot. If acceleration due to gravity is g, what is the instantaneous angular speed of the rod when it makes an angle 60° with the vertical

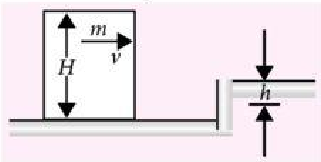
(a) $\left(\frac{g}{L}\right)^{1/2}$ (b) $\left(\frac{3g}{4L}\right)^{1/2}$

(c) $\left(\frac{3\sqrt{3}g}{2L}\right)^{1/2}$ (d) $\left(\frac{3g}{2L}\right)^{1/2}$

19. A solid sphere is rotating abouit a diameter at an angular velocity ω. If it cools so that its radius reduces to $\frac{1}{n}$ of its original value, its angular velocity becomes

(a) $\frac{\omega}{n}$ (b) $\frac{\omega}{n^2}$ (c) $n\omega$ (d) $n^2\omega$

20. A cube of mass m and height H slides with a speed v. It strikes the obstacle of height $h = \frac{H}{4}$. The speed of the CM of the cube just after the collision is



(a) $\frac{\sqrt{5}}{6}v$ (b) $\frac{\sqrt{3}}{6}v$ (c) $\frac{\sqrt{5}}{4}v$ (d) $\frac{\sqrt{3}}{4}v$

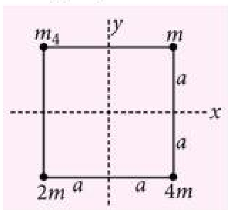
SOLUTIONS

1. (d): $x_{CM} = 0$

or $\frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4} = 0$

or $(2m)(-a) + 4m(a) + m(a) + m_4(-a) = 0$

or $m_4 = 3m$



Similarly $y_{CM} = 0$

or $(2m)(-a) + 4m(-a) + m(a)$ $+ m_4(a) = 0$ or $m_4 = 5m$

Since, value of m_4 are different to satisfy by both $x_{CM} = 0$ and $y_{CM} = 0$

Hence, it is not possible.

2. (d): $I_{ring} = Ml^2$

$I_{Disc} = \frac{Ml^2}{2}$

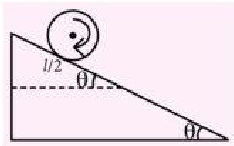
$I_{square} = \frac{M(2l)^2}{6} = \frac{2}{3}Ml^2$ (Using $\perp r$ axis theorem)

$I_{four\ rods} = 4\left[\left(\frac{M}{4}\frac{(2l)^2}{12} + \frac{M}{4}(l)^2\right)\right] = \frac{4Ml^2}{12} + Ml^2$

$= \frac{Ml^2}{3} + Ml^2 = \frac{4}{3}Ml^2$.

Thus, $I_{four\ rods}$ has the largest moment of inertia.

3. (b): Loss in PE = $Mg\left(\frac{l}{2}\sin\theta\right)$



Gain in KE

$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

For ring, $I = MR^2$, $\omega = \frac{v}{R}$

$\Rightarrow \frac{1}{2}I\omega^2 = \frac{1}{2}MR^2 \times \frac{v^2}{R^2} = \frac{1}{2}Mv^2$

Hence gain in K.E. = $\frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = Mv^2$

$\Rightarrow Mv^2 = Mg\frac{l}{2}\sin\theta \Rightarrow v = \sqrt{\frac{gl\sin\theta}{2}}$.

4. (c): The acceleration of disc rolling down on an inclined plane is given by,

$a_{roll} = \frac{g\sin\theta}{1 + \frac{I_{disc}}{MR^2}} = \frac{g\sin\theta}{1 + \frac{MR^2/2}{MR^2}} \Rightarrow a_{roll} = \frac{2}{3}g\sin\theta$

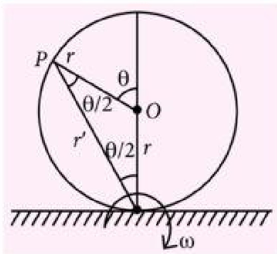
Also, $a_{slide} = g\sin\theta$

$\frac{a_{roll}}{a_{slide}} = \frac{2}{3}$

$s = \frac{1}{2}(a_{slide})t_1^2 = \frac{1}{2}(a_{roll})t_2^2 \Rightarrow a_{slide} \times t_1^2 = a_{roll} \times t_2^2$

$\Rightarrow \frac{t_2^2}{t_1^2} = \frac{a_{slide}}{a_{roll}} = \frac{3}{2}$

5. (b): $v_P = \omega r'$



As is evident, $r' = 2\left(r\cos\frac{\theta}{2}\right)$

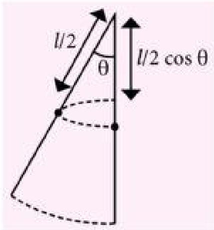
$\Rightarrow v_p = \omega\left(2r\cos\frac{\theta}{2}\right) = 2\cos\frac{\theta}{2}(\omega r)$

$= 2\cos\frac{\theta}{2}(v_{CM}) \Rightarrow v_P = 2v\cos\frac{\theta}{2}$

MONTHLY TUNE UP CLASS XII ANSWER KEY

1. (b)	2. (b)	3. (c)	4. (b)	5. (d)
6. (a)	7. (b)	8. (a)	9. (c)	10. (a)
11. (b)	12. (b)	13. (b)	14. (a)	15. (b)
16. (c)	17. (a)	18. (a)	19. (b)	20. (a,c,d)
21. (a,b,d)	22. (a,b,c)	23. (a,c)	24. (30)	25. (8)
26. (3.2)	27. (d)	28. (a)	29. (c)	30. (b)

6. (b) : The CM of the rod has fallen by $\frac{l}{2}(1-\cos\theta)$.



$\therefore \quad \Delta U = -\text{Mg}\left[\frac{l}{2}(1-\cos\theta)\right];$

$\Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{Ml^2}{3}\right)\cdot\omega^2$

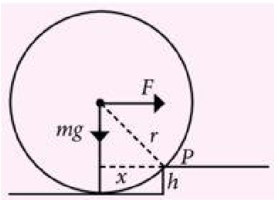
$\Delta U + \Delta K = 0$ or $\Delta K = -\Delta U$

$\Rightarrow \frac{1}{2}\left(\frac{Ml^2}{3}\right)\cdot\omega^2 = \text{Mg}\left(\frac{l}{2}\cdot(1-\cos\theta)\right)$

$\omega^2 = \frac{3g(1-\cos\theta)}{l}$

$\omega^2 = \frac{6g\sin^2\left(\frac{\theta}{2}\right)}{l} \Rightarrow \omega = \sqrt{\frac{6g}{l}}\cdot\sin\left(\frac{\theta}{2}\right)$

7. (c) : Torque due to F must be more than torque due to mg about P.



$F_{\min}(r-h) = mgx$

$F_{\min}(r-h)$

$= mg\sqrt{r^2-(r-h)^2}$

$F_{\min} = \frac{mg\sqrt{r^2-(r^2+h^2-2rh)}}{(r-h)} = \frac{mg\sqrt{h(2r-h)}}{(r-h)}.$

8. (c) : For string to be wound around the nail, $v \geq \sqrt{5gr}$

$\Rightarrow \quad v \geq \sqrt{5g(10-L)}$

Now $\frac{1}{2}mv^2 = mg(10) \Rightarrow v^2 = 2g\,(10)$

$\Rightarrow \quad \sqrt{2g\cdot 10} \geq \sqrt{5g(10-L)}$

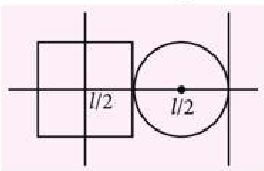
$20g \geq 5g(10-L) \Rightarrow 20 \geq 50 - 5L \Rightarrow 5L \geq 30$

$L \geq 6$

$L_{\min} = 6$ cm.

9. (a) : Mass of square plate $M_1 = l^2\sigma$

Mass of circular disc $M_2 = \pi\frac{l^2}{4}\sigma$



$x_{CM} = \frac{M_1\times 0 + \frac{\pi l^2\sigma l}{4}}{l^2\sigma\left[1+\frac{\pi}{4}\right]} \Rightarrow x_{CM} = \frac{\pi l}{4\frac{(\pi+4)}{4}} = \frac{\pi l}{\pi+4}$

$x_{CM} < \frac{l}{2}$, *i.e.*, COM lies in the square

10. (a) : Acceleration of system

$a = \frac{mg\sin 60° - mg\sin 30°}{2m}$

Here m = mass of each block

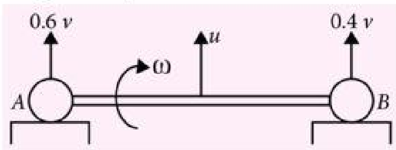
or $\quad a = \left(\frac{\sqrt{3}-1}{4}\right)g$

Now $\vec{a}_{CM} = \frac{m\vec{a}_1 + m\vec{a}_2}{2m}$

Here $\vec{a}_1$ and $\vec{a}_2$ are $\left(\frac{\sqrt{3}-1}{4}\right)g$ at the right angles.

Hence, $|\vec{a}_{CM}| = \frac{\sqrt{2}}{2}a = \left(\frac{\sqrt{3}-1}{4\sqrt{2}}\right)g$

11. (a) : If v is the velocity by which balls strike the bases, then their velocities after collision are 0.6 v and 0.4 v as shown in the figure 'u' is the velocity of COM just after collision and ω is the angular velocity acquired by the rod.



For end A : $0.6v = u + \frac{l}{2}.\omega$ ...(i)

For end B : $0.4v = u - \frac{l}{2}.\omega$ ...(ii)

Subtracting (i) and (ii) we get

$0.2v = \omega l \Rightarrow \omega = \frac{v}{5l}$

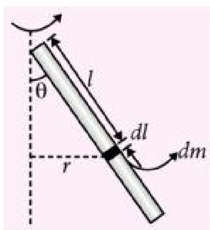
Now using $[v^2 = u^2 + 2as]$

$v^2 = 0 + 2\times 10\times 0.2 = 4 \Rightarrow v = 2$ m s^{-1}

$\omega = \frac{2}{5\times 1} = \frac{2}{5}$ rad s^{-1}

12. (d) : We can observe each and every element of rod in rotating with different radius about the axis of rotation.

Take an elementary mass dm of the rod.



$dm = \frac{m}{l_0} dl$

The moment of inertia of the elementary mass is given as

$dI = (dm)r^2$

The moment of inertia of the rod,

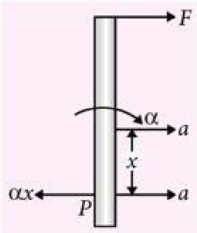
$I = \int dI \Rightarrow I = \int r^2 dm$

Substituting $r = l \sin\theta$ and $dm = \frac{m}{l_0}.dl$ we obtain

$$I = \int (l^2 \sin^2\theta)\frac{m}{l_0} dl = \frac{m\sin^2\theta}{l_0}\int_0^{l_0} l^2 dl = \frac{ml_0^3}{3l_0}\sin^2\theta$$

$$\Rightarrow \quad I = \frac{ml_0^2 \sin^2\theta}{3}$$

13. (a) : Acceleration of the rod $a = \frac{F}{m}$



Angular acceleration of the rod

$\alpha = \frac{\tau}{I} = \frac{Fl/2}{ml^2/12} = \frac{6F}{ml}$

Net acceleration of point P,

$a_P = a - \alpha x = 0$

$\Rightarrow \quad \frac{F}{m} - \frac{6F}{ml}x = 0 \Rightarrow x = \frac{l}{6} = 1\text{ m}$

14. (d): The total KE is $K = K_{rod} + K_{hollow\ shere} + K_{solid\ sphere}$

where $K_{rod} = \frac{1}{2}mv^2$ ...(i)

Since the CM of each sphere moves with a velocity $v_{CM} = \frac{v}{2}$,

$$K_{hollow\,sphere} = \frac{1}{2}mv_{CM}^2\left(1+\frac{K^2}{R^2}\right)$$

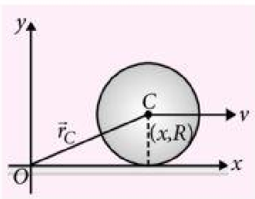
$$= \frac{1}{2}m\left(\frac{v}{2}\right)^2\left(1+\frac{2}{3}\right) = \frac{5mv^2}{24} \quad \text{... (ii)}$$

$$K_{solid\,sphere} = \frac{1}{2}m\left(\frac{v}{2}\right)^2\left(1+\frac{2}{5}\right) = \frac{7}{40}mv^2 \quad \text{...(iii)}$$

Adding equation (i), (ii) and (iii)

$$k = \frac{1}{2}mv^2 + \frac{5}{24}mv^2 + \frac{7}{40}mv^2 = \frac{53}{60}mv^2$$

15. (b): The angular momentum of the disc about O is



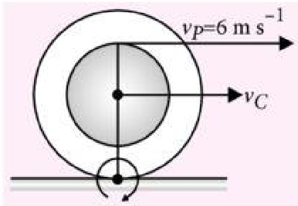
$$\vec{L}_O = m\vec{r}_C \times \vec{v}_C + I_C\vec{\omega}$$

$$= m(x\hat{i}+R\hat{j})\times v\hat{i} + \frac{1}{2}mR^2\left(\frac{3v}{R}\hat{k}\right)$$

$$= m(x\hat{i}+R\hat{j})\times v\hat{i} + \frac{1}{2}mR^2\left(\frac{3v}{R}\hat{k}\right)mvR(\hat{j}\times\hat{i})$$

$$+\frac{3}{2}mvR\hat{k} - mvR\hat{k} + \frac{3}{2}mvR\hat{k} = \frac{mvR}{2}\hat{k}$$

16. (c) : As cotton reel rolls point of contact of reel with ground will act as instantaneous axis of rotation.



$v_P = \omega(R + r)$

$\Rightarrow \quad \omega = \frac{v_P}{R+r} = 20\text{ rad s}^{-1}$

Velocity of centre of reel

$$v_C = \omega R = \frac{v_p R}{(R+r)} = \frac{6\times 20}{(20+10)} = 4\text{ m s}^{-1}$$

17. (d): Let after collision velocity of rod be v and angular velocity be ω then

$$mv_0 = mv + MV \Rightarrow mv_0 x = mvx + \frac{Ml^2}{12}\omega$$

$$mv_0 = mv + \frac{M\omega\eta^2 x}{12}$$

From above equations,

$$\frac{M\omega\eta^2 x}{12} = MV \Rightarrow \omega\eta^2 x = 12V$$

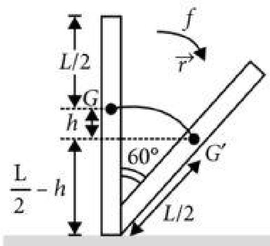
Velocity of father end of the rod $= V - \frac{l\omega}{2}$

$$= \frac{\omega\eta^2 x}{12} - \frac{\omega\eta x}{2} = \frac{\omega\eta x}{2}\left(\frac{\eta}{6}-1\right)$$

If this velocity is opposite to v_0, then

$$\frac{\eta}{6} - 1 < 0 \Rightarrow \eta < 6$$

18. (d): The fall of centre of gravity h is given by



$$\frac{\left(\frac{L}{2}-h\right)}{\left(\frac{L}{2}\right)} = \cos 60°,$$

or $\quad h = \frac{L}{2}(1-\cos 60°)$

$\therefore$ Decrease in potential energy

$= \quad Mgh = Mg\dfrac{L}{2}(1-\cos 60°)$

Kinetic energy of rotation $= \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\times\dfrac{ML^2}{3}\omega^2$

[$I = \dfrac{ML^2}{3}$ (Because rod is rotating about an axis passing through its one end)]

According to law of conservation of energy,

$$Mg\frac{L}{2}(1-\cos 60°) = \frac{ML^2}{6}\omega^2 \Rightarrow \omega = \sqrt{\frac{3g}{2L}}.$$

19. (d): On applying law of conservation of angular momentum

$I_1\omega_1 = I_2\omega_2$

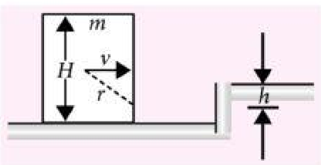
For solid sphere,

$$I = \frac{2}{5}mr^2 \Rightarrow \frac{2}{5}mr_1^2\omega_1 = \frac{2}{5}mr_2^2\omega_2$$

$$r^2\omega = \left(\frac{r}{n}\right)^2\omega_2 \Rightarrow \omega_2 = n^2\omega$$

20. (a): $mv\left(\dfrac{H}{2}-h\right)$

$$= m\left[\frac{(H^2+H^2)}{12}+\left(\frac{H}{2}\right)^2+\left(\frac{H}{2}-h\right)^2\right]\omega$$



$$\omega = \frac{\left(\frac{H}{2}-h\right)v}{\frac{H^2}{6}+\frac{H^2}{4}+\left(\frac{H}{2}-h\right)^2}$$

$$v = r\omega = \frac{\left(\frac{H}{2}-h\right)v.\sqrt{\frac{H^2}{4}+\left(\frac{H}{2}-h\right)^2}}{\frac{5}{12}H^2+\left(\frac{H}{2}-h\right)^2}$$

If $h = \dfrac{H}{4}$, we have

$$v' = \frac{\frac{H}{4}v.\sqrt{\frac{H^2}{4}+\frac{H^2}{16}}}{\left(\frac{5}{12}+\frac{1}{16}\right)H^2} = \frac{\frac{v}{4}.\frac{\sqrt{5}}{14}}{\frac{15+3}{48}} = \frac{3\sqrt{5}}{18}v \Rightarrow v' = \frac{\sqrt{5}}{6}v$$

◈◈

PHYSICS AROUND US

Understand the Science behind Real Life Phenomena

The Sun the only star that can be seen during the day.

Stars glow during the day, but they are impossible to see because of the brightness and closeness of the Sun. Also, the light coming from the Sun is scattered into the bright blue colour that we are familiar with as being the colour of the sky. This blue-tinted atmosphere plays a part in blocking out the stars, which in fact shine very faintly. On the other hand, if you lived on the moon, its lack of atmosphere would allow you to see the stars both day and night.

The radius of the Earth can be calculated by observing the sunset at the seashore.

Imagine you are located at the equator of the Earth and it is mid-summer. You note the exact time the Sun sets for you. You ask a friend living a distance S due west of you to note the time when the Sun sets for him. If the delay in his observing the Sunset is y hours then the circumference of the Earth C is given by $C = (24 \times S) / y$. The justification for this simple equation is: If a point on the Earth moves through a distance S in y hours, then the distance it would move in 24 hours, must be equal to the circumference of the Earth. If you are not on the equator, some modification of this equation may be required, but the principle remains the same.

What constitutes the Earth's magnetic field?

There is no bar magnet sitting at the center of the Earth. The Earth's magnetic field arises precisely because the inside of the Earth is hot enough to melt iron and there is iron in the core of the Earth. The heat inside the Earth, generated as a result of decay of radioactive elements at the core produces convection cells which transport liquid conducting material upward and then down again, very much like what happens when we put a pot of water to boil. The motion of the conducting material, in rather regular loops along with the rotation of the Earth, produces a dynamo effect - in other words, loops of electric currents. The Earth's magnetic field is supposed to be a result of these currents.

Is there any truth to the saying, "Red skies at night, sailors' delight; red skies in the morning, sailors take warning"?

The saying may have some truth where storms usually approach from the west and so in what are called storm systems. If the western sky is red at sunset, then the area west is free of storm cloud that would block the sunlight skirting Earth's curve, and you would probably have good weather for a few day. However, if the eastern sky is red at sunrise, then the area east is free of storm clouds, and the next storm may be coming soon from the west.

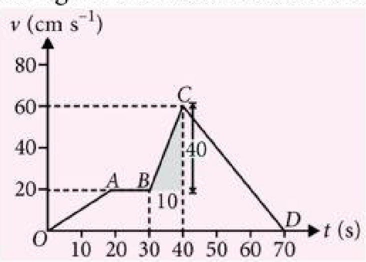
Class XI

Be NEET READY

with Exclusive and brain storming MCQs

Practicing these MCQs help to Strengthen your concepts and give you extra edge in your NEET Preparations

1. The velocity versus time curve of a moving particle is given in figure. The maximum acceleration is



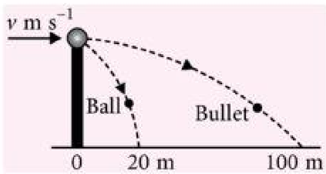
(a) $1\ \text{cm s}^{-2}$ (b) $2\ \text{cm s}^{-2}$
(c) $3\ \text{cm s}^{-2}$ (d) $4\ \text{cm s}^{-2}$

2. A body dropped from top of a tower falls through 40 m during the last two seconds of its fall. The height of the tower (in m) is (Take $g = 10\ \text{m s}^{-2}$)
(a) 60 m (b) 45 m (c) 80 m (d) 50 m

3. The velocity at the maximum height of a projectile is half its initial velocity of projection v_0. Its range on the horizontal plane is
(a) $3v_0^2/g$ (b) $3v_0^2/2g$
(c) $v_0^2/3g$ (d) $\sqrt{3}v_0^2/2g$

4. A balloon of mass m is descending down with an acceleration a (where $a < g$). How much mass be removed from it so that it starts moving up with an acceleration a ?
(a) $\dfrac{2ma}{g+a}$ (b) $\dfrac{2ma}{g-a}$ (c) $\dfrac{ma}{g+a}$ (d) $\dfrac{ma}{g-a}$

5. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg travelling with a velocity $v\ \text{m s}^{-1}$ in a horizontal direction, hits the centre of the ball. After the collision, the ball and the bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post shown in figure.



The initial velocity v of the bullet is
(a) $250\ \text{m s}^{-1}$ (b) $250\sqrt{2}\ \text{m s}^{-1}$
(c) $400\ \text{m s}^{-1}$ (d) $500\ \text{m s}^{-1}$

6. Starting from rest, a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The coefficient of friction between the body and the inclined plane is
(a) 0.33 (b) 0.25 (c) 0.75 (d) 0.80

7. If the linear momentum is increased by 50%, then KE will be increased by
(a) 50% (b) 100% (c) 125% (d) 200%

8. A block of mass 0.50 kg is moving with a speed of $2.00\ \text{m s}^{-1}$ on a smooth surface. It strikes another mass of 1.00 kg and then they move as a single body. The energy loss during the collision is
(a) 0.16 J (b) 1.00 J (c) 0.67 J (d) 0.34 J

9. A uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

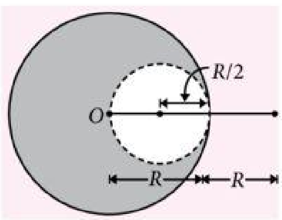
(a) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l\omega}{g}$

(c) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l^2\omega^2}{g}$

10. The ratio of the radii of gyration of a circular disc and a circular ring of the same radii about a tangential axis is

(a) $1:\sqrt{2}$ (b) $\sqrt{5}:\sqrt{6}$ (c) $\sqrt{2}:\sqrt{3}$ (d) $\sqrt{2}:1$

11. A solid sphere of uniform density and radius R applies a gravitational force of attraction F_1 on a particle placed at a distance $2R$ from the centre of the sphere. A spherical cavity of radius $(R/2)$ is now made in the sphere as shown in figure. The sphere with cavity now applies a gravitational force F_2 on the same particle. The ratio (F_2/F_1) is



(a) 1/2 (b) 3/4 (c) 7/8 (d) 7/9

12. A capillary tube is attached horizontally to a constant pressure head arrangement. If the radius of the tube is increased by 10%, the rate of flow of liquid will change nearly by

(a) +10% (b) –10% (c) +46% (d) +40%

13. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0 while box B contains one mole of helium at temperature $(7/3)T_0$. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature. (Ignore the heat capacity of boxes). Then the final temperature of the gases, T_f, in terms of T_0, is

(a) $T_f = \frac{5}{2}T_0$ (b) $T_f = \frac{3}{7}T_0$

(c) $T_f = \frac{7}{3}T_0$ (d) $T_f = \frac{3}{2}T_0$

14. Two vibrating tuning forks producing progressive waves given by $y_1 = 4\sin(500\pi t)$ and $y_2 = 2\sin(506\pi t)$ are held near the ear of a person. The person will hear

(a) 3 beats/s with intensity ratio between maxima and minima equal to 2

(b) 3 beats/s with intensity ratio between maxima and minima equal to 9

(c) 6 beats/s with intensity ratio between maxima and minima equal to 2

(d) 6 beats/s with intensity ratio between maxima and minima equal to 9

15. If 2 moles of an ideal monoatomic gas at temperature T_0 is mixed with 4 moles of another ideal monoatomic gas at temperature $2T_0$, then the temperature of the mixture is

(a) $(5/3)T_0$ (b) $(3/2)T_0$ (c) $(4/3)T_0$ (d) $(5/4)T_0$

SOLUTIONS

1. (d): Maximum acceleration = slope of BC

$$= \frac{40\,\text{cm s}^{-1}}{10\,\text{s}} = 4\text{ cm s}^{-2}$$

2. (b): Height through which a body falls in t seconds,

$$h = \frac{1}{2}gt^2 \quad \ldots\text{(i)}$$

Height through which the body falls in $(t-2)$ seconds,

$$h' = \frac{1}{2}g(t-2)^2 \quad \ldots\text{(ii)}$$

From eq (i) and (ii), we get

Thus, $h - h' = \frac{1}{2}gt^2 - \frac{1}{2}g(t-2)^2 = 40\,\text{m}$

or $2g(t-1) = 40$ m

hence, $t = 3$ s (on putting $g = 10$ m s^{-2})

Clearly, $h = \frac{1}{2} \times (10) \times (3)^2$ m = 45 m

3. (d): At the highest point, $v_{x0} = \frac{v_0}{2}$ and $v_{y0} = 0$

Clearly, $v_0\cos\theta = \frac{v_0}{2}$ or $\theta = 60°$

$$R = \frac{v_0^2\sin 2\theta}{g} = \frac{v_0^2\sin 120°}{g} = \frac{v_0^2(\sqrt{3}/2)}{g} = (\sqrt{3}v_0^2)/2g$$

4. (a): When the balloon descends with acceleration a,

$$mg - F = ma \quad \ldots\text{(i)}$$

When the balloon ascends with acceleration a (with mass m' removed),

$$F - (m - m')g = (m - m')a \quad \ldots\text{(ii)}$$

From eqns. (i) and (ii), we get

$(mg - ma) - mg + m'g = ma - m'a$

or $m'(g + a) = 2ma$

or $m' = \frac{2ma}{(g+a)}$

5. **(d):** As $y = \frac{1}{2}gt^2$, $5 = \frac{1}{2}(10)t^2$ or $t = 1\text{ s}$

Further, as $v_{ball}\, t = 20$ m or $v_{ball} = \frac{20\text{ m}}{1\text{ s}} = 20\text{ m s}^{-1}$

and as $v_{bullet}\, t = 100$ m, $v_{bullet} = \frac{100\text{ m}}{1\text{ s}} = 100\text{ m s}^{-1}$

Applying the law of conservation of momentum, $(0.01\text{ kg})\, v = (0.01\text{ kg})\,(100\text{ m s}^{-1}) + (0.2\text{ kg})\,(20\text{ m s}^{-1})$, hence $v = 500\text{ m s}^{-1}$

6. **(c) :** When there is no friction, $a = g\sin\theta$.
When there is friction, net downwards force acting on the body, *i.e.*,
$F = mg\sin\theta - f = mg\sin\theta - \mu R = mg\sin\theta - \mu\, mg\cos\theta$
Acceleration acting on the body,
$a' = \frac{F}{m} = \frac{mg(\sin\theta - \mu\cos\theta)}{m}$ or $a' = g(\sin\theta - \mu\cos\theta)$
If s is the distance through which the block slides,
$s = \frac{1}{2}at^2 = \frac{1}{2}a't'^2$ or $\frac{a}{a'} = \frac{t'^2}{t^2} = 4$ (as $t' = 2t$)

or $\frac{g\sin\theta}{g(\sin\theta - \mu\cos\theta)} = 4$ or $4\sin\theta - 4\mu\cos\theta = \sin\theta$

or $\mu = \frac{3}{4}\tan\theta = \frac{3}{4}(1) = 0.75$ $[\because \theta = 45°]$

7. **(c) :** $p_f = p_i + (1/2)p_i = (3/2)p_i$ or $\frac{p_f}{p_i} = \frac{3}{2}$

Thus, $\frac{K_f}{K_i} = \left(\frac{p_f}{p_i}\right)^2 = \frac{9}{4}$

or $K_f = \frac{9}{4}K_i$ or $K_f - K_i = \frac{5}{4}K_i$

Thus, percentage increase in kinetic energy
$= \frac{K_f - K_i}{K_i} \times 100 = 125\%$

8. **(c) :** As $0.5 \times 2 + 1 \times 0 = (0.5 + 1)v$
(assuming the body of mass 1.00 kg to be rest)
$v = \frac{2}{3}\text{ m s}^{-1} = 0.67\text{ m s}^{-1}$

Energy loss $= K_i - K_f = \frac{1}{2}(0.5)(2)^2 - \frac{1}{2}(1.5)(0.67)^2$ J
$= 1\text{ J} - 0.33\text{ J} = 0.67\text{ J}$

9. **(d):** When the centre of mass of the rod rises through h, increase in $PE = mgh$.

Kinetic energy of the rod $= \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$.

Thus, $mgh = \left(\frac{ml^2}{6}\right)\omega^2$ or $h = \frac{1}{6}\frac{l^2\omega^2}{g}$

10. **(b):** For the given axis,
$I_{disc} = \frac{5}{4}MR^2, I_{ring} = \frac{3}{2}MR^2$,

$k_{disc} = \sqrt{\frac{5}{4}R^2} = \frac{\sqrt{5}}{2}R$; $k_{ring} = \sqrt{\frac{3}{2}R^2} = \sqrt{\frac{3}{2}}R$

Thus, $\frac{k_{disc}}{k_{ring}} = \frac{(\sqrt{5}/2)R}{\sqrt{3/2}R} = \frac{\sqrt{5}}{\sqrt{6}} = \sqrt{5} : \sqrt{6}$

11. **(d):** $F_1 = \frac{GMm}{(2R)^2} = \frac{x}{4}$, where $\frac{GMm}{R^2} = x$

$F' = \frac{GM'm}{\left(\frac{3}{2}R\right)^2} = \frac{G(M/8)m}{(9/4)R^2} = \frac{x}{18}$

(as M' is the mass of the cavity)

Thus, $F_2 = F_1 - F' = \frac{x}{4} - \frac{x}{18} = \frac{7x}{36}$

or $\frac{F_2}{F_1} = \frac{7x}{36} \times \frac{4}{x} = \frac{7}{9}$

12. **(c) :** $\frac{V'}{V} = \frac{\pi P r'^4 / 8\eta l}{\pi P r^4 / 8\eta l}$

$= \left(\frac{r'}{r}\right)^4 = \left(\frac{1.1r}{r}\right)^4 = 1.4641$

Thus, $\frac{V' - V}{V} \times 100 = \left(\frac{V'}{V} - 1\right)100$
$= (1.4641 - 1)100 = 46\%$

13. **(d):** As $dU = C_V\, dT$,
$dU = \frac{5}{2}R(T_f - T_0) + \frac{3}{2}R(T_f - \frac{7}{3}T_0)$ as C_V for nitrogen is $(5/2)R$ and for helium its value is $(3/2)R$.
Put $dU = 0$ and obtain $T_f = \frac{3}{2}T_0$

14. **(b):** $\omega_1 = 500\pi$, $v_1 = 250$; $\omega_2 = 506\pi$, $v_2 = 253$
Number of beats/s = 3
Further, $A_1 = 4$ and $A_2 = 2$
Amplitude ratio, $r = \frac{A_1}{A_2}$ 2 =

$\therefore \frac{I_{max}}{I_{min}} = \frac{(r+1)^2}{(r-1)^2} = 9$

15. **(a) :** Let T be the temperature of the mixture. As
$U = U_1 + U_2$,
$\frac{f}{2}(n_1 + n_2)RT = \frac{f}{2}n_1RT_0 + \frac{f}{2}n_2R(2T_0)$
or $(2 + 4)T = 2T_0 + 8T_0$ (as $n_1 = 2$, $n_2 = 4$)
or $T = (5/3)T_0$

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TIPS corner

15 Ways to master the art of self-discipline

The art of Self-discipline is the set of actions that help you develop a total control over life by employing various perspectives. No one is born with the ability to have a complete control over oneself. It is something that you succeed at only after making certain sacrifices from your end. Self-discipline is a habit that matures with practice and ultimately shapes you to lead a happy and healthy lifestyle. With self discipline, you will understand self potential. Another benefit from self discipline is that you will learn to focus more and pay attention to what is a priority to you without getting distracted. You might get frustrated in the process of maturing as a self-disciplined person, but, trust it's totally worth it.

If you are interested in developing a mastery over the art of self-discipline, then here's a list of things that you'd have to do.

1 Respect yourself and fill your mind with positive information.

Self discipline involves self respect. A human body is created with great mastery and you will know when something is off balance. Understand your body messages and treat it well. Exercise or do meditation to get the balance of mind and body. Also, you must nurture your mind by soaking it with positive and engaging information. Read books, engage in meaningful discussions, avoid negative conversations and people, and listen to soothing, joyful and boosting music rather than watching television and filling your mind with all the negativity and commercialism.

It is essential to feel connected to something greater than you. By meditation, praying or spending time with nature you can connect with the Almighty. It helps in making decisions for greater good and not just for something that feels good in that moment.

2 Eat healthy food at regular intervals.

People don't know that your hunger has a direct link with your self-discipline and it's generally observed that when you're hungry, your ability to think and operate is adversely affected. In that situation, you generally won't have a control over the brain, and as a result, you will certainly find it difficult to execute even the simplest of the task with utmost perfection.

So, whenever you're on the receiving end of such problems, do remember to eat healthy food, that too at regular intervals that lets you help you get a control over oneself.

You become disciplined from the time you start taking care for yourself. When you treat your body well, it helps you in reaching and accomplishing amazing goals.

3 Value your time.

You must value your time with your work, family and self. Make generous space in your life for each of the zones. Schedule your family time and personal time similar to how you schedule your work time and other responsibilities.

4 Prepare written schedule.

When you prepare a written schedule and add everything to it, you generate priorities and sticking to your priorities will help you develop self discipline. Writing down helps you to remember your priority and not miss out anything.

5 Build the will to succeed, one step at a time.

Many people want to develop self discipline by bringing drastic changes in their lives. However one must know that this can lead to failure and can have damaging consequences. You must make small changes in your life with things and not completely change everything at once. Simple and small is the key to mastering the art of self discipline. If you are struggling and don't understand how to make small and simple goals, take one thing at a time and work on it.

6 **Set realistic and achievable goals.**

People make the biggest mistake by setting goals that are next to impossible to achieve for them, just for the sake of developing self discipline. You need to learn to set realistic goals and set milestones for the long term goals to improve self discipline. It will help you to easily monitor your progress and boosts you up for further goals.

7 **Stop procrastination.**

Your delayed actions can cause you to miss out on crucial opportunities. Procrastination is one fatal habit that can get you totally messed up and close the doors for your future development. Instead of delaying the activities, start executing them and try to finish them off within the given duration.

8 **Make a daily checklist.**

It is helpful to create a daily to do list, to help you exercise self discipline. You never miss a thing in your routine and it helps you to make sure you are doing everything without your mood coming in the way. Ensure each day that you complete each of the mentioned tasks.

9 **Reward yourself on success.**

You must reward yourself when you master self discipline in a thing and accomplished that goal. Watch a movie in the weekend or treat yourself with a delightful meal. To encourage yourself more, think of a reward that is very stimulating for you and which you would do anything for.

10 **Understand your priorities.**

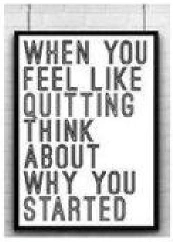
You will need to prioritize your activities in order to develop a total control over yourself. Prioritizing helps you in multitasking as well, by letting you assign proper time for each task until its completion. Having a definite set of action plan contributes a lot to the self-discipline. Mind you, prioritizing isn't an easy task. You will have to give up on a lot of activities in order to execute your plans in a timely manner. You should be able to understand the biggest priority in your life in that moment to help you motivate properly. Contemplate what is important to you in your life and rectify your priorities list.

11 **Keep a check on your emotions and Speak to yourself.**

No matter how strenuous your situation might be, always keep your emotions in check. Do not let your emotions get the better of you under any circumstances. It might seem as a humongous move at first, but, you will have to keep your emotions under control amidst the most gruelling situations. Don't hurt yourself when you fail. Never curse yourself or let yourself be swamped with hurtful feelings when you fail in your quest to develop self discipline. There are always bad times and mistakes can happen by any person, no one is perfect. You are only going to learn self discipline when you gather your senses and learn from your mistakes.

When you self communicate just to criticize yourself, it can be self destroying. But when you use the technique to boost up your energy, motivate and encourage going on with something, you will find many advantages of this technique.

12 **Read inspiring stories.**



Many times, when you read success stories or the inspirational account of success, you receive motivation and drive to go on. Engross in other's success stories and you are more likely to follow your own goals with courage and determination.

13 **Remove temptation and get rid of destructive habits.**

All have those vulnerability triggers that make them weak and lead to failure. Perhaps it could be some distraction, hunger or sleepiness. Recognize and get rid of the triggers. If you stay up late, spend lot of time surfing then it is time for you to get rid of these habits or your self-discipline strive is going to suffer. A lot of lack of discipline come from giving up to simple temptations.

14 **Recheck your goals.**

If you are stuck with the same situation then perhaps your goal is not motivating or exciting enough for you. May be you need to make changes in your goals or change a goal completely. Recheck each goal and settle for what works for you.

15 **Be consistent with your daily steps.**

You must always persist looking forward and moving in a forward direction. Never look back or take any step backwards. Make all your steps effective in reaching a step forward to your ultimate goal.

The above tips can be very useful and effective with proper implementation and you will ultimately begin seeing changes in your life that are leading you to self discipline.

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CLASS XI

CBSE DRILL

Chapterwise practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2018-19.

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 12 are short answer questions and carry 2 marks each.
(iv) Q. no. 13 to 24 are also short answer questions and carry 3 marks each.
(v) Q. no. 25 to 27 are long answer questions and carry 5 marks each.
(vi) Use log tables if necessary, use of calculator is not allowed.

Time Allowed : 3 hours | Maximum Marks : 70

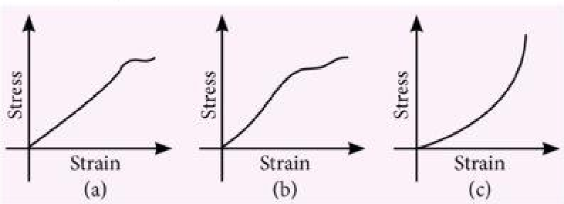
Mechanical Properties of Solids | Mechanical Properties of Fluids

SECTION A

1. A hard wire is broken by bending it repeatedly in alternating directions. Why ?

2. How does the boiling point of a liquid vary with pressure ?

3. Following are the graph of elastic materials. Which one corresponds to that of brittle material ?



4. Two balls A and B have radii in the ratio 1 : 4. What will be the ratio of their terminal velocities in a liquid ?

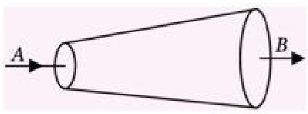
5. Water rises in a capillary tube, whereas mercury falls in the same tube. Why?

SECTION B

6. The breaking force for a wire is F. What will be the breaking force for
   (a) two parallel wires of the same size and
   (b) for a single wire of double the thickness ?

7. By how much a rubber string of length 10 m increases in length under its own weight when suspended vertically? Density of rubber $= 1.5 \times 10^3\ \text{kg m}^{-3}$; $Y = 5 \times 10^8\ \text{N m}^{-2}$; $g = 10\ \text{m s}^{-2}$.

8. An ideal fluid flows through a pipe of circular cross section with diameters 5 cm and 10 cm as shown in the figure. Find the ratio of velocities of fluid at A and B.



9. A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ_0. After some time, the ball falls with a constant velocity. Calculate the viscous force on the ball.

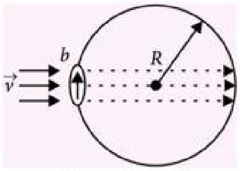
10. When the tension in a metal wire is T_1, its length is l_1. When the tension is T_2, its length is l_2. Find the natural length of wire.

11. Why does modulus of elasticity of most of the materials decrease with the increase of temperature?

OR

If n identical water droplets falling under gravity with terminal velocity v coalesce to form a single drop which has the terminal velocity $4v$, find the number n.

12. A bubble having surface tension S and radius R is formed on a ring of radius b ($b<<R$). Air is blown inside the tube with velocity v as shown in figure. The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring.



SECTION C

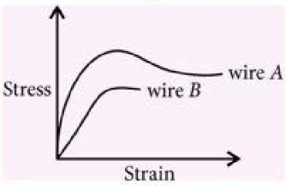
13. A square lead slab of side 50 cm and thickness 10 cm is subjected to a shearing force (on its narrow face) of 9.0×10^4 N. The lower edge is riveted to the floor. How much will the upper edge be displaced? If modulus of rigidity (G) is 5.6×10^9 N m^{-2}.

14. State and prove Torricelli's theorem.

15. **(i)** A ball floats on the surface of water in a container exposed to the atmosphere. Will the ball remain immersed at its initial depth or will it sink or rise somewhat if the container is shifted to the moon?

(ii) The viscous force 'F' acting on a body of radius 'r' moving with a velocity 'v' in a medium of coefficient of viscosity 'η' is given by $F = 6\pi\eta rv$. Check the correctness of the formula.

16. Stress strain curve for two wires of material A and B are as shown in figure.



(a) Which material is more ductile?

(b) Which material has greater value of Young's modulus?

(c) Which of the two is stronger material?

(d) Which material is more brittle?

17. Explain why

(i) a balloon filled with helium does not rise in air indefinitely but halts after a certain height (Neglect winds)?

(ii) the force required by a man to raise his limbs immersed in water is smaller than the force for the same movement in air?

18. Water from a tap emerges vertically downwards with an initial velocity v_0. Assume pressure is constant throughout the stream of water and the flow is steady, find the distance from the tap at which cross-sectional area of stream is half of the cross-sectional area of stream at the tap.

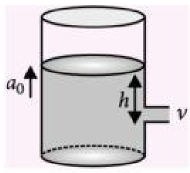
19. A cable is replaced by another cable of the same length and material but of twice the diameter.

(a) How does this affect its elongation under a given load?

(b) How many times will be the maximum load it can now support without exceeding the elastic limit?

20. **(i)** For the area of the hole is much lesser than the area of the base of a vessel of liquid, find velocity of efflux v of the liquid if vessel is accelerating as shown in figure.

(a_0 = vertical acceleration)

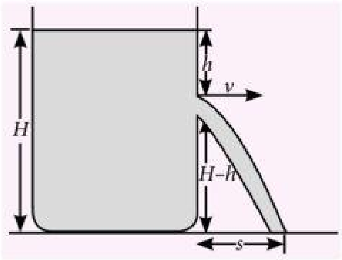


(ii) A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, what is the value of R?

MONTHLY TUNE UP CLASS XI ANSWER KEY

1.	(a)	2.	(b)	3.	(a)	4.	(a)	5.	(d)
6.	(b)	7.	(b)	8.	(b)	9.	(d)	10.	(c)
11.	(b)	12.	(a)	13.	(a)	14.	(d)	15.	(c)
16.	(a)	17.	(b)	18.	(a)	19.	(c)	20.	(a, b, c)
21.	(a, b, c, d)			22.	(b, d)	23.	(a, b, c)		
24.	(4)	25.	(123)	26.	(2)	27.	(d)	28.	(b)
29.	(a)	30.	(c)						

21. Water stands at a depth H in a tank whose side walls are vertical as shown in the figure. A hole is made on one side of the walls at a depth h below the water surface.



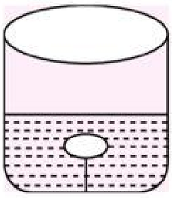
(a) At what distance s from the foot of the wall does the emerging stream of water strike the floor ?

(b) For what value of h this range is maximum?

22. (i) The breaking stress for aluminium is 7.5×10^7 N m^{-2}. Find the greatest length of aluminium wire that can hang vertically without breaking under its own weight. Density of aluminium is 2.7×10^3 kg m^{-3}.

(ii) The shear modulus of a material is always considerably smaller than the Young's modulus for it. What does it signify ?

23. A solid sphere of mass m = 2 kg and density of 0.5×10^3 kg m^{-3} is held stationary relative to a tank filled with water as shown in figure. The tank is accelerating vertically upward with acceleration 2 m s^{-2}.



(i) Calculate the tension in the thread connecting the sphere and the bottom of the tank.

(ii) If the thread snaps, calculate the acceleration of the sphere with respect to the tank. (Density of water is ρ = 1000 kg m^{-3} and g = 10 m s^{-2}.)

24. (a) Explain why should the beams used in the construction of bridges have large depth and small breadth.

(b) Why are girders given I shape ?

SECTION D

25. Explain what happens when the load on a metal wire suspended from a rigid support is gradually increased. Illustrate your answer with a suitable stress-strain graph.

OR

(a) Pressure decreases as one ascends the atmosphere. If the density of air is ρ, what is the change in pressure dP over a differential height dh ?

(b) Considering the pressure P to be proportional to the density, find the pressure P at a height h if the pressure on the surface of the earth is P_0.

(c) If $P_0 = 1.013 \times 10^5$ N m^{-2}, $\rho_0 = 1.29$ kg m^{-3} and g = 9.8 m s^{-2}, at what height will the pressure drop to (1/10) the value at the surface of the earth?

(d) This model of the atmosphere works for relatively small distances. Identify the underlying assumption that limits the model.

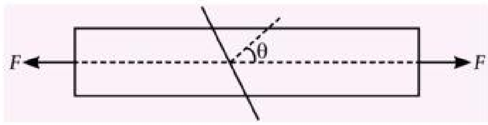
26. A bar of cross-section A is subjected to equal and opposite tensile forces at its ends. Consider a plane section of the bar whose normal makes an angle θ with the axis of the bar.

(i) What is the tensile stress on this plane?

(ii) What is the shearing stress on this plane?

(iii) For what value of θ is the tensile stress maximum?

(iv) For what value of θ is the shearing stress maximum?



OR

State and prove Bernoulli's theorem.

27. A light rod of length 2 m is suspended horizontally by means of two vertical wires of equal lengths tied to its ends. One of the wires is made of steel and is of cross-section A_1 = 0.1 cm^2 and the other is of brass and is of cross-section A_2 = 0.2 cm^2. Find out the position along the rod at which a weight must be suspended to produce (i) equal stress in both wires, (ii) equal strain in both wires. For steel, $Y = 20 \times 10^{10}$ N m^{-2} and for brass $Y = 10 \times 10^{10}$ N m^{-2}.

OR

Explain how does a body attain a terminal velocity when it is dropped from rest in a viscous medium. Derive an expression for the terminal velocity of a small spherical body falling through a viscous medium.

SOLUTIONS

1. When the wire is subjected to repeated alternating strains, the strength of its material decreases and the wire breaks.

2. The boiling point of a liquid increases with pressure. For example, if the pressure is more than the atmospheric pressure, water boils at a temperature higher than 100° C.

3. Graph (a) represents a brittle material as it indicates a very small plastic range of extension.

4. Terminal velocity, $v \propto r^2$

$$\therefore \quad \frac{v_A}{v_B} = \left(\frac{r_A}{r_B}\right)^2 = \left(\frac{1}{4}\right)^2 = 1:16$$

5. In a capillary tube, a liquid rises to a height h given by $\left(h = \frac{2\sigma\cos\theta}{r\rho g}\right)$

For water, θ is positive and hence h is positive. So water rises in the capillary tube. For mercury θ is obtuse, cos θ is negative and hence h is negative. So mercury gets depressed in the capillary tube.

6. (a) When two wires of same size are suspended in parallel, a force F equal to the breaking force will act on each wire if a breaking force of $2F$ is applied on the parallel combination.

(b) $F = \frac{YA\,\Delta l}{l} = \frac{Y.\pi r^2\,\Delta l}{l}$ *i.e.*, $F \propto r^2$

Thus for a single wire of double the thickness, the breaking force will be $4F$.

7. Let A be the area of cross-section of the string in SI units.

Mass of string, m = Volume × Density

$= (A \times 10) \times 1.5 \times 10^3 = 1.5 \times 10^4 A$ kg

Stress in the string

$$= \frac{mg}{A} = \frac{1.5\times10^4 \times A \times 10}{A} = 1.5\times10^5 \text{ N m}^{-2}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} \text{ or } 5\times10^8 = \frac{1.5\times10^5}{\Delta L/5}$$

$$\therefore \quad \Delta L = \frac{1.5\times10^5\times5}{5\times10^8} = 1.5\times10^{-3}\text{ m} = 1.5\text{ mm}$$

8. Using continuity equation, $A_1v_1 = A_2v_2$

$$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2} = \left(\frac{r_2}{r_1}\right)^2 = \frac{4}{1}$$

9. Volume of the ball, $V = \frac{m}{\rho}$

Mass of the liquid displaced, $m' = V\rho_0 = \frac{m}{\rho}.\rho_0$

When the body falls with a constant velocity,

Viscous force = Effective weight of the ball

F = Weight of the ball – Upthrust

$= mg - m'g$

$$F = mg - \frac{m\rho_0}{\rho}.g = mg\left(1 - \frac{\rho_0}{\rho}\right).$$

10. $Y = \frac{Fl}{A\Delta l}$

∵ Y, l and A constants,

$\therefore \quad \frac{F}{\Delta l} =$ constant or $\Delta l \propto F$

∴ If tension is T_1, then

$l_1 - l \propto T_1$ ...(i)

If tension is T_2, then

$l_2 - l \propto T_2$ ...(ii)

Dividing eqn. (i) by eqn. (ii), we get

$$\frac{l_1 - l}{l_2 - l} = \frac{T_1}{T_2} \text{ or } l_1T_2 - lT_2 = l_2T_1 - lT_1$$

or $l(T_1 - T_2) = l_2T_1 - l_1T_2$

or $l = \frac{l_2T_1 - l_1T_2}{T_1 - T_2} = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}$

11. As the temperature increases, the interatomic forces of attraction become weaker. For given stress, a larger strain or deformation is produced at a higher temperature. Hence the modulus of elasticity (stress/strain) decreases with the increase of temperature.

OR

Radius of each small drop = r

Volume of each small drop = $\frac{4}{3}\pi r^3$

Volume of bigger drop = $\frac{4}{3}\pi R^3$

$R^3 = nr^3 \Rightarrow R = n^{\frac{1}{3}}r$

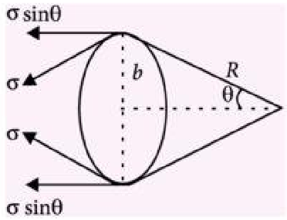
As terminal velocity, $v \propto r^2$

$\frac{v}{v'} = \left(\frac{r}{R}\right)^2$, where v' is the terminal velocity of bigger drop.

$$\frac{v}{v'} = n^{\frac{-2}{3}} \Rightarrow v' = n^{\frac{2}{3}}v$$

Since $v' = 4v$, ∴ $4v = n^{\frac{2}{3}}v$ or $n = (4)^{\frac{3}{2}} = 8$

12. The bubble will separate from the ring when



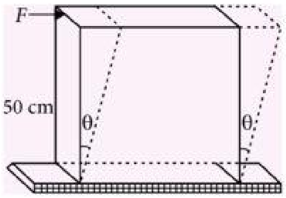
$2\pi b \times 2\sigma \sin\theta = \rho A v^2$

or $4\pi b\sigma \times \frac{b}{R} = \rho \times \pi b^2 \times v^2$

or $R = \frac{4\sigma}{\rho v^2}$

13. The lead slab is fixed and force is applied parallel to narrow face as shown in figure. The area of the face parallel to which force is applied is

A = 50 cm × 10 cm = 0.5 m × 0.1 m = 0.05 m^2



Therefore, the stress applied $= \frac{9.0\times10^4\,\text{N}}{0.05\,\text{m}^2}$

$= 1.80 \times 10^6$ N m^{-2}

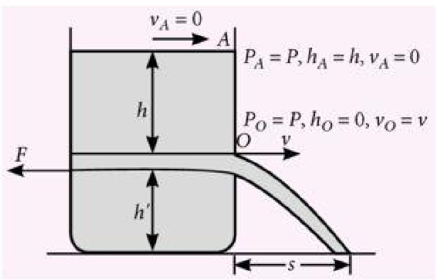
We know that shearing strain, $\frac{\Delta x}{L} = \frac{\text{Stress}}{G}$

Therefore, the displacement, $\Delta x = \frac{\text{Stress}\times L}{G}$

$\Delta x = \frac{1.8\times10^6\,\text{N m}^{-2}\times0.5\,\text{m}}{5.6\times10^9\,\text{N m}^{-2}} = 1.6 \times 10^{-4}$ m

Δx = 0.16 mm

14. Torricelli's theorem : It states that velocity of liquid flowing out of a narrow hole (*i.e.*, velocity of efflux) at a depth h below the free surface of liquid in a vessel is $v = \sqrt{2gh}$. Let us consider a tank containing an ideal liquid and having a hole at O as shown in figure. Further, let the flow of the liquid be streamlined.



Let h = height of the liquid surface above the hole,
h' = height of the hole from the ground,
s = horizontal distance travelled by the liquid,
v = speed with which the liquid comes out of O,
ρ = density of the liquid,
P = atmospheric pressure.

According to Bernoulli's theorem,
total energy of the liquid at A = total energy of the liquid at O,

i.e., $P_A + \rho g h_A + \frac{1}{2}\rho v_A^2 = P_O + \rho g h_O + \frac{1}{2}\rho v_O^2$

or $P + \rho g h + 0 = P + 0 + \frac{1}{2}\rho v^2$

(as pressure at A = pressure at O = atmospheric pressure = P. At A, $v_A = 0$ and at O, $h_O = 0$)

Thus, $gh = \frac{1}{2}v^2$ or $v = \sqrt{2gh}$

v is called the speed of efflux of the liquid.

15. (i) The gravity on moon is about one-sixth of that on the earth. But gravity has equal effect both on weight of the body and the upthrust. So equilibrium of the floating body is not affected. On the earth, weight of the floating body is balanced by upthrust due to both water and air.

∴ $W = mg = V_w\rho_w g + V_a\rho_a g$

or $m = V_w\rho_w + V_a\rho_a$ ...(i)

But the moon has no atmosphere. So

$W = mg = V'_w\rho_w g$

or $m = V'_w\rho_w$ ...(ii)

From eqn. (i) and (ii), we note that $V'_w = V_w + \frac{V_a\rho_a}{\rho_w}$

Clearly, $V'_w > V_w$

That is, the volume of ball immersed in water on the moon is greater than that on earth. Hence ball will sink slightly more in water when taken to the moon.

(ii) $[F]$ = [MLT^{-2}]

$[6\pi\eta r v]$ = [ML^{-1}T^{-1}] [L] [LT^{-1}] = [MLT^{-2}]

∴ Dimensions of LHS = Dimensions of RHS.

Hence the given formula for viscous force F is dimensionally correct.

16. (a) Wire of material A with larger plastic region is more ductile.

(b) Young's modulus is $\frac{\text{Stress}}{\text{Strain}}$ ∴ $Y_A > Y_B$

(c) For given strain, larger stress is required for A than that for B.

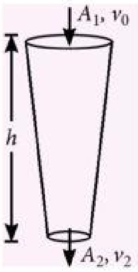
∴ A is stronger than B.

(d) Material with smaller plastic region is more brittle, therefore B is more brittle than A.

17. (i) A balloon filled with helium goes on rising in air as long as the weight of the air displaced by it (*i.e.*, upthrust) is greater than the weight of filled balloon. We know that the density of air decreases with height. Therefore, the balloon halts after attaining a height at which density of air is such that the weight of air displaced just equals the weight of helium filled balloon.

(ii) Water exerts much more upthrust on the limbs of man than air. So the net weight of limbs in water is much less than that in air. Hence the force required by a man to raise his limbs immersed in water is smaller than the force for the same movement in air.

18. In the shown figure, $v_2^2 = v_0^2 + 2gh$ and $A_1 v_0 = A_2 v_2$



Solving, $\dfrac{A_2}{A_1} = \dfrac{v_0}{\sqrt{v_0^2 + 2gh}}$

$\dfrac{A_2}{A_1} = \dfrac{A_2}{2A_2} = \dfrac{v_0}{\sqrt{v_0^2 + 2gh}}$

$4{v_0}^2 = {v_0}^2 + 2gh \quad \Rightarrow \quad h = \dfrac{3v_0^2}{2g}$

19. (a) Young's modulus,

$$Y = \frac{Mgl}{\pi r^2 . \Delta l} = \frac{Mgl}{\pi\left(\frac{D}{2}\right)^2 . \Delta l} = \frac{4\,Mgl}{\pi D^2 . \Delta l}$$

where D is the diameter of the wire.

Elongation, $\Delta l = \dfrac{4\,Mgl}{\pi D^2 Y}$ *i.e.*, $\Delta l \propto \dfrac{1}{D^2}$

Clearly, if the diameter is doubled, the elongation will become one fourth.

(b) Load, $Mg = \dfrac{\pi D^2 \cdot \Delta l \cdot Y}{4l}$ *i.e.*, $Mg \propto D^2$

Clearly, if the diameter is doubled, the wire can support 4 times the original load.

20. (i) Effective value of acceleration due to gravity becomes $(g + a_0)$.

Required velocity of efflux, $v = \sqrt{2(g + a_0)h}$

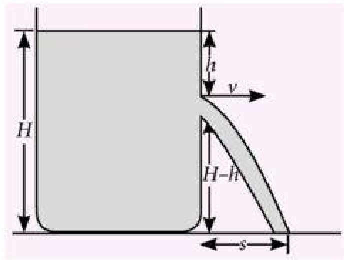
(ii) Equating the rate of flow, $v_1 A_1 = v_2 A_2$

But $v_1 = \sqrt{2gy}, A_1 = L^2, v_2 = \sqrt{2g \times 4y},\ A_2 = \pi R^2$

$\therefore \quad \sqrt{2gy} \times L^2 = \sqrt{2g \times 4y} \times \pi R^2$ (using Torricelli's law)

or $\quad L^2 = 2\pi R^2 \quad$ or $\quad R = \dfrac{L}{\sqrt{2\pi}}$

21. (a) Velocity of efflux, $v = \sqrt{2gh}$



Time taken by the liquid to touch the ground, *i.e.*, to travel a vertical distance $(H - h)$ is given by,

$$(H - h) = \frac{1}{2}gt^2 \qquad \text{or} \qquad t = \sqrt{\frac{2(H - h)}{g}}$$

Thus, $s = vt = \sqrt{2gh} \times \sqrt{\dfrac{2(H - h)}{g}}$ or $s = 2\sqrt{h(H - h)}$

(b) For s to be maximum, $\dfrac{ds}{dh} = 0$

or $\quad \dfrac{d}{dh}[2\sqrt{h(H - h)}] = 0$

or $\quad 2 \times (1/2)\,(Hh - h^2)^{-1/2}\,(H - 2h) = 0$

or $\quad \dfrac{H - 2h}{\sqrt{Hh - h^2}} = 0 \quad$ or $\quad h = \dfrac{H}{2}$

22. (i) Let L be the greatest length of aluminium wire that can hang without breaking.

Mass of the wire, $M = AL\rho$

Stress $= \dfrac{Mg}{A} = \dfrac{(AL\rho)g}{A} = L\rho g$

As breaking stress $= 7.5 \times 10^7$ N m^{-2},

$L\rho g = 7.5 \times 10^7$ N m^{-2}

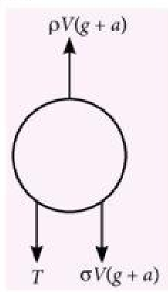
or $\quad L = \dfrac{7.5 \times 10^7}{\rho g} = \dfrac{7.5 \times 10^7}{2.7 \times 10^3 \times 9.8}$

$= 2.8 \times 10^3$ m $= 2.8$ km

(ii) For a material, shear modulus is smaller than the Young's modulus. This shows that it is easier to slide layers of atoms of solid over one another than to pull them apart or to squeeze them close together.

23. Density of sphere, $\sigma = 0.5 \times 10^3$ kg m^{-3}

(i) Here $\rho = 10^3$ kg m^{-3}, $g = 10$ m s^{-2}, $a = 2$ m s^{-2}, $m = 2$ kg



Volume of sphere, $V = \dfrac{m}{\sigma}$

Weight of sphere in accelerating medium $= \sigma V(g + a)$

Upthrust on sphere due to liquid $= \rho V(g + a)$

As the block is held stationary

$T + \sigma V(g+a) = \rho V(g+a)$

$$T = (\rho - \sigma)\frac{m}{\sigma}(g+a) = \frac{0.5\times10^3\times2\times12}{0.5\times10^3} = 24\text{ N}$$

(ii) If the thread snaps, $T = 0$.

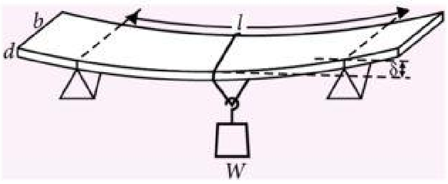
$F_{net} = F_{up} - W = (\rho - \sigma)V(g+a)$

$$= 0.5\times10^3\times\frac{2}{0.5\times10^3}\times12 = 24\text{ N}$$

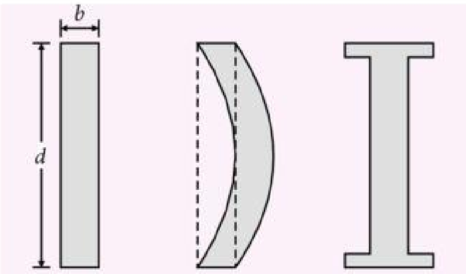
acceleration of sphere $= \frac{24}{2} = 12\text{ m s}^{-2}$

24. (a) The knowledge of elasticity is applied in designing a bridge such that it does not bend too much or break under the load of traffic, the force of wind and under its own weight. Consider a rectangular bar length l, breadth b and thickness d supported at both ends, as shown in figure. When a load W is suspended at its middle, the bar gets depressed by an amount given by

$$\delta = \frac{Wl^3}{4Y\,bd^3}$$



(b) Bending can be reduced by using a material with a large Young's modulus Y. As δ is proportional to d^{-3} and to b^{-1}, so depression can be decreased more effectively by increasing the depth d rather than the breadth b. But a deep bar has a tendency to bend under the weight of a moving traffic, as shown in figure (b). This bending is called buckling. Hence a better choice is to have a bar of I-shaped cross-section, as shown in figure (c). This section provides a large load bearing surface and enough depth to prevent bending. Also, this shape reduces the weight of the beam without sacrificing its strength and hence reduces the cost.



(a) Rectangular cross-section of bar (b) Buckling of a deep bar (c) I-shaped cross-section of bar

Joint Entrance Examination (Main) 2019

The JEE (Main)-2019 will be conducted by National Testing Agency (NTA) twice before admissions in the next academic session. The NTA will conduct the first JEE (Main)-January 2019 for admission to Undergraduate Programs in NITs, IIITs and other Centrally Funded Technical Institutions etc. on the weekends (Saturdays / Sundays) during **6th January 2019 - 20th January 2019**. The test details are given below:

Paper	Subjects	Mode of Examination	Timing of Examination	
			First Shift	Second Shift
Paper-1 (For B.E./B. Tech.)	Mathematics, Physics & Chemistry	"Computer Based Test (CBT)" mode only	09.30 a.m. to 12.30 p.m.	02.30 p.m. to 05.30 p.m.
Paper - 2 (For B. Arch/ B.Planning) (to be attempted in one sitting)	Mathematics Part I Aptitude Test Part II	"Computer Based Test (CBT)" mode only	Will be held in one shift only	
	Drawing Test Part III	"Pen & Paper Based" (offline) mode to be attempted on Drawing sheet		

The exact date and shift allotted to candidates for JEE (Main) - January 2019 shall be displayed by the 5th October 2018 on the NTA websites A candidate may appear in Paper 1 and/or Paper 2 depending upon the course/s he/she wishes to pursue. All the candidates aspiring to take admission to the undergraduate programs at IITs for the year 2019 will have to appear in the Paper-1 (B. E. /B. Tech.) of JEE (Main)-2019. Based on the performance in Paper-1 (B. E. /B. Tech.) of JEE (Main)-2019, a number of top candidates as per the requirement of JEE (Advanced) (including all categories) will be eligible to appear in JEE (Advanced)-2019.

Similarly, the second JEE (Main) - April 2019 will be conducted between **6th April 2019 - 20th April 2019** for which a separate notice will be issued later on and the candidates will be required to apply separately.

However, candidates are not required to compulsorily appear in both the tests i.e. JEE (Main) - January 2019 and JEE (Main) - April 2019. In case, a candidate appears in both the tests, the better of the two scores will be used for the admissions and eligibility for JEE (Advanced)-2019.

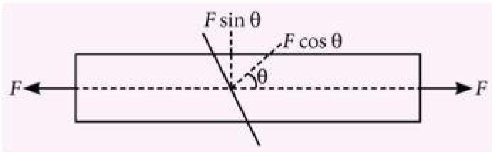
Candidates who desire to appear in the JEE (Main) - January 2019 may see the detailed Information Bulletin for JEE (Main) - January 2019 which is available on the website www.nta.ac.in and www.jeemain.nic.in w.e.f. 1st September 2018.

25. *Refer to answer* 26, page no. 305 (MTG CBSE Champion Physics class 11)

OR

Refer to answer 20, page no. 336 (MTG CBSE Champion Physics class 11)

26. **(i)** The resolved part of F along the normal is the tensile force on this plane and the resolved part parallel to the plane is the shearing force on the plane.



$\therefore$ Area of plane section = A sec θ

$$\text{Tensile stress} = \frac{\text{Force}}{\text{Area}} = \frac{F\cos\theta}{A\sec\theta} = \frac{F}{A}\cos^2\theta$$

(ii) Shearing stress applied on the top face

So, $F = F \sin\theta$

$$\text{Shearing stress} = \frac{\text{Force}}{\text{Area}} = \frac{F\sin\theta}{A\sec\theta} = \frac{F}{A}\sin\theta\cos\theta$$

$$= \frac{F}{2A}\sin 2\theta, \qquad [\because \sin 2\theta = 2\sin\theta\cos\theta]$$

(iii) Tensile stress will be maximum when $\cos^2\theta$ is maximum *i.e.*, $\cos\theta = 1$ or $\theta = 0°$

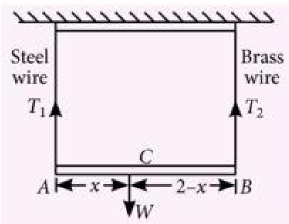
(iv) Shearing stress will be maximum when $\sin 2\theta$ is maximum *i.e.* $\sin 2\theta = 1$

or $2\theta = 90°$ or $\theta = 45°$.

OR

Refer to answer 62, page no. 342 (MTG CBSE Champion Physics class 11)

27. The situation is shown in figure. Let AB be the rod of length 2 m. Suppose a weight W is hung at C at distance x from A. Let T_1 and T_2 be the tensions in the steel and brass wire respectively.



(i) Stress in steel wire = $\dfrac{T_1}{A_1}$,

Stress in brass wire = $\dfrac{T_2}{A_2}$

As both the stresses are equal, so

$$\therefore \quad \frac{T_1}{A_1} = \frac{T_2}{A_2} \quad \text{or} \quad \frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{0.1}{0.2} = \frac{1}{2}$$

Now moments about C are equal as the system is equilibrium

$$\therefore \quad T_1 x = T_2 (2-x) \quad \text{or} \quad \frac{T_1}{T_2} = \frac{2-x}{x}$$

$$\text{or} \quad \frac{1}{2} = \frac{2-x}{x} \qquad \left(\because \frac{T_1}{T_2} = \frac{1}{2}\right)$$

$x = 4 - 2x$

$\therefore \quad 3x = 4 \quad$ or $\quad x = \dfrac{4}{3} = 1.33$ m

(ii) Now Y = Stress/Strain

$\therefore$ Strain = Stress/Y

$$\text{Strain in steel wire} = \frac{T_1/A_1}{Y_1}$$

$$\text{Strain in brass wire} = \frac{T_2/A_2}{Y_2}$$

$$\text{Now} \quad \frac{T_1}{A_1 Y_1} = \frac{T_2}{A_2 Y_2}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{A_1 Y_1}{A_2 Y_2} = \frac{0.1\text{ cm}^2 \times 20 \times 10^{10}\text{ Nm}^{-2}}{0.2\text{ cm}^2 \times 10 \times 10^{10}\text{ Nm}^{-2}} = 1$$

$$\text{Again, } T_1 x = T_2 (2-x) \quad \text{or} \quad \frac{T_1}{T_2} = \frac{2-x}{x}$$

$$\text{or} \quad 1 = \frac{2-x}{x} \qquad \left[\because \frac{T_1}{T_2} = 1\right]$$

$\therefore \quad x = 2 - x$

or $\quad 2x = 2 \quad$ or $\quad x = 1$ m.

OR

Refer to answer 95, page no. 346 (MTG CBSE Champion Physics class 11)

Class XI

MONTHLY TUNE UP!

PRACTICE PROBLEMS

These practice problems enable you to self analyse your extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Performance analysis table given at the end will help you to check your readiness.

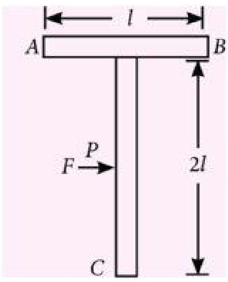
System of Particles and Rotational Motion

Total Marks : 120 **Time Taken : 60 Min.**

NEET / AIIMS

Only One Option Correct Type

1. A 'T' shaped object with dimension shown in the figure, is lying on a smooth floor. A force F is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.



(a) $\frac{4}{3}l$
(b) l
(c) $\frac{2}{3}l$
(d) $\frac{3}{2}l$

2. A particle of mass m moving with velocity u makes an elastic one dimensional collision with a stationary particle of mass m. They are in contact for a brief time T. Their force of interaction increases from zero to F_0 linearly in time $\frac{T}{2}$ and decreases linearly to zero in further time $\frac{T}{2}$. The magnitude of F_0 is

(a) $\frac{mu}{T}$ (b) $\frac{2mu}{T}$
(c) $\frac{mu}{2T}$ (d) None of these

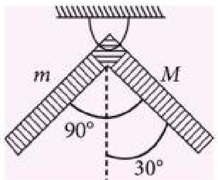
3. A circular ring of mass 6 kg and radius a is placed such that its centre lies at the origin. Two particles of masses 2 kg each are placed at the intersecting points of the circle with positive X-axis and positive Y-axis. Then, the angle made by the position vector of centre of mass of entire system with X-axis is

(a) 45° (b) 60°
(c) $\tan^{-1}\left(\frac{4}{5}\right)$ (d) 30°

4. The motor of an engine is rotating about its axis with an angular velocity of 100 rpm. It comes to rest in 15 s, after being switched off. Assuming constant angular deceleration. What are the numbers of revolutions made by it before coming to rest ?

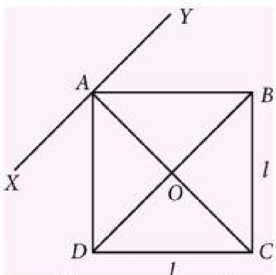
(a) 12.5 (b) 40 (c) 32.6 (d) 15.6

5. Two uniform rods of equal length but different masses are rigidly joined to form an L-shaped body, which is then pivoted as shown in figure. If in equilibrium the body is in the shown configuration, ratio M/m will be



(a) 2 (b) 3 (c) $\sqrt{2}$ (d) $\sqrt{3}$

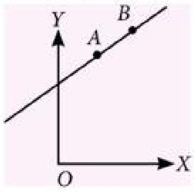
6. Four point masses each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of the system about an axis passing through A and parallel to BD is



(a) $\sqrt{3}\ ml^2$ (b) $3\ ml^2$ (c) ml^2 (d) $2ml^2$

7. A fly wheel of moment of inertia 3×10^2 kg m^2 is rotating with uniform angular speed of 4.6 rad s^{-1}. If a torque of 6.9×10^2 N m retards the wheel, then the time in which the wheel comes to rest is
(a) 1.5 s (b) 2 s (c) 0.5 s (d) 1s

8. A particle of mass m moves in the XY plane with a velocity v along the straight line AB. If the angular momentum of the particle with respect to origin O is L_A when it is at A and L_B when it is at B, then



(a) $L_A > L_B$
(b) $L_A = L_B$
(c) The relationship between L_A and L_B depends upon the slope of the line AB
(d) $L_A < L_B$

9. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement is correct ?
(a) Both cylinders P and Q reach the ground at the same time.
(b) Cylinder P has larger linear acceleration than cylinder Q.
(c) Both cylinder P and Q reaches the ground with same translational kinetic energy.
(d) Cylinder Q reaches the ground with larger angular speed.

10. A circular platform is mounted on a frictionless vertical axle. Its radius $R = 2$ m and its moment of inertia about the axle is 200 kg m^2. It is initially at rest. A 50 kg man stands on the edge of the platform and begins to walk along the edge at the speed of 1 m s^{-1} relative to the ground. Time taken by the man to complete one revolution is
(a) π s (b) $\frac{3\pi}{2}$ s (c) 2π s (d) $\frac{\pi}{2}$ s

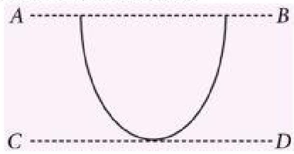
11. Two spherical bodies of mass M and $5M$ and radii R and $2R$ released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is
(a) $4.5R$ (b) $7.5R$ (c) $1.5R$ (d) $2.5R$

12. Two masses m_1 and m_2 ($m_1 > m_2$) are connected by massless flexible and inextensible string passed over massless and frictionless pulley. The acceleration of centre of mass is
(a) $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$ (b) $\frac{m_1 - m_2}{m_1 + m_2} g$
(c) $\frac{m_1 + m_2}{m_1 - m_2} g$ (d) Zero

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as:
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If assertion is false but reason is true.

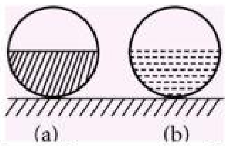
13. **Assertion :** Two axes AB and CD are as shown in figure. Given figure is of a semi-circular ring. As the axis moves from AB towards CD, moment of inertia first decreases, then increases.



Reason : Centre of mass lies somewhere between AB and CD.

14. **Assertion :** A ring and a disc of same mass and radius begin to roll without slipping from the top of an inclined surface at $t = 0$. The ring reaches the bottom of incline in time t_1 while the disc reaches the bottom in time t_2, then $t_1 < t_2$.
Reason : Disc will roll down the plane with more acceleration because of its lesser value of moment of inertia.

15. **Assertion :** Two identical spherical spheres are half filled with two liquids of densities ρ_1 and ρ_2 ($> \rho_1$).

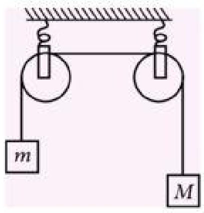


The centre of mass of both the spheres lie at same level.
Reason : The centre of mass will lie at centre of the sphere.

JEE MAIN / ADVANCED

Only One Option Correct Type

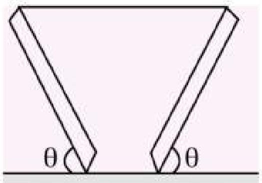
16. Each pulley in figure has radius r and moment of inertia I. The acceleration of the block is



(a) $\dfrac{(M-m)g}{\left(M+m+\dfrac{2I}{r^2}\right)}$ (b) $\dfrac{(M-m)g}{\left(M+m-\dfrac{2I}{r^2}\right)}$

(c) $\dfrac{(M-m)g}{\left(M+m+\dfrac{I}{r^2}\right)}$ (d) $\dfrac{(M-m)g}{\left(M+m-\dfrac{I}{r^2}\right)}$

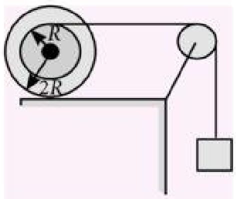
17. Two uniform boards, tied together with the help of a string, are balanced on a surface as shown in figure.



The coefficient of static friction between boards and surface is 0.5. The minimum value of θ, for which this type of arrangement is possible is
(a) 30°
(b) 45°
(c) 37°
(d) It is not possible to have this type of balanced arrangement.

18. A solid cylinder is placed on the end of an inclined plane. It is found that the plane can be tipped at an angle θ before the cylinder starts to slide. When the cylinder turns on its sides and is allowed to roll, it is found that the steepest angle at which the cylinder performs pure rolling is ϕ. The ratio of tan ϕ/tan θ is
(a) 3 (b) 1/3 (c) 1 (d) 1/2

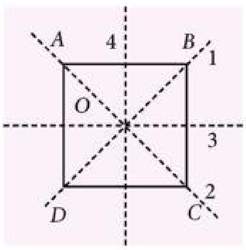
19. In the figure shown mass of both, the spherical body and block is m. Moment of inertia of the spherical body about centre of mass is $2mR^2$. The spherical body rolls on the horizontal surface. There is no slipping at any surfaces in contact. The ratio of kinetic energy of the spherical body to that of block is



(a) 3/4 (b) 1/3 (c) 2/3 (d) 1/2

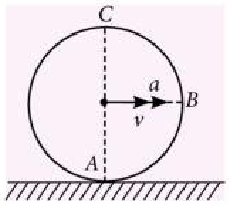
More than one Options Correct Type

20. The moments of inertia of a thin square plate $ABCD$ of uniform thickness about an axis passing through the centre O and perpendicular to the plate are



(a) $I_1 + I_2$ (b) $I_3 + I_4$
(c) $I_1 + I_3$ (d) $I_1 + I_2 + I_3 + I_4$
(Where I_1, I_2, I_3 and I_4 are respectively, the moments of inertia about axes 1, 2, 3 and 4, where axes are in the plane of the plate).

21. A wheel is rolling on a horizontal plane. At a certain instant, it has a velocity 'v' and acceleration 'a' of CM as shown in figure. Acceleration of
(a) A is vertically upwards
(b) B may be vertically downwards
(c) C cannot be horizontal
(d) some point on the rim may be horizontal leftwards

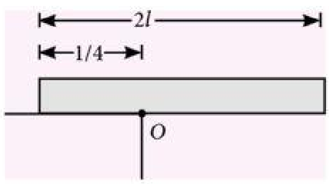


22. A particle of mass m is projected with a velocity v making an angle of 45° with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection, when the particle is at its maximum height h, is
(a) zero (b) $\dfrac{mv^3}{4\sqrt{2}g}$ (c) $\dfrac{mv^3}{\sqrt{2}g}$ (d) $m\sqrt{2gh^3}$

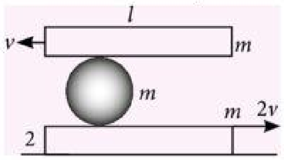
23. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A}$ and $\vec{L}$ where $\vec{A}$ is a constant vector, and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that
(a) $\dfrac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instants of time
(b) the component of $\vec{L}$ in the direction of A does not change with time.
(c) the magnitude of $\vec{L}$ does not change with time.
(d) $\vec{L}$ does not change with time.

Numerical Value Type

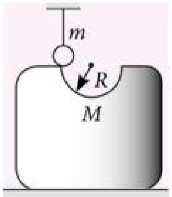
24. One fourth length of a uniform rod of length $2l$ and mass m is placed on a horizontal table and the rod is held horizontal. The rod is released from rest. The normal reaction on the rod as soon as the rod is released is $\dfrac{x\,mg}{31}$. Find the value of x.



25. Two plates 1 and 2 move with velocities $-v$ and $2v$ respectively. If the sphere does not slide relative to the plates, assuming the masses of each body as m. The kinetic energy of the system (plates + sphere) is $\frac{x}{40} mv^2$. The value of x is



26. A semi-circular track of radius $R = 62.5$ cm is cut in a block. Mass of block, having track, is $M = 1$ kg and rests over a smooth horizontal floor. A cylinder of radius $r = 10$ cm and mass $m = 0.5$ kg is hanging by a thread such that axes of cylinder and track are in same level and surface of cylinder is in contact with the track as shown in the figure. When the thread is burnt, cylinder starts to move down the track. Sufficient friction exists between surface of cylinder and track, so that cylinder does not slip. Calculate velocity (in ms^{-1}) of axis of cylinder when it reaches bottom of the track ($g = 10$ m s^{-2}).



Comprehension Type

A uniform rod of length L lies on a smooth horizontal table. The rod has a mass M. A particle of mass m moving with speed v strikes the rod perpendicularly at one of the ends of the rod and sticks to it after collision.

27. Find the velocity of the centre of mass C of the system constituting 'the rod plus the particle'.

(a) $\frac{2Mv}{M-m}$ (b) $\frac{2mv}{M+m}$ (c) $\frac{Mv}{M+m}$ (d) $\frac{mv}{M+m}$

28. Find the velocity of the rod with respect to C before the collision

(a) $\frac{Mv}{M+m}$ (b) $\frac{mv}{M+m}$ (c) $\frac{2Mv}{M+m}$ (d) $\frac{2mv}{m+M}$

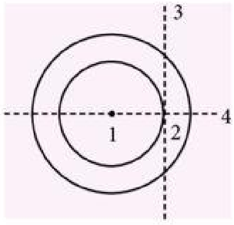
Matrix Match Type

29. From a uniform disc of mass M and radius R, a concentric disc of radius $R/2$ is cut out.

For the remaining annular disc : I_1 is the moment of inertia about axis '1', I_2 about '2', I_3 about '3' and I_4 about '4'.

Axes '1' and '2' are perpendicular to the disc and '3' and '4' are in the plane of the disc.

Axes '2', '3' and '4' intersect at a common point.



Match the following columns:

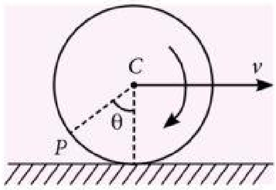
	Column I		Column II
(A)	I_1 is equal to	(P)	$\frac{21}{32} MR^2$
(B)	I_2 is equal to	(Q)	$I_1/2$
(C)	$I_3 + I_4$ is equal to	(R)	$\frac{15}{32} MR^2$
(D)	$I_2 - I_3$ is equal to	(S)	none of these

	A	B	C	D
(a)	R	P	P	Q
(b)	P	Q	R	S
(c)	Q	P	R	P
(d)	R	P	R	S

30. Match column I with column II.

A disc rolls on ground without slipping. Velocity of centre of mass is v.

There is a point P on circumference of disc at angle θ. Suppose v_P is the speed of this point. Then, match the following columns.



	Column I		Column II
(A)	If $\theta = 60°$	(P)	$v_P = \sqrt{2}v$
(B)	If $\theta = 90°$	(Q)	$v_P = v$
(C)	If $\theta = 120°$	(R)	$v_P = 2v$
(D)	If $\theta = 180°$	(S)	$v_P = \sqrt{3}v$

	A	B	C	D
(a)	P	P	S	Q
(b)	S	R	R	S
(c)	Q	P	S	R
(d)	R	S	Q	P

Keys are published in this issue. Search now! ☺

CHECK YOUR PERFORMANCE

No. of questions attempted ......

No. of questions correct ......

Marks scored in percentage ......

If your score is

> 80%	Your preparation is going good, keep it up to get high score.
60-80%	Need more practice, try hard to score more next time.
<60%	Stress more on concepts and revise thoroughly.

FOCUS NEET/JEE 2019

Class XII

Focus more to get high rank in NEET, JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

UNIT 4 : ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

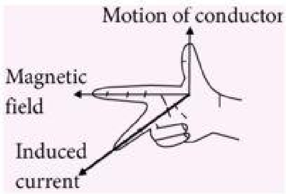
Faraday's Law of Induction

- According to Faraday's first law of electromagnetic induction, whenever the magnetic flux linked with a circuit changes, an emf is induced in the circuit. The SI unit of induced emf is volt.
- **Magnetic Flux :** The total number of magnetic field lines crossing through the surface is known as the magnetic flux of the any surface held in magnetic field $\vec{B}$.
  If a loop enclosing an area A is placed in a magnetic field $\vec{B}$, then the magnetic flux through the loop is
  $$\phi_B = \int \vec{B} \cdot d\vec{A}$$
- According to Faraday's second law of electromagnetic induction, the magnitude of the induced emf in a circuit is equal to the time rate of magnetic flux through the circuit.
  i.e. $\varepsilon = -\dfrac{d\phi}{dt}$
  In the case of a closely wound coil of N turns, change of flux associated with each turn is the same.
  $\therefore \quad \varepsilon = -N\dfrac{d\phi}{dt}$

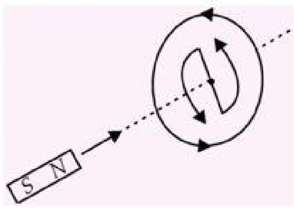
Lenz's Law and energy conservation

According to Lenz's law, the induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current. In other words, the polarity of the induced emf/current is such that it always opposes the change in magnetic flux which produces the induced emf/current. Lenz's law is in accordance with the principle of conservation of energy.

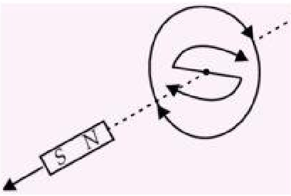
- **Fleming's Right Hand Rule :** Fleming's right hand rule also gives us the direction of induced emf or current, in a conductor moving in a magnetic field.



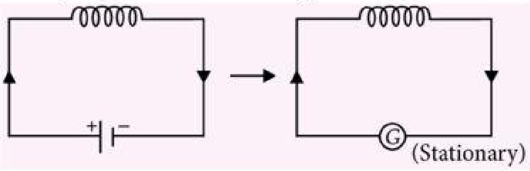
  According to this rule, if we stretch the fore finger, central finger and thumb of our right hand in mutually perpendicular directions such that fore finger points along the direction of the field and thumb is along the direction of motion of the conductor, then the central finger would give us the direction of induced current or emf. The direction of induced current or emf given by Lenz's law and Fleming's right hand rule is the same.
- **Applications of Lenz's Law :** When a north pole of a bar magnet is moved towards a coil, the current induced in the coil will be in anticlockwise direction as shown in the figure.



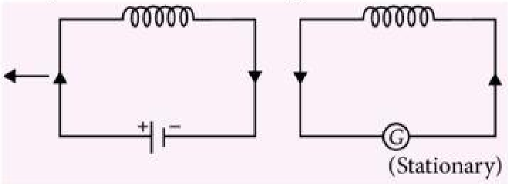
When a north pole of bar magnet is moved away from the coil, the current induced in the coil will be in clockwise direction as shown in the figure.



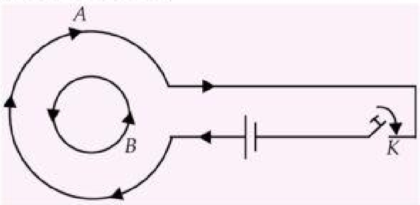
When a current carrying coil is moved towards a stationary coil, the direction of current induced in stationary coil is as shown in figure.



When a current carrying coil is moved away from a stationary coil, the direction of current induced in stationary coil is as shown in figure.

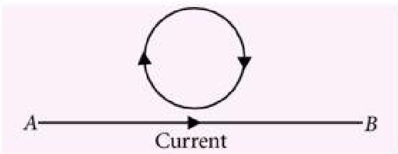


When two coils A and B are arranged as shown in figure, then on pressing K, current in A increases in clockwise direction. Therefore, induced current in B will be in anticlockwise direction.



However, when key K is released, current in A decreases in clockwise direction. Therefore, induced current in B will be in clockwise direction.

When current in a straight conductor AB is increased, induced current in loop will be in clockwise direction as shown in the figure. If current in AB is decreasing, the induced current in the loop will be in anticlockwise direction.



MOTIONAL EMF

If a conductor is moving with velocity $\vec{v}$ in a magnetic field, electrons inside it experience a force $\vec{F} = e(\vec{v} \times \vec{B})$ and accumulate at the end of the conductor. Very soon, an electric field is established. Eventually component of magnetic force along the conductor length is balanced by the electric field force and the drifting of electrons stops and an emf is established.

Now, $\varepsilon = -\int \vec{E} \cdot d\vec{l} = \int (\vec{v} \times \vec{B}) \cdot d\vec{l}$

This is general expression for induced emf in a conducting wire. If $\vec{v}, \vec{B}$ and $\vec{l}$ are mutually perpendicular to each other then, $\varepsilon = Bvl$.

Motional emf in Loop

If a conducting rod moves on two parallel conducting rails then an emf is induced whose magnitude is $|\varepsilon| = Blv$ and direction is given by Fleming's right hand rule. Induced current $|I| = \frac{|\varepsilon|}{R} = \frac{Blv}{R}$

Rotational emf

If a conducting rod of length l rotates about an axis passing through one of its ends (that end may be fixed) with an angular velocity ω in a plane perpendicular to the magnetic field $\vec{B}$. Then an induced emf is set up between ends of rod whose magnitude is given by $|\varepsilon| = \frac{1}{2} Bl^2\omega$

Eddy Currents

The current induced in bulk pieces of conductors when the magnetic flux linked with the conductor changes are known as Eddy currents.

INDUCTANCE

An electric current can be induced in a coil by flux change produced by another coil in its vicinity or flux change produced by the same coil. The flux through a coil is proportional to the current *i.e.* $\phi \propto I$

For a closely wound coil of N turns, $N\phi \propto I$

or $\phi = \frac{LI}{N}$

where L is the constant of proportionality called inductance. SI unit of inductance is Tm^2A^{-1} or henry (H) and its dimensional formula is $[ML^2T^{-2}A^{-2}]$.

Self Inductance

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with it will also change. As a result of this, an emf is induced in the coil or the circuit which opposes the change that causes it.

This phenomenon is known as self induction and the emf induced is known as self induced emf or back emf.
When a current I flows through a coil and ϕ is the magnetic flux linked with the coil, then

$\phi \propto I$ or $\phi = LI$

where L is coefficient of self induction or self inductance of the coil.

The self induced emf is $\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

Self inductance of a solenoid is $L = \mu_0 n^2 lA$

where l is length of the coil solenoid, n is number of turns per unit length of a solenoid and A is area of cross section of the solenoid.

Self inductance of a circular coil is $L = \frac{\mu_0 N^2 \pi R}{2}$

where R is the radius of a coil and N is the number of turns.

Mutual Inductance

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence an emf will be induced in the neighbouring coil or circuit. This phenomenon is known as mutual induction. The coil or circuit in which the current changes is known as primary, while the other in which emf is set up is known as secondary.

Let I_P be the current flowing through primary coil at any instant. If ϕ_S is the flux linked with secondary coil then $\phi_S \propto I_P$ or $\phi_S = MI_P$

where M is coefficient of mutual inductance of two coils.

The emf induced in the secondary coil is given by

$$\varepsilon_S = -M\frac{dI_P}{dt}$$

Combination of Inductances

When two coils having self inductance L_1 and L_2 are connected in series keeping them far apart (so that mutual inductance between them is negligible), the equivalent inductance of the combination is given by

$$L = L_1 + L_2$$

When two coils are connected in parallel, the equivalent inductance is given by

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

Energy Stored by an Inductor

A current-carrying inductor stores energy in the magnetic field associated with it and energy stored for current I and an inductance L are related as $\frac{1}{2}LI^2$.

AC Generator

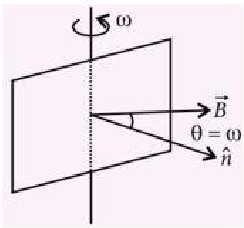
It is a device used to obtain a supply of alternating emf by converting rotational mechanical energy into electrical energy. It is based on the phenomenon of electromagnetic induction. *i.e.* when a coil is rotated in uniform magnetic field, an induced emf is produced in it.

The instantaneous value of the e.m.f. produced is given by $\varepsilon = NBA\omega\sin\omega t$, where N is number of turns of the coil, A is the area of coil and ω is angular frequency of rotation of the coil in a magnetic field strength B.

Basic Principle of AC Generation

Alternating voltage is generated by rotating a coil of conducting wire in a strong magnetic field. The magnetic flux linked with the coil changes with time and an alternating emf is thus induced. Instantaneous flux linked with coil is

$$\phi = (\vec{A}\cdot\vec{B})n$$
$$= ABn\cos(\omega t + \theta_0)$$



where,

A = area of the coil (in m^2)

B = magnetic field (in tesla)

n = number of turns

ω = angular frequency $= \frac{2\pi}{T} = 2\pi\nu$ (in rad s^{-1})

ν = frequency (in hertz)

θ_0 = initial phase angle.

and the alternating voltage is given by

$$V = -\frac{d\phi}{dt} = V_0 \sin\omega t$$

Where $V_0 = ABn\omega$

The instantaneous value of an ac is given by

$I = I_0 \sin\omega t$

Here, ω is the angular frequency of ac and $(\omega/2\pi)$ is the frequency of ac. $(2\pi/\omega)$ represents the time period of ac.

In one cycle of ac, current increases from zero to a maximum, then decreases to zero and reverses in direction and then decreases to zero. Thus current is zero twice in one cycle and is numerically maximum also twice in one cycle, once in the forward direction and once in the backward direction in one cycle. Time taken to complete one cycle is called Time Period.

The frequency of ac represents the number of cycles of ac completed in one second. Ac supplied in India has a frequency of 50 Hz.

Alternating Current

It is the current which varies continuously in magnitude and periodically in direction. It can be represented by $I = I_0 \sin\omega t$ or $I = I_0\cos\omega t$.
where I_0 is peak value of current and is known as amplitude of ac and I is the instantaneous value of ac.

$\omega = \frac{2\pi}{T} = 2\pi\upsilon$ where T is period of ac and υ is frequency of ac.

Voltages and currents that vary symmetrically in magnitude and direction with time are very common. The electric mains supply in our homes and offices is a voltage that varies like a sine function with time. Such a voltage is called alternating voltage (ac voltage) and the current driven through the appliances is called the alternating current (ac current)

Average Values of AC Voltage and AC Current

AC voltage or current are commonly sinusoidal (sine or cosine function) and their mean values for complete cycle is zero. The average values for half cycles are equally positive and negative

- **Average value for one half cycle (or rectified average value)**

  $V = V_0 \sin \omega t$

  $$\therefore\ (V)_{av} = \frac{\int_0^{T/2} V\,dt}{\int_0^{T/2} dt} = \frac{2}{\pi}V_0 = 0.637\,V_0.$$

  This is also known as the rectified average value of a sinusoidal voltage and is represented as V_{av}.

- **Root Mean Square Value (V_{rms} or I_{rms})**

  Since V or I are equally negative and positive, their squares will always be positive and the square root of the average of their square will give the rms values.

  $\therefore\ V = V_0 \sin\omega t$

  $$(V^2)_{av} = \frac{1}{T}\int_0^T V_0^2 \sin^2 \omega t\,dt = \frac{V_0^2}{2T}\int_0^T (1-\cos 2\omega t)dt = \frac{V_0^2}{2}$$

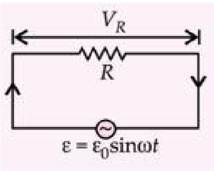
  Thus $V_{\text{rms}} = \sqrt{(V^2)_{av}} = \frac{V_0}{\sqrt{2}}$

  and $I_{\text{rms}} = \sqrt{(I^2)_{av}} = \frac{I_0}{\sqrt{2}}$

  or RMS value $= \frac{\text{Peak value}}{\sqrt{2}}$

Series AC Circuits

When only Resistance is in AC Circuit



Consider a simple ac circuit consisting of a resistor of resistance R and an ac generator, as shown in figure.
According to Kirchhoff's loop law at any instant, the algebraic sum of the potential difference around a closed loop in a circuit must be zero.

$\varepsilon - V_R = 0$; $\varepsilon - I_R R = 0$; $\varepsilon_0 \sin\omega t - I_R R = 0$

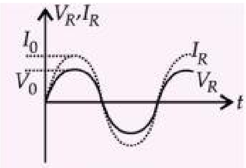
$$I_R = \frac{\varepsilon_0}{R}\sin\omega t = I_0 \sin\omega t \qquad \text{...(i)}$$

where I_0 is the maximum current, $I_0 = \frac{\varepsilon_0}{R}$

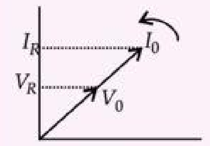
From above equations, we see that the instantaneous voltage drop across the resistor is

$$V_R = I_0 R \sin\omega t \qquad \text{...(ii)}$$

- **Phasor Diagram :** We see in equation (i) and (ii), I_R and V_R both vary as $\sin\omega t$ and reach their maximum values at the same time as shown in figure (a), they are said to be in phase. A phasor diagram is used to represent phase relationships. The lengths of the arrows correspond to V_0 and I_0. The projections of the arrows onto the vertical axis gives V_R and I_R.



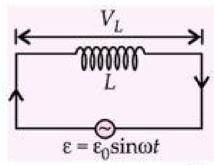
Wave diagram
(a)



Phasor diagram
(b)

In case of the single-loop resistive circuit, the current and voltage phasors lie along the same line, as shown in figure (b), because I_R and V_R are in phase.

When only Inductor is in an AC Circuit



Now consider an ac circuit consisting only of an inductor of inductance L connected to the terminals of an ac generator, as shown in the figure.

The induced emf across the inductor is given by $L\frac{dI}{dt}$.
On applying Kirchhoff's loop rule to the circuit

$$\varepsilon - V_L = 0 \quad \Rightarrow \quad \varepsilon - L\frac{dI}{dt} = 0$$

When we rearrange this equation and substitute $\varepsilon = \varepsilon_0 \sin\omega t$, we get

$$L\frac{dI}{dt}=\varepsilon_0 \sin\omega t \quad \text{...(iii)}$$

Integration of this expression gives the current as a function of time

$$I_L=\frac{\varepsilon_0}{L}\int \sin\omega t dt=-\frac{\varepsilon_0}{\omega L}\cos\omega t+C$$

For average value of current over one time period to be zero, $C = 0$

$$\therefore \quad I_L=-\frac{\varepsilon_0}{\omega L}\cos\omega t$$

When we use the trigonometric identity $\cos\omega t = -\sin(\omega t - \pi/2)$, we can express equation as

$$I_L=\frac{\varepsilon_0}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right) \quad \text{...(iv)}$$

From equation (iv), we see that the current reaches its maximum values when $\cos \omega t = 1$

$$I_0=\frac{\varepsilon_0}{\omega L}=\frac{\varepsilon_0}{X_L} \quad \text{...(v)}$$

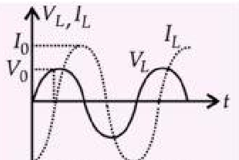
where the quantity X_L, called the inductive reactance, is $X_L = \omega L$

The expression for the rms current is similar to equation (v), with ε_0 replaced by ε_{rms}. Inductive reactance, like resistance, has unit of ohm.

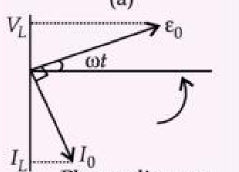
$$V_L=L\frac{dI}{dt}=\varepsilon_0\sin\omega t=I_0X_L\sin\omega t$$

- **Phasor Diagram :** We can think of equation (v) as ohms law for an inductive circuit. On comparing result of equation (iv) with equation (iii), we can see that the current and voltage are out of phase with each other by $\pi/2$ rad, or 90°.

  A plot of voltage and current versus time is given in figure (a). The voltage reaches its maximum value one quarter of an oscillation period before the current reaches its maximum value. The corresponding phasor diagram for this circuit is shown in figure (b). Thus, we see that for a sinusoidal applied voltage, the current in an inductor always lags bel the inductor by 90°.



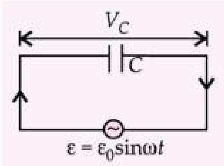
Wave diagram
(a)



Phasor diagram
(b)

When only Capacitor is in an AC circuit

Figure shows an ac circuit consisting of a capacitor of capacitance C connected across the terminals of an ac generator. On applying Kirchhoff's loop rule to this circuit gives.



$$\varepsilon - V_C = 0$$

$$V_C = \varepsilon = \varepsilon_0 \sin\omega t \quad \text{...(vi)}$$

where V_C is the instantaneous voltage drop across the capacitor.

From the definition of capacitance, $V_C = Q/C$, and this value for V_C substituted into equation (vi) gives

$$Q = C\varepsilon_0 \sin\omega t$$

Since $I = dQ/dt$, on differentiating above equation gives the instantaneous current in the circuit.

$$I_C=\frac{dQ}{dt}=C\varepsilon_0\omega\cos\omega t$$

Here again we see that the current is not in phase with the voltage drop across the capacitor, given by equation (vi). Using the trigonometric identity $\cos\omega t = \sin(\omega t + \pi/2)$, we can express this equation in the alternative form

$$I_C=\omega C\varepsilon_0\sin\left(\omega t+\frac{\pi}{2}\right) \quad \text{...(vii)}$$

From equation (vii), we see that the current in the circuit reaches its maximum value when $\cos\omega t = 1$.

$$i_0=\omega C\varepsilon_0=\frac{\varepsilon_0}{X_C}$$

where X_C is called the capacitive reactance

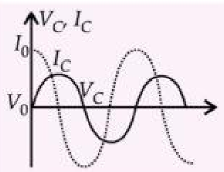
$$X_C=\frac{1}{\omega C}$$

The SI unit of X_C is also ohm. The rms current is given by an expression similar to equation with V_0 replaced by V_{rms}.

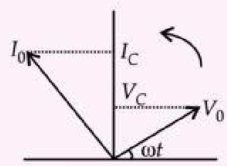
- **Phasor Diagram :** Combining equation (vi) and (vii), we can express the instantaneous voltage drop across the capacitor as, $V_C = V_0 \sin\omega t = I_0X_C \sin\omega t$

  Comparing the result of equation (v) with equation (vi), we see that the current is $\frac{\pi}{2}$ rad = 90° out of phase with the voltage across the capacitor.

  A plot of current and voltage versus time, shows that the current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value. The corresponding phasor diagram is shown in the figure (b). Thus we see that for a sinusoidally applied emf,the current always leads the voltage across a capacitor by 90°.



Wave diagram
(a)



Phasor diagram
(b)

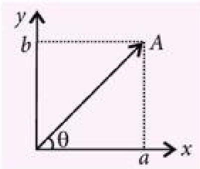
Vector Analysis (Phasor Algebra)

The complex quantities normally employed in ac circuit analysis, can be added and subtracted like coplanar vectors. Such coplanar vectors, which represent sinusoidally time varying quantities, are known as phasors.

In cartesian form, a phasor A can be written as

$A = a + jb$

where a is the x-component and b is the y-component of phasor A.



The magnitude of A is, $|A| = \sqrt{a^2 + b^2}$ and the angle between the direction of phasor A and the positive x-axis is,

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

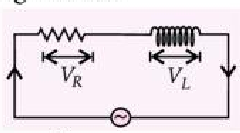
when a given phasor A, the direction of which is along the x-axis is multiplied by the operator j, a new phasor jA is obtained which will be 90° anticlockwise from A, *i.e.*, along y-axis. If the operator j is multiplied now to the phasor jA, a new phasor $j^2 A$ is obtained which is along x-axis and having same magnitude as of A. Thus, $j^2 A = -A$

$$j^2 = -1 \quad \text{or} \quad j = \sqrt{-1}$$

Now using the j operator, let us discuss different circuits of an ac.

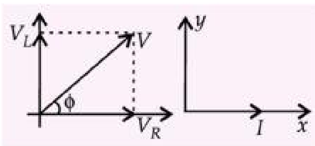
Series *L-R* Circuit

Now consider an ac circuit consisting of a resistor of resistance R and an inductor of inductance L in series with an ac source generator.



Suppose in phasor diagram, current is taken along positive x-direction. Then V_R is also along positive x-direction and V_L along positive y-direction.

As we know potential difference across a resistance in ac is in phase with current, and it leads current in phase by 90° across the inductor, so we can write



$V = V_R + jV_L = IR + j(IX_L) = IR + j(I\omega L) = IZ$

- **Impedance and Phase Difference :** Here, $Z = R + jX_L = R + j(\omega L)$ is called impedance of the circuit. Impedance plays the same role in ac circuits as the ohmic resistance does in dc circuits The modulus of impedance is,

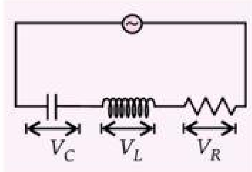
$$|Z| = \sqrt{R^2 + (\omega L)^2}$$

The potential difference leads the current by an angle,

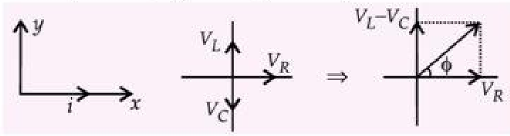
$$\phi = \tan^{-1}\left|\frac{V_L}{V_R}\right| = \tan^{-1}\left(\frac{X_L}{R}\right) \Rightarrow \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Series *L-C-R* Circuit and Resonance

Now consider an ac circuit consisting of a resistor of resistance R, a capacitor of capacitance C and an inductor of inductance L, in series with an ac source generator.



Suppose in a phasor diagram, current is taken along positive x-direction. Then V_R is along positive x-direction. V_L along positive y- direction and V_C along negative y-direction, as potential difference across an inductor leads the current by 90° in phase while that across a capacitor, lags it in phase by 90°.



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

So, we can write, $V = V_R + jV_L - jV_C$

$= IR + j(IX_L) - j(IX_C) = IR + j[I(X_L - X_C)] = IZ$

- **Impedance and Phase Difference :** Here impendance is,

$$Z = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

The modulus of impedance is,

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$ and the potential difference leads the current by an angle.

$$\phi = \tan^{-1}\left|\frac{V_L - V_C}{V_R}\right| = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \quad ...(viii)$$

$$\phi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

The steady current in the circuit is given by

$$I = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t + \phi)$$

where ϕ is given from equation (viii)

The peak current is $I_0 = \dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

It depends on angular frequency ω of ac source.

- **Resonance Condition :** The current will be maximum when $\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega = \sqrt{\frac{1}{LC}}$ and corresponding frequency is $\upsilon = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

  This frequency is known as resonant frequency of the given circuit. At this frequency peak current will be $I_0 = \frac{V_0}{R}$

  If the resistance R in the LCR circuit is zero, the peak current at resonance is $I_0 = \frac{V_0}{0}$

  It means, there can be a finite current in pure LC circuit even without any applied emf,

  This current in the circuit is at frequency,

  $$\upsilon = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Quality Factor

It is a measure of sharpness of resonance. It is defined as the ratio of reactance of either the inductance or capacitance at the resonant angular frequency to the total resistance of the circuit.

$$Q = \frac{X_L}{R} = \frac{\omega_r L}{R} \; ; \; Q = \frac{X_C}{R} = \frac{1}{\omega_r CR} \; ; \; Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Quality factor is also expressed in terms of bandwidth

$$Q = \frac{\text{Resonant frequency}}{\text{Bandwidth}}$$

Power in AC Circuit

In an ac circuit we may define three types of power.

- **Instantaneous power :** The power in the ac circuit at any instant of time is known as instantaneous power. It is equal to the product of values of alternating voltage and alternating current at that time.
- **Average power (P_{av}) :** The power averaged over one full cycle of ac is known as average power. It is also known as true power.

  $$P_{av} = V_{rms}\, I_{rms} \cos\phi = \frac{V_0 I_0}{2}\cos\phi$$
- **Apparent power :** The product of virtual voltage (V_{rms}) and virtual current (I_{rms}) in the circuit is known as virtual power.

  $$P_v = V_{rms}\, I_{rms} = \frac{V_0 I_0}{2}$$

Power Factor

It is defined as the ratio of true power to apparent power of an ac circuit

$$\cos\phi = \frac{\text{True power}}{\text{Apparent power}}$$

- Power factor is also defined as the ratio of the resistance to the impedance of an ac circuit

  $$\cos\phi = \frac{R}{Z}$$

  It is unitless and dimensionless quantity.
- In pure resistive circuit,

  $\phi = 0°$; $\cos\phi = 1$.
- In pure inductive or capacitive circuit

  $\phi = \frac{\pi}{2}$; $\cos\phi = 0$.
- In RL circuit,

  $Z = \sqrt{R^2 + X_L^2}$ and $\cos\phi = \frac{R}{Z}$
- In RC circuit,

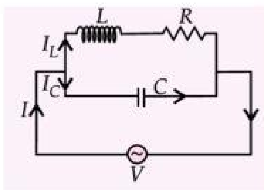
  $Z = \sqrt{R^2 + X_C^2}$ and $\cos\phi = \frac{R}{Z}$
- In series LCR circuit,

  $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and $\cos\phi = \frac{R}{Z}$
- At resonance, $X_L = X_C$

  $\therefore \; Z = R$ and $\phi = 0°$

  $\cos\phi = 1$

Parallel AC Circuits



Let us consider an alternating source connected across an inductance L in parallel with a capacitor C. The resistance in series with the inductance is R and with the capacitor is zero.

Let the instantaneous value of emf applied be V and the corresponding current is I, I_L and I_C. Then,

$$I = I_L + I_C$$

or $$\frac{V}{Z} = \frac{V}{R + j\omega L} - \frac{V}{j/\omega C} = \frac{V}{R + j\omega L} - \frac{(\omega C)V}{j}$$

$$= \frac{V}{R + j\omega L} - \frac{j(\omega C)V}{j^2} = \frac{V}{R + j\omega L} + j(\omega C)V \quad (\text{as } j^2 = -1)$$

- **Admittance :**

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$\frac{1}{Z}$ is known as admittance (Y). Therefore,

$$Y=\frac{1}{Z}=\frac{R-j\omega L}{R^2+\omega^2L^2}+j\omega C=\frac{R+j(\omega CR^2+\omega^3L^2C-\omega L)}{R^2+\omega^2L^2}$$

- **Resonance Condition**

  The magnitude of the admittance,

  $$Y=|Y|=\frac{\sqrt{R^2+(\omega CR^2+\omega^3L^2C-\omega L)^2}}{R^2+\omega^2L^2}$$

  The admittance will be minimum, when

  $\omega CR^2+\omega^3L^2C-\omega L=0$

  $$\omega=\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$$

  It gives the condition of resonance and the corresponding frequency, $\nu=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{L^2}}$ is known as resonance frequency. At resonance frequency admittance is minimum or the impedance is maximum. Thus, the parallel circuit does not allow this frequency from the source to pass in the circuit. Due to this reason the circuit with such a frequency is known as rejector circuit.

  If R = 0, resonance frequency is $\frac{1}{2\pi\sqrt{LC}}$ same as resonance frequency in series circuit.

  At resonance, the reactive component of Y is real. The reciprocal of the admittance is called the parallel resistor or the dynamic resistance. The dynamic resistance is thus, reciprocal of the real part of the admittance.

  $$\text{Dynamic resistance}=\frac{R^2+\omega^2L^2}{R}$$

  Substituting $\omega^2=\frac{1}{LC}-\frac{R^2}{L^2}$

  we have, dynamic resistance $=\frac{L}{CR}$

  $\therefore$ peak current through the supply $=\frac{V_0}{L/CR}=\frac{V_0CR}{L}$

  The peak current through capacitor $=\frac{V_0}{1/C\omega}=\omega V_0C.$

  The ratio of the peak current through capacitor and through the supply is known as Q-factor.

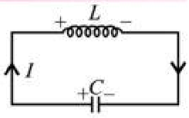
  $$\text{Thus, } Q\text{-factor}=\frac{V_0\omega C}{V_0CR/L}=\frac{\omega L}{R}$$

  This is basically the measure of current magnification. The rejector circuit at resonance exhibits current magnification of $\frac{\omega L}{R}$, similar to the voltage magnification of the same ratio exhibited by the series acceptor circuit at resonance.

  At resonance the current through the supply and voltage are in phase, while the current through the capacitor leads the voltage by 90°.

LC OSCILLATIONS

- $L\frac{d^2q}{dt^2}+\frac{q}{C}=0 \quad \left(\because\ I=-\frac{dq}{dt}\right)$



- Frequency, $\upsilon=\frac{1}{2\pi\sqrt{LC}}$
- Oscillation of charge, $q=q_m\cos(\omega t+\phi)$

  (where ω = natural frequency = $\frac{1}{\sqrt{LC}}$)
- Oscillation of current, $I=I_m\sin(\omega t+\phi)$

  (where $I_m=\omega q_m$)
- Electric energy, $U_E=\frac{q^2}{2C}=\frac{q_m^2}{2C}\cos^2(\omega t+\phi)$
- Magnetic energy,

  $$U_B=\frac{1}{2}LI^2=\frac{1}{2}L\omega^2q_m^2\sin^2(\omega t+\phi)=\frac{q_m^2}{2C}\sin^2(\omega t+\phi)$$

TRANSFORMERS

- A transformer consists of an iron core on which a primary coil of turns N_p and a secondary coil of turns N_S are bound. If the primary coil is connected to an AC source, the primary and secondary voltages and current are related by

  $$V_s=\left(\frac{N_s}{N_p}\right)V_p\ ;\ I_s=\left(\frac{N_p}{N_s}\right)I_p$$
- If the secondary coil has a greater number of turns than the primary, the voltage is stepped-up ($V_s > V_p$). This type of arrangement is called a step-up transformer. If the secondary coil has turns less than the primary, we have a step-down transformer.
- Efficiency of transformer,

  $$\eta=\frac{\text{Output power}}{\text{Input power}}=\frac{V_sI_s}{V_pI_p}$$

  For an ideal transformer, η = 1
- The ratio N_P/N_S is known as turns ratio of the transformer. Transformers are mainly characterized by the turns ratio.

BRAIN MAP

CURRENT ELECTRICITY

ELECTRIC CURRENT

Electric Current

- Rate of flow of electric charge (positive), $I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = \oint \vec{J}.d\vec{S}$
- J is the current density which is the current passing through a cross-section of the wire.
- $\vec{J}.d\vec{S}$ shows current is scalar inspite of this we represent current with an arrow.
- Its SI unit ampere (A) and C.G.S. unit is emu. It is called biot (Bi)

Flow of Charge

$\Delta Q = I\Delta t$ (Constant I)

$= \int I dt$ ($I = f(t)$)

Area under I-t graph gives flow of charge.

OHM'S LAW

Basic Features of Ohm's Law

- Vector form of Ohm's law, $\vec{J} = \sigma\vec{E}$
  where conductivity $\sigma = \frac{1}{\rho}$ and $\vec{J}$ is the current density.
- Graph between V and I for a metallic conductor

Current and Drift Speed

- Drift speed, $v_d = \frac{eE}{m}\tau$
- Mobility, $\mu_e = \frac{v_d}{E} = \frac{e\tau}{m}$
- Current in terms of drift velocity, $I = neAv_d$
- In terms of relaxation time τ, $R = \frac{ml}{ne^2\tau A}$ and $\rho = \frac{m}{ne^2\tau}$

OHM's Law

If the physical conditions of the conductor (length, temperature, mechanical strain etc.) remain same, then the current flowing through the conductor is directly proportional to the potential difference across it's two ends i.e., $I \propto V \Rightarrow V = IR$

Limitations of Ohm's Law

It is not a universal law that applies everywhere under all conditions.

= Ohm's law is not followed for ⇒

- **Materials :** Crystal rectifiers, thermistors, thyristors, semi-conductors.
- **Conditions :**
  (i) At very high temperatures
  (ii) At very low temperatures
  (iii) At very high potential differences.

ELECTRICAL RESISTANCE

Resistance

The property of a substance by virtue of which it opposes the flow of current through it.

$R = \rho\frac{l}{A} = \frac{m}{ne^2\tau}\cdot\frac{l}{A}$

= Specific resistance of the material of conductor ⇒

Resistivity (ρ)

It is numerically equal to the resistance of a substance having unit area of cross-section and unit length.

Temperature Dependence

- For a conductor then
  $R_t = R_0(1 + \alpha t + \beta t^2)$, $R_t = R_0(1 + \alpha t)$
- Temperature coefficient $\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$
- Also for resistivity, $\rho_t = \rho_0(1 + \alpha t)$

R_0 = resistance at 0°C
R_t = resistance at t°C
α, β = temperature co-efficients

COMBINATION OF RESISTANCE AND ELECTRICAL POWER

Electrical Energy and Power

- Heat energy developed across a resistor $H = I^2Rt$
- Power $P = I^2R = \frac{V^2}{R}$

Equivalent Power of Bulb

- Resistance of bulb, $R = \frac{V^2}{P}$ or $R \propto \frac{1}{P}$
- In parallel $P = P_1 + P_2$
- In series $\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$ or $P = \frac{P_1P_2}{P_1 + P_2}$
- In $R = \frac{V^2}{P}$, V and P are rated values for that bulb.
- In parallel, a bulb having more rated power glows more brightly. In series, a bulb having less rated power glows more brightly.

Series Combination

- In series, equivalent resistance
  $R_S = R_1 + R_2 + R_3 + ...$
- In series connection, the current through each resistance is same but voltage divides.

Parallel Combination

- In parallel, equivalent resistance
  $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$
- In parallel connection, voltage is same but current divides.

KIRCHHOFF'S LAW

Junction Rule

At any junction of circuit, the sum of currents entering and leaving must be zero. $\Sigma I = 0$.
It is based on conservation of charge.

Loop Rule

The algebraic sum of changes in potential around any closed loop must be zero. $\Sigma\varepsilon - \Sigma IR = 0$
It is based on conservation of energy.

Problem Solving Strategies

- Distribute current at various junctions in the circuit starting from positive terminal.
- Pick a point and begin to walk around a closed loop.
- Write down the voltage change for that element according to the sign convention.
- By applying KVL, select the required number of loops as many as unknowns are available and apply KVL across each loop.
- Solve the set of simultaneous equation to find the unknowns.

Wheatstone Bridge

- In balanced condition,
  $\frac{R_1}{R_2} = \frac{R_3}{R_4}$
- It provides a parallel method to determine the value of an unknown resistance.

Meter Bridge

- To measure resistance of unknown resistor using balance condition of wheatstone bridge

$R = \frac{Sl_1}{(100 - l_1)}$

R = unknown resistance
S = shunt resistance
l_1 = balanced length

Potentiometer

- Principle : Fall of potential across any portion of the wire, $V \propto l$, $V = Kl$
- Comparision of emfs of two cells, $\frac{\varepsilon_1}{\varepsilon_2} = \frac{l_1}{l_2}$
- Internal resistance of a cell,
  $r = R\left(\frac{\varepsilon}{V} - 1\right) = R\left(\frac{l_1}{l_2} - 1\right)$

GROUPING OF BATTERIES

Series Grouping

- For n identical batteries
  $I = \frac{n\varepsilon}{nr + R}$ $\left\{ \begin{array}{l} \varepsilon = \text{emf} \\ r = \text{internal resistance} \end{array} \right.$
- If polarity of m batteries is reversed
  $I = (n - 2m)\varepsilon/(m + R)$

Parallel Grouping

- With identical batteries :
  $I = \frac{\varepsilon_{net}}{R_{net}}, \varepsilon_{net} = \varepsilon$
  $R_{net} = \frac{r}{N} + R$
- With unidentical batteries :
  $\varepsilon_{net} = \frac{\Sigma(\varepsilon/r)}{\Sigma(1/r)}, I = \frac{\varepsilon_{net}}{R_{net}}$

Combining series and parallel grouping

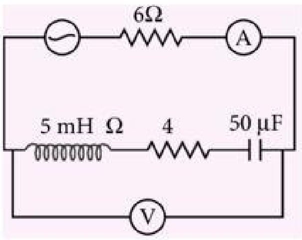
Mixed Grouping

- For n rows of identical batteries with m cells in each row. Then,
  $R_{net} = \frac{mr}{n} + R$,
  $\varepsilon_{net} = m\varepsilon$, $I = \frac{\varepsilon_{net}}{R_{net}}$
- In this case current through the external resistance is maximum when $R = \frac{mr}{n}$

SPEED PRACTICE

1. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1.0 A at approximately the time
(a) 500 s (b) 20 s (c) 35 ms (d) 1 ms

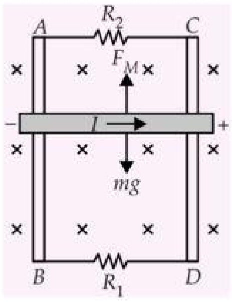
2. In the circuit shown in figure, the A.C. source gives a voltage $V = 20 \cos(2000\,t)$. Neglecting source resistance, the voltmeter and ammeter readings will be



(a) 0 V, 0.47 A (b) 1.68 V, 0.47 A
(c) 0 V, 1.4 A (d) 5.6 V, 1.4 A

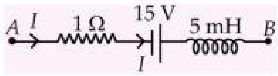
3. A resistance of 20 Ω is connected to a source of an alternating potential $V = 220 \sin(100\,\pi t)$ V. The time taken by the current to change from its peak value to rms value, is
(a) 0.2 s (b) 0.25 s
(c) 25×10^{-3} s (d) 2.5×10^{-3} s

4. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistances R_1 and R_2 as shown in figure. A horizontal metallic rod of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. When the terminal velocity is attained, the power dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively. Find the terminal velocity of the rod and the resistance R_1 and R_2.



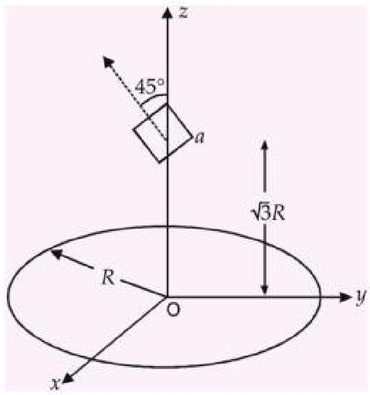
(a) 1 m s^{-1}, 0.47 Ω, 0.30 Ω (b) 2 m s^{-1}, 0.30 Ω, 0.30 Ω
(c) 1 m s^{-1}, 0.47 Ω, 0.50 Ω (d) 2 m s^{-1}, 0.30 Ω, 0.47 Ω

5. The network shown in Figure, is a part of a complete circuit. What is the potential difference $V_B - V_A$, when the current I is 5A and is decreasing at a rate of 10^3 (A s^{-1})?



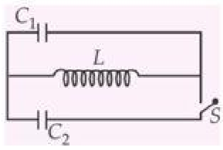
(a) 5 V (b) 15 V (c) 10 V (d) 0.5 V

6. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a ($a << R$) having two turns is placed with its center at $z = \sqrt{3}\,R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then find the value of p.



(a) 1 (b) 3 (c) 5 (d) 7

7. At a moment ($t = 0$) when charge on capacitor C_1 is zero, the switch is closed. If i_0 be the current through inductor at that instant, for $t > 0$, find the maximum charge on C_1.



(a) $i_0 C_1 \sqrt{\frac{L}{C_1 + C_2}}$ (b) $i_0 C_2 \sqrt{\frac{L}{C_1 + C_2}}$
(c) $i_0 \sqrt{\frac{L}{C_1 + C_2}}$ (d) $i_0 C_1 \sqrt{\frac{C_1 + C_2}{L}}$

8. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. Find the time at which the energy is stored equally between the electric and the magnetic fields.
(a) $\frac{\pi}{2}\sqrt{LC}$ (b) $\frac{\pi}{4}\sqrt{LC}$ (c) $\frac{\pi}{8}\sqrt{LC}$ (d) $\pi\sqrt{LC}$

9. A simple LR circuit is connected to a battery at time $t = 0$. The energy stored in the inductor reaches half its maximum value at time

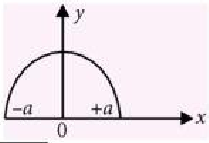
(a) $\frac{R}{L}\ln\left[\frac{\sqrt{2}}{\sqrt{2}-1}\right]$ (b) $\frac{L}{R}\ln\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$

(c) $\frac{L}{R}\ln\left[\frac{\sqrt{2}}{\sqrt{2}-1}\right]$ (d) $\frac{R}{L}\ln\left[\frac{\sqrt{2}-1}{\sqrt{2}}\right]$

10. Power factor of an LR series circuit is 0.6 and that of a CR series circuit is 0.5. If the element (L, C, and R) of the two circuits are joined in series the power factor of this circuit is found to be 1. The ratio of the resistance in the LR circuit to the resistance in the CR circuit is

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{4}{3\sqrt{3}}$ (d) $\frac{3\sqrt{3}}{4}$

11. Determine the rms value of a semi-circular current wave which has a maximum value of a.



(a) $(1/\sqrt{2})a$ (b) $\sqrt{(3/2)}a$

(c) $\sqrt{(2/3)}a$ (d) $(1/\sqrt{3})a$

12. A 50 W, 100 V lamp is to be connected to an ac mains of 200 V, 50 Hz. What capacitor is essential to be put in series with the lamp?

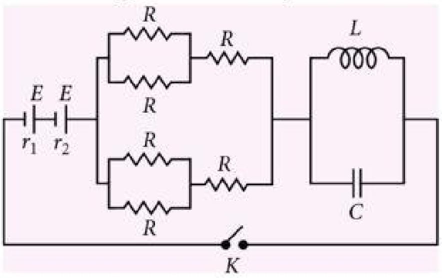
(a) $\frac{25}{\sqrt{2}}\mu F$ (b) $\frac{50}{\pi\sqrt{3}}\mu F$

(c) $\frac{50}{\sqrt{2}}\mu F$ (d) $\frac{100}{\pi\sqrt{3}}\mu F$

13. A coil having 500 square loops each of side 10 cm is placed normal to a magnetic field which increased at a rate of 1.0 T s^{-1}. Find the induced e.m.f. (in volt).

(a) 50 V (b) 1 V (c) 10 V (d) 5 V

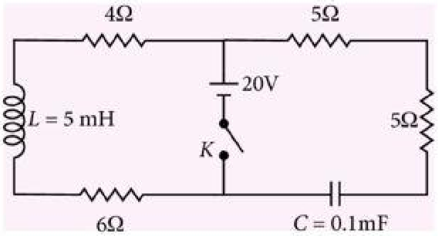
14. In a circuit shown in figure, A and B are two cells of same emf E but different internal resistances r_1 and r_2 ($r_1 > r_2$), respectively. Find the value of R such that potential difference across the terminals of cell A is zero a long time after key K is closed.



(a) $\frac{2}{3}(r_1 - r_2)$ (b) $\frac{3}{2}(r_1 - r_2)$

(c) $\frac{3}{4}(r_1 - r_2)$ (d) $\frac{4}{3}(r_1 - r_2)$

15. In the circuit shown in figure, key K is closed at $t = 0$, the current through the key at the instant $t = 10^{-3}$ ln 2s is



(a) 2A (b) 3.5A (c) 2.5A (d) 0

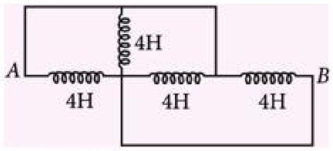
16. In an LR series circuit, a sinusoidal $V = V_0 \sin \omega t$ is applied. It is given that L = 35 mH, R = 11Ω, V_{rms} = 220 V, $\omega/2\pi$ = 50 Hz and π = 22/7. Find the amplitude of the current in steady state and obtain the phase difference between the current and the voltage.

(a) 20A, $\frac{\pi}{4}$ (b) 2A, $\frac{\pi}{6}$ (c) 2A, $\frac{\pi}{4}$ (d) 2A, $\frac{\pi}{3}$

17. A bulb is rated at 100V, 100W. It can be treated as a resistor. Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz.

(a) $\frac{\pi}{\sqrt{3}}$H (b) $\frac{\sqrt{3}}{\pi}$H (c) 2H (d) $\sqrt{3}$H

18. The equivalent inductance between A and B is



(a) 1 H (b) 4 H (c) 0.8 H (d) 16 H

19. An alternating emf of angular frequency ω is applied across an inductance. The instantaneous power developed in the circuit has an angular frequency

(a) $\frac{\omega}{4}$ (b) $\frac{\omega}{2}$ (c) 2ω (d) 4ω

20. The magnetic flux through a coil varies with time as $\phi = 5t^2 + 6t + 9$. The ratio of emf at t = 3 s to t = 0 s will be

(a) 1 : 9 (b) 1 : 6 (c) 6 : 1 (d) 9 : 1

SOLUTIONS

1. **(d):** $i = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right); i = \left(\frac{12\text{ V}}{6\,\Omega}\right)\left[1 - e^{-\left(\frac{6}{8.4\times10^{-3}}\right)t}\right]$

$$|1\text{ A}| = (2\text{ A})\left[1 - e^{-\left[\frac{6t}{8.4\times10^{-3}}\right]}\right] \Rightarrow -e^{\frac{6t}{8.4\times10^{-3}}} = \frac{1}{2}$$

or $\frac{6t}{8.4\times10^{-3}} = \ln 2 \Rightarrow t = \frac{(0.693)(8.4\times10^{-3})}{6}$

$= 0.00097\text{ s} = 0.97\text{ ms} \approx 1\text{ ms}.$

2. **(d):** $V_{rms} = \frac{20}{\sqrt{2}}\text{ V}$

$X_L = \omega L = (5\times10^{-3}\text{ H})\left(2000\frac{\text{rad}}{\text{s}}\right) = 10\,\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{(50\times10^{-6}\text{F})\cdot\left(2000\frac{\text{rad}}{\text{s}}\right)} = 10\,\Omega$

$R = R_R + R_L \Rightarrow R = 6\,\Omega + 4\,\Omega = 10\,\Omega$

$Z = \sqrt{(X_L - X_C)^2 + R^2}$

$= \sqrt{(10\,\Omega - 10\,\Omega)^2 + (10\,\Omega)^2} = 10\,\Omega$

$I_{rms} = \frac{V_{rms}}{Z} = \frac{20\text{ V}}{\sqrt{2}(10\,\Omega)} = (\sqrt{2})\text{ A} = 1.4\text{ A}$

$V_{voltmeter} = I_{rms}\cdot\left(\sqrt{(X_L - X_C)^2 + R_L^2}\right)$

$= (\sqrt{2}\text{ A})\cdot\left(\sqrt{(10\,\Omega - 10\,\Omega)^2 + (4\,\Omega)^2}\right)$

$= 4\sqrt{2}\text{ V} = 5.6\text{ V}$

3. **(d):** $I = \frac{V}{R} = \frac{220\sin(100\pi t)}{20}$

$\Rightarrow I = 11\sin(100\pi t)$

Here peak value = 11 A

It happens for the first time when $(100\pi t_1) = \frac{\pi}{2}$

$\Rightarrow t_1 = \frac{1}{200}\text{ s}$

The rms value of $I = \frac{11}{\sqrt{2}}$.

It happens when $(100\pi t_2) = \frac{3\pi}{4}$

$t_2 = \frac{3}{400}$

$(t_2 - t_1) = \frac{3}{400} - \frac{1}{200} = \frac{1}{400}\text{ s} = 2.5\times10^{-3}\text{ s}.$

4. **(a):** For terminal velocity net force on the rod must be zero

$BIl = mg$, *i.e.*, $I = \frac{0.2\times9.8}{0.6\times1} = \frac{9.8}{3}\text{ A}$

Now if e is the emf induced in the rod,

$e\times I = P = P_1 + P_2$

So, $e = \frac{(0.76+1.20)}{(9.8/3)} = 0.6\text{ V}$

Now as this e is generated due to motion of rod with terminal velocity in the magnetic field, *i.e.*,

$e = Bv_T l$ so $v_T = \frac{e}{Bl} = \frac{0.6}{0.6\times1} = 1\text{ ms}^{-1}$

Power dissipated across a conductor $P = \frac{V^2}{R}$

i.e., $R = \frac{V^2}{P}$

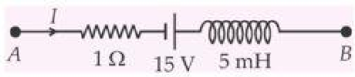
The resistance are in parallel combination so,

$V_1 = V_2 = e$

So, $R_1 = \frac{e^2}{P_1} = \frac{(0.6)^2}{0.76} = 0.47\,\Omega$

And, $R_2 = \frac{e^2}{P_2} = \frac{(0.6)^2}{1.2} = 0.3\,\Omega$.

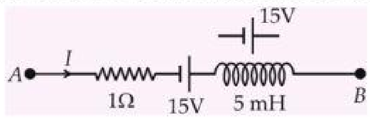
5. **(b):** $\frac{di}{dt} = 10^3\text{ A s}^{-1}$



$\therefore$ Induced emf across inductance, $|e| = L\frac{di}{dt}$

$|e| = (5\times10^{-3})(10^3)\text{ V} = 5\text{ V}$

Since, the current is decreasing, the polarity of this emf would be so as to increase the existing current. The circuit can be redrawn as shown in the figure.



Now, $V_A - 5 + 15 + 5 = V_B$

$\therefore V_A - V_B = -15\text{ V}$ or $V_B - V_A = 15\text{ V}$

6. **(d):** $B = \frac{\mu_0 iR^2}{2(R^2 + X^2)^{3/2}}$

$B = \frac{\mu_0 iR^2}{2(R^2 + 3R^2)^{3/2}} = \frac{\mu_0 iR^2}{2(4R^2)^{3/2}} = \frac{\mu_0 iR^2}{2\,.\,2^3\,.\,R} = \frac{\mu_0 i}{16R}$

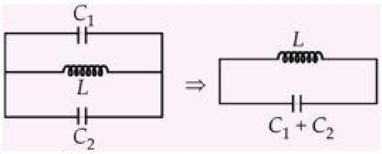
$\phi = NBA\cos45° = 2\frac{\mu_0 i}{16R}a^2\frac{1}{\sqrt{2}}$

$\phi = \frac{\mu_0 ia^2}{8\sqrt{2}R}$

$M = \frac{\phi}{i} \Rightarrow M = \frac{\mu_0 a^2}{2^{7/2} R} = \frac{\mu_0 a^2}{2^{p/2} R}$

$\therefore \; p = 7$

7. (a) :



We know that $q = q_m \cos \omega t$

$i = q_m \omega \cos\left[\omega t + \frac{\pi}{2}\right], \; i = i_0 \cos\left(\omega t + \frac{\pi}{2}\right)$

Maximum current $i_0 = q_m \omega$

where $\left[\omega = \frac{1}{\sqrt{L(C_1 + C_2)}}\right]$

Maximum charge on $(C_1 + C_2)$

$q_m = \frac{i_0}{\omega} = i_0 \sqrt{L(C_1 + C_2)}$

Maximum charge on C_1

$= \frac{C_1}{C_1 + C_2} \times i_0 \sqrt{L(C_1 + C_2)} = i_0 C_1 \sqrt{\frac{L}{C_1 + C_2}}$

8. (b): In LC oscillation energy is transferred C to L or L to C.

Maximum energy in $L = \frac{1}{2} L I_{max}^2$

Maximum energy in $C = \frac{q_{max}^2}{2C}$

Equal energy will be when $\frac{1}{2} L I^2 = \frac{1}{2}\left[\frac{1}{2} L I_{max}^2\right]$

$I = \frac{1}{\sqrt{2}} I_{max} \Rightarrow I = I_{max} \sin\omega t = \frac{1}{\sqrt{2}} I_{max}$

$\omega t = \frac{\pi}{4}$ or $\frac{2\pi}{T} t = \frac{\pi}{4}$ or $t = \frac{T}{8}$

$t = \frac{1}{8} 2\pi \sqrt{LC} = \frac{\pi}{4}\sqrt{LC}$

9. (c) : $U_{max} = \frac{1}{2} L I_0^2, \; U = \frac{U_{max}}{2}$

$\Rightarrow \frac{1}{2} L I^2 = \frac{1}{2}\left[\frac{1}{2} L I_0^2\right] \Rightarrow I_0^2 [1 - e^{-t/\tau}]^2 = \frac{I_0^2}{2}$

$\Rightarrow e^{-t/\tau} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2} - 1}{\sqrt{2}} \Rightarrow -\frac{t}{\tau} = \ln\left(\frac{\sqrt{2} - 1}{\sqrt{2}}\right)$

$\Rightarrow t = \tau \ln\left(\frac{\sqrt{2}}{\sqrt{2} - 1}\right) \Rightarrow t = \frac{L}{R} \ln\left(\frac{\sqrt{2}}{\sqrt{2} - 1}\right)$

10. (c) : For (LR) circuit

$\cos\theta = \frac{3}{5} \Rightarrow \theta = 53° \Rightarrow X_L = \frac{4}{3} R_1$

For (CR) circuit, $\cos\theta = \frac{1}{2} \Rightarrow \theta = 60 \Rightarrow X_C = \sqrt{3} R_2$

In LCR, power factor is 1 *i.e.*, $(X_C - X_L)$ is zero.

$\therefore \; X_L = X_C \Rightarrow \frac{4}{3} R_1 = \sqrt{3} R_2 \Rightarrow \frac{R_2}{R_1} = \frac{4}{3\sqrt{3}}$

11. (c) : The equation of a semi-circular wave is

$x^2 + y^2 = a^2$ or $y^2 = a^2 - x^2$

$I_{rms} = \sqrt{\frac{1}{2a}\int_{-a}^{+a} y^2 dx}$

$I^2_{rms} = \frac{1}{2a}\int_{-a}^{+a} (a^2 - x^2) dx$

$= \frac{1}{2a}\int_{-a}^{+a} (a^2 - x^2) dx = \frac{1}{2a}\left| a^2 x - \frac{x^3}{3} \right|_{-a}^{+a}$

$= \frac{1}{2a}\left(a^3 - \frac{a^3}{3} + a^3 - \frac{a^3}{3}\right) = \frac{2a^2}{3}$

$I_{rms} = \sqrt{\frac{2a^2}{3}} = \sqrt{\frac{2}{3}} a$

12. (b): As resistance of the lamp

$R = \frac{V_s^2}{P_0} = \frac{100^2}{50} = 200\,\Omega$

The rms value of current is

$I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}$ A.

So when the lamp is put in series with a capacitance and run at 200 V ac, from $V = IZ$, we have

$Z = \frac{V}{I} = \frac{200}{(1/2)} = 400\,\Omega$

Now as in case of CR circuit, $Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

i.e., $R^2 + \left(\frac{1}{\omega C}\right)^2 = 160000$

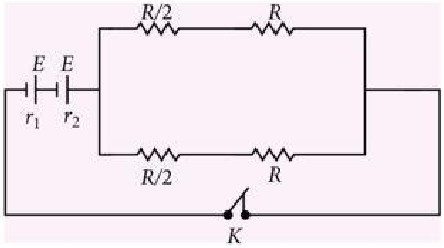
or, $\left(\frac{1}{\omega C}\right)^2 = 16 \times 10^4 - (200)^2 = 12 \times 10^4$

$\frac{1}{\omega C} = \sqrt{12} \times 10^2$

$C = \frac{1}{100\pi \times \sqrt{12} \times 10^2}\,\text{F} = \frac{100}{\pi\sqrt{12}}\,\mu\text{F} = \frac{50}{\pi\sqrt{3}}\,\mu\text{F}$

13. (d): $e = \left| -\frac{d\phi}{dt} \right| = Na^2 \frac{dB}{dt} = 5$ volt

14. (d): After a long time, steady state is reached in which impedance due to inductor (ωL for dc) is zero and that due to capacitance $\left(\frac{1}{\omega C}\right)$ becomes infinite, so equivalent circuit is shown in figure.



Net external resistance $R_{ext} = \frac{\frac{R}{2}+R}{2} = \frac{3}{4}R$

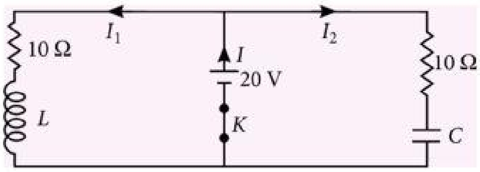
Net internal resistance $R_{int} = r_1 + r_2$

$\therefore$ Current in circuit $I = \frac{2E}{\frac{3}{4}R + r_1 + r_2}$

The potential difference across the terminals of cell A is zero; so $E - Ir_1 = 0$

$$\Rightarrow E - \frac{2Er_1}{\frac{3}{4}R + r_1 + r_2} = 0 \Rightarrow R = \frac{4}{3}(r_1 - r_2)$$

15. (c) : Current in branches containing L and C will flow independently



$$I_1 = \frac{20}{10}\left(1 - e^{-\frac{t}{5\times10^{-4}}}\right) = \frac{3}{2} = 1.5\text{ A}$$

$$I_2 = \frac{20}{10}e^{-\frac{t}{10^{-3}}} = 1.0\text{ A}$$

$\therefore \quad I = I_1 + I_2 = 2.5$ A

16. (a) : Inductive reactance,

$X_L = \omega L = 2\pi \times 50 \times 35 \times 10^{-3}$

$= 2 \times \frac{22}{7} \times 50 \times 35 \times 10^{-3} = 11\ \Omega$

Therefore, impedance of circuit

$Z = \sqrt{R^2 + X_L^2} = \sqrt{11^2 + 11^2} = 11\sqrt{2}\ \Omega$

Given $V_{rms} = 220$ V

Amplitude of voltage $V_0 = \sqrt{2}V_{rms} = 220\sqrt{2}$ V

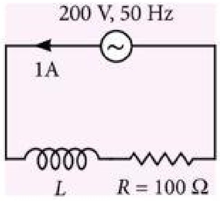
Amplitude of current $i_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{11\sqrt{2}} = 20$ A

Phase lag of current over voltage

$$\phi = \tan^{-1}\frac{\omega L}{R} = \tan^{-1}\frac{11}{11} = \tan^{-1}(1) = \frac{\pi}{4}$$

17. (b): From the rating of the bulb, the resistance of the bulb is

$R = \frac{V^2}{P} = 100\ \Omega$



For the bulb to be operated at its rated value, the rms current through it should be 1A.

Also, $I_{rms} = \frac{V_{rms}}{Z}$

$$\therefore \quad 1 = \frac{200}{\sqrt{100^2 + (2\pi 50L)^2}} \Rightarrow L = \frac{\sqrt{3}}{\pi}\text{ H}$$

18. (a)

19. (c) : $V = V_0 \sin \omega t$

If it is applied across an inductor, then I lag by $\frac{\pi}{2}$.

Thus, $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

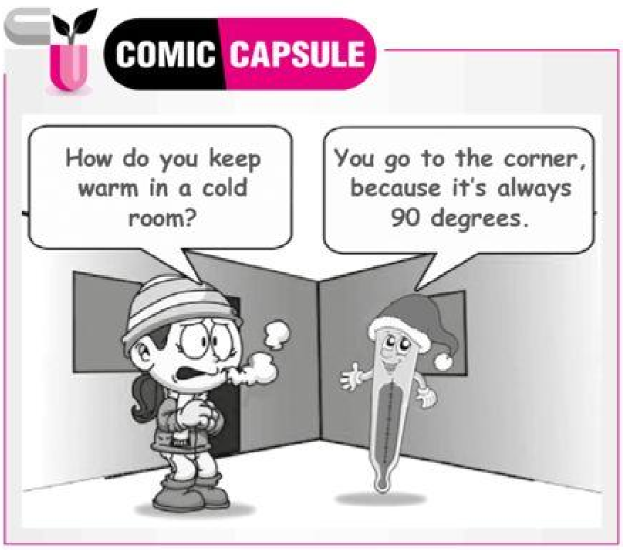
$P = VI = V_0 I_0 \sin \omega t \sin\left(\omega t - \frac{\pi}{2}\right)$

$= -\left(\frac{V_0 I_0}{2}\right) 2 \sin \omega t \cos \omega t$

$P = -\left(\frac{V_0 I_0}{2}\right)\sin(2\omega t) = \frac{V_0 I_0}{2}\sin(2\omega t + \pi)$

$\Rightarrow P$ has an angular frequency of '2ω'.

20. (c)



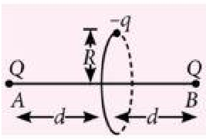
Class XII

Be JEE READY

with exclusive and brain storming MCQs

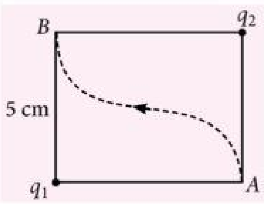
Practicing these MCQs help to Strengthen your concepts and give you extra edge in your JEE Preparation

1. Two positive point charges each of magnitude 10 C are fixed at positions A and B at separation $2d$ = 6m. A negatively charged particle of mass m = 90 gm and charge of magnitude 10×10^{-6} C is revolving in a circular path of radius 4 m in the plane perpendicular to the line AB and bisecting the line AB. Neglect the effect of gravity. Find the angular velocity of the particle.



(a) 200 rad s^{-1} (b) 400 rad s^{-1}
(c) 250 rad s^{-1} (d) 100 rad s^{-1}

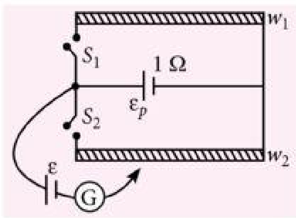
2. In rectangle shown in the figure, the two corners have charges $q_1 = -5\ \mu C$ and $q_2 = +2.0\ \mu C$ The work done in moving a charge of + 3.0 μC from B to A is



$\left(\text{take } \frac{1}{4\pi\varepsilon_0} = 10^{10}\right)$

(a) 2.8 J (b) 3.5 J (c) 4.5 J (d) 5.5 J

3. Two identical potentiometer wires w_1 and w_2 of equal length l, connected to a battery of emf ε_p and internal resistance 1 Ω through two switches S_1 and S_2. A battery of emf ε is balanced on these potentiometer wires one by one. If potentiometer wire w_1 is of resistance 2 Ω and balancing length is $l/2$ on it, when only S_1 is closed and S_2 is open. On closing S_2 and opening S_1 the balancing length on w_2 is found to be $2l/3$, then find the resistance of potentiometer wire w_2.



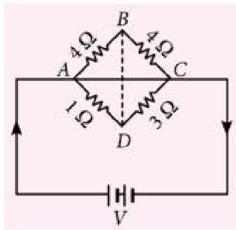
(a) 0.5 Ω (b) 1 Ω (c) 0.25 Ω (d) 0.75 Ω

4. Assume that each atom of copper contributes one electron. If the current flowing through a copper wire of 1 mm diameter is 1.1 A, the drift velocity of electrons will be

(a) 0.3 mm s^{-1} (b) 0.5 mm s^{-1}
(c) 0.1 mm s^{-1} (d) 0.2 mm s^{-1}

(Density of Cu = 9 g cm^{-3}, atomic weight of Cu = 63 g)

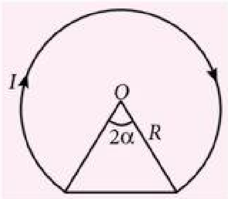
5. In the circuit shown in figure, if a conducting wire is connected between the points B and D, the current in this wire will



(a) flow from B to D
(b) flow from D to B
(c) flow in the direction which will be decided by the value of V
(d) be zero

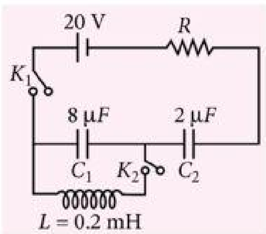
6. A particle of charge -16×10^{-18} C moving with velocity 10 m s^{-1} along the X-axis enters a region where a magnetic field of induction B is along the y-axis and an electric field of magnitude of 10^4 V m^{-1} is along the negative Z-axis. If the charged particle continues moving along the X-axis, the magnitude of B is
(a) 10^3 Wb m^{-2} (b) 10^5 Wb m^{-2}
(c) 10^{16} Wb m^{-2} (d) 10^{-3} Wb m^{-2}

7. The magnetic field intensity due to a thin wire carrying current I shown in figure is



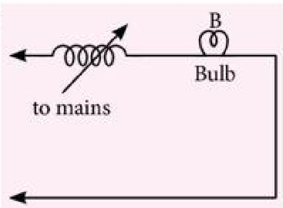
(a) $\frac{\mu_0 I}{2\pi R}(\pi - \alpha + \tan\alpha)$
(b) $\frac{\mu_0 I}{2\pi R}(\pi - \alpha)$
(c) $\frac{\mu_0 I}{2\pi R}(\pi + \alpha)$
(d) $\frac{\mu_0 I}{2\pi R}(\pi + \alpha - \tan\alpha)$

8. A circuit containing capacitors C_1 and C_2, shown in the figure, is in the steady state with key K_1 close. At the instant $t = 0$, K_1 is opened and K_2 is closed. The angular frequency of oscillation of the circuit and charge across C_1 at $t = 125$ μs are respectively.



(a) 25×10^3 rad s^{-1}, 32 μC
(b) 5×10^4 rad s^{-1}, 16 μC
(c) 5×10^4 rad s^{-1}, −16 μC
(d) 25×10^3 rad s^{-1}, −32 μC

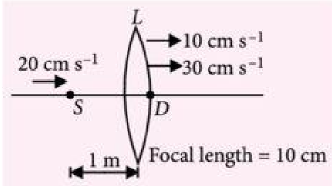
9. A typical light dimmer used to dim the stage lights in a theatre consists of a variable induction L (where inductance is adjustable between zero and L_{max}) connected in series with a light bulb B as shown in figure. The mains electrical supply is 220 V at 50 Hz, the light bulb is rated at 220 V, 1100 W. What L_{max} is required if the rate of energy dissipation in the light bulb is to be varied by a factor of 5 from its upper limit of 1100 W ?



(a) 0.69 H (b) 0.28 H
(c) 0.38 H (d) 0.56 H

10. Two thin symmetrical lenses of different nature and of different material have equal radii of curvature R = 15 cm. The lenses are put close together and immersed in water(μ_w= 4/3). The focal length of the system in water is 30 cm. The difference between refractive indices of the two lenses is
(a) 1/2 (b) 1/4 (c) 1/3 (d) 3/4

11. The figure shows the initial position of a point source of light S, a detector D and a lens L. Now at t = 0, all the three start moving towards right with different velocities as shown. The time(s) at which the detector receives the maximum light is



(a) 0.56 s and 8.94 s (b) 3.8 s
(c) 8.94 s and 19.62 s (d) 0.56 s

12. If three times of λ_{min} of continuous X ray spectrum of target metal at 40 kV is same as the wavelength of K_α line of this metal at 30 kV, then find the atomic number of the target metal.
(a) 67 (b) 47 (c) 37 (d) 27

13. Transitions between three energy levels in a particular atom give rise to three spectral lines of wavelengths, in increasing magnitudes, λ_1, λ_2 and λ_3. Which one of the following equations correctly relates λ_1, λ_2 and λ_3?
(a) $\lambda_1 = \lambda_2 - \lambda_3$ (b) $\lambda_1 = \lambda_3 - \lambda_2$
(c) $\frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$ (d) $\frac{1}{\lambda_1} = \frac{1}{\lambda_3} - \frac{1}{\lambda_2}$

14. Half-lives of two radioactive substances A and B are, respectively, 20 min and 40 min. Initially, the samples of A and B have equal number of nuclei. After 80 min the ratio of the remaining number of A and B nuclei is
(a) 1:16 (b) 4:1 (c) 1:4 (d) 1:1

15. In semiconductor, the concentrations of electrons and holes are 8×10^{18} m^{-3} and 5×10^{18} m^{-3}, respectively. If the mobilities of electrons and hole are 2.3 m^2 V^{-1} s^{-1} and 0.01 m^2 V^{-1} s^{-1}, respectively, then semiconductor is
(a) N-type and its resistivity is 0.34 Ω m
(b) P-type and its resistivity is 0.034 Ω m
(c) N-type and its resistivity is 0.034 Ω m
(d) P-type and its resistivity is 3.40 Ω m

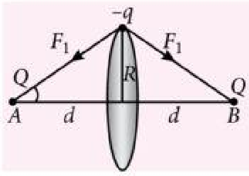
SOLUTIONS

1. **(b):** Net force on $-q$ towards the centre.

$F = (2F_1 \cdot \sin\theta) = 2.\dfrac{KQq}{d^2+R^2} \times \dfrac{R}{\sqrt{d^2+R^2}}$

For particle to move in a circle

$F = m\omega^2 R$



$\Rightarrow \dfrac{2KQqR}{(d^2+R^2)^{3/2}} = m\omega^2 R$

$\Rightarrow \omega = \sqrt{\dfrac{2KQq}{m(d^2+R^2)^{3/2}}}$

$= \sqrt{\dfrac{2\times 9\times 10^9 \times 10\times 10\times 10^{-6}}{90\times 10^{-3}\times (3^2+4^2)^{3/2}}} = 400 \text{ rad s}^{-1}$

2. **(a):** Work done for moving a charge q_0 from B to A.

$W_{B\to A} = q_0\,(V_A - V_B)$

Also, V_A = potential at point A

$= \dfrac{kq_1}{r_1} + \dfrac{kq_2}{r_2} = k\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2}\right)$

$= 10^{10}\left\{\dfrac{-5\times 10^{-6}}{15\times 10^{-2}} + \dfrac{2\times 10^{-6}}{5\times 10^{-2}}\right\} = \dfrac{10^6}{15}\text{ V}$

Similarly,

$V_B = 10^{10}\left\{\dfrac{2\times 10^{-6}}{15\times 10^{-2}} - \dfrac{5\times 10^{-6}}{5\times 10^{-2}}\right\} = \dfrac{-13}{15}\times 10^6\text{ V}$

$\Rightarrow W_{B\to A} = 3\times 10^{-6}\left\{\dfrac{1}{15}\times 10^6 - \left(\dfrac{-13}{15}\times 10^6\right)\right\} = 2.8\text{ J}$

3. **(b):** When S_1 is closed and S_2 is open, w_1 is in circuit. Current through w_1.

$I_1 = \dfrac{\varepsilon_p}{(R+r)} \Rightarrow I_1 = \dfrac{\varepsilon_p}{2+1} = \dfrac{\varepsilon_p}{3}$

Potential difference across $\dfrac{l}{2}$ length of w_1

$= \dfrac{1}{2}[2I_1] = \dfrac{\varepsilon_p}{3}$

This should be equal to ε. So, $\dfrac{\varepsilon_p}{3} = \varepsilon \Rightarrow \varepsilon_p = 3\varepsilon$

Similarly for second case : $I_2 = \dfrac{\varepsilon_p}{R+1} = \dfrac{3\varepsilon}{R+1}$

Potential difference across $2l/3$ length of w_2

$= \dfrac{2}{3}I_2 R = \dfrac{2}{3}\left(\dfrac{3\varepsilon}{R+1}\right)R = \dfrac{2\varepsilon R}{R+1}$

This should be equal to ε. So, $\dfrac{2\varepsilon R}{R+1} = \varepsilon \Rightarrow R = 1\ \Omega$

4. **(c):** Number density of electrons,

$n = \dfrac{6\times 10^{23}}{(63\text{ g})/(9\text{ g cm}^{-3})} = \left(\dfrac{6}{7}\right)10^{23}\text{ cm}^{-3} = \left(\dfrac{6}{7}\right)10^{29}\text{ m}^{-3}$

$A = \pi r^2 = \pi(0.5\times 10^{-3}\text{m})^2 = 0.25\pi \times 10^{-6}\text{ m}^2$

$v_d = \dfrac{I}{neA}$

$= \dfrac{1.1\text{ A}}{[(6/7)10^{29}\text{ m}^{-3}](1.6\times 10^{-19}\text{ C})(0.25\pi\times 10^{-6}\text{ m}^2)}$

$= 0.1\text{ mm s}^{-1}$

5. **(b):** As $I_{ABC} = \dfrac{V}{4\,\Omega + 4\,\Omega} = \dfrac{V}{8\,\Omega}$,

$(V_A - V_B) = \left(\dfrac{V}{8\,\Omega}\right)4\,\Omega = \dfrac{V}{2}$

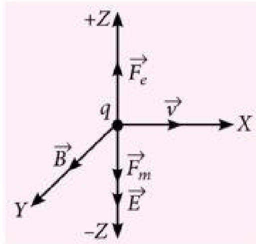
Similarly, $(V_A - V_D) = \dfrac{V}{4}$

Thus, $(V_D - V_B) = (V_D - V_A) + (V_A - V_B)$

$= \dfrac{-V}{4} + \dfrac{V}{2} = \dfrac{V}{4}$

Since, $(V_D - V_B) > 0$, current flows from D to B.

6. **(a):** $\vec{F}_e = q\vec{E}$



$= (-16\times 10^{-18})(-10^4\hat{k})$

$= 16\times 10^{-14}\hat{k}$

$\vec{F}_m = q(\vec{v}\times\vec{B})$

$= (-16\times 10^{-18})(10\hat{i}\times B\hat{j})$

$= (-16\times 10^{-17}B\,\hat{k})$

From figure, since $\vec{F}_m + \vec{F}_e = \vec{0},\ \vec{F}_m = -\vec{F}_e$

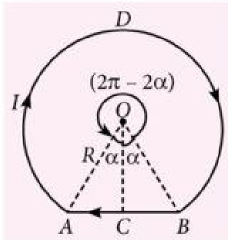
or $16\times 10^{-17}B = 16\times 10^{-14}$

Hence, $B = 10^3\text{ Wb m}^{-2}$

7. **(a):** Refer to figure. $B_{AB} = \dfrac{\mu_0 I}{4\pi(OC)}[\sin\alpha - \sin(-\alpha)]$

$= \dfrac{\mu_0 I}{4\pi R\cos\alpha}(2\sin\alpha)$ (as $OC = R\cos\alpha$)

or $B_{AB} = \left(\dfrac{\mu_0 I}{2\pi R}\tan\alpha\right)_{\otimes}$



$B_{ADB} = \left[\left(\dfrac{\mu_0 I}{4\pi R}\right)(2\pi - 2\alpha)\right]_{\otimes}$

$= \left[\dfrac{\mu_0 I}{2\pi R}(\pi - \alpha)\right]_{\otimes}$

Thus, $B = B_{ADB} + B_{AB} = \dfrac{\mu_0 I}{2\pi R}(\pi - \alpha + \tan\alpha)$

8. **(a):** Angular frequency of oscillation

$\omega = \dfrac{1}{\sqrt{LC_1}} = \dfrac{1}{\sqrt{0.2\times 10^{-3}\times 8\times 10^{-6}}}$

$= 2.5\times 10^4\text{ rad s}^{-1} = 25\times 10^3\text{ rad s}^{-1}$

$\dfrac{Q}{C_1} - L\dfrac{dI}{dt} = 0 \Rightarrow \dfrac{Q}{C_1} + L\dfrac{d^2Q}{dt^2} = 0$

The solution of this equation is $Q = Q_0 \sin(\omega t + \phi)$, where $t = 0$, $Q = Q_0$

$$Q_0 = C_{eq}V = \frac{C_1C_2}{C_1+C_2}V = \frac{8\times2\times10^{-12}\times20}{10\times10^{-6}}$$

$=32 \times 10^{-6}$C $\quad \because \phi = \frac{\pi}{2}$

$\therefore\ Q = (32\ \mu\text{C})\cos(2.5\times10^4\times125\times10^{-6}) \simeq 32\ \mu\text{C}$

9. **(b) :** For power to be consumed at the rate of $\frac{1100}{5} = 220$ W, we have $P = E_vI_v\cos\theta$

$$220 = \frac{220\times220}{\sqrt{R^2+L^2\omega^2}}\times\frac{R}{\sqrt{R^2+L^2\omega^2}}$$

where $R = \frac{V^2}{P} = \frac{220^2}{1100} = 44\ \Omega$

$$220 = \frac{(220)^2\times44}{44^2+(L\omega)^2} \Rightarrow 44^2+(L\omega)^2 = 220\times44$$

$$L\omega = \sqrt{220\times44-44^2} = 88\Omega$$

$$L = \frac{88}{2\pi\times f} = \frac{88}{2\pi\times50} = \frac{88}{2\times22}\times\frac{7}{50} = 0.28\text{ H}$$

10. **(c) :** Let f_1 and f_2 be the focal length in water.

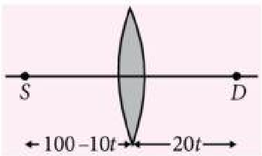
Then, $\frac{1}{f_1} = \left(\frac{\mu_1}{\mu_w}-1\right)\left(\frac{1}{R}+\frac{1}{R}\right) = \left(\frac{\mu_1}{\mu_w}-1\right)\left(\frac{2}{R}\right)$ ...(i)

$\frac{1}{f_2} = \left(\frac{\mu_2}{\mu_w}-1\right)\left(-\frac{1}{R}-\frac{1}{R}\right) = \left(\frac{\mu_2}{\mu_w}-1\right)\left(-\frac{2}{R}\right)$ ...(ii)

Adding (i) and (ii), we get $\frac{1}{f_1}+\frac{1}{f_2} = \frac{2(\mu_1-\mu_2)}{\mu_wR}$

or $\quad \frac{1}{30} = \frac{2(\mu_1-\mu_2)}{\mu_wR} \quad \therefore\ (\mu_1-\mu_2) = \frac{\mu_wR}{60} = \frac{1}{3}$

11. **(a) :** The detector will receive the maximum light when the image of point source of light coincides with the position of detector.



Let at any time t, image of S and detector coincides then,

for $\quad u = -(100-10t)$cm, $v = 20t$ cm, $f = 10$ cm

The detector receives maximum light.

From, $\frac{1}{v}-\frac{1}{u} = \frac{1}{f} \quad \Rightarrow \frac{1}{20t}-\frac{1}{-(100-10t)} = \frac{1}{10}$

$\Rightarrow \quad 2t^2 - 19t + 10 = 0 \quad \Rightarrow t = 0.56$ s and 8.94 s

12. **(c) :** $\lambda_{min} = \frac{hc}{eV} = \frac{12400}{40000} = 0.31$ Å

At 40 kV : $\lambda_{min} = \frac{12400}{40000} = 0.31$ Å

Wavelength of K_α is independent of applied potential.

For K_α, X-ray : $\frac{3}{4}(13.6)(Z-1)^2 = E = \frac{hc}{\lambda_{K_\alpha}}$

Given that $\lambda_{K_\alpha} = 3\lambda_{min} \Rightarrow \frac{1216}{(Z-1)^2} = 3\times0.31$

$$\Rightarrow (Z-1)^2 = \frac{1216}{0.93} \simeq 1308 \Rightarrow Z-1 = 36 \Rightarrow Z = 37$$

13. **(c)**

14. **(c) :** We know that $N = N_0\left(\frac{1}{2}\right)^n$

For A, $N_A = N_0\left(\frac{1}{2}\right)^{n_A} = N_0\left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$

$$\left[\because n_A = \frac{t}{T_A} = \frac{80}{20} = 4\right]$$

For B, $N_B = N_0\left(\frac{1}{2}\right)^{n_B} = N_0\left(\frac{1}{2}\right)^2 = \frac{N_0}{4}$

$$\left[\because n_B = \frac{t}{T_B} = \frac{80}{40} = 2\right]$$

$\therefore \quad \frac{N_A}{N_B} = \frac{1}{4} \quad$ or $\quad N_A : N_B = 1:4$

15. **(a)**

CLASS XII

CBSE DRILL

Chapterwise practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2018-19.

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 12 are short answer questions and carry 2 marks each.
(iv) Q. no. 13 to 24 are also short answer questions and carry 3 marks each.
(v) Q. no. 25 to 27 are long answer questions and carry 5 marks each.
(vi) Use log tables if necessary, use of calculator is not allowed.

Time Allowed : 3 hours | Maximum Marks : 70

Electromagnetic Waves | Optics

SECTION A

1. Is the speed of light in glass independent of the colour of light? If not, which of the two colours; red and violet, travels slower in a glass prism?

2. What would the diver see when looking up at the water at angle of $i > i_C$?

3. State the reason why microwaves are best suited for long distance transmission of signals.

4. A converging lens is kept coaxially in contact with a diverging lens, both lenses being of equal focal lengths. What is the focal length of the combination?

5. Why can light waves be polarised while sound waves cannot be polarised ?

SECTION B

6. A convex lens of focal length 25 cm is placed coaxially in contact with a concave lens of focal length 20 cm. Determine the power of the combination. Will the system be converging or diverging in nature?

7. Use the mirror equation to show that an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.

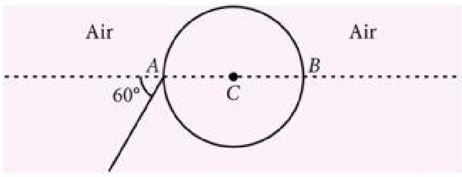
8. Which constituent radiation of the electromagnetic spectrum is used
   (a) in radar,
   (b) to take photograph of internal parts of a human body and
   (c) for taking photographs of the sky during night and foggy conditions?
   Give one reason for your answer in each case.

9. Identify the electromagnetic waves whose wavelengths vary as
   (a) 10^{-11} m $< \lambda <$ 10^{-14} m
   (b) 10^{-4} m $< \lambda <$ 10^{-6} m
   Write one use of each.

10. Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 7.2 mm. Calculate the wavelength of another source of laser light which produces interference fringes separated by 8.1 mm using same pair of slits.

11. (a) A ray of light falls on a transparent sphere with centre C as shown in the figure. The ray emerges from the sphere parallel to the line AB. Find the

angle of refraction at A if refractive index of the material of the sphere is $\sqrt{3}$.



(b) When red light passing through a convex lens is replaced by light of blue colour, how will the focal length of the lens change?

OR

(a) Write the conditions under which light sources can be said to be coherent.

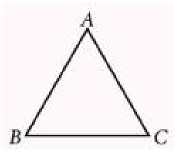
(b) Why is it necessary to have coherent sources in order to produce an interference pattern?

12. (a) State the necessary conditions for producing total internal reflection of light.

(b) Draw ray diagrams to show how specially designed prisms make use of total internal reflection to obtain inverted image of the object by deviating rays (i) through 90° and (ii) through 180°.

SECTION C

13. (a) A ray of light incident on face AB of an equilateral glass prism, shows minimum deviation of 30°. Calculate the speed of light through the prism.



(b) Find the angle of incidence at face AB so that the emergent ray grazes along the face AC.

14. (a) An equiconvex lens of focal length 'f' is cut into two identical plane convex lenses. How will the power of each part be related to the focal length of the original lens?

(b) A double convex lens of +5 D is made of glass of refractive index 1.55 with both faces of equal radii of curvature. Find the value of its radius of curvature.

15. Consider a plane e.m. wave travelling with speed c in the positive z-direction.

(a) Use Faraday's law to show that $E = cB$

(b) Use modified Ampere's circuital law to show that $c = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}}$

16. Use Huygen's principle to show how a plane wavefront propagates from a denser to rarer medium. Hence verify Snell's law of refraction.

17. Answer the following questions :

(a) In a double slit experiment using light of wavelength 600 nm, the angular width of the fringe formed on a distant screen is 0.1°. Find the spacing between the two slits.

(b) Light of wavelength 500 Å propagating in air gets partly reflected from the surface of water. How will the wavelengths and frequencies of the reflected and refracted light be affected?

18. Write Maxwell's generalization of Ampere's Circuital Law. Show that in the process of charging a capacitor, the current produced within the plates of the capacitor is

$$i = \varepsilon_0 \frac{d\phi_E}{dt}$$

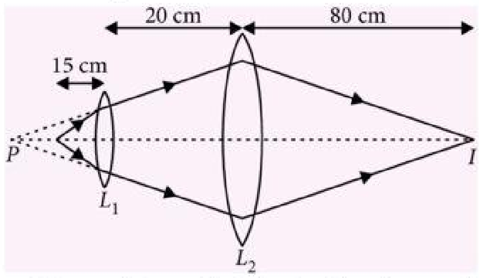
where ϕ_E is the electric flux produced during charging of the capacitor plates.

19. Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120\ \text{NC}^{-1}$ and that its frequency is $\upsilon = 50.0$ MHz.

(a) Determine, B_0, ω, k and λ.

(b) Find expressions for $\vec{E}$ and $\vec{B}$.

20. In the following diagram, an object 'O' is placed 15 cm in front of a convex lens L_1 of focal length 20 cm and the final image is formed at I at a distance of 80 cm from the second lens L_2. Find the focal length of the lens L_2.



21. An object of 3 cm height is placed at a distance of 60 cm from a convex mirror of focal length 30 cm. Find the nature, position and size of the image formed.

OR

(a) When the oscillating electric and magnetic fields are along the x-and y-direction respectively.

(i) point out the direction of propagation of electromagnetic wave.

(ii) express the velocity of propagation in terms of the amplitudes of the oscillating electric and magnetic fields.

(b) How do you show that the e.m. wave carries energy and momentum?

22. Two wavelengths of sodium light 590 nm and 596 nm are used, in turn to study the diffraction taking place at a single slit of aperture 2×10^{-4} m. The distance between the slit and the screen is 1.5 m. Calculate the separation between the positions of the first maxima of the diffraction pattern obtained in the two cases.

23. (a) Draw a ray diagram showing the formation of image by a reflecting telescope.

(b) Write two advantages of a reflecting telescope over a refracting telescope.

24. (a) Good quality sun-glasses made of polaroids are preferred over ordinary coloured glasses. Justifying your answer.

(b) Two polaroids P_1 and P_2 are placed in crossed postions. A third polaroid P_3 is kept between P_1 and P_2 such that pass axis of P_3 is parallel to that of P_1. How would the intensity of light (I_2) transmitted through P_2 vary as P_3 is rotated? Draw a plot of intensity 'I_2" versus the angle 'θ' between pass axes of P_1 and P_3.

SECTION D

25. (a) Derive the mathematical relation between refractive indices n_1 and n_2 of two media and radius of curvature R for refraction at a convex spherical surface. Consider the object to be a point source lying on the principle axis in rarer medium of refractive index n_1 and a real image formed in the denser medium of refractive index n_2. Hence, derive lens maker's formula.

(b) Light from a point source in air falls on a convex spherical glass surface of refractive index 1.5 and radius of curvature 20 cm. The distance of light source from the glass surface is 100 cm. At what position is the image formed ?

OR

(a) A monochromatic source of light of wavelength λ illuminates a narrow slit of width d to produce a diffraction pattern on the screen. Obtain the conditions when secondary wavelets originating from the slit interfere to produce maxima and minima on the screen.

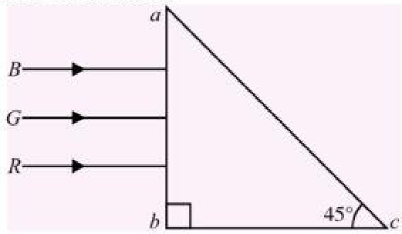
(b) How would the diffraction pattern be affected when

(i) the width of the slit is decreased?

(ii) the monochromatic source of light is replaced by white light?

26. (a) A ray of monochromatic light is incident on one of the faces of an equilateral triangular prism of refracting angle A. Trace the path of ray passing through the prism. Hence, derive an expression for the refractive index of the material of the prism in terms of the angle of minimum deviation and its refracting angle.

(b) Three light rays red (R), green (G) and blue (B) are incident on the right angled prism abc at face ab. The refractive indices of the material of the prism for red, green and blue wavelengths are respectively 1.39, 1.44 and 1.47. Trace the paths of these rays reasoning out the difference in their behaviour.



OR

In Young's double slit experiment, deduce the condition for (i) constructive, and (ii) destructive interference at a point on the screen. Draw a graph showing variation of intensity in the interference pattern against position 'x' on the screen.

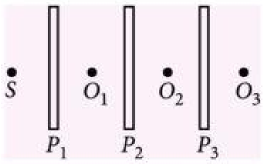
27. Draw a ray diagram to show the working of a compound microscope. Deduce an expression for the total magnification when the final image is formed at the near point.

In a compound microscope, an object is placed at a distance of 1.5 cm from the objective of focal length 1.25 cm. If the eye piece has a focal length of 5 cm and the final image is formed at the near point, estimate the magnifying power of the microscope.

OR

(a) Describe briefly how an unpolarised light get linearly polarized when it passes through a polaroid.

(b) Three identical polaroid sheets P_1, P_2 and P_3 are oriented so that the pass axis of P_2 and P_3 are inclined at angle of 60° and 90° respectively with respect to the pass axis of P_1. A monochromatic source S of unpolarised light of intensity I_0 is kept in front of the polaroid sheet P_1 as shown in the figure. Determine the intensities of light as observed by the observers O_1, O_2 and O_3 as shown.



SOLUTIONS

1. No, the speed of light in glass (*i.e.*, v) depends upon the colour of light (*i.e.*, λ).

As $\mu = c/v$, $v = \frac{c}{\mu}$, where c is the speed of light in a vacuum.

According to Cauchy's formula.

$\mu = a + b/\lambda^2$, where a and b are constant and their values depend upon the nature of the substance.

Thus, $v = \frac{c}{a + b/\lambda^2}$

As $\lambda_r > \lambda_v$, $v_v < v_r$ *i,e.*, violet component of white light travels slower than the red component.

2. Because of total internal reflections, the diver will not see anything above water. Instead, he would see the reflection of something on the sides and/or bottom of the pool.

3. Because of their smaller wavelength, microwaves are not bent by objects of normal dimensions. So they can be used to beam signal in a particular direction.

4. As $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}, \frac{1}{F} = \frac{1}{f} - \frac{1}{f} = 0$ or $F = \infty$

(as $f_1 = f$ and $f_2 = -f$)

5. Only those waves can be polarised which are transverse in nature. Thus, light waves, which are transverse in nature, can be polarised whereas sound waves, which are longitudinal cannot be polarised.

6. Given that focal length of convex lens, $f_1 = +25$ cm and focal length of concave lens, $f_2 = -20$ cm

Equivalent focal length,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{25} + \frac{1}{-20} = -\frac{1}{100}$$

$\therefore$ $F = -100$ cm

Power of the combination, $P = \frac{1}{F(\text{m})} = \frac{1}{-1\,\text{m}} = -1\,\text{D}$

The focal length of the combination= 1 m = 100 cm.

The system will be diverging in nature as the focal length is negative.

7. From mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

Now for a concave mirror, $f < 0$ and for an object is placed on the left side of the mirror, the object distance $u < 0$

$\therefore$ $2f < u < f$ or $\frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$

or $-\frac{1}{2f} < -\frac{1}{u} < -\frac{1}{f}$

or $\frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < \frac{1}{f} - \frac{1}{f}$ or $\frac{1}{2f} < \frac{1}{v} < 0$

This implies that $v < 0$ so that image is formed on left. Also the above inequality implies

$2f > v$

or $|2f| < |v|$ [$\because$ $2f$ and v are negative]

i.e., the real image is formed beyond $2f$.

8. **(a)** Microwaves are used in radar. Because of their small wavelengths, microwaves are not diffracted or bent by objects of normal dimensions. So these waves can be used to beam signal in a particular direction.

(b) X-rays are used to photograph internal parts of a body, X-rays have high penetrating power.

(c) Infrared rays are used for taking photographs of the sky during night and foggy conditions. This is because these radiations are readily absorbed by water molecules present in most materials. After absorption they heat up the materials and their surroundings.

9. **(a)** Gamma rays lie between 10^{-11} m-10^{-14} m. These rays are used in radiotherapy to treat certain cancers and tumors.

(b) Infrared waves lie between 10^{-4} m-10^{-6} m. These waves are used in taking photographs during conditions of fog, smoke etc as these waves are scattered less than visible rays.

10. Fringe width, $\beta = \frac{D\lambda}{d}$

When D and d are kept fixed, $\frac{\beta}{\beta_1} = \frac{\lambda}{\lambda_1}$

or $\lambda_1 = \frac{\lambda\beta_1}{\beta} = \frac{630 \times 8.1}{7.2} = \frac{5103}{7.2} = 708.75$ nm

11. **(a)** From Snell's law, we have : $\frac{\sin(i)}{\sin(r)} = \mu$

At A, $i = 60°$; $\mu = \sqrt{3}$

Now, $\sin(r) = \frac{\sin(i)}{\mu}$

$\Rightarrow \quad \sin(r) = \dfrac{\sin(60°)}{\sqrt{3}} = \dfrac{1}{2}$

$\Rightarrow \quad r = \sin^{-1}\left(\dfrac{1}{2}\right)$

$\therefore \quad r = 30°$

(b) Focal length of the lens decrease when red light is replaced by blue light.

OR

(a) The essential condition, which must satisfied sources to be coherent are :

(i) the two light waves should be of same wavelength.

(ii) the two light waves should either be in phase or should have a constant phase difference.

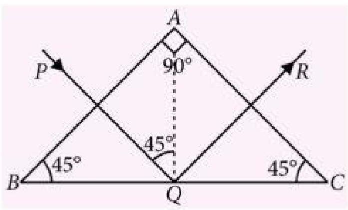
(b) Because coherent sources emit light waves of same frequency or wavelength and of a stable phase difference.

12. (a) Essential conditions for total internal reflection :

(i) Light should travel from a denser medium to a rarer medium.

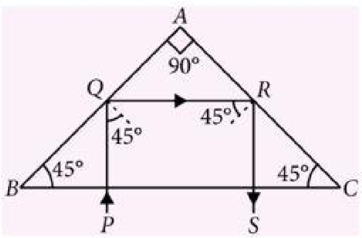
(ii) Angle of incidence in denser medium should be greater than the critical angle for the pair of media in contact.

(b) (i) To deviate a ray of light through 90° :



A totally reflecting prism is used to deviate the path of the ray of light through 90°, when it is inconvenient to view the direct light. In Michelson's method to find velocity of light, the direct light from the octagonal mirror is avoided from direct viewing by making use of totally reflecting prism.

(ii) To deviate a ray of light through 180° : When the ray of light comes to meet the hypotenuse face *BC* at right angles to it, it is refracted out of prism as such along the path *RS*. The path of the ray of light has been turned through 180° due to two total internal reflections.



13. (a) The refractive index of the material of prism,

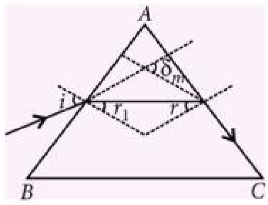
$$\mu = \frac{\sin\left[\dfrac{A+\delta_m}{2}\right]}{\sin\dfrac{A}{2}}$$

Given : $A = 60°, \delta_m = 30°$

$$\mu = \frac{\sin 45°}{\sin 30°} = \frac{1}{\sqrt{2}} . 2 \Rightarrow \mu = \sqrt{2}$$

$$\because \quad \mu = \frac{c}{v} \Rightarrow v = \frac{c}{\mu} = \frac{3\times 10^8}{1.414} = 2.12 \times 10^8 \text{ m s}^{-1}$$

(b) $\sin i_C = \dfrac{1}{\mu} = \dfrac{1}{\sqrt{2}}$



$i_C = r = 45°$

$A = r_1 + r$

$\Rightarrow \quad r_1 = 15°$

$\dfrac{\sin i}{\sin r_1} = \sqrt{2}$

$\sin i = \sqrt{2}\,\sin 15° = \dfrac{(\sqrt{3}-1)}{2\sqrt{2}} \times \sqrt{2}$

$\sin i = \dfrac{\sqrt{3}-1}{2}$

$i = \sin^{-1}\left(\dfrac{\sqrt{3}-1}{2}\right)$

14. (a) The focal length of original equiconvex lens is *f*. Let the focal length of each part after cutting be *F*.

Here, $\dfrac{1}{f} = \dfrac{1}{F} + \dfrac{1}{F} \Rightarrow \dfrac{1}{f} = \dfrac{2}{F}$

$\Rightarrow f = \dfrac{F}{2} \Rightarrow F = 2f$

Power of each part will be given by

$P = \dfrac{1}{F} \Rightarrow P = \dfrac{1}{2f}$

(b) From lens maker formula, we have

$P = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

where

P = Power of lens = +5 *D*

μ = Refractive index of the lens = 1.55

R_1 = Radius of curvature of first face (+ve)

R_2 = Radius of curvature of second face (–ve)

Given : $R_1 = R_2 = R$

$$\Rightarrow \quad 5 = (1.55 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) \Rightarrow 5 = (1.55-1)\left(\frac{2}{R}\right)$$

$$\Rightarrow \quad 5 = 0.55\left(\frac{2}{R}\right) \Rightarrow R = \frac{0.55 \times 2}{5} \Rightarrow R = 0.22 \text{ m}$$

The radius of curvature of the lens is 22 cm.

15. Let $\vec{E}$ be in the x-direction and $\vec{B}$ in the y-direction.
(a) Consider a rectangular loop in the x-z plane with one side of length l parallel to $\vec{E}$. Suppose at any instant, the rectangle is partially on the left of the wavefront and partially on the right of the wave front.
Rate of change of magnetic flux,

$$\frac{d\phi}{dt} = Blc$$

Line integral of $\vec{E} = \oint \vec{E} \cdot dl = El$

From Faraday's law,

$$\oint \vec{E} \cdot dl = \frac{d\phi_B}{dt}$$

$\therefore \quad El = Blc \quad$ or $\quad E = cB$

(b) Consider a similar loop in the y-z plane

Rate of change of electric flux, $\frac{d\phi_E}{dt} = Elc$

Line integral of $\vec{B} = \oint \vec{B} \cdot dl = Bl$

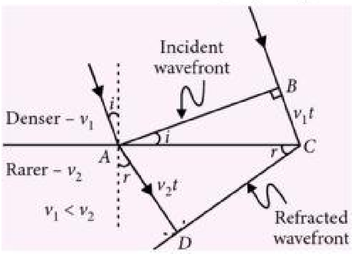
From Ampere's law, $\oint \vec{B} \cdot dl = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt} = \mu_0 \varepsilon_0 Elc$

$\therefore \quad Bl = \mu_0 \varepsilon_0 Elc$

or $\quad B = \mu_0 \varepsilon_0 Ec = \mu_0 \varepsilon_0 cB.c \qquad [\because E = cB]$

or $\quad c^2 = \frac{1}{\mu_0 \varepsilon_0}$ or $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

16. Given figure shows the refraction of a plane wavefront at a rarer medium *i.e.*, $v_2 > v_1$



The incident and refracted wavfronts are shown in figure.
Let the angles of incidence and refraction be i and r respectively.
From right ΔABC, we have,

$$\sin \angle BAC = \sin i = \frac{BC}{AC}$$

From right ΔADC, we have,

$$\sin \angle DCA = \sin r = \frac{AD}{AC}$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1 t}{v_2 t} \text{ or } \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = {}^1\mu_2 \quad \text{(a constant)}$$

This verifies Snell's law of refraction. The constant ${}^1\mu_2$ is called the refractive index of the second medium with respect to first medium.

17. (a) Angular width, $\theta = \frac{\lambda}{d}$ or $d = \frac{\lambda}{\theta}$

Here, $\lambda = 600 \text{ nm} = 6 \times 10^{-7}$ m

$\theta = 0.1° = \frac{0.1 \times \pi}{180}$ rad $= \frac{\pi}{1800}$ rad, $d = ?$

$$\therefore \quad d = \frac{6 \times 10^{-7} \times 1800}{\pi} = 3.44 \times 10^{-4} \text{ m}$$

(b) Frequency of a light depends on its source only. So, the frequencies of reflected and refracted light will be same as that of incident light.
Reflected light is in the same medium (air) so its wavelength remains same as 500 Å.

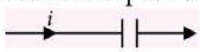
Wavelength of refracted light, $\lambda_r = \frac{\lambda}{\mu_w}$

μ_w = refractive index of water.
So, wavelength of refracted wave will be decreased.

18. Maxwell's generalization of Ampere's circuital Law,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 (i + i_d) = \mu_0 \left(i + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

In the process of charging the capacitor there is change in electric flux between the capacitor plates.



$$\frac{d\phi_E}{dt} = \frac{d}{dt}(EA)$$

$E \rightarrow$ Electric field between the plates $= \frac{q}{A\varepsilon_0}$

$A \rightarrow$ Area of the plate

So, $\frac{d\phi_E}{dt} = \frac{d}{dt}\left(\frac{q}{A\varepsilon_0} \times A \right) = \frac{1}{\varepsilon_0} \frac{dq}{dt} = \frac{i_d}{\varepsilon_0}$

$$\therefore \quad i_d = i = \varepsilon_0 \frac{d\phi_E}{dt}$$

19. Here $E_0 = 120 \text{ NC}^{-1}$,
$\upsilon = 50.0 \text{ MHz} = 50 \times 10^6$ Hz

(a) $B_0 = \frac{E_0}{c} = \frac{120 \text{ NC}^{-1}}{3 \times 10^8 \text{ ms}^{-1}} = 4 \times 10^{-7}$ T

$\omega = 2\pi\upsilon = 2 \times 3.14 \times 50 \times 10^6$
$= 3.14 \times 10^8 \text{ rad s}^{-1}$.

$$k = \frac{\omega}{c} = \frac{3.14 \times 10^8}{3 \times 10^8} = 1.05 \text{ m}^{-1}$$

$\lambda = \frac{c}{\upsilon} = \frac{3\times10^8}{50\times10^6} = 6.00$ m

(b) If the wave is propagating along x-axis, then field $\vec{E}$ will be along y-axis and field $\vec{B}$ along z-axis.

$\therefore \quad \vec{E} = E_0 \sin(kx - \omega t)\hat{j}$

or $\quad \vec{E} = 120\sin(1.05x - 3.14\times10^8 t)\hat{j}$ NC^{-1}

where x is in metre and t in second.

$\vec{B} = B_0 \sin(kx - \omega t)\hat{k}$

$= 4\times10^{-7}\sin(1.05x - 3.14\times10^8 t)\hat{k}$ tesla

20. As per the figure,

The image formed by lens L_1 is at P. Therefore, using lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

As per the parameters given in the question

$u = -15$ cm, $f_{L_1} = 20$ cm

So, the image distance will be

$$\frac{1}{v} - \frac{1}{(-15)} = \frac{1}{20}$$

$v = -60$ cm

Now, this image is acting as an object for the lens L_2. We can again use the lens formula and other parameters given in the question and question figure to find the focal length of lens L_2.

$$\frac{1}{v_{L_2}} - \frac{1}{u_{L_2}} = \frac{1}{f_{L_2}}$$

Here, $u_{L_2} = v + (-20) = -60 - 20 = -80$ cm

$v_{L_2} = 80$ cm

$$\frac{1}{80} - \frac{1}{(-80)} = \frac{1}{f_{L_2}}$$

$f_{L2} = 40$ cm

So, the focal length of the lens $L_2 = 40$ cm.

21. Here, height of object $h = 3$ cm

$u = -60$ cm, $f = +30$ cm

Using the mirror formula, we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{-60} = \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{1}{30} + \frac{1}{60}$$

$$\frac{1}{v} = \frac{2+1}{60} \Rightarrow \frac{1}{v} = \frac{3}{60} \quad \therefore \; v = 20 \text{ cm}$$

The image is virtual and erect.

The image is at a distance of 20 cm from the mirror on the opposite side of the object.

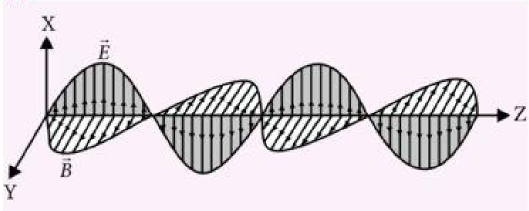
$$\frac{h'}{h} = -\frac{v}{u} \Rightarrow \frac{h'}{3} = -\left(\frac{20}{-60}\right) \Rightarrow \frac{h'}{3} = \frac{1}{3}$$

$\Rightarrow h' = 1$ cm

$\therefore$ Image is diminished and its size is 1 cm.

OR

(a) (i)



(ii) Speed of e.m. wave can be given as the ratio of magnitude of electric field (E_0) to the magnitude of magnetic field (B_0), *i.e.*, $c = \frac{E_0}{B_0}$

(b) Electromagnetic waves or photons transport energy and momentum. When an electromagnetic wave interacts with a small particle, it can exchange energy and momentum with the particle. The force exerted on the particle is equal to the momentum transferred per unit time. Optical tweezers use this force to provide a non-invasive technique for manipulating microscopic-sized particles with light.

22. Given that: Wavelength of the light beam,

$\lambda_1 = 590$ nm $= 5.9\times10^{-7}$ m

Wavelength of another light beam,

$\lambda_2 = 596$ nm $= 5.96\times10^{-7}$ m

Distance of the slits from the screen $= D = 1.5$ m

Slits width $= a = 2\times10^{-4}$ m

For the first secondary maxima,

$$\sin\theta = \frac{3\lambda_1}{2a} = \frac{x_1}{D}$$

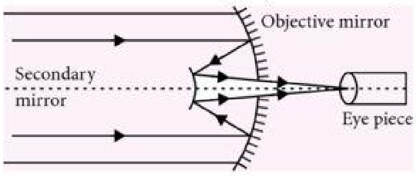
$$x_1 = \frac{3\lambda_1 D}{2a} \text{ and } x_2 = \frac{3\lambda_2 D}{2a}$$

$\therefore$ Separation between the positions of first secondary maxima of two sodium lines,

$$x_2 - x_1 = \frac{3D}{2a}(\lambda_2 - \lambda_1)$$

$$= \frac{3\times1.5}{2\times2\times10^{-4}}(5.96\times10^{-7} - 5.9\times10^{-7}) = 6.75\times10^{-5} \text{ m}$$

23. (a) The formation of image by a reflecting telescope,



(b) Advantage of reflecting telescope, over a refracting type:

(i) In refracting telescope the final image is formed after two times of partial refraction through the lenses, major losses in the intensity take place due to partial reflection and refractions. In reflecting telescope, all the light intensity incident forms the final image as no loss of intensity can be ensured in reflection.

(ii) In refracting telescope glass of lens offers different refractive indices to different colours and hence chromatic abberation due to which coloured image is formed take place. Reflecting telescope is free from chromatic abberation.

24. (a) Sun glasses filled with polaroid sheets protect our eyes from glare. Polaroids reduce head light glare of motor car being driven at night. Polaroids are used in three dimensional pictures, *i.e.*, in holography.

(b) Let two polaroids P_1 and P_2 are placed in crossed positions. Let P_3 be the polaroid sheet placed between P_1 and P_2 making an angle θ with pass axis of P_1

If I_1 = intensity of polarised light after passing through P_1 then intensity of light after passing through P_3 will be

$I = I_1 \cos^2\theta$ ...(i)

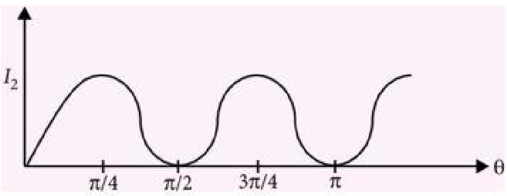
Now angle between P_2 and P_3

$= \left(\frac{\pi}{2} - \theta\right)$ [∴ P_1 and P_2 are in crossed position]

∴ Outcoming intensity after P_2 is,

$I_2 = I \cos^2 (90° - \theta)$

or $I_2 = I_1 \cos^2\theta \sin^2\theta = I_1\left(\frac{1}{2}\sin 2\theta\right)^2$



25. (a) *Refer to answer 60(a), Page no.225* (MTG CBSE Champion Physics Class 12)

(b) $R = 20$ cm, $n_2 = 1.5$, $n_1 = 1$, $u = -100$ cm

$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ or $\frac{1.5}{v} + \frac{1}{100} = \frac{1.5-1}{20}$

or $\frac{1.5}{v} = \frac{0.5}{20} - \frac{1}{100} = \frac{1}{40} - \frac{1}{100} = \frac{3}{200}$

or $v = \frac{200}{3} \times 1.5 = 100$ cm

⇒ So, a real image is formed on the other side, 100 cm away from the surface.

OR

Refer to answer 73, Page no. 264 (MTG CBSE Champion Physics Class 12)

26. (a) *Refer to answer 85(a), Page no. 228* (MTG CBSE Champion Physics Class 12)

(b) Critical angle for

(i) Red light is $\sin c_r = \frac{1}{1.39} = 0.7194$

or $c_r = 46°$

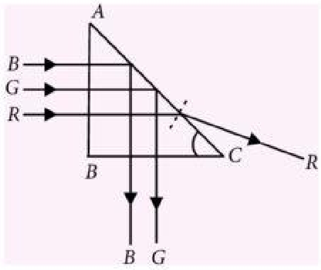
(ii) Green light is $\sin c_g = \frac{1}{1.44} = 0.6944$

or $c_g = 44°$

(iii) Blue light is $\sin c_b = \frac{1}{1.47} = 0.6802$

or $c_b = 43°$

As angle of incidence $i = 45°$ of red light ray on face AC is less that its critical angle of 46°, so red light ray will emerge out of face AC.



OR

Refer to answer 38, Page no. 257 (MTG CBSE Champion Physics Class 12)

27. (a) *Refer to answer 130, Page no. 236* (MTG CBSE Champion Physics Class 12)

OR

(a) *Refer to answer 104 (a), Page no. 270* (MTG CBSE Champion Physics Class 12)

(b) Intensity observed by observer $O_1 = \frac{I_0}{2}$

Intensity observed by observer

$O_2 = \frac{I_0 \cos^2 60°}{2} = \frac{I_0}{2} \times \frac{1}{4} = \frac{I_0}{8}$

Intensity observed by observer O_3

$= \frac{I_0}{8}\cos^2(90° - 60°) = \frac{I_0}{8}\sin^2 30° = \frac{I_0}{8} \times \left(\frac{1}{2}\right)^2 = \frac{I_0}{32}$

◈◈

Class XII

MONTHLY TUNE UP!

PRACTICE PROBLEMS

These practice problems enable you to self analyse your extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Performance analysis table given at the end will help you to check your readiness.

Electromagnetic Induction and Alternating Current

Total Marks : 120 **Time Taken : 60 Min.**

NEET / AIIMS

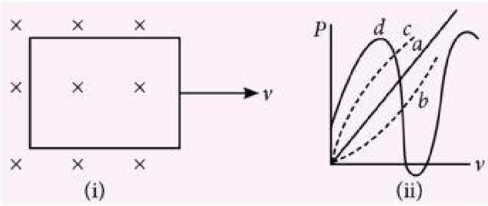
Only One Option Correct Type

1. An emf of 15 V is applied in a circuit coil containing 5 H inductance and 10 Ω resistance. The ratio of the currents at time $t = \infty$ and $t = 1$ s is
(a) $\frac{e^{1/2}}{e^{1/2}-1}$ (b) $\frac{e^2}{e^2-1}$
(c) $1-e^{-1}$ (d) e^{-1}

2. The resistance and inductance of series circuit are 5 Ω and 20 H, respectively. At the instant of closing the switch, the current is increasing at the rate 4 A s^{-1}. The supply voltage (in V) is
(a) 20 (b) 80 (c) 120 (d) 100

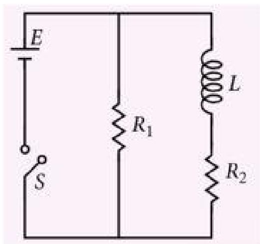
3. A pair of coils of turns n_1 and n_2 are kept close together. Current passing through the first is reduced at the rate r, and emf 3 mV is developed across the other coil. If the second coil carries current which is then reduced at the rate $2r$, then the emf produced across the first coil will be
(a) $\frac{6n_1}{n_2}$ mV (b) $\frac{6n_2}{n_1}$ mV
(c) 6 mV (d) $\frac{3}{2}$ mV

4. A conducting loop being pulled out of magnetic field with a speed v. Which of the following plots may represent the power delivered by the pulling agent as a function of the speed v?



(a) a (b) b (c) c (d) d

5. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2$ Ω and $R_2 = 2$ Ω are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is



(a) $6e^{-5t}$ V
(b) $\frac{12}{t}e^{-3t}$ V
(c) $6\left(1-e^{\frac{-1}{0.2}}\right)$ V
(d) $12e^{-5t}$ V

6. The horizontal component of the earth's magnetic field at a place is 4.0×10^{-4} T and the dip is 45°. A metal rod of length 20 cm is placed in the north-south direction and is moved at a constant speed of 5 cm s^{-1} towards east. Calculate the emf induced in the rod.
(a) 4×10^{-6} V (b) 2×10^{-6} V
(c) 6×10^{-4} V (d) 4×10^{-8} V

7. If the rms current in a 50 Hz AC circuit is 5 A, the value of the current $\frac{1}{300}$ s after its value becomes zero is
(a) $5\sqrt{2}$ A (b) $5\sqrt{3/2}$ A
(c) 5/6 A (d) $5/\sqrt{2}$ A

8. In a series *LCR* circuit, resistance $R = 10\ \Omega$ and the impedance $Z = 10\ \Omega$. The phase difference between the current and the voltage is
(a) 0° (b) 30° (c) 45° (d) 60°

9. An inductor of reactance 1 Ω and a resistor of 2 Ω are connected in series to the terminals of a 6 V (rms) AC source. The power dissipated in the circuit is
(a) 8 W (b) 12 W (c) 14.4 W (d) 18 W

10. An *LCR* series circuit consists of a resistance of 10 Ω, a capacitor of reactance 60 Ω and an inductor coil. The circuit is found to resonate when put across a 300 V, 100 Hz supply. The inductance of coil (in H) is (take, $\pi = 3$)
(a) 0.1 (b) 0.01 (c) 0.2 (d) 0.02

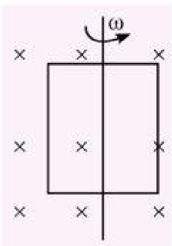
11. A circuit draws 330 W from a 110 V, 60 Hz AC line. The power factor is 0.6 and the current lags the voltage. The capacitance of a series capacitor that will result in a power factor of unity is equal to
(a) 31 μF (b) 54 μF (c) 151 μF (d) 201 μF

12. A 200 km telephone wire has capacity of 0.014 μF km^{-1}. If it carries an alternating current of frequency 50 kHz, what should be the value of an inductance required to be connected in series so that impedance is minimum?
(a) 3.6×10^{-8} H (b) 0.36×10^{-5} H
(c) 3.6×10^{-10} H (d) 3.6×10^{-12} H

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as:
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** A conducting loop is rotated in a uniform magnetic field with constant angular velocity ω as shown in figure. At time $t = 0$, plane of the loop is perpendicular to the magnetic field. Induced emf produced in the loop is maximum when plane of loop is parallel to magnetic field.



Reason : When plane of loop is parallel to magnetic field, then magnetic flux passing through the loop is zero.

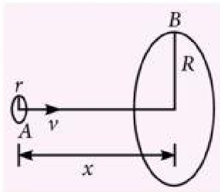
14. **Assertion :** An induced emf of 2 V is developed in a circular loop, if current in the loop is changed at a rate of 4 A s^{-1}. If 4 A of current is passed through this loop, then flux linked with this coil will be 2 Wb.
Reason : Flux linked with the coil is
$$|\phi| = \left|\frac{e}{di/dt}\right| i$$

15. **Assertion :** In *LCR* series *AC* circuit $X_L = X_C = R$ at a given frequency. When frequency is doubled the impedance of the circuit is $\frac{\sqrt{13}}{2}R$.
Reason : The given frequency is resonance frequency.

JEE MAIN / ADVANCED

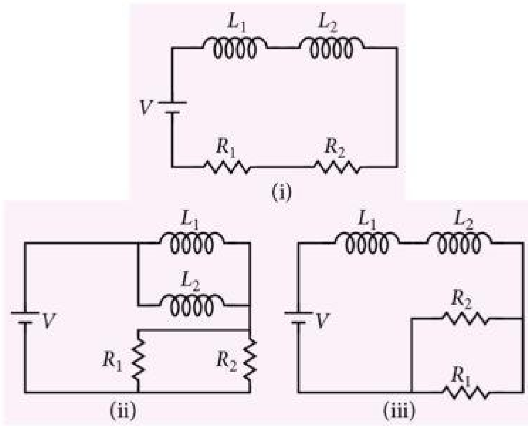
Only One Option Correct Type

16. Loop *A* of radius $r << R$ moves toward loop *B* with a constant velocity *v* in such a way that their planes are always parallel. What is the distance between the two loops (x) when the induced emf in loop *A* is maximum ?



(a) R (b) $\frac{R}{\sqrt{2}}$
(c) $\frac{R}{2}$ (d) $R\left(1-\frac{1}{\sqrt{2}}\right)$

17. Given $L_1 = 1$ mH, $R_1 = 1\ \Omega$, $L_2 = 2$ mH, $R_2 = 2\ \Omega$

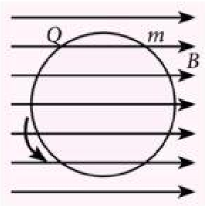


Neglecting mutual inductance, the time constants (in ms) for circuits (i), (ii), and (iii) are

(a) $1, 1, \frac{9}{2}$ (b) $\frac{9}{4}, 1, 1$

(c) 1, 1, 1 (d) $1, \frac{9}{4}, 1$

18. A thin uniform ring of mass m and electric charge Q uniformly distributed rotates around an axis perpendicular to its plane and going through its center. The angular momentum of the ring is 7.5×10^{-4} kg m^2 s^{-1}. The ring is in a homogeneous magnetic field of a field strength of 0.1 T and the lines of the magnetic induction are parallel with the plane of the ring. Torque exerted on the ring is [The specific charge (charge-mass ratio) of the ring is $Q/m = 10^{-5}$ C kg^{-1}].



(a) 3.75×10^{-10} N m upward along the plane
(b) 3.75×10^{-10} N m downward along the plane
(c) 7.5×10^{-10} N m upward along the plane
(d) none of the above

19. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be

(a) halved (b) the same
(c) doubled (d) quadrupled

More than One Options Correct Type

20. Two different coils have self-inductances $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power is given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then

(a) $\frac{i_1}{i_2} = \frac{1}{4}$ (b) $\frac{i_1}{i_2} = 4$

(c) $\frac{W_1}{W_2} = \frac{1}{4}$ (d) $\frac{V_1}{V_2} = 4$

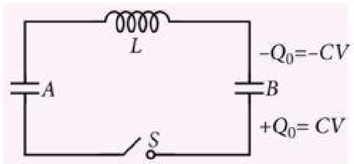
21. The uniform magnetic field perpendicular to the plane of a conducting ring of radius a changes at the rate of α, then

(a) all the points on the ring are at the same potential
(b) the emf induced in the ring is $\pi a^2 \alpha$
(c) electric field intensity E at any point on the ring is zero
(d) $E = (a\alpha)/2$

22. A choke coil of resistance 5 Ω and inductance 0.6 H is in series with a capacitance of 10 μF. If a voltage of 200 V is applied and the frequency is adjusted to resonance, the current and voltage across the inductance and capacitance are I_0, V_0 and V_1 respectively. Then we have,

(a) $I_0 = 40$ A (b) $V_0 = 9.8$ kV
(c) $V_1 = 9.8$ kV (d) $V_1 = 19.6$ kV

23. An inductor and two capacitors are connected in the circuit as shown in figure. Initially capacitor A has no charge and capacitor B has CV charge. Assume that the circuit has no resistance at all. At $t = 0$, switch S is closed, then [given $LC = \frac{2}{\pi^2 \times 10^4}$ s^2 and $CV = 100$ mC]



(a) when current in the circuit is maximum, charge on each capacitor is same
(b) when current in the circuit is maximum, charge on capacitor A is twice the charge on capacitor B
(c) $q = 50\,(1 + \cos 100\,\pi t)$ mC, when q is the charge on capacitor B at time t
(d) $q = 50\,(1 - \cos 100\,\pi t)$ mC, where q is the charge on capacitor B at time t

Numerical Value Type

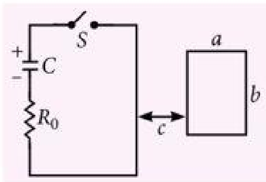
24. A copper rod of length 0.19 m is moving with uniform velocity 10 m s^{-1} parallel to a long straight wire carrying a current of 5.0 A. The rod is perpendicular to the wire with its ends at distances 0.01 m and 0.2 m from it. The emf induced in the rod is NμV. Find the value of N.

25. A long solenoid of diameter 0.1 m has 2×10^4 turns per metre. At the center of the solenoid, a 100-turn coil of radius 0.01 m is placed with its axis coinciding with the solenoid axis. The current in the solenoid is decreased at a constant rate from +2 A to –2 A in 0.05 s. Find the total charge (in μC) flowing through the coil during this time when the resistance of the coil is $40\,\pi^2\,\Omega$.

26. A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz ac supply. Time lag between maximum and minimum current is $N \times 10^{-3}$ s. Find the value of N.

Comprehension Type

In the circuit shown in figure, the capacitor has capacitance C = 20 μF and is initially charged to 100 V with the polarity shown. The resistor R_0 has resistance 10 Ω. At time t = 0, the switch is closed. The smaller circuit is not connected in any way to the larger one. The wire of the smaller circuit has a resistance of 1.0 $\Omega\,m^{-1}$ and contains 25 loops. The larger circuit is a rectangle 2.0 m by 4.0 m, while the smaller one has dimensions a = 10.0 cm and b = 20.0 cm. The distance c is 5.0 cm (The figure is not drawn to scale.) Both circuits are held stationary. Assume that only the wire nearest to the smaller circuit produces an appreciable magnetic field through it.



27. The current (in A) in the larger circuit 200 ms after closing S is

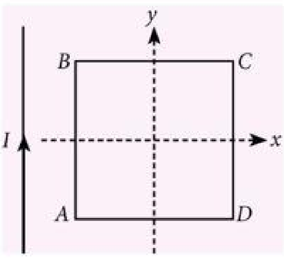
(a) $5/e$ (b) $2/e$ (c) $15/e$ (d) $10/e$

28. The current (in μA) in the smaller circuit 200 μs after closing S is

(a) 54 (b) 10 (c) 15 (d) 36

Matrix Match Type

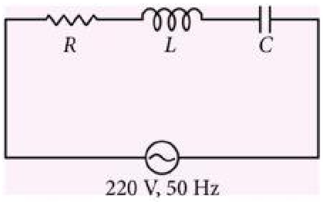
29. A long current carrying wire and a loop made of conducting wire are placed in x-y plane, such that the long wire is parallel to y-axis. Column I is regarding some changes made in the position of loop and Column II indicates the resulting effects. Match the columns.



	Column I		Column II
i.	If loop is moved away from the wire while keeping in x-y plane,	p.	current is induced in the loop in anticlockwise direction.
ii.	If loop is moved towards the wire while keeping in x-y plane	q.	current is induced in the loop in clockwise direction.
iii.	If loop is rotated about x-axis, then just after this	r.	no emf is induced in the loop.
iv.	If loop is rotated about y-axis, then just after this	s.	the wire will attract or repel the loop.

	(i)	(ii)	(iii)	(iv)
(a)	p,s	r,s	q	r
(b)	q,r	p,q	s	r,s
(c)	q,s	p,s	r	r
(d)	p,r	p,q	p,s	r

30. In series RLC circuit, R = 100 Ω, $C = \dfrac{100}{\pi}\,\mu F$, and $L = \dfrac{100}{\pi}$ mH are connected to an source as shown in figure. The rms value of ac voltage is 220 V and its frequency is 50 Hz. In column I, some physical quantities are mentioned, while in column II, information about quantities are provided. Match the entries of column I with the entries of column II.



	Column I		Column II
i.	Average power dissipated in the resistor is	p.	zero
ii.	Average power dissipated in the inductor is	q.	non-zero
iii.	Average power dissipated in the capacitor is	r.	163 SI units
iv.	The rms voltage across the capacitor is	s.	265.7 SI units

	(i)	(ii)	(iii)	(iv)
(a)	q,s	q,r	p	q
(b)	q,s	p	p	q,r
(c)	r	p,q	s	r,s
(d)	p,q	q	r,s	p

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PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / NEET / AIIMS / JIPMER with additional study material.
In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / NEET. The detailed solutions of these problems will be published in next issue of Physics For You.
The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.
We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

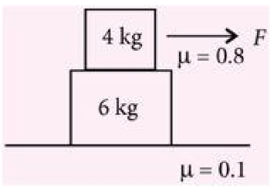
PROBLEM Set 63

SINGLE OPTION CORRECT TYPE

1. In a vernier callipers 1 main scale division is 1 mm and there are 10 divisions on vernier scale which match with 9 divisions on main scale. When there is no object between the jaws (when they are in contact) the zero mark of the vernier scale is to the left of zeroth mark of main scale and 8^{th} division of vernier scale coincided with one of the main scale divisions. When a plate is kept between the jaws zeroth mark of vernier scale crossed 12 mm mark on the main scale and vernier coincidence is 3. The correct thickness of the plate is

(a) 121.1 mm (b) 12.5 mm
(c) 13.1 mm (d) 11.5 mm

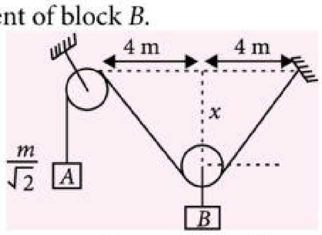
2. A block of mass 4 kg is placed on an another block of mass 6 kg which is placed on a rough horizontal surface as shown in the figure.



What maximum force that could be applied on 4 kg block in horizontal direction so that there is no relative slipping between the blocks? (g = 10 m s^{-2})

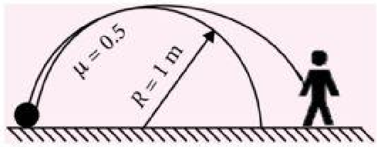
(a) 46.6 N (b) 32 N (c) 42 N (d) 38.8 N

3. The system of mass A and B Shown in figure is released from rest from x = 0. Find maximum displacement of block B.



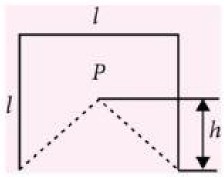
(a) $8\sqrt{2}$ m (b) $4\sqrt{2}$ m (c) $16\sqrt{2}$ m (d) 8 m

4. A person is pulling a particle of mass m = 2 kg from ground on a fixed rough hemispherical surface upto the top of the hemisphere with the help of a light inextensible string. Find the work done by the person. [R = 1, μ = 0.5 and g = 10 m s^{-2}]



(a) 10 J (b) 30 J (c) 25.5 J (d) 72 J

5. An isosceles triangle is to be cut from one edge of a square lamina of uniform mass density (as shown in the figure) such that the remaining portion when suspended from the apex P of the cut will remain in equilibrium in any position. The value of h is



(a) $\frac{(2-\sqrt{2})l}{2}$ (b) $\frac{(3+\sqrt{3})l}{2}$

(c) $\frac{(3-\sqrt{3})l}{2}$ (d) $\frac{(2+\sqrt{2})l}{2}$

6. A particle is projected at time t = 0 from a point P on the ground with a speed v_0, at an angle of 45° to the horizontal. Find the magnitude and direction of the angular momentum of the particle about P at time $t = v_0/g$.

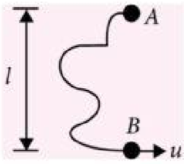
(a) $-\frac{mv_0^3}{2\sqrt{2}g}$ (b) $\frac{v_0^3}{2g}$

(c) $\frac{mv_0}{\sqrt{2}g}$ (d) $\frac{v_0^2}{2\sqrt{2}g}$

By Akhil Tewari, Author Rank up Physics for JEE Main & Advanced, Professor, IITians PACE, Mumbai.

MORE THAN ONE OPTION CORRECT TYPE

7. The particles A and B each of mass 'm' are attached by a light inextensible string of length $2l$. The whole system lies on a smooth horizontal table with B initially at a distance l from A. B is given velocity 'u' as shown in figure. Then just after the jerk



(a) $v_A = \frac{u\sqrt{3}}{4}$ (b) $v_B = \frac{u\sqrt{3}}{4}$

(c) component of v_B along AB is $\frac{u\sqrt{3}}{4}$

(d) component of v_B perpendicular to $AB = \frac{u}{2}$

8. The uniform speed of a body is the same as seen from any point in the body. A light cord is wrapped around the rim of the disc of mass 2 kg and mass of 1 kg is tied to the free end. If it is released from rest,

(a) the tension in the cord is 5 N

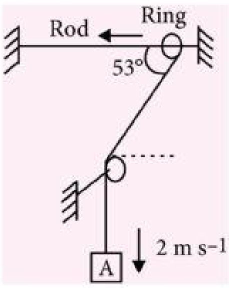
(b) in the first 4 s the angular displacement of the disc is 40 rad

(c) the work done by the torque on the disc in the first 4 s is 200 J

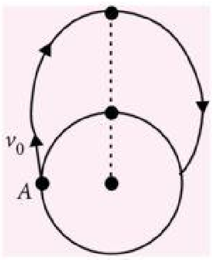
(d) the increase in kinetic energy of the disc in the first 4 s is 200 J.

NUMERICAL VALUE TYPE

9. Consider surfaces as frictionless for the diagram drawn if velocity of block A is 2 m s^{-1} downward. Velocity of ring is $\frac{n \times 2}{3}$ m s^{-1} towards left. Find n. [String is ideal]



10. A projectile projected tangentially from the surface of a planet of radius R from a point A, follows the indicated trajection during its motion. If it is at a height of $3R$ at the farthest point of its trajectory, then the velocity of projection at A, v_0 is given by $\sqrt{NgR}$. Find value of N.



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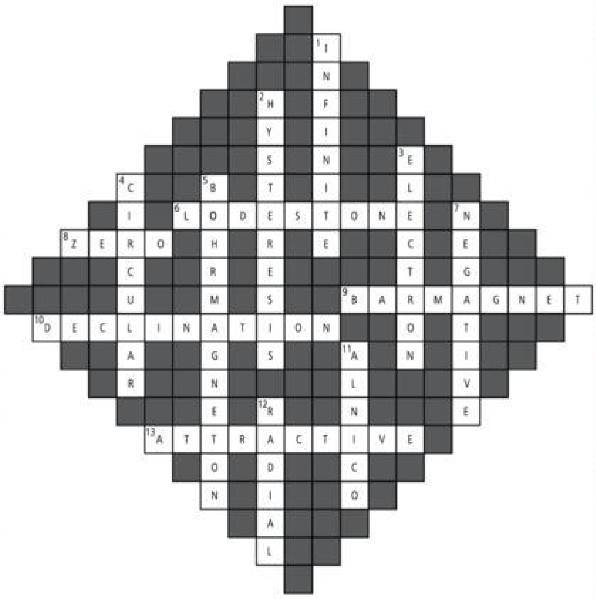
CROSSWORD SOLUTION SEPTEMBER 2018



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SET-62

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GEAR UP FOR JEE MAIN 2019

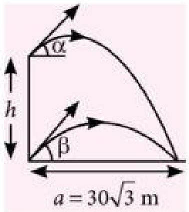
JEE Main I between 6^{th} to 20^{th} January and JEE Main II between 6^{th} to 20^{th} April 2019

1. In two systems of relations among velocity, acceleration, and force are, respectively, $v_2 = \frac{\alpha^2}{\beta} v_1$, $a_2 = \alpha\beta a_1$, and $F_2 = \frac{F_1}{\alpha\beta}$. If α and β are constants, then correct relation among mass, length, and time in two systems is

(a) $T_2 = T_1 \frac{\alpha}{\beta^2}$ (b) $M_2 = \frac{M_1}{\alpha^2\beta^2}$

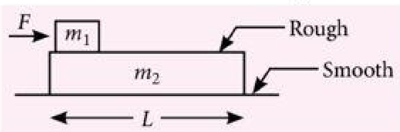
(c) $L_2 = L_1 \frac{\alpha}{\beta^2}$ (d) Both (a) and (b)

2. Shots are fired simultaneously from the top and bottom of a vertical cliff with the elevation $\alpha = 30°$, $\beta = 60°$, respectively figure. The shots strike an object simultaneously at the same point. If $a = 30\sqrt{3}$ m is the horizontal distance of the object form the cliff, then the height h of the cliff is



(a) 30 m (b) 45 m (c) 60 m (d) 90 m

3. In the figure shown a plank of length $l = 64$ m, mass $m_2 = 5$ kg rests on a smooth surface. Upper surface of block is rough with coefficient of kinetic friction $\mu_k = 0.5$ and $\mu_s = 0.6$. A force F of magnitude 30 N is applied on block m_1. What is displacement of plank till the small block falls over from plank?



(a) 0.6 m (b) 10 m (c) 16 m (d) 2.5 m

4. A lead bullet just melts when stopped by an obstacle. Assuming that 25 percent of the heat is absorbed by the obstacle, find the velocity of the bullet. Its initial temperature is 27°C.
(Melting point of lead = 327°C, specific heat of lead = 0.03 cal g^{-1} $°C^{-1}$, latent heat of fusion of lead = 6 cal g^{-1} $°C^{-1}$, J = 4.2 joule $calorie^{-1}$).

(a) 300 m s^{-1} (b) 409.8 m s^{-1}

(c) 1100 m s^{-1} (d) 25 m s^{-1}

5. A carpet of mass M, made of inextensible, material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to $R/2$.

(a) $\sqrt{\frac{14Rg}{3}}$ (b) $\sqrt{gR}$

(c) $\sqrt{3gR}$ (d) 5 gR

6. A body is projected vertically upwards from the bottom of a crater of moon of depth $R/100$ where R is the radius of moon with a velocity equal to the escape velocity on the surface of moon. Calculate maximum height attained by the body from the surface of the moon.

(a) 150 R (b) 99 R (c) 50 R (d) 199 R

7. A drop of water of mass 0.2 g is placed between two glass plates. The distance between them is 0.01 cm. Find the force of attraction between the plates if surface tension of water = 0.07 N m^{-1}.

(a) 2.8 N (b) 3.5 N (c) 0.7 N (d) 1.25 N

8. A column of mercury of length 10 cm is contained in the middle of a horizontal tube of length 1 m which is closed at both the ends. The two equal lengths contain air at standard atmospheric pressure of 0.76 m of mercury. The tube is now turned to vertical position. By what distance will the column of mercury be displaced ?

(a) 3.14 cm (b) 1.28 cm

(c) 2.95 cm (d) 4.83 cm

9. A thermostated chamber at a height h above earth's surface maintained at 30°C has a clock fitted with uncompensated pendulum. The maker of the clock for chamber mistakenly designed it to maintain correct time at 20°C. It is found that when the chamber is brought to earth's surface the clock in it clicked correct time. R_e is the radius of Earth. The linear coefficient of the material of pendulum is

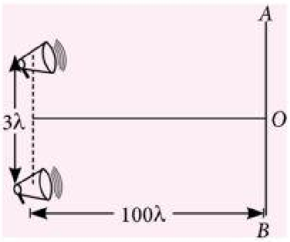
(a) $\frac{h}{R_e}$ (b) $\frac{h}{5R_e}$ (c) $\frac{5R_e}{h}$ (d) $\frac{R_e}{h}$

10. Two particles move parallel to x-axis about the origin with same amplitude 'a' and frequency ω. At a certain instant they are found at a distance $a/3$ from the origin on opposite sides but their velocities are in the same direction. What is the phase difference between the two?

(a) $\cos^{-1}\frac{7}{9}$ (b) $\cos^{-1}\frac{5}{9}$

(c) $\cos^{-1}\frac{4}{9}$ (d) $\cos^{-1}\frac{1}{9}$

11. Two loudspeakers are emitting sound waves of wavelength λ with an initial phase difference of $\pi/2$. At what minimum distance from O on line AB will one hear a maxima?



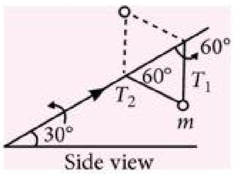
(a) 25λ

(b) $\frac{100\lambda}{\sqrt{15}}$

(c) $\frac{25\lambda}{3}$

(d) 50λ

12. A rod is arranged at an angle of 30° from the horizontal. Attached to the rod with two strings is the mass m, as shown. The rod is rotated, maintaining its direction in space, so that m travels in a circular path. The strings are of equal length, and make angles of 60° with the rod as shown. Take the length of the strings as 2.4 m. Calculate the minimum value of the tangential speed (in m s^{-1}) of the mass such that the string with tension T_2 does not becomes slack when the mass directly above the rod.



(a) 6

(b) 0.2

(c) $\sqrt{6}$

(d) 2.7

13. An electron of mass m_e, initially at rest, moves through a certain distance in a uniform electric field in time t_1. A proton of mass m_p, also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity, the ratio of t_2/t_1 is nearly equal to

(a) 1 (b) $(m_p/m_e)^{1/2}$

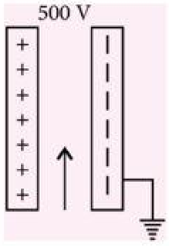
(c) $(m_e/m_p)^{1/2}$ (d) 1836

14. In the circuit shown in fig the heat produced in the 5 ohm resistor due to the current flowing through it is 10 calorie per second. The heat generated in the 4 ohm resistor is

(a) 1 cal s^{-1} (b) 2 cal s^{-1}

(c) 3 cal s^{-1} (d) 4 cal s^{-1}

15. A potential difference of 500 V is applied across the plates of a parallel plate condenser. The separation between the plates is 3 mm. An electron projected vertically, parallel to the plates, with a velocity of 3×10^6 m s^{-1} moves undeflected between the plates.



Find the magnitude of the magnetic field in the region between the condenser plates. (Neglect the edge effects).

(Charge of the electron = – 1.6×10^{-19} coulomb)

(a) 0.055 T (b) 0.83 T (c) 1.49 T (d) 3.58 T

16. A proton, a deuteron and an α-particle having the same kinetic energy are moving in circular trajectories in a constant magnetic field. If r_p, r_d and r_α denote respectively the radii of the trajectories of these particles, then

(a) $r_\alpha = r_p < r_d$ (b) $r_\alpha > r_d > r_p$

(c) $r_\alpha = r_d > r_p$ (d) $r_p = r_d = r_\alpha$

17. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12 V battery. The current in the coil is 1.0 A at approximately the time

(a) 500 s (b) 25 s (c) 35 ms (d) 1 ms

18. A ray of light is incident at an angle of 60° on one face of a prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. The emergent ray is perpendicular to the face through which it emerges calculate the refractive index of the material of the lens.

(a) $\sqrt{3}$ (b) $\sqrt{5}$ (c) $\sqrt{7}$ (d) $\sqrt{14}$

19. A telescope has an objective of focal length 50 cm and an eyepiece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focussed for distinct vision on a scale 200 cm away from the objective. Calculate the separation between the objective and the eyepiece

(a) 70.8 cm (b) 88 cm

(c) 95 cm (d) 109 cm

20. An alpha particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of

(a) 1Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm

21. A triode has plate characteristics in the form of parallel lines in the region of our interest. At a grid voltage of –1 volt the anode current I (in milli ampere) is given in terms of plate voltage V (in volt) by the algebraic relation: $I = 0.125V - 7.5$ For grid voltage of –3 volt, the current at anode voltage of 300 V is 5 mA. Determine the plate resistance (r_P) and the transconductance (g_m) for the triode.

(a) $8 \times 10^3\,\Omega$, 22.5×10^{-3} A V^{-1}
(b) $4 \times 10^8\,\Omega$, 12.5×10^{-6} A V^{-1}
(c) $8 \times 10^3\,\Omega$, 12.5×10^{-3} A V^{-1}
(d) None of these

22. A radioactive material decays by simultaneous emission of two particles with respective half-lives 1620 and 810 year. The time, in year, after which one-fourth of the material remains is

(a) 1080 (b) 2430 (c) 3240 (d) 4860

23. The two rails of a railway track, insulated from each other and the ground, are connected to a millivolt meter. What is the reading of the milli voltmeter when a train travels at a speed of 170 km h^{-1} along the track, given that the horizontal components of earth's magnetic field is 0.2×10^{-4} wb m^{-2} and the rails are separated by 1 m ?

(a) 2.5 mV (b) 3.56 mV
(c) 1.8 mV (d) 0.944 mV

24. A copper wire is stretched to make it 0.2 % longer. What is the percentage change in its resistance ?

(a) 0.3 (b) 0.4 (c) 0.8 (d) 0.2

25. In an experiment of simple pendulum time period measured was 50 s for 25 vibrations when the length of the simple pendulum was taken 100 cm. If the least count of stop watch is 0.1 s and that of meter scale is 0.1 cm. Calculate the maximum possible error in the measurement of value of g.

(a) 1.5% (b) 0.75% (c) 2.5% (d) 0.5%

26. The following observations were taken for determining surface tension of water by capillary tube method. Diameter of capillary $D = 1.25 \times 10^{-2}$m and rise of water in capillary, $h = 1.45 \times 10^{-2}$ m. Taking $g = 9.80$ m s^{-2} and using the relation $T = \left(\frac{rgh}{2}\right) \times 10^3$ N m^{-1}, what is the possible error in measurement of surface tension T ?

(a) 2.4% (b) 15% (c) 1.6% (d) 0.15%

27. A student performs an experiment to determine the Young's modulus of a wire, exactly 2 m long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ± 0.01 mm. Take $g = 9.8$ m s^{-2} (exact). The Young's modulus obtained from the reading is

(a) $(2.0 \pm 0.3) \times 10^{11}$ N m^{-2}
(b) $(2.0 \pm 0.2) \times 10^{11}$ N m^{-2}
(c) $(2.0 \pm 0.1) \times 10^{11}$ N m^{-2}
(d) $(2.0 \pm 0.05) \times 10^{11}$ N m^{-2}.

28. In a screw gauge, the zero of main scale coincides with fifth division of circular scale in figure (i). The circular division of screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball in figure (ii) is

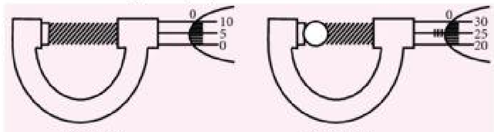
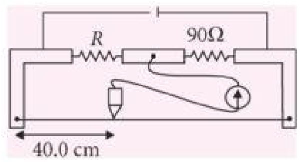


Figure (i) Figure (ii)

(a) 2.25 mm (b) 2.20 mm
(c) 1.20 mm (d) 1.25 mm

29. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90 Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1 mm. The unknown resistance is

(a) 60 ± 0.15 Ω
(b) 135 ± 0.56 Ω
(c) 60 ± 0.25 Ω
(d) 135 ± 0.23 Ω



30. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air column is the second resonance. Then,

(a) the intensity of the sound heard at the first resonance was more than that at the second resonance

(b) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube
(c) the length of the air-column at the first resonance was somewhat shorter than $1/4^{th}$ of the wavelength of the sound in air
(d) Both (a) and (c)

SOLUTIONS

1. **(d):** $v_2 = v_1 \frac{\alpha^2}{\beta} \Rightarrow [L_2T_2^{-1}] = [L_1T_1^{-1}]\frac{\alpha^2}{\beta}$ ...(i)

$a_2 = a_1\alpha\beta \Rightarrow [L_2T_2^{-2}] = [L_1T_1^{-2}]\alpha\beta$ ...(ii)

$v_2 = v_1 \frac{\alpha^2}{\beta} \Rightarrow [L_2T_2^{-1}] = [L_1T_1^{-1}]\frac{\alpha^2}{\beta}$

and $F_2 = \frac{F_1}{\alpha\beta} \Rightarrow [M_2L_2T_2^{-2}] = [M_1L_1T_1^{-2}] \times \frac{1}{\alpha\beta}$ ...(iii)

Dividing eqn (iii) by eqn (ii), we get

$$M_2 = \frac{M_1}{(\alpha\beta)\alpha\beta} = \frac{M_1}{\alpha^2\beta^2}$$

Squaring eqn (i) and dividing by eqn (ii), we get

$$L_2 = L_1\frac{\alpha^3}{\beta^3}$$

Dividing eqn (i) by eqn (ii), we get $T_2 = T_1\frac{\alpha}{\beta^2}$

2. **(c):** $u_1 \cos\alpha = u_2 \cos\beta$

$$30\sqrt{3} = \frac{u_2^2 \sin 2\beta}{g} \Rightarrow 30\sqrt{3} = \frac{u_2^2\sqrt{3}}{2g}$$

$$\Rightarrow u_2^2 = 600 \text{ or } u_2 = 10\sqrt{6}$$

$$t_2 = \frac{2u_2 \sin 60°}{g} = \frac{2(10\sqrt{6})\sqrt{3}/2}{10} \Rightarrow t_2 = \sqrt{18}$$

$$u_1 = \frac{10\sqrt{6}\cos 60°}{\cos 30°} = 10\sqrt{2}$$

$$-h = 10\sqrt{2}\sin 30°\sqrt{18} - \frac{1}{2}10(\sqrt{18})^2$$

$$\Rightarrow -h = 30 - 90 \Rightarrow h = 60 \text{ m}$$

3. **(c):** Block slips on plank

$a_1 = 10$ m s^{-2}, $a_2 = 2$ m s^{-2}
$a_{rel} = 8$ m s^{-2}
Time taken by m_1 to fall over plank

$$s_{rel} = \frac{1}{2}a_{rel}t^2 \Rightarrow 64 = \frac{1}{2}\times 8t^2 \text{ or } t = 4 \text{ s}$$

Displacement of plank $= \frac{1}{2}\times 2\times 4\times 4 = 16$ m

4. **(b):** Kinetic energy of bullet is converted into heat. 25% of heat is absorbed by obstacle.
75% of heat is absorbed by lead. This heat raises temperature of lead to 327° C and then melts it.

$$75\% \times \left(\frac{1}{2}Mv^2\right) = ms\,\Delta T + mL$$

or $\frac{75}{100}\times\frac{mv^2}{2} = m(s\Delta T + L)$

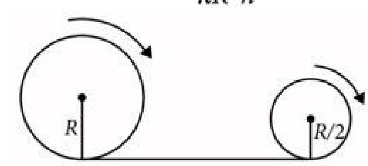
$$s = \frac{0.03 \text{ cal}}{\text{g}\times°\text{C}} = \frac{0.03\times 4.2 \text{ J}}{(10^{-3}\text{kg})\times°\text{C}} = \frac{0.03\times 4.2\times 1000 \text{ J}}{°\text{C}\times kg}$$

$$L = \frac{6 \text{ cal}}{2\times°\text{C}} = \frac{6\times 4.2 \text{ J}}{10^{-3} \text{ kg}} = 6\times 4.2\times 10^3 \text{ J kg}^{-1}$$

$$\therefore \frac{3}{4}\times\frac{v^2}{2} = 126\times(327-27) + 25200 = 37800 + 25200$$

or $v^2 = \frac{8}{3}\times 63000$ $v = 168000$ or $v = 409.8$ m s^{-1}

5. **(a):** Density of carpet $= \frac{M}{\pi R^2 h}$ or $D = M/(\pi R^2 h)$



$\therefore$ Mass of unrolled carpet $= M'$

$$\therefore M' = \pi\left(\frac{R}{2}\right)^2 h\times\text{density} = \frac{\pi R^2 h}{4}\times\frac{M}{\pi R^2 h} = \frac{M}{4}$$

Energy is conserved.

$\therefore$ Loss in potential energy $= MgR - M'g\left(\frac{R}{2}\right)$

$$= MgR - \frac{MgR}{4\times 2} = \frac{7MgR}{8} \quad \text{...(i)}$$

Gain in KE = Translational KE + Rotational KE

$$= \frac{1}{2}\left(\frac{M}{4}\right)v_c^2 + \frac{1}{2}I\omega^2 \text{ where } \omega = \frac{v_c}{R/2}$$

$$= \frac{M}{8}v_c^2 + \frac{1}{2}\left[\frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2\right]\frac{v_c^2}{(R/2)^2}$$

$$= \frac{M}{8}v_c^2 + \frac{Mv_c^2}{16} = \frac{3Mv_c^2}{16} \quad \text{...(ii)}$$

$$\therefore \frac{3Mv_c^2}{16} = \frac{7MgR}{8} \Rightarrow v_c^2 = \frac{7\times gR\times 2}{3} = \frac{14gR}{3}$$

or $v_c^2 = \frac{14gR}{3}$ or $v_c = \sqrt{\frac{14Rg}{3}}$

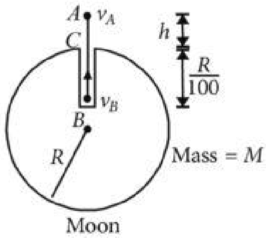
6. **(b):** BC is the crater of moon. B denotes bottom of crater.
Depth of crater $= R/100$

Radius of moon = R

Speed of particle at $B = v_B$

Escape velocity on surface of moon = v_e

$\therefore \quad v_e = \sqrt{\frac{2GM}{R}}$



At highest point $v_A = 0$

Mechanical energy is conserved in the process.

Decrease in kinetic energy = $\frac{1}{2}mv_B^2$

Increase in PE = $U_A - U_B$

V_A = Potential at $A = -\frac{GM}{(R+h)}$

V_B = Potential at $B = -\frac{GM}{R^3}\left[1.5\,R^2 - 0.5\left(R - \frac{R}{100}\right)^2\right]$

$= -\frac{GM}{R^3}\left[\frac{3R^2}{2} - \left(\frac{1}{2}\right)\left(\frac{99R}{100}\right)^2\right]$

$-\frac{GM}{R^3} \times \frac{R^2}{2}[3 - 0.98] = -\frac{GM}{R} \times \frac{2.02}{2}$

$\therefore \quad$ Increase in $PE = -\frac{GMm}{R+h} + \frac{GMm}{R} \times (1.01)$

Equate KE and PE.

$\frac{1}{2}mv_B^2 = -\frac{GMm}{(R+h)} + \frac{GMm \times 1.01}{R}$

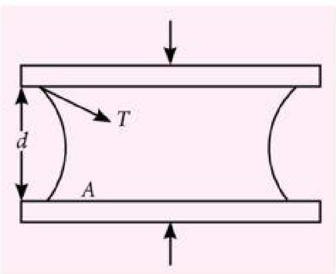
$\frac{GM}{R} = GM\left[-\frac{1}{R+h} + \frac{1.01}{R}\right]$

or $\quad \frac{1}{R} = -\frac{1}{R+h} + \frac{1.01}{R}$

or $\quad \frac{1}{R+h} = \frac{0.01}{R} = \frac{1}{100R}$

or $100\,R = R + h$ or $h = 99R$

7. **(a):** Let R be the radius of circular layer of water. Then $\pi R^2 d\rho = m$



Pressure at $A = P_0 - \frac{2T}{d}$ (meniscus is cylindrical in shape)

Pressure between the plates is less than the atmospheric pressure and so the plates are pressed together.

F = Force of attraction = $\Delta\rho \times$ area

$$F = \frac{2T}{d} \times \pi R^2 = \frac{2T}{d} \times \frac{m}{d.\rho} = \frac{2Tm}{d^2\rho}$$

$$= \frac{2 \times 0.2 \times 10^{-3} \times 0.07}{0.01^2 \times 10^{-4} \times 1000} = 2.8\,\text{N}$$

8. **(c)**

9. **(b):** Variation of acceleration due to gravity with altitude

$$g_h = g\left(1 - \frac{2h}{R_e}\right);\ \Delta g = \frac{2gh}{R_e}$$

Variation of l with temperature = Δl

Linear expansivity = $\frac{\Delta l}{l(\Delta\theta)}$

$$T_h = 2\pi\sqrt{\frac{l}{g - \Delta g}} = 2\pi\sqrt{\frac{l}{g}}\left(1 - \frac{\Delta g}{g}\right)^{-1/2} = T\left(1 + \frac{\Delta g}{2g}\right)$$

$$T_\theta = 2\pi\sqrt{\frac{l + \Delta l}{g}} = T\left(1 + \frac{\Delta l}{2l}\right)$$

The clock shows correct time if $T_h = T_\theta$, $\frac{\Delta l}{2l} = \frac{\Delta g}{2g}$

Linear expansivity = $\frac{\Delta l}{10l} = \frac{\Delta g}{10g} = \frac{2h}{10R_e} = \frac{h}{5R_e}$

10. (a): Let $x_1 = a \sin \omega t$ and $x_2 = a \sin(\omega t + \delta)$ be two S.H.M.

$$\frac{a}{3} = a \sin \omega t \text{ and } \frac{-a}{3} = a \sin(\omega t + \delta)$$

$$\sin \omega t = \frac{1}{3} \text{ and } \sin(\omega t + \delta) = \frac{-1}{3}$$

Eliminating t, $\frac{1}{3}\cos\delta + \sqrt{1 - \frac{1}{9}}\sin\delta = \frac{-1}{3}$

$9\cos^2\delta + 2\cos\delta - 7 = 0$

$\cos\delta = -1$ or $\frac{7}{8}$ *i.e.*, $\delta = 180°$ or $\cos^{-1}\left(\frac{7}{9}\right)$

Now $v_1 = a\omega \cos \omega t$ and $v_2 = a\,\omega \cos(\omega t + \delta)$

If we put $\delta = 180°$

We find that v_1 and v_2 are of opposite signs.

Hence $\delta = 180°$ is not applicable.

$$\therefore\ \delta = \cos^{-1}\left(\frac{7}{9}\right)$$

11. (c): $\Delta\phi = 2n\pi \Rightarrow \frac{\pi}{2} + \frac{2\pi}{\lambda} d\sin\theta = 2n\pi$

$$\frac{2\pi}{\lambda} d\sin\theta = \left(2n - \frac{1}{2}\right)\pi$$

$$\sin\theta = \left(2n - \frac{1}{2}\right)\frac{\lambda}{2d} = \frac{1}{2} \times \frac{\lambda}{2 \times 3\lambda} = \frac{1}{12}$$

$$\Rightarrow \frac{y}{\sqrt{(100\lambda)^2}} = \frac{1}{12}$$

$$144y^2 = (100\lambda)^2\ ;\ y \approx \frac{100\lambda}{12} = \frac{25\lambda}{3}$$

12. (a): $r = \cos 30° = \frac{2\sqrt{3}}{2}$

$T_2 \cos 30° + T_1 \cos 30° + mg \cos 30° = \frac{mv^2}{r}$

$T_2 \sin 30° + mg \sin 30° = T_1 \sin 30°$

Just slack $\Rightarrow T_0 = 0° \Rightarrow T_1 = mg$

$2mg \cos 30° \times L \cos 30° = mv^2$

$$v^2 = 2\frac{3}{4} L \times 10$$

$$v^2 = \frac{3}{2} \times 10 \times 2.4 = 36 \Rightarrow v = 6\ \text{m s}^{-1}$$

13. (b): Acceleration of electron $= a_e$

$$\therefore\ a_e = \frac{\text{Force on electron}}{\text{Mass of electron}} = \frac{eE}{m_e}$$

Similarly, acceleration of proton, $a_p = \frac{eE}{m_p}$

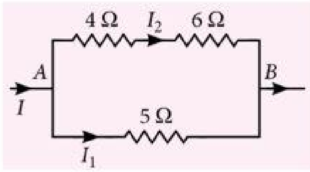
$$\because\ S = ut + \frac{1}{2}at^2$$

$$\therefore\ S = 0 + \frac{1}{2}a_e t_1^2 \text{ and } S = 0 + \frac{1}{2}a_p t_2^2$$

$$\therefore\ \frac{1}{2}a_e t_1^2 = \frac{1}{2}a_p t_2^2$$

or $\left(\frac{t_2}{t_1}\right)^2 = \frac{a_e}{a_p} = \frac{eE}{m_e} \times \frac{m_p}{eE} = \frac{m_p}{m_e}$ $\therefore$ $\frac{t_2}{t_1} = \left(\frac{m_p}{m_e}\right)^{1/2}$

14. (b): $V_{AB} = I_2(4 + 6)$



$\therefore$ $V_{AB} = 10\,I_2$

Also $V_{AB} = I_1\,(5)$

$\therefore$ $5\,I_1 = 10\,I_2$

or $I_1 = 2\,I_2$ ...(i)

Let $H = \frac{\text{Heat generated}}{\text{second}} = I^2R$

$$\therefore\ \frac{H_4}{H_5} = \frac{I_2^2 \times 4}{I_1^2 \times 5}$$

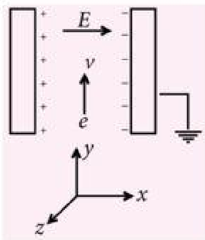
$$\frac{H_4}{10} = \frac{I_2^2 \times 4}{(2I_2)^2 \times 5} \text{ or } H_4 = \frac{10 \times 4}{4 \times 5} = 2\ \text{cal s}^{-1}.$$

15. (a): Since the electron moves undeflected between the plates of condenser, there should be equal and opposite magnetic and electric forces upon it.

Electric force is to the left, due to positively charged plate.

Magnetic force on electron should be towards right. The magnetic field should therefore be directed perpendicular to plane of paper inwards, according to Fleming's left hand rule.

Magnetic force = Electrical force



$evB = eE$

or $B = \frac{E}{v} = \left(\frac{V}{d}\right)\frac{1}{v}$

$$B = \left(\frac{500}{3 \times 10^{-3}}\right)\left(\frac{1}{3 \times 10^6}\right)$$

or $B = 0.055$ T

16. (a): The magnetic force on the charged particles provides necessary centripetal force for circular motion of the particles.

$$\frac{mv^2}{r} = qvB \text{ or } r = \frac{mv}{qB}$$

$$r = \frac{p}{qB} \text{ or } r = \frac{\sqrt{2mK}}{qB} \qquad (\because p^2 = 2mK)$$

$$r_p : r_d : r_\alpha = \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2}$$

or $r_p : r_d : r_\alpha = 1 : \sqrt{2} : 1$ or $r_\alpha = r_p < r_d$

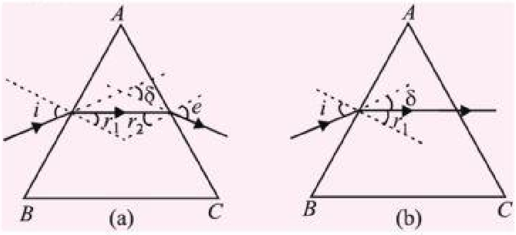
17. (d): $I = I_0\,(1 - e^{-t/\tau})$ where $I_0 = \frac{V}{R}$ and $\tau = \frac{L}{R}$

$\therefore \quad I = \frac{V}{R}[1 - e^{-Rt/L}]$, when current grows in L-R circuit.

or $\quad 1 = \frac{12}{6}[1 - e^{-6t/(8.4 \times 10^{-3})}]$

or $\quad t = 0.97 \times 10^{-3}$ s = 0.97 ms or $t = 1$ ms.

18. (a) : Refer the figure (a). Given $i = 60°$, $\delta = 30°$ and $A = 30°$.



We have

$\delta = i + e - A$ ...(i)

From equation (i), we get $30° = 60° + e - 30°$ or $e = 0$, *i.e.* the emergent ray is perpendicular to the face AC through which it emerges. This is shown in figure (b). Here also $i = 60°$ and $\delta = 30°$

Therefore, $r_1 = i - \delta = 60° - 30° = 30°$. Hence

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} = 1.732$$

19. (a) : Given $u = -200$ cm, $f_o = 50$ cm. The distance of the image I formed by the objective is given by

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_0} \text{ or } \frac{1}{v} = \frac{1}{f_o} + \frac{1}{u} = \frac{1}{50} - \frac{1}{200}$$

which gives $v = \frac{200}{3}$ cm. Therefore,

The image I serves as the object for the eye-piece. Thus $v' = -25$ cm, $f_e = 5$ cm. The object distance u' is given by

$$\frac{1}{u'} = \frac{1}{v'} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5}$$

which gives $u' = -\frac{25}{6}$ cm.

Separation between objective and eye-piece

$$|v| = |u'| = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = 70.8 \text{ cm.}$$

20. (c) : Energy is conserved.

Loss in kinetic energy = Gain in potential energy

$$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_{min}} = 5 \times (1.6 \times 10^{-13})\text{ J}$$

$$\therefore \quad r_{min} = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{5 \times 1.6 \times 10^{-13}}$$

$$r_{min} = \frac{(9 \times 10^9)(2)(92)(1.6 \times 10^{-19})^2}{5 \times 1.6 \times 10^{-13}}$$

or $\quad r_{min} = 5.3 \times 10^{-14}$ m or $r_{min} = 5.3 \times 10^{-12}$ cm

The distance of closest approach is of the order of 10^{-12} cm.

21. (c) : At $V_g = -1$ V, $I_P = (0.125\ V_P - 7.5) \times 10^{-3}$ A

$$\therefore \quad \frac{dI_P}{dV_P} = 0.125 \times 10^{-3}\ \frac{\text{A}}{\text{V}}$$

or $\quad r_p = \frac{dV_p}{dI_p} = \frac{1}{0.125 \times 10^{-3}}\frac{\text{V}}{\text{A}}$

or $\quad r_P = 8 \times 10^3\ \Omega$ ...(i)

Again, $I_P = (0.125\ V_P - 7.5)$mA

$I_P = (0.125 \times 300 - 7.5)$ at $V_P = 300$ V, $V_g = -1$ V.

or $\quad I_P = (37.5 - 7.5)$ mA or $I_P = 30$ mA

In second case, $V_g = -3$ V, $V = 300$ V, $I = 5$ mA

$\therefore \quad g_m = \frac{\Delta I_P}{\Delta V_g}$ at V_P constant

or $\quad g_m = \frac{(30-5) \times 10^{-3}}{[-1-(-3)]}$

or $\quad g_m = \frac{25 \times 10^{-3}}{2} = 12.5 \times 10^{-3}\text{ A V}^{-1}$ ...(ii)

22. (a) : $\because \quad \frac{N}{N_0} = e^{-\lambda t}$

There is a simultaneous emission of two particles.

$\therefore \quad \frac{N}{N_0} = e^{-(\lambda_1 + \lambda_2)t}$ or $\frac{N_0}{4N_0} = e^{-(\lambda_1 + \lambda_2)t}$

or $\quad \log 4 = (\lambda_1 + \lambda_2)\, t \log e$

Now $\lambda_1 = \frac{0.693}{1620}$ and $\lambda_2 = \frac{0.693}{810}$

$$\therefore \quad 2.303[2 \times 0.3] = 0.693\left[\frac{1}{1620} + \frac{1}{810}\right]t$$

or $\quad t = \frac{2.303 \times 0.6 \times 1620}{0.693 \times 3}$ or $t = 1080$ year

23. (d): EMF developed due to motion = ε

$\therefore \quad \varepsilon = vBl$ where $v = \frac{170 \times 1000}{60 \times 60} = 47.2\text{ m s}^{-1}$

$\varepsilon = 47.2 \times (0.2 \times 10^{-4}) = 9.44 \times 10^{-4} = 0.944$ mV

24. (b): The mass of a wire length l, cross-sectional area A and density d is given by

$m = Ald$ or $A = \frac{m}{ld}$

$\therefore \quad$ The resistance of wire of resistivity ρ is

$$R = \frac{\rho l}{A} = \frac{\rho d l^2}{m} = kl^2 \quad \text{...(i)}$$

where $k = \frac{\rho d}{m}$ is a constant of the wire. Taking logarithm of both sides of (i) we have

$\log R = \log k + 2 \log l$

Differentiating

$$\frac{\delta R}{R} = 0 + \frac{2\delta l}{l} = \frac{2\delta l}{l}$$

Given $\frac{\delta l}{l} = 0.2\,\%$.

Therefore, $\frac{\delta R}{R} = 2 \times 0.2\,\% = 0.4\,\%$.

Thus, the resistance of the increases by 0.2 %.

25. (d): The time period of simple pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

or $T^2 = \frac{4\pi^2 l}{g}$ or $g = \frac{4\pi^2 l}{T^2}$

As 4 and π are constants, maximum permissible error in g is given by $\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$

Here $\Delta L = 0.1$ cm, $L = 1$ m $= 100$ cm,

$\Delta T = 0.1$ s $T = 50$ s

$$\therefore \frac{\Delta g}{g} = \frac{0.1}{100} + 2\left(\frac{0.1}{50}\right) = \frac{0.1}{100} + \left(\frac{0.1}{25}\right)$$

$$\text{or } \frac{\Delta g}{g} \times 100 = \left[\frac{0.1}{100} + \frac{0.1}{25}\right] \times 100$$

$$= 0.1 + 0.4 = 0.5\%$$

26. (c) : Percentage error in T,

$$\frac{\Delta T}{T}\% = \frac{\Delta r}{r}\% + \frac{\Delta g}{g}\% + \frac{\Delta h}{h}\%$$

$$= \frac{0.01}{1.25} \times 100 + \frac{0.01}{9.80} \times 100 + \frac{0.01}{1.45} \times 100$$

$$= 0.8 + 0.1 + 0.7 = 1.6$$

27. (b): Young's modulus $= \frac{\text{stress}}{\text{strain}}$

$$\text{or } Y = \frac{4Fl}{\pi d^2 \Delta l} = \frac{4 \times (1 \times 9.8) \times 2}{\pi \times (0.4 \times 10^{-3})^2 \times (0.8 \times 10^{-3})}$$

$= 2 \times 10^{11}$ N m^{-2}.

Now, $\frac{\Delta Y}{Y} = \frac{2\Delta d}{d} + \frac{\Delta(\Delta l)}{\Delta l}$

$$\text{or } \Delta Y = \left(\frac{2 \times 0.01}{0.4} + \frac{0.05}{0.8}\right) \times 2 \times 10^{11}\,\text{N m}^{-2}$$

$= 0.2 \times 10^{11}$ N m^{-2}

Hence the Young's modulus obtained from the reading is $(2.0 \pm 0.2) \times 10^{11}$ N m^{-2}.

28. (c) : Least count of screw gauge $= \frac{\text{Pitch}}{n}$

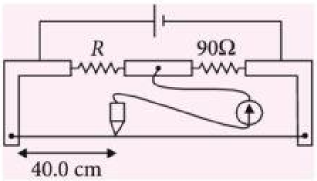
$= \frac{0.5}{50} = 0.01$ mm

$\therefore$ Diameter of ball

$= (2 \times 0.5\text{ mm}) + (25 - 5) \times 0.01$ mm

$= 1 + (0.01 \times 20) = 1.2$ mm.

29. (c) :



When a metre bridge is balanced, then

$$\frac{R_1}{x} = \frac{R_2}{(100 - x)} \quad \text{...(i)}$$

From figure,

$R_1 = R$, $R_2 = 90\ \Omega$, $x = 40$ cm

Then, $\frac{R}{40} = \frac{90}{(100 - 40)} = \frac{90}{60}$

$\Rightarrow R = 60\ \Omega$

Now, for ΔR taking natural log on both sides of eqn. (i),

$\ln R_1 = \ln R_2 + \ln x - \ln(100 - x)$

or $\ln R = \ln x - \ln(100 - x) + \ln(90)$

On differentiating,

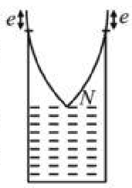
$$\frac{\Delta R}{R} = \frac{\Delta x}{x} - \frac{\Delta(100 - x)}{(100 - x)} \Rightarrow \frac{\Delta R}{R} = \frac{\Delta x}{x} + \frac{\Delta x}{(100 - x)}$$

$$\Rightarrow \Delta R = \left(\frac{0.1}{40} + \frac{0.1}{60}\right) \times 60 = \frac{0.5}{120} \times 60 = 0.25\ \Omega$$

$\therefore$ Required value of $R = (60 \pm 0.25)\ \Omega$

30. (d) : (a) Intensity of the fundamental is more than that of the overtones. Therefore the 1st resonance was having more intensity.

(b) The prongs should not be in the horizontal position but vertical over the resonance tube.



(c) The antinodes are formed always a little above the open end of the tube.

This is called end correction.

This effect will be there for overtones also.

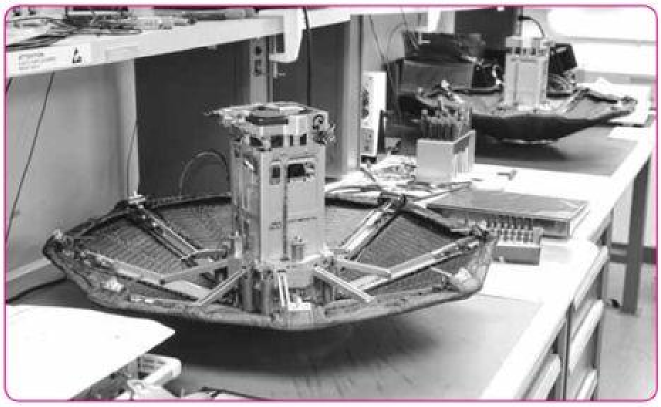
$\therefore$ Length of the air column is less than l/4.

Live Physics

Exploring the Solar System? You May Need to Pack an Umbrella

Gearing up for its first flight test, NASA's Adaptable Deployable Entry Placement Technology, or ADEPT, is no ordinary umbrella. ADEPT is a foldable device that opens to make a round, rigid heat shield, called an aeroshell. This game-changing technology could squeeze a heat shield into a rocket with a diameter larger than the rocket itself. The design may someday deliver much larger payloads to planetary surfaces than is currently possible.

Spacecraft typically approach planets at speeds tens of thousands of miles per hour —screaming fast. Entering a planet's atmosphere at those speeds compresses atmospheric gas, creating pressure shock and generating intense heat right in front of the spacecraft.



Aeroshells slow spacecraft during entry and shield them from heat. ADEPT could be key to future NASA missions that require extra-large aeroshells to protect spacecraft destined to land on the surface of other planets, all without requiring larger rockets.

ADEPT's first flight test is scheduled for Sept. 12 from Spaceport America in New Mexico aboard an UP Aerospace suborbital SpaceLoft rocket. ADEPT will launch in a stowed configuration, resembling a folded umbrella, and then separate from the rocket in space and unfold 60 miles above Earth.

The test will last about 15 minutes from launch to Earth return. The peak speed during the test is expected to be three times the speed of sound, about 2,300 miles per hour. That is not fast enough to generate significant heat during descent, but the purpose of the test is to observe the initial sequence of ADEPT's deployment and assess aerodynamic stability while the heat shield enters Earth's atmosphere and falls to the recovery site.

"For a deployable like ADEPT, you can do ground-based testing, but ultimately, a flight test demonstrates end-to-end functionality – surviving launch environments, deploying in zero gravity and the vacuum of space, holding that rigid shape and then entering, in our case, Earth's atmosphere," said Paul Wercinski, ADEPT project manager at NASA's Ames Research Center in California's Silicon Valley.

This umbrella-like mechanical aeroshell design uses flexible 3D woven carbon fabric skin stretched over deployable ribs and struts, which become rigid when fully flexed. The carbon fabric skin covers its structural surface, and serves as the primary component of the entry, descent and landing thermal protection system.

"Carbon fabric has been the major recent breakthrough enabling this technology, as it utilizes pure carbon yarns that are woven three-dimensionally to give you a very durable surface," said Wercinski. "Carbon is a wonderful material for high temperature applications."

The next steps for ADEPT are to develop and conduct a test for an Earth entry at higher "orbital" speeds, roughly 17,000 miles per hour, to support maturing the technology with an eye towards Venus, Mars or Titan, and also returning lunar samples back to Earth.

The ADEPT aeroshell heat shield technology was developed at Ames. The center leads the agency in the development and innovation of thermal protection system technologies.

Through such programs, NASA supports promising technologies from government, industry and academia for development and/or testing. UP Aerospace, based in Highlands Ranch, Colorado, is the flight provider.

(Source : NASA website)

PHYSICS MUSING

SOLUTION SET-62

1. **(c):** According to question

$Mg\sin\theta - f = ma$ ...(i)

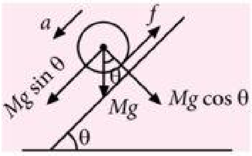
Now, Torque acting about centre of mass is due to friction force

$$\tau = fR = I\frac{\alpha}{R}$$

$$\Rightarrow f = \frac{2}{5}MR^2\frac{a}{R^2} = \frac{2}{5}Ma \quad ...(ii)$$

From eqn (i) and (ii)

$$a = \frac{Mg\sin\theta}{\frac{7}{5}M} = \frac{5g\sin\theta}{7}$$



If slides without friction. Let the acceleration be a'

$$\therefore \; a' = g\sin\theta = \frac{7}{5}a$$

2. **(a) :** The perpendicular distance of the weight acting through P from OR = $\frac{l}{2}\cos\theta$.

Again the perpendicular distance of the weight acting through Q from O is,

$$OQ = BQ - BO = \frac{l}{2} - l\cos\theta$$

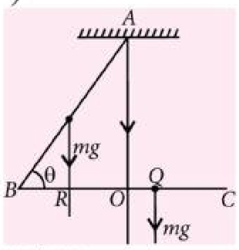
At equilibrium of two weights

$$mg\times\frac{l}{2}\cos\theta = mg\times\left(\frac{l}{2} - l\cos\theta\right)$$

$$\Rightarrow \frac{3}{2}\cos\theta = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \frac{1}{3}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$$



3. **(b) :** Conserving angular momentum

$$m.(v_1\cos 60°).4R = m.v_2R; \; \frac{v_2}{v_1} = 2$$

Conserving energy of the system

$$-\frac{GMm}{4R} + \frac{1}{2}mv_1^2 = -\frac{GMm}{R} + \frac{1}{2}mv_2^2$$

$$\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 = \frac{3}{4}\frac{GM}{R} \text{ or } v_1^2 = \frac{1}{2}\frac{GM}{R}$$

$$v_1 = \frac{1}{\sqrt{2}}\sqrt{64\times10^6} = \frac{8}{\sqrt{2}}\text{km s}^{-1}$$

4. **(b) :** Let mass of the ball is 'm'.

$$\frac{1}{2}mv^2 = m(V_A - V_B)$$

$$\frac{1}{2}mv^2 = m\left[-\frac{GM}{R} - \left(-\frac{3}{2}\frac{GM}{R}\right)\right]$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2R} \Rightarrow v = \sqrt{\frac{GM}{R}}$$

Velocity of ball just after collision,

$$v' = ev = \frac{1}{5}\sqrt{\frac{GM}{R}}$$

Let r be the distance from the centre upto where the ball reaches after collision. Then,

$$\frac{1}{2}mv'^2 = m[V(r) - V_{(centre)}]$$

$$\frac{1}{50}\frac{GMm}{R} = m\left[\frac{3}{2}\frac{GM}{R} - \frac{GM}{R^3}\left(\frac{3R^2}{2} - \frac{r^2}{2}\right)\right]$$

$$\Rightarrow \frac{1}{50} = \frac{3}{2} - \frac{3}{2} + \frac{r^2}{2R^2} \quad \Rightarrow \frac{1}{25} = \frac{r^2}{R^2} \Rightarrow r = \frac{R}{5}$$

The desired distance,

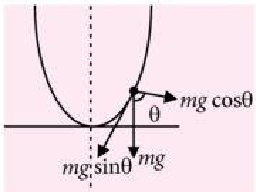
$$S = R + \frac{R}{5} + \frac{R}{5} = \frac{7R}{5}$$

5. **(c):** $F = mg\sin\theta \approx mg\tan\theta$, $(\because \theta \text{ is small})$

$x^2 = 40y$

$$2x = 40\left(\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = \frac{2x}{40}$$



$$F = mg\frac{dy}{dx} = -mg\times\frac{2x}{40} \Rightarrow a = -\frac{x}{2}$$

$$\Rightarrow \omega = \frac{1}{\sqrt{2}} \quad (\because a = -\omega^2 x)$$

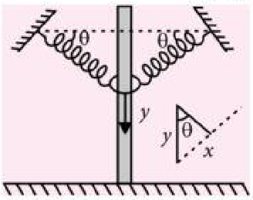
6. (d) : For small displacement, $y = \frac{x}{\sin\theta}$

$2kx_0 \sin\theta = mg$ ...(i)

If ring is displaced y

$mg - 2k(x_0 + y\sin\theta)\sin\theta = ma$

$a = -\frac{2k\sin^2\theta}{m}y$ ...(ii)



On comparing with the standard equation

$a = -\omega^2 y$

$\omega^2 = \frac{2k\sin^2\theta}{m}$

$T = 2\pi\sqrt{\frac{m}{2k\sin^2\theta}}$

7. (d) : Velocity of a particle in S.H.M. is given by

$v = \omega\sqrt{A^2 - x^2}$

Taking square on both the sides

$v^2 = \omega^2(A^2 - x^2) = 3\omega^2 a^2 \Rightarrow \omega = \frac{v}{\sqrt{3}a} = 1 \text{ rad s}^{-1}$

$x = 2a\sin(\omega t + \phi)$ on differentiation with respect to t

$v = \frac{dx}{dt} = 2a\omega\cos(\omega t + \phi)$

$\frac{1}{2} = \sin\phi \Rightarrow \phi = n\pi + (-1)^n\alpha = n\pi + (-1)^n\frac{\pi}{6}$

v at $\phi = \frac{\pi}{6} = +\text{ve}$ and v at $\phi = \frac{5\pi}{6} = -\text{ve}$

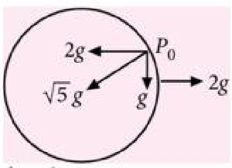
So, $\phi = \frac{\pi}{6}$

$x = 2a\sin(\omega t + \phi) = 2a[\sin\omega t\cos\phi + \cos\omega t\sin\phi]$

$= 2a\left[\sin\omega t\frac{\sqrt{3}}{2} + \frac{1}{2}\cos\omega t\right]$ $(\omega = 1 \text{ rad s}^{-1})$

$x = a(\sqrt{3}\sin t + \cos t)$

8. (c): $g_{eff} = \sqrt{5}g$.



Therefore pressure at centre

$= P_0 + \sqrt{5}\rho Rg$

9. (c): When a spherical object of radius r moving at velocity v through a fluid of viscosity coefficient η will experience a viscous force F given by

$F = 6\pi\eta rv$

$ma = 6\pi\eta rv$

$m\left(-\frac{dv}{dt}\right) = 6\pi\eta rv$

$-m\left(\frac{dv}{v}\right) = 6\pi\eta r.dt$

Taking integration on both the sides

$-\int_2^{0.5}\frac{dv}{v} = 6\pi\eta r\int_0^t dt$

$-\int_2^{0.5}\frac{dv}{v} = 6\pi\left(\frac{1}{18\pi}\right)\int_0^t dt$

$-[\ln v]_2^{0.5} = \frac{t}{3} \Rightarrow -\ln\left(\frac{0.5}{2}\right) = \frac{t}{3}$

$\ln\frac{2}{0.5} = \frac{t}{3} \Rightarrow t = 3\ln 4 \text{ s}$

10. (b) : v = velocity of rod, $e = Blv$

$q = C \times V$

Now, $q = C \times Blv$

(Where q is the amount of charged stored in a capacitor)

$i = \frac{dq}{dt} = \frac{d}{dt}(BlvC)$ $\left(\frac{dv}{dt} = a\right)$

$= CBla$

$\therefore$ $mg - ilB = ma$

$mg - (CBla)lB = ma$; $a = \frac{mg}{m + CB^2l^2}$

CBSE board question paper pattern to change from 2020

According to media reports, CBSE is planning to make some changes in the examination pattern of class 10 and class 12 from 2020 as part of a revamp that would include changes in the examination schedule for vocational subjects as well as for the main subjects - among other changes. Citing MHRD sources, it is reported that the initiative of this pattern change is being taken to discourage students from rote learning. The new pattern would test students on their analytical skills and reasoning abilities instead of blind copy pasting of textbook text. The board has also claimed that this step will produce a better result and the academic quality of institutions will be renewed

Following are some of the major changes expected to be introduced in the new CBSE exam pattern 2020

- Question papers to be designed to check the problem-solving and analytical thinking of students
- Paper pattern to be revamped to include more short answer-type questions like those ranging from 1 to 5 marks
- Vocational course exams to be held in February, and the final board exams to conclude by March in around 15 days.
- Following the early wrapping up of boards, the results are likely to be declared earlier than the schedule followed in the current structure. This will give evaluators more time to check papers. Results, too, will likely be declared earlier.

Under the new plan, there is a vision for putting more emphasis on improved quality of academics in institutions such as teachers, learning outcomes and pedagogy. Respective state governments would be responsible for evaluating their schools' working criteria, infrastructure, and facilities, and their report will be the guide for CBSE to arrive at a decision. CBSE's renewed paper pattern also aims to simplify and shorten the rules of affiliation and renewal for schools.

5 MIND BLOWING FACTS

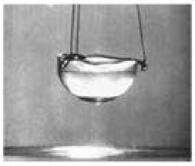
1. It can take a photon 40,000 years to travel from the core of the sun to its surface, but only 8 minutes to travel the rest of the way to earth.

The sunlight goes to the surface of the earth in the speed of light, while photons have to travel through many masses of gases and vacuum spaces to reach the earth. Sunlight takes around 8 minutes to reach the earth because Earth orbit the Sun at a gap of roughly 150 million km. The light travels at a speed of 300,000 km/s. The answer by dividing the digits comes to 500 seconds which is 8 minutes and 20 seconds. The photons are the large particles of electromagnetic emission that define the atom properties of an electromagnetic movement. A photon is formed by synthesis responses inside the Sun's center part. Sun's center is very dense, so the photons have to travel through huge hydrogen masses and other particles. When a photon runs into the fragments like hydrogen it gets enthralled, the electrons in the atom get agitated, and then it gets fired back out. As soon as these photons are away from sun's atmosphere, they travel at a great speed to reach the earth.

2. Polar bears can't be detected by infrared cameras.

Polar bears are heated perfectly due to a thick layer of blubber under their skin and a dense fur coat covering their body. But their outer layer stays the same temperature as the snow around them, that's why they are invisible in infrared cameras that are aimed to detect the heat lost by a living subject.

3. Superfluid Helium Can Climb Walls

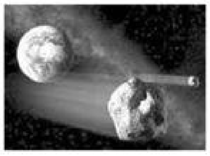


Researchers have known for decades that if you cool liquid helium few degrees below its boiling point of – 452 degrees Fahrenheit (– 269 degrees Celsius) it will suddenly be able to do things that other fluids can't, like dribble through molecule-thin cracks, climb up and over the sides of a dish, and remain motionless when its container is spun.

4. It can rain diamonds on other planets.

Extra-terrestrial diamonds are very common. Microscopic diamonds not much larger than molecules are abundant in meteorites and some of them retain a record of their formation in stars before the Solar System existed. High pressure experiments suggest large quantities of diamonds are formed from methane on the ice giant planets Uranus and Neptune, while some extrasolar planets may be composed almost entirely of diamond. Diamonds are also found in stars and may have been the first mineral ever to have formed. The atmospheres of some planets have such high pressure that they can crystalize carbon atoms and turn them into diamonds.

5. The largest asteroid ever recorded is a mammoth piece of space rock named Ceres.



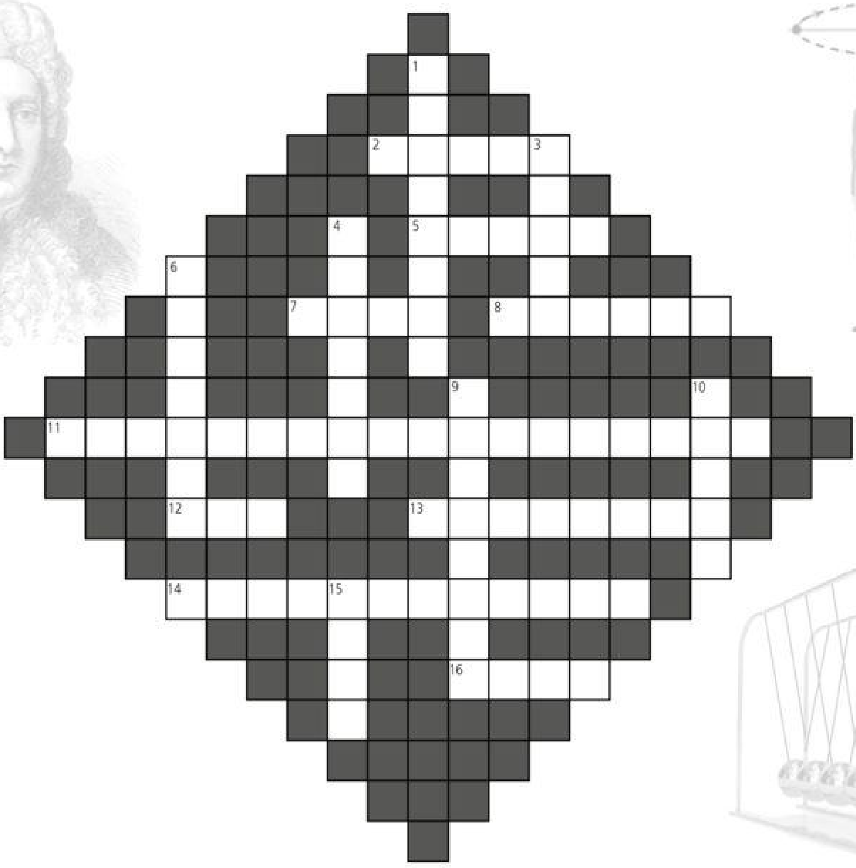
The asteroid is almost 600 miles in diameter. It's by far the largest in the asteroid belt and accounts for a whole third of the belt's mass. The surface area is approximately equal to the land area of India. There is actually some debate over whether to refer to it as a dwarf planet instead of an asteroid, even if it has mostly asteroid-like qualities.

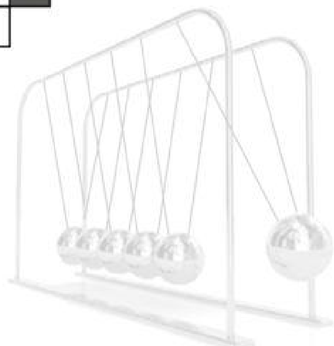
CROSS WORD

CLASS XI

Laws of Motion, Work, Energy and Power

Readers can send solutions at editor@mtg.in or post us with their complete address by 10th of every month to get their names published in next issue.





ACROSS

2. The law of motion that gives the definition of force **(5)**
5. The law of motion that represents a certain symmetry in nature **(5)**
7. The work done by centripetal force **(4)**
8. The law of motion that gives the measure of force **(6)**
11. The type of collision in which kinetic energy is not conserved **(9, 9)**
12. The CGS unit of work **(3)**
13. The sign of work done by friction on a body sliding down an inclined plane **(8)**
14. The commercial unit of energy **(8, 4)**
16. The measure of inertia of an object **(4)**

DOWN

1. An example of a non-conservative force **(8)**
3. The number of different types of inertia **(5)**
4. The natural tendency of an object to resist a change in its state of motion or of rest **(7)**
6. The change in momentum **(7)**
9. The physical quantity which is conserved in all collisions **(8)**
10. The physical quantity which is expressed as force times velocity **(5)**
15. The physical quantity whose dimensional formula in $[ML^2T^{-2}]$ **(4)**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