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PHYSICS for you

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Class XI

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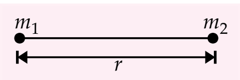
Unit 5 GRAVITATION

NEWTON'S LAW OF GRAVITATION

It states that every body in the universe attracts every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two bodies.

According to Newton's law of Gravitation,

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = \frac{G m_1 m_2}{r^2}$$



where G is a constant of proportionality and is called universal gravitational constant.

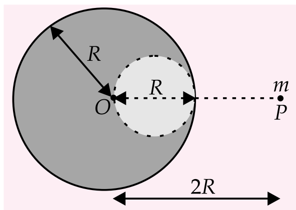
- Universal gravitational constant G is a scalar quantity.
- Its value is same throughout the universe and is independent of the nature and size of the bodies as well as the nature of the medium between the bodies.
  - Value of G in SI system is 6.67×10^{-11} N m^2 kg^{-2} and in CGS system is 6.67×10^{-8} dyne cm^2 g^{-2}.
  - Dimensional formula for G is $[M^{-1}L^3T^{-2}]$.

Gravitational Force

The law of gravitation is universal as it is applicable to all bodies, independent of nature of the intervening medium, temperature and other physical condition of the bodies.

- It forms action-reaction pair, *i.e.*, the gravitational forces between two bodies are equal in magnitude but opposite in direction and hence they obey Newton's third law of motion.
- It is a conservative force, *i.e.* work done by it is path independent.
- It is a central force, *i.e.*, it acts along the line joining the centre of the two interacting bodies.

Illustration 1 : A uniform sphere of mass M and radius R exerts a force F on a small mass m situated at a distance of $2R$ from the centre O of the sphere. A spherical portion of diameter R is cut from the sphere as shown in figure. What will be the force of attraction between the remaining part of the sphere and the mass m?



Soln.: Gravitational force of attraction on mass m at P due to solid sphere is

$$F = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2} \quad \text{or} \quad \frac{GMm}{R^2} = 4F \qquad \text{...(i)}$$

Mass of the spherical portion removed from sphere

$$M' = \frac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

Gravitational force of attraction on mass m at point P if mass of the spherical portion removed is present there, is

$$F' = \frac{G(M/8)m}{\left(\frac{3R}{2}\right)^2} = \frac{GMm}{R^2} \times \frac{1}{8} \times \frac{4}{9}$$

$$= 4F \times \frac{1}{8} \times \frac{4}{9} = \frac{2F}{9} \qquad \text{(Using (i))}$$

$\therefore$ Gravitational force of attraction on mass m at P due to remaining part of the sphere is

$$F'' = F - F' = F - \frac{2F}{9} = \frac{7F}{9}$$

ACCELERATION DUE TO GRAVITY

Acceleration produced in a body due to the force of gravity is called as acceleration due to gravity. It is denoted by symbol 'g' and mathematically it is given by

$$g = \frac{GM_e}{r^2}$$

where M_e is the mass of the earth and r is the distance of the body from the centre of the earth.

- If the body is on the surface of the earth, *i.e.*, $r = R_e$ (radius of the earth), then

  $$g = \frac{GM_e}{R_e^2}$$

- It is also defined as the acceleration produced in the body while falling freely under the effect of gravity alone.
- It is a vector quantity. Its SI unit is m s^{-2} and its dimensional formula is $[M^0LT^{-2}]$.
- The value of g on the surface of earth is taken to be 9.8 m s^{-2}.
- The value of g varies with altitude, depth, shape and the rotation of earth.
- The acceleration due to gravity (g) is related with gravitational constant (G) by the relation

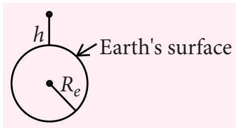
  $$g = \frac{GM_e}{R_e^2} = \frac{G\frac{4}{3}\pi R_e^3 \rho}{R_e^2} = \frac{4}{3}\pi G R_e \rho$$

  where M_e is the mass of the earth, R_e is the radius of the earth and ρ is the uniform density of the material of the earth.
- The value of acceleration due to gravity is independent of the shape, size, mass etc. of the body but depends upon mass and radius of the earth or planet due to which there is a gravity pull.
- If the radius of a planet decreases by n%, keeping its mass unchanged, the acceleration due to gravity on its surface increases by $2n$%.
- If the mass of a planet increases by n%, keeping its radius unchanged, the acceleration due to gravity on its surface increases by n%.
- If the density of a planet decreases by n%, keeping its radius unchanged, the acceleration due to gravity decreases by n%.

Variation of Acceleration due to Gravity

- **Due to altitude (h)**

  The acceleration due to gravity at height h above the earth's surface is given by



  $$g_h = \frac{GM_e}{(R_e + h)^2} = g\left(1 + \frac{h}{R_e}\right)^{-2} \quad \left(\because g = \frac{GM_e}{R_e^2}\right)$$

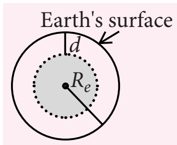
  For $h << R_e$,

  $$\therefore \quad g_h = g\left(1 - \frac{2h}{R_e}\right)$$

  When we move above the earth's surface, the value of acceleration due to gravity goes on decreasing.

- **Due to depth (d)**

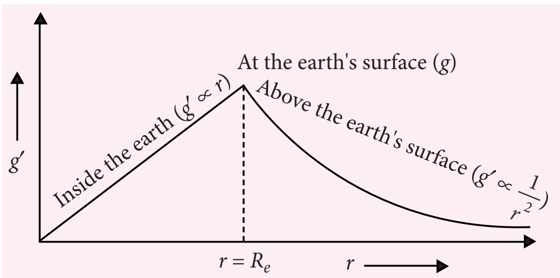
  The acceleration due to gravity at a depth d below the earth's surface is given by



  $$g_d = \frac{GM_e}{R_e^3}(R_e - d) = g\left(\frac{R_e - d}{R_e}\right) = g\left(1 - \frac{d}{R_e}\right)$$

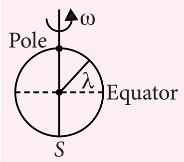
  At the centre of the earth, $d = R_e$. $\therefore$ $g_d = 0$.

  - When we move below the earth's surface, the value of acceleration due to gravity also decreases. The value of acceleration due to gravity is maximum at the earth's surface and becomes zero at the centre of the earth.
  - The variation of acceleration due to gravity (g') with distance from the centre of the earth (r) is as shown in the figure.



- **Due to rotation of the earth about its axis**

  The acceleration due to gravity at latitude λ is given by $g_\lambda = g - R_e\,\omega^2\cos^2\lambda$



  where ω is the angular speed of rotation of the earth about its axis and its value is 7.3×10^{-5} rad s^{-1}.

  - At the equator, $\lambda = 0°$

    $g_\lambda = g_e = g - R_e\omega^2\cos^2 0° = g - R_e\omega^2$.
  - At the poles, $\lambda = 90°$

    $g_\lambda = g_p = g - R_e\omega^2\cos^2 90° = g$
  - The value of acceleration due to gravity increases from equator to the pole due to rotation of the earth.

    $g_p - g_e = g - (g - R_e\omega^2) = R_e\omega^2$
  - If the earth stops rotating about its axis ($\omega = 0$), the value of g will increase everywhere, except at the poles. On the contrary, if there is increase in the angular speed of the earth, then except at the poles the value of g will decrease at all places.

- **Due to shape of the earth**

  Earth is not a perfect sphere but it is an ellipsoid. The earth's radius is 21 km larger at the equator than at the poles. Thus, the earth has an equatorial bulge and is flattened at the poles. Both, rotation and equatorial bulge contribute additively to keep the g smaller at the equator than at the poles.

Illustration 2 : The weight of an object at the equator is the same as its weight at height h above the pole. If ω is the rotational speed of Earth, then what is the value of h?

Soln.: $g' = g_p - \omega^2 R\cos^2\lambda$

At equator, $\lambda = 0$

$\Rightarrow\ g_{eq} = g_p - \omega^2 R\cos^2 0 = g_p - \omega^2 R$

Now, g' above the pole is

$$g' = g_p\left(1 - \frac{2h}{R}\right)$$

As $g_{eq} = g'$

$$g_p - \omega^2 R = g_p\left(1 - \frac{2h}{R}\right)$$

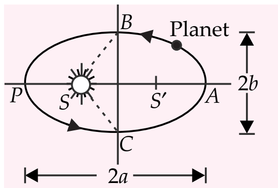
$$\text{So, } \omega^2 R = \frac{2h\,g_p}{R} \quad \Rightarrow \quad h = \frac{\omega^2 R^2}{2g_p} = \frac{\omega^2 R^2}{2g}.$$

KEPLER'S LAWS OF PLANETARY MOTION

To explain the motion of the planets, Kepler formulated the following three laws:

Law of Orbits (First Law)

Each planet revolves around the sun in an elliptical orbit with the sun situated at one of the two foci.



As shown in figure the planets move around the sun in an elliptical orbit. An ellipse has two foci S and S' the sun remains at one of the line foci.

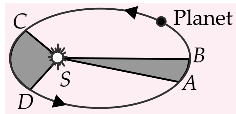
The points P and A on the orbit are called the perihelion and the aphelion and represent the closest and farthest distances from the sun respectively.

Law of areas (Second Law)

The radius vector drawn from the sun to the planet sweeps out equal areas in equal intervals of time *i.e.*, the areal velocity of the planet (or the area swept out by the planet per unit time) around the sun is constant *i.e.*,

areal velocity $= \dfrac{d\vec{A}}{dt} =$ a constant, for a planet.

Kepler's second law follows from the law of conservation of angular momentum.



Suppose a planet takes same time to go from position A to B as in going from C to D. From Kepler's second law, the areas ASB and CSD (covered in equal time) must be equal.

Law of periods (Third law)

The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semi major axis of the elliptical orbit.

i.e., $T^2 \propto a^3$ [where a is the semi major axis of the elliptical orbit of the planet around the sun.]

This law is also known as harmonic law.

Illustration : 3 A planet moving along an elliptical orbit is closest to the sun at a distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are the linear velocities at these points respectively, then the ratio $\dfrac{v_1}{v_2}$ is

Soln.: (b) : According to the law of conservation of angular momentum

$L_1 = L_2$

$mv_1r_1 = mv_2r_2$ or $v_1r_1 = v_2r_2$ or $\dfrac{v_1}{v_2} = \dfrac{r_2}{r_1}$

GRAVITATIONAL FIELD

- The space around a material body in which its gravitational pull can be experienced is called its gravitational field.
- The intensity of gravitational field (E) at a point due to a body of mass M at a distance r from the centre of body is
  $E = \frac{GM}{r^2}$ = acceleration due to gravity.
- The intensity of gravitational field is a vector quantity. It is always directed towards the centre of the mass producing the gravitational field.
- The value of intensity of gravitational field is zero at infinity.
- The total intensity of gravitational field at a point is the vector sum of the intensities of the gravitational fields due to individual mass bodies present in that field, *i.e.*,
  $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

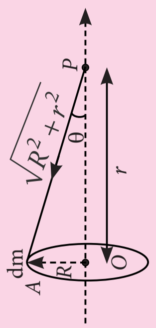
GRAVITATIONAL POTENTIAL

The amount of work done in bringing a body having unit mass from infinity to a point without acceleration in the gravitational field.

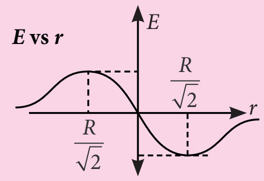
- Gravitational potential at a point in the gravitational field due to a body of mass M at a distance r is given by $V = -\frac{GM}{r}$
- Gravitational potential V is a scalar quantity but potential gradient (dV/dr) is a vector quantity.
- Gravitational potential V is related with gravitational intensity I by a relation.
  $E = -dV/dr$
- In a gravitational field due to number of mass bodies, the total gravitational potential at a point is the sum of the gravitational potential due to individual mass bodies at that point, *i.e.*,
  $V = V_1 + V_2 + V_3 + ...$

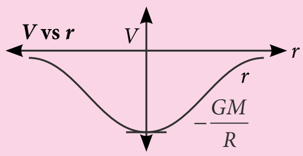
Gravitational field and Potential of some continuous mass Distribution

- Uniform circular ring of mass M and radius R.

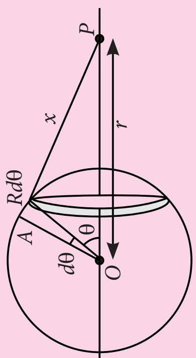


Intensity of Gravitational Field	Gravitational Potential
(i) $E = -\frac{GMr}{(r^2+R^2)^{3/2}}$ on the axis passing through centre of the ring	(i) $V = -\frac{GM}{\sqrt{R^2+r^2}}$ on the axis passing through centre of the ring
(ii) $E = -\frac{GM}{r^2}$ for $r >> R$	(ii) $V = -\frac{GM}{R}$ for $r >> R$
(iii) $E = -\frac{GMr}{R^3}$ for $r << R$	(iii) $V = -\frac{GM}{R}$ for $r << R$

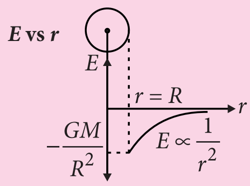


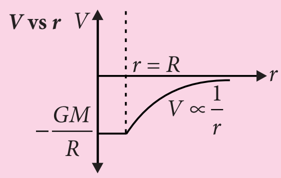


- Thin spherical shell of mass M and radius R

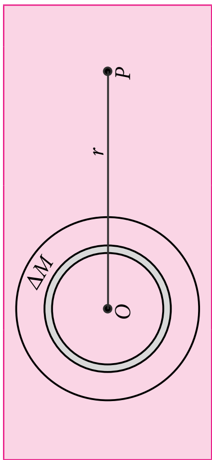


Intensity of Gravitational Field	Gravitational Potential
(i) $E = 0$ for $r < R$	(i) $V = -\frac{GM}{R}$ for $r < R$
(ii) $E = -\frac{GM}{R^2}$ for $r = R$	(ii) $V = -\frac{GM}{R}$ for $r = R$
(iii) $E = -\frac{GM}{r^2}$ $r > R$	(iii) $V = -\frac{GM}{r}$ for $r > R$

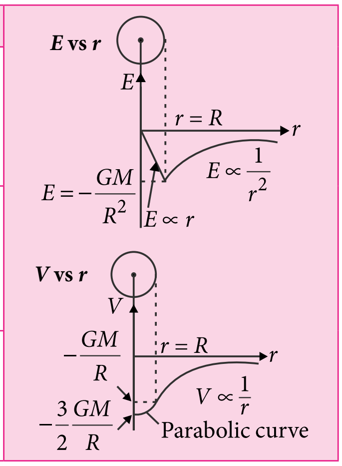




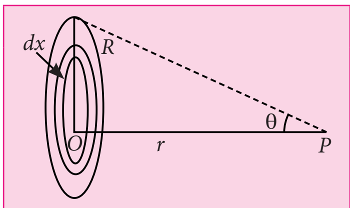
- A solid sphere of mass M and radius R, with uniform mass density.



Intensity of Gravitational Field	Gravitational Potential
(i) $E = -\frac{GMr}{R^2}$ for $r < R$	(i) $V = -\frac{GM}{2R^3}(3R^2 - r^2)$ for $(r < R)$
(ii) $E = -\frac{GM}{R^2}$ for $r = R$	(ii) $V = -\frac{GM}{R}$ for $r = R$
(iii) $E = -\frac{GM}{r^2}$ for $r > R$	(iii) $V = -\frac{GM}{r}$ for $r > R$



- Uniform Disc of mass m and radius R

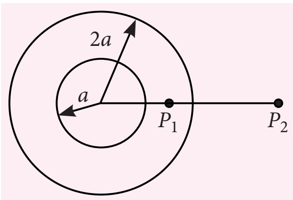


Identity of Gravitational Field	Gravitational Potential
(i) $E = \frac{2GMr}{R^2}\left[\frac{1}{r} - \frac{1}{\sqrt{r^2 + R^2}}\right]$	(i) $V = -\frac{2GM}{R^2}\left[r - \sqrt{R^2 + r^2}\right]$

Illustration 4 : A uniform solid sphere of mass M and radius a is surrounded symmetrically by a thin spherical shell of equal mass and radius $2a$. Find the gravitational field at a distance.

(a) $\frac{3}{2}a$ from centre (b) $\frac{5}{2}a$ from centre.

Soln.: (a) : The situation is indicated in figure in the two cases. The point P_1 is a distance $\frac{3}{2}$ a from centre and P_2 is at a distance $\frac{5}{2}$ a from centre. As P_1 is inside the cavity of the thin spherical shell the field here due to the shell is zero. The field due to the solid sphere is



$$E = \frac{GM}{\left(\frac{3}{2}a\right)^2} = \frac{4\,GM}{9a^2}$$

This also represents the resultant field at P_1.

Resultant field = $\frac{4GM}{9a^2}$. The direction is towards centre.

(b) In this case P_2 is outside the sphere as well as the shell. Both may be replaced by single particles of same mass at the centre.

The field due to each of them at

$$P_2 = \frac{GM}{\left(\frac{5}{2}a\right)^2} = \frac{4\,GM}{25a^2}$$

Resultant field = $\frac{4\,GM}{25a^2} + \frac{4\,GM}{25a^2} = \frac{8\,GM}{25a^2}$

This is also acting towards the common centre.

GRAVITATIONAL POTENTIAL ENERGY

Gravitational potential energy of a body at a point in gravitational field of another body is equal to the amount of work done in bringing the given body from infinity to that point without acceleration.

- The gravitational potential energy of a body of mass m placed at a distance r from the centre of mass M is

  U = gravitational potential × mass of body

  $= -\frac{GM}{r} \times m$

- Work done against the gravitational force in taking a body of mass m from surface of earth to a height h is the change in potential energy of the body and is given by

  $$W = \Delta U = \frac{GM\,mh}{R(R+h)}$$

(a) If $h << R$, then $dU = \frac{GM\,mh}{R^2} = gmh$

(b) If $h = R$, then $dU = \frac{GM\,mh}{2R^2} = \frac{1}{2}gmh$

- Gravitational potential energy is a scalar quantity.

ESCAPE VELOCITY

The escape speed on the earth (or any planet) is defined as the minimum speed with which a body has to be projected vertically upwards from the surface of earth (or any other planet) so that it just crosses the gravitational field of earth (or of that planet) and never returns on its own.

Escape speed v_e is given by

$$v_e = \sqrt{\frac{2GM}{R}} \quad \left(\begin{array}{l}\text{where } M = \text{Mass of the earth or planet}\\ \qquad R = \text{Radius of the earth or planet}\end{array}\right)$$

$$v_e = \sqrt{\frac{2G \times \text{volume} \times \text{density}}{R}}$$

or $$v_e = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \rho} = \sqrt{\frac{8\pi\rho G R^2}{3}}$$

- For earth, $v_e = 11.2$ km s^{-1}
- To throw an ant or an elephant out of the gravitational field, the required velocity of projection is same.
- If ratio of the radii of two planets is r and the ratio of the acceleration due to gravity on their surface is a, then ratio of escape velocities is $\sqrt{ar}$.
- If the radius of the earth is doubled keeping the mass unchanged, the escape velocity will becomes $(1/\sqrt{2})$ times the present value.

Illustration 6 : A rocket starts vertically upwards with speed v_0. Show that its speed v at a height is given by $v_0^2 - v^2 = \frac{2gh}{1+\frac{h}{R}}$, where R is the radius of earth. Hence deduce the maximum height reached by the rocket fired with a speed of 90% to escape velocity.

Soln.: Kinetic energy on the surface of earth $= \frac{1}{2}mv_0^2$

Potential energy on the surface of earth $= \frac{-GMm}{R}$

Total energy $= \frac{1}{2}mv_0^2 - \frac{GMm}{R}$

Kinetic energy at a height $h = \frac{1}{2}mv^2$

Potential energy at such height $= \frac{-GMm}{(R+h)}$

Total energy $= \frac{1}{2}mv^2 - \frac{GMm}{R+h}$

By the principle of conservation of energy,

$$\frac{1}{2}mv^2 - \frac{GMm}{R+h} = \frac{1}{2}mv_0^2 - \frac{GMm}{R}$$

$$\frac{1}{2}(v_0^2 - v^2) = \frac{GM}{R} - \frac{GM}{(R+h)}$$

But $GM = gR^2$

$$\frac{1}{2}(v_0^2 - v^2) = \frac{gR^2h}{R(R+h)}, \quad v_0^2 - v^2 = \frac{2gRh}{R+h} = \frac{2gh}{1+\frac{h}{R}}$$

For maximum height $v = 0$,

$v_0 = 90\%$ of escape velocity $= 0.9\sqrt{2gR}$

$$\therefore \quad \left(0.9\sqrt{2gR}\right)^2 - 0 = \frac{2gh_{max}}{1+\frac{h_{max}}{R}}$$

$$0.81R = \frac{h_{max}}{1+\frac{h_{max}}{R}}$$

$0.81R = 0.81\,h_{max} - h_{max}$, or $0.19\,h_{max} = 0.81R$

$$h_{max} = \frac{0.81R}{0.19} = 4.26\,\text{R}$$

SATELLITE

Consider a satellite of mass m revolving in a circle around the earth. If the satellite is at a height h above the earth's surface, the radius of its orbit is $r = R + h$, where R is the radius of the earth. The gravitational force between m and M provides the necessary centripetal force for circular motion.

Orbital Velocity (v_0)

The minimum speed required to put the satellite into a given orbit around the earth is called the orbital speed of a satellite.

- Orbital velocity is independent of the mass of the orbiting body and is always along the tangent to the orbital.

  Close to the surface of the earth, $r = R$ as $h = 0$

  $$\therefore \quad v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR} = \sqrt{10 \times 6.4 \times 10^6} \approx 8 \text{ km/s}$$

  Close to the surface of the planet,

  $$v_0 = \sqrt{\frac{GM}{R}} = \frac{v_e}{\sqrt{2}}, \; i.e., \; v_e = \sqrt{2}v_0$$

- Orbital velocity vary near the surface of the earth and is about 7.92 km s^{-1}.

- If the altitude of the satellite is n times the radius of the earth, then the orbital velocity will be $(1/\sqrt{1+n})$ times the orbital velocity near the surface of the earth.
- If the radius of the orbit of a satellite is n times the radius of the earth, then its orbital velocity will be $(1/\sqrt{n})$ the orbital velocity near the surface of the earth.

Time Period of a Satellite

The time taken by a satellite to complete one revolution is called the time period (T) of the satellite.

It is given by, $T = \dfrac{2\pi r}{v} = 2\pi r\sqrt{\dfrac{r}{GM}}$

or, $T = \dfrac{2\pi r\sqrt{r}}{\sqrt{GM}}$ or, $T^2 = \left(\dfrac{4\pi^2}{GM}\right)r^3 \Rightarrow T^2 \propto r^3$

Angular Momentum of a Satellite (*L*)

In case of satellite motion, angular momentum will be given by

$$L = mvr = mr\sqrt{\frac{GM}{r}}, \ L = (m^2GMr)^{1/2}$$

In case of satellite motion, the net force on the satellite is centripetal force. The torque of this force about the centre of the orbit is zero. Hence, angular momentum of the satellite is conserved, *i.e.*, L = constant.

Energy of a Satellite

The PE of a satellite is

$$U = mV = -\frac{GMm}{r} \qquad \left[\because V = -\frac{Gm}{r}\right]$$

The kinetic energy of the satellite is

$$K = \frac{1}{2}mv_0^2 = \frac{GMm}{2r} \qquad \left[\because v_0 = \sqrt{\frac{GM}{r}}\right]$$

Total mechanical energy of the satellite

$$= -\frac{GMm}{r} + \frac{GMm}{2r} = -\frac{GMm}{2r}$$

Total energy of a satellite in its orbit is negative. Negative energy means that the satellite is bound to the central body by an attractive force and energy must be supplied to remove it from the orbit to infinity.

Binding Energy of the Satellite

The energy required to remove the satellite from its orbit to infinity is called binding energy of the satellite.

Binding energy = $-E = \dfrac{GMm}{2r}$

Types of Satellites

Geostationary satellite

If there is a satellite rotating in the direction of earth's rotation, *i.e.*, from west to east, then for an observer on the earth, the relative angular velocity of the satellite will be $(\omega_S - \omega_E)$.

However, if $\omega_S - \omega_E = 0$, satellite will appear stationary relative to the earth. Such a satellite is called 'geo-stationary satellite' The orbit of a geostationary satellite is called 'Parking orbit'.

- Time Period : T = 24 hours.
- Height of the geostationary satellite :

$$h = \left[\frac{T^2GM_E}{4\pi^2}\right]^{\frac{1}{3}} - R_E \cong 36,000 \text{ km approx}$$

- **Uses :**
  - Radio communication
  - Transmission of TV and telephone signals.

Polar satellite

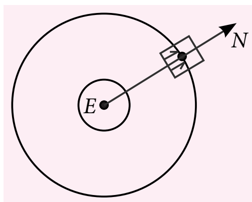
The satellites revolve around the earth in a circles north to south orbit which passes over the north and south poles.

- Low altitude satellite ≈ 500 – 800 km.
- Angle of inclinations of orbit with equatorial plane is 90°.
- They crosses equator of the earth daily at same time hence they are also called sun-synchronous satellites.
- **Uses :**
  - Remote sensing
  - Meteorology and environmental studies

WEIGHTLESSNESS IN A SATELLITE

The radial acceleration of the satellite is given by

$$a_r = \frac{F_r}{m} = \frac{GMm}{r^2 \times m} = \frac{GM}{r^2}$$



For a astronaut inside the satellite, we have

$$\frac{GMm_a}{r^2} - N - m_a a_r = 0$$

Where m_a is mass of astronaut, a_r is radial acceleration of satellite and N is normal reaction on the astronaut

or, $\dfrac{GMm_a}{r^2} - N - \dfrac{GMm_a}{r^2} = 0 \Rightarrow N = 0$

Hence, the astronaut feels weightlessness.

SPEED PRACTICE

1. What is the height at which the weight of body will be the same as at the same depth from the surface of the earth? (Radius of earth is R.)
(a) $\frac{R}{2}$ (b) $\sqrt{5}R - R$
(c) $\frac{\sqrt{5}R - R}{2}$ (d) $\frac{\sqrt{3}R - R}{2}$

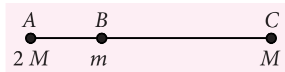
2. Two astronauts have deserted their spaceship in a region of space far from the gravitational attraction of any other body. Each has a mass of 60 kg and they are 60 m apart. They are initially at rest relative to one another. How long will it be before the gravitational attraction brings them 2.2 cm closer together?
(Given $G = 6.66 \times 10^{-11}$ N m^2 kg^{-2})
(a) 10^5 s (b) $10^5 \times \sqrt{2}$ s
(c) 10^6 s (d) $10^6 \times \sqrt{2}$ s

3. The minimum and maximum distances of a satellite from the centre of the earth are 2 R and 4 R respectively, where R is the radius of the earth. The radius of curvature of the satellite orbit of the point of maximum distance is
(a) $\frac{2R}{3}$ (b) $\frac{4R}{3}$ (c) $\frac{8R}{3}$ (d) $\frac{16R}{3}$

4. A uniform solid sphere of radius R produces a gravitational acceleration a_g on its surface. At what two distances from the centre of the sphere the acceleration due to gravity is $a_g/4$?
(a) $4R$, $0.50R$ (b) $2R$, $0.25R$
(c) $3R$, $0.33R$ (d) $2R$, $0.50R$

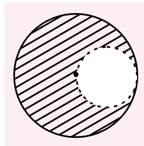
5. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape veloctiy from earth. The height of satellite above earth's surface is nR. Then the value of n is
(a) 2 (b) 1 (c) 3 (d) 5

6. Particles of masses $2M$, m and M are, respectively, at points A, B and C with $AB = \frac{1}{2}(BC)$ as shown in figure. m is very much smaller than M and at time $t = 0$, they are all at rest as given in figure. At subsequent times before any collision takes place,



(a) m will remain at rest.
(b) m will move towards M.
(c) m will move towards 2 M.
(d) m will have oscillatory motion.

7. From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is (G = gravitational constant)



(a) $\frac{-2GM}{3R}$ (b) $\frac{-2GM}{R}$
(c) $\frac{-GM}{2R}$ (d) $\frac{-GM}{R}$

8. If the earth's satellite of mass m orbiting at a distance $2R$ from the centre of earth has to be transferred into the orbit of radius $3R$, the amount of energy required is (R is the radius of earth)
(a) mgR (b) $\frac{mgR}{3}$ (c) $\frac{mgR}{2}$ (d) $\frac{mgR}{12}$

9. What is a period of revolution of earth satellite? (Ignore the height of satellite above the surface of earth. Given : $g = 10$ m s^{-1}, $R_E = 6400$ km, $\pi = 3.14$)
(a) 83.73 minutes (b) 85 minutes
(c) 90 minutes (d) 156 minutes

10. A projectile is fired vertically upward from the surface of earth with a velocity of kv_e, where v_e is the escape velocity and $k < 1$. Neglecting air resistance, the maximum height to which it will rise, measured from the centre of the earth, is (R is the radius of the earth)
(a) $\frac{R}{1-k^2}$ (b) $\frac{R}{k^2}$ (c) $\frac{1-k^2}{R}$ (d) $\frac{k^2}{R}$

11. A satellite revolves in elliptical orbit around a planet of mass M situated at the centre of the path. Its time period is T. The length of the major axis of the path is (neglect the gravitational effect of other objects in space)

(a) $2\left[\frac{GMT^2}{4\pi^2}\right]^{1/3}$ (b) $\left[\frac{GMT^2}{4\pi^2}\right]^{1/3}$

(c) $\frac{1}{2}\left[\frac{GMT^2}{4\pi^2}\right]^{1/3}$ (d) none of these

12. Two satellites S_1 and S_2 are revolving round a planet in coplanar and concentric circular orbit of radii R_1 and R_2 in the same direction respectively. Their respective periods of revolution are 1 hr and 8 hr. The radius of the orbit of satellite S_1 is equal to 10^4 km. Find the relative speed in km h^{-1} when they are closest.

(a) $2\pi \times 10^4$ km h^{-1} (b) $\pi \times 10^4$ km h^{-1}

(c) $\frac{\pi}{2} \times 10^4$ km h^{-1} (d) $\frac{\pi}{3} \times 10^4$ km h^{-1}

13. The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then

(a) $d = 1$ km (b) $d = \frac{3}{2}$ km

(c) $d = 2$ km (d) $d = \frac{1}{2}$ km

14. Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will

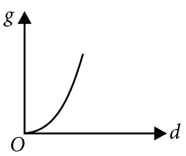
(a) move towards each other.
(b) move away from each other.
(c) will become stationary.
(d) keep floating at the same distance between them.

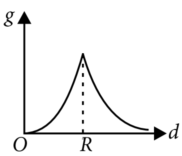
15. A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface, is

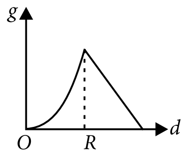
(a) $\frac{mg_0R^2}{2(R+h)}$ (b) $-\frac{mg_0R^2}{2(R+h)}$

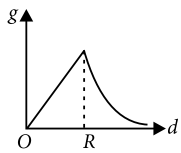
(c) $\frac{2mg_0R^2}{R+h}$ (d) $-\frac{2mg_0R^2}{R+h}$

16. The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius)

(a) 

(b) 

(c) 

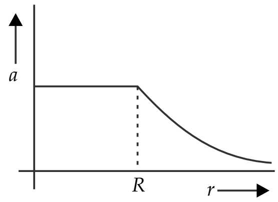
(d) 

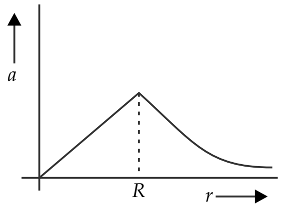
17. If the Earth has no rotational motion, the weight of a person on the equator is W. Determine the speed with which the earth would have to rotate about its axis so that the person at the equator will weigh $\frac{3}{4}W$. Radius of the Earth is 6400 km and $g = 10$ m s^{-2}.

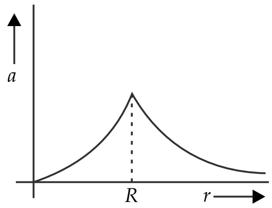
(a) 0.63×10^{-3} rad s^{-1} (b) 0.83×10^{-3} rad s^{-1}

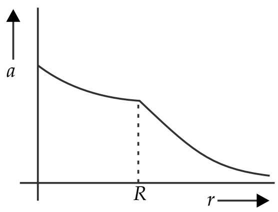
(c) 0.28×10^{-3} rad s^{-1} (d) 1.1×10^{-3} rad s^{-1}

18. The mass density of a spherical body is given by $\rho(r) = \frac{k}{r}$ for $r \le R$ and $\rho(r) = 0$ for $r > R$, where r is the distance from the centre. The correct graph that describes qualitatively the acceleration, a of a test particle as a function of r is

(a) 

(b) 

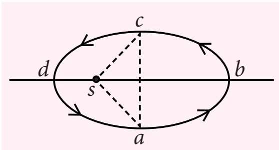
(c) 

(d) 

19. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth.

The escape velocity from Earth's gravitational field is v_e = 11.2 km s^{-1}. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet)
(a) $v_s = 62$ km s^{-1} (b) $v_s = 22$ km s^{-1}
(c) $v_s = 72$ km s^{-1} (d) $v_s = 42$ km s^{-1}

20. Figure shows elliptical path *abcd* of a planet around the sun *S* such that the area of triangle *csa* is $\frac{1}{4}$ the area of the ellipse. In figure, *db* is the semi-major axis, and *ca* as the semi-minor axis. If t_1 is the time taken for planet to go over path *abc* and t_2 for path taken over *cda* then



(a) $t_1 = 4t_2$ (b) $t_1 = 2t_2$
(c) $t_1 = 3t_2$ (d) $t_1 = t_2$

SOLUTIONS

1. (c) : $\dfrac{gR^2}{(R+h)^2} = g\left(1-\dfrac{h}{R}\right)$

or $\left(1-\dfrac{h}{R}\right)\left(1+\dfrac{h^2}{R^2}+\dfrac{2h}{R}\right)=1$

or $\dfrac{h^3}{R^3}+\dfrac{h^2}{R^2}-\dfrac{h}{R}=0$ or $\dfrac{h}{R}\left(\dfrac{h^2}{R^2}+\dfrac{h}{R}-1\right)=0$

or $\dfrac{h}{R}=\dfrac{-1\pm\sqrt{1+4}}{2}=\dfrac{\sqrt{5}-1}{2}$ or $h=\dfrac{\sqrt{5}R-R}{2}$

2. (b): Acceleration of first astronaut,

$a_1 = \dfrac{Gm_1m_2}{r^2}\times\dfrac{1}{m_1}=\dfrac{Gm_2}{r^2}$

Acceleration of second astronaut,

$a_2 = \dfrac{Gm_1m_2}{r^2}\times\dfrac{1}{m_2}=\dfrac{Gm_1}{r^2}$

Net acceleration of approach is

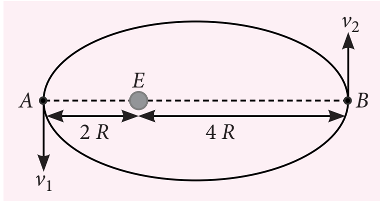
$a = a_1 + a_2 = \dfrac{G}{r^2}(m_1+m_2)$

$= \dfrac{6.66\times10^{-11}}{(60)^2}\times(60+60) = 2.22\times10^{-12}$ m s^{-2}

Now, $s=\dfrac{1}{2}at^2$

or $t=\sqrt{\dfrac{2s}{a}}=\sqrt{\dfrac{2\times2.2\times10^{-2}}{2.22\times10^{-12}}}=10^5\sqrt{2}\ s$

3. (c) : Refer to figure and using law of conservation of angular momentum, we have



$mv_1(2R) = mv_2(4R)$ or $v_1 = 2v_2$ ...(i)

According to law of conservation of mechanical energy,

$\dfrac{1}{2}mv_1^2-\dfrac{GMm}{2R}=\dfrac{1}{2}mv_2^2-\dfrac{GMm}{4R}$ ...(ii)

Solving (i) and (ii), we get

$v_1=\sqrt{\dfrac{2GM}{3R}}$ and $v_2=\sqrt{\dfrac{GM}{6R}}$

If r is the radius of curvature of the orbital path of satellite at B, then

$\dfrac{mv_2^2}{r}=\dfrac{GMm}{(4R)^2}$

or $r=\dfrac{16R^2v_2^2}{GM}=\dfrac{16R^2}{GM}\times\dfrac{GM}{6R}=\dfrac{8R}{3}$

4. (b): At a height h above the surface,

$g_h=g\left(\dfrac{R}{R+h}\right)^2$; Here, $g_h=\dfrac{a_g}{4}$ and $g=a_g$

$\therefore\ \dfrac{a_g}{4}=a_g\left[\dfrac{R}{R+h}\right]^2$ or $\dfrac{R}{R+h}=\dfrac{1}{2}$

or $R+h=2R$ or $h=R$

$\therefore$ Distance from the centre = $R+h=R+R=2R$

At a depth d below the surface, $g_d = g\left(1-\dfrac{d}{R}\right)$

Here, $g_d=\dfrac{a_g}{4}$ and $g=a_g$

$\therefore\ \dfrac{a_g}{4}=a_g\left(1-\dfrac{d}{R}\right)$ or $d=\dfrac{3}{4}R$

$\therefore$ Distance from the centre = $R-d$

$=R-\dfrac{3}{4}R=\dfrac{R}{4}=0.25R$

5. (b): Escape velocity, $v_e=\sqrt{2gR}$, where g is acceleration due to gravity on surface of earth and R the radius of earth.

Orbital velocity, $v_0=\dfrac{1}{2}v_e=\dfrac{1}{2}\sqrt{2gR}=\sqrt{\dfrac{gR}{2}}$

If h is the height of satellite above earth then

$$\frac{mv_0^2}{R+h}=\frac{GMm}{(R+h)^2} \text{ or } v_0^2=\frac{GM}{R+h}=\frac{gR^2}{(R+h)}$$

$$\therefore \quad \left(\frac{1}{2}v_e\right)^2=\frac{gR^2}{R+h}$$

$$\frac{gR}{2}=\frac{gR^2}{R+h}$$

$R + h = 2R \Rightarrow h = R$. Hence $n = 1$

6. **(c) :** Force on B due to A = $F_{BA} = \frac{G(2Mm)}{(AB)^2}$ towards BA

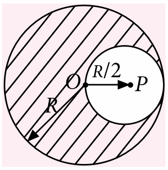
Force on B due to $C = F_{BC} = \frac{GMm}{(BC)^2}$ towards BC

$$\Rightarrow F_{BC}=\frac{GMm}{(2AB)^2}=\frac{GMm}{4(AB)^2}<F_{BA}\ (\because BC=2AB)$$

Hence, m will move towards the mass $2M$.

7. **(d):** Potential at point P (centre of cavity) before removing the spherical portion,

$$V_1=\frac{-GM}{2R^3}\left(3R^2-\left(\frac{R}{2}\right)^2\right)=\frac{-11GM}{8R}$$



Mass of spherical portion to be removed,

$$M'=\frac{MV'}{V}=\frac{M\frac{4\pi}{3}\left(\frac{R}{2}\right)^3}{\frac{4\pi}{3}R^3}=\frac{M}{8}$$

Potential at point P due to spherical portion to be removed,

$$V_2=\frac{-3GM'}{2R'}=\frac{-3G(M/8)}{2(R/2)}=\frac{-3GM}{8R}$$

$\therefore$ Potential at the centre of cavity formed,

$$V_P=V_1-V_2=\frac{-11GM}{8R}-\left(\frac{-3GM}{8R}\right)=\frac{-GM}{R}$$

8. **(d):** Energy of the satellite in an orbit of radius r is

$E=-\frac{GMm}{2r}$ where M and m are the masses of the earth and satellite respectively.

Initial energy of the satellite is $E_i=-\frac{GMm}{2(2R)}$

Final energy of the satellite is $E_f=-\frac{GMm}{2(3R)}$

Amount of energy required is

$\Delta E = E_f - E_i$

$$=-\frac{GMm}{2(3R)}-\left(-\frac{GMm}{2(2R)}\right)$$

$$=\frac{GMm}{2R}\left[\frac{1}{2}-\frac{1}{3}\right]=\frac{GMm}{12R}=\frac{mgR}{12}\quad\left(\because g=\frac{GM}{R^2}\right)$$

9. **(a) :** The period of revolution of earth satellite is

$$T=2\pi\sqrt{\frac{(R_E+h)^3}{gR_E^2}}\text{ ; Neglecting } h\text{, then } T=2\pi\sqrt{\frac{R_E}{g}}$$

Substituting the given values, we get

$$T=2\times3.14\sqrt{\frac{6400\times10^3}{10}}\text{ s}=5024\text{ s}=83.73\text{ minutes}$$

10. **(a) :** Let the projectile be fired vertically upward from the earth's surface with velocity v and it reaches a maximum height h. By the law of conservation of mechanical energy,

$$\frac{1}{2}mv^2-\frac{GMm}{R}=-\frac{GMm}{R+h}$$

$$\text{or}\quad \frac{1}{2}mv^2=GMm\left[\frac{1}{R}-\frac{1}{R+h}\right]=\frac{GMmh}{(R)(R+h)}$$

$$\text{or}\quad \frac{1}{2}mv^2=\frac{mghR}{(R+h)}\qquad\left(\because g=\frac{GM}{R^2}\right)$$

As per question,

$$v=kv_e=k\sqrt{2gR}\text{ and }h=r-R\qquad(\because v_e=\sqrt{2gR})$$

where r is the distance from the centre of the earth.

$$\therefore\quad \frac{1}{2}mk^2 2gR=\frac{mg(r-R)R}{r}$$

$$\text{or}\quad k^2=\frac{r-R}{r}\quad\text{or}\quad(1-k^2)r=R\quad\text{or}\quad r=\frac{R}{1-k^2}$$

11. **(a) :** Since the planet is at the center, the focus and centre of the elliptical path coincide, and the elliptical path becomes circular and the major axis is nothing but the diameter. For a circular path,

$$\frac{mv^2}{r}=\frac{GM}{r^2}m\quad\text{or}\quad v=\sqrt{\frac{GM}{r}}$$

$$\text{Also } T=\frac{2\pi r}{v}=\frac{2\pi r^{3/2}}{\sqrt{GM}}\quad\text{or}\quad T^2=\frac{4\pi^2r^3}{GM}$$

$$\Rightarrow\quad \text{Radius, } r=\left(\frac{GMT^2}{4\pi}\right)^{1/3}$$

$$\Rightarrow\quad \text{Diameter (major axis)} = 2\left(\frac{GMT^2}{4\pi}\right)^{1/3}$$

12. (b) : By Kepler's 3rd law, $\frac{T^2}{R^3}$ = constant

$\therefore \quad \frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3}$ or $\frac{1}{(10^4)^3} = \frac{64}{R_2^3}$ or $R_2 = 4 \times 10^4$ km

Distance travelled in one revolution,

$S_1 = 2\pi r_1 = 2\pi \times 10^4$ and $S_2 = 2\pi R_2 = 2\pi \times 4 \times 10^4$

$$v_1 = \frac{S_1}{t_1} = \frac{2\pi\times10^4}{1} = 2\pi\times10^4 \text{ km h}^{-1}$$

and $v_2 = \frac{S_2}{t_2} = \frac{2\pi\times4\times10^4}{8} = \pi\times10^4$ km h^{-1}

$\therefore$ Relative velocity

$v_1 - v_2 = 2\pi \times 10^4 - \pi \times 10^4 = \pi \times 10^4$ km h^{-1}

13. (c) : The acceleration due to gravity at a height h is given as $g_h = g(1 - 2h/R_e)$ where R_e is radius of earth. The acceleration due to gravity at a depth d is given as $g_d = g(1 - d/R_e)$

Given, $g_h = g_d$

$$\therefore \quad g\left(1-\frac{2h}{R_e}\right) = g\left(1-\frac{d}{R_e}\right)$$

$\therefore \quad d = 2h = 2 \times 1 = 2$ km $\qquad (\because\ h = 1$ km$)$

14. (a) : Since two astronauts are floating in gravitational free space. The only force acting on the two astronauts is the gravitational pull of their masses, *i.e.*, $F = \frac{Gm_1m_2}{r^2}$, which is attractive in nature.

Hence they move towards each other.

15. (b)

16. (d) : Variation of g inside the earth's surface at depth h is given by

$$g' = g\left(1-\frac{h}{R}\right) = g\left(\frac{R-h}{R}\right) = \frac{gd}{R}$$

where d is the distance from the centre of the Earth.

i.e., $g' \propto d$ (inside the earth's surface)

Acceleration due to gravity outside the Earth's surface at height h is

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{gR^2}{d^2} \quad \textit{i.e.,} \quad g' \propto \frac{1}{d^2}$$

17. (a) : Here, the weight of person on the equator = W

If earth rotates about its axis, then weight = $\frac{3W}{4}$

Radius of the earth = 6400 km

The acceleration due to gravity at the equator,

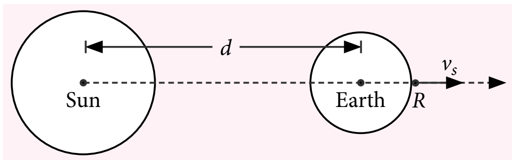
$g_e = g - \omega^2 R \cos^2\theta$

$\frac{3}{4}g = g - \omega^2 R \cos^2 0°$ or $\omega^2 R = \frac{g}{4}$

$$\omega = \sqrt{\frac{g}{4R}} = \sqrt{\frac{10}{4\times6400\times10^3}} = 0.63\times10^{-3} \text{ rad s}^{-1}$$

18. (a)

19. (d) :



Given : Mass of the sun, $M_s = 3\times10^5 \times$ mass of the earth $= 3 \times 10^5 M_e$

Distance between the sun and the earth,

$d = 2.5 \times 10^4 \times$ radius of the earth $= 2.5 \times 10^4 R_e$,

Escape speed, $v_e = 11.2$ km s^{-1}

$$\therefore \quad v_e = \sqrt{\frac{2GM_e}{R_e}} = 11.2 \text{ km s}^{-1}$$

Minimum velocity required for the rocket to escape from the given earth – sun system = v_s

Using energy conservation for the given situation,

$$\frac{1}{2}mv_s^2 = \frac{GM_em}{R_e} + \frac{GM_sm}{d+R_e}$$

$$= \frac{GM_em}{R_e} + \frac{G\times3\times10^5 M_em}{(2.5\times10^4 R_e + R_e)}$$

$\therefore \quad v_s^2 = \frac{2GM_e}{R_e} + \frac{24GM_e}{R_e} \qquad (\because\ 2.5 \times 10^4 >> 1)$

$\Rightarrow \quad v_s^2 = v_e^2 + 12v_e^2 = 13v_e^2$

or $\quad v_s = \sqrt{13}\, v_e = 40.38$ km s$^{-1} \approx 42$ km s^{-1}

20. (c) : Let the area of the ellipse be A.

As per Kepler's 2nd law, areal velocity of a planet around the sun is constant, *i.e.*, $\frac{dA}{dt}$ = constant.

$$\therefore \quad \frac{t_1}{t_2} = \frac{\text{Area of } abcsa}{\text{Area of } adcsa} = \frac{\frac{A}{2}+\frac{A}{4}}{\frac{A}{2}-\frac{A}{4}} = \frac{\frac{3A}{4}}{\frac{A}{4}} = 3$$

$\Rightarrow \quad t_1 = 3t_2$

Note : Here db is the major axis of the ellipse, not semi-major axis and ca is the minor axis of the ellipse, not semi-minor axis.

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EXAM PREP 2018

CLASS XI

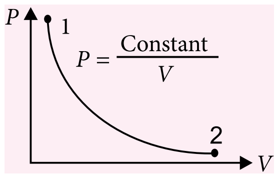
Useful for Medical/Engg. Entrance Exams

CHAPTERWISE MCQs FOR PRACTICE

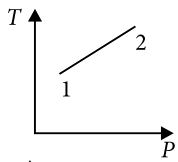
THERMODYNAMICS

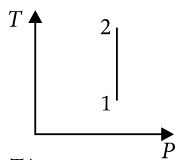
1. Starting with the same initial conditions, an ideal gas expands from volume V_1 to V_2 in three different ways. The work done by the gas is W_1 if the process is purely isothermal, W_2 if purely isobaric and W_3 if purely adiabatic. Then
   (a) $W_2 > W_1 > W_3$ (b) $W_2 > W_3 > W_1$
   (c) $W_1 > W_2 > W_3$ (d) $W_1 > W_3 > W_2$

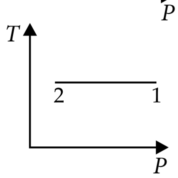
2. Consider P-V diagram for an ideal gas shown in figure.

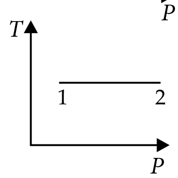


Out of the following diagrams, which represents the T-P diagram?

(a)


(b)


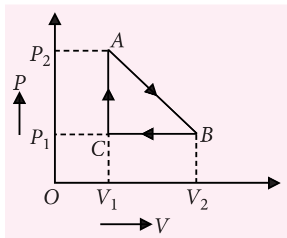
(c)


(d)


3. An ideal gas is made to undergo a thermodynamic process given by $V \propto T^2$; the molar heat capacity of the gas for the above process is.
   (a) $\left(\frac{\gamma-1}{2\gamma-1}\right)R$ (b) $\left(\frac{2\gamma-1}{\gamma-1}\right)R$
   (c) $\left(\frac{\gamma-1}{2\gamma-1}\right)$ (d) $(2\gamma - 1)R$

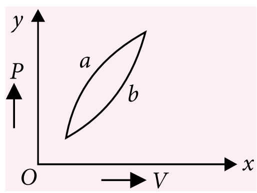
4. A polyatomic gas ($\gamma = 4/3$) is compressed to $\left(\frac{1}{8}\right)^{th}$ of its volume adiabatically. If the initial pressure is P, then the final pressure will be
   (a) $4P$ (b) $18P$ (c) $16P$ (d) $2P$

5. Work done by the system in closed path $ABCA$, as shown in figure is



   (a) zero (b) $(V_1 - V_2)(P_1 - P_2)$
   (c) $\frac{(P_2 - P_1)(V_2 - V_1)}{2}$ (d) $\frac{(P_2 + P_1)(V_2 - V_1)}{2}$

6. Figure shows two processes a and b for a given sample of a gas. If ΔQ_1, ΔQ_2 are the amounts of heat absorbed by the system in the two cases ; and ΔU_1, ΔU_2 are changes in internal energies respectively, then



   (a) $\Delta Q_1 = \Delta Q_2 ; \Delta U_1 = \Delta U_2$
   (b) $\Delta Q_1 > \Delta Q_2 ; \Delta U_1 > \Delta U_2$
   (c) $\Delta Q_1 < \Delta Q_2 ; \Delta U_1 < \Delta U_2$
   (d) $\Delta Q_1 > \Delta Q_2 ; \Delta U_1 = \Delta U_2$

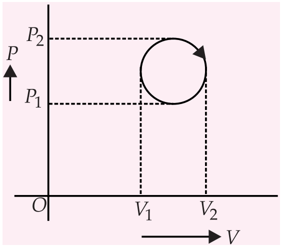
7. In a refrigerator, the low temperature coil of evaporator is at –23 °C and the compressed gas in the condenser has a temperature of 77 °C. How

much electrical energy is spent in freezing 1 kg of water already at 0 °C?

(a) 134400 J (b) 1344 J

(c) 80000 J (d) 32000 J

8. In the cyclic process shown in the *P*-*V* diagram, calculate the work done.



(a) $\pi\left(\frac{V_2 - V_1}{2}\right)^2$

(b) $\pi\left(\frac{P_2 - P_1}{2}\right)^2$

(c) $\frac{\pi}{4}(P_2 - P_1)(V_2 - V_1)$

(d) $\pi(P_2V_2 - P_1V_1)$

9. 70 calories of heat are required to raise the temperature of 2 moles of an ideal gas at constant pressure from 30 °C to 35 °C. The amount of heat required to raise the temperature of the same sample of the gas through the same range at constant volume is (Take $R = 2$ cal mol^{-1} K^{-1})

(a) 30 cal (b) 50 cal (c) 70 cal (d) 90 cal

10. An ideal Carnot engine whose efficiency is 40% receives heat at 500 K. If its efficiency were 50%, then intake temperature for same exhaust temperature would be

(a) 700 K (b) 900 K (c) 800 K (d) 600 K

11. A closed gas cylinder is divided into two parts by a piston held tight. The pressure and volume of gas in two parts respectively are (P, 5V) and (10P, V). If now the piston is left free and the system undergoes isothermal process, then the volume of the gas in two parts respectively are

(a) 2V, 4V (b) 3V, 3V (c) 5V, V (d) 4V, 2V

12. Ideal gas undergoes an adiabatic change in its state from (P_1, V_1, T_1) to (P_2, V_2, T_2). The work done (W) in the process is (μ = number of moles, C_P and C_V are molar specific heats of gas)

(a) $W = \mu(T_1 - T_2)C_P$ (b) $W = \mu(T_1 - T_2)C_V$

(c) $W = \mu(T_1 + T_2)C_P$ (d) $W = \mu(T_1 + T_2)C_V$

13. The work of 146 kJ is performed in order to compress one kilomole of a gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is ($R = 8.3$ J mol^{-1} K^{-1})

(a) diatomic

(b) triatomic

(c) a mixture of monatomic and diatomic

(d) monatomic

14. In a refrigerator one removes heat from a lower temperature and deposits to the surroundings at a higher temperature. In this process, mechanical work has to be done, which is provided by an electric motor. If the motor is of 1 kW power, and heat is transferred from –3 °C to 27 °C, find the heat taken out of the refrigerator per second assuming its efficiency is 50% of a perfect engine.

(a) 19 kJ (b) 20 kJ (c) 21 kJ (d) 22 kJ

15. One mole of an ideal gas undergoes a process $P = \frac{P_0}{1+\left(\frac{V_0}{V}\right)^2}$. Here, P_0 and V_0 are constants. Change in temperature of the gas when volume is changed from $V = V_0$ to $2V_0$ is

(a) $\frac{2P_0V_0}{5R}$ (b) $\frac{11P_0V_0}{10R}$ (c) $\frac{5P_0V_0}{4R}$ (d) P_0V_0

KINETIC THEORY

16. Three closed vessels, A, B and C at the same temperature T and contain gases which obey the Maxwellian distribution of velocities. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of the O_2 molecules in vessel A is v_1 and that of the N_2 molecules in vessel B is v_2, the average speed of the O_2 molecules in vessel C is

(a) $(v_1 + v_2)/2$ (b) v_1

(c) $(v_1 v_2)^{1/2}$ (d) $\sqrt{3kT/M}$

17. A vessel contains 32 g of O_2 at a temperature T. The pressure of the gas is P. An identical vessel containing 4 g of H_2 at a temperature 2T has a pressure of

(a) 8P (b) 4P (c) P (d) P/8

18. Ten small planes are flying at a speed of 150 km h^{-1} in total darkness in an air space that is 20 × 20 × 1.5 km^3 in volume. You are in one of the planes, flying at random within this space with no way of knowing where the other planes are. On the average about how long a time will elapse between near collision with your plane. Assume for this rough computation that a safety region around the plane can be approximated by a sphere of radius 10 m.

(a) 20 h (b) 125 h (c) 167 h (d) 225 h

19. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by

(a) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2$ K (b) $\frac{(\gamma-1)}{2\gamma R}Mv^2$ K

(c) $\frac{\gamma Mv^2}{2R}$ K (d) $\frac{(\gamma-1)}{2R}Mv^2$ K

20. Which one of the following is not an assumption of kinetic theory of gases?
(a) The volume occupied by the molecules of the gas is negligible.
(b) The force of attraction between the molecules is negligible.
(c) The collision between the molecules are elastic.
(d) All molecules have same speed.

21. A gas at absolute temperature 300 K has pressure 4×10^{-10} N m^{-2}. The number of molecules per cm^3 is of the order of
(Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J K^{-1})
(a) 100 (b) 10^5 (c) 10^8 (d) 10^{11}

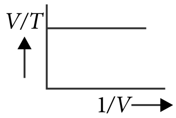
22. If γ be the ratio of specific heats of a perfect gas, the number of degrees of freedom of a molecule of the gas is
(a) $\frac{25}{2}(\gamma-1)$ (b) $\frac{3\gamma-1}{2\gamma-1}$
(c) $\frac{2}{\gamma-1}$ (d) $\frac{9}{2}(\gamma-1)$

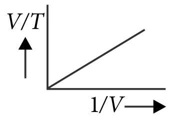
23. 1 mole of H_2 gas is contained in a box of volume V = 1.00 m^3 at T = 300 K. The gas is heated to a temperature of T = 3000 K and the gas gets converted to a gas of hydrogen atoms. The final pressure would be (considering all gases to be ideal)
(a) same as the pressure initially
(b) 2 times the pressure initially
(c) 10 times the pressure initially
(d) 20 times the pressure initially

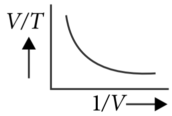
24. Mean free path of a gas molecule is
(a) inversely proportional to number of molecules per unit volume
(b) inversely proportional to diameter of the molecule
(c) directly proportional to the square root of the absolute temperature
(d) directly proportional to the molecular mass

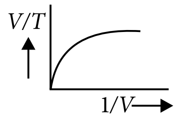
25. Hydrogen is at temperature T and helium is at temperature $2T$. The internal energy of both gases is the same. The ratio of number of moles of hydrogen and helium gases is
(a) 6/5 (b) 5/6 (c) 3/2 (d) 2/3

26. Which one of the following graphs is correct at constant pressure?

(a) 

(b) 

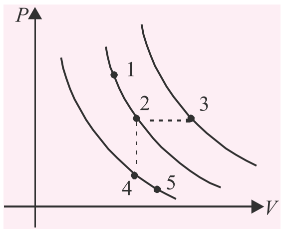
(c) 

(d) 

27. The certain amount of gas is sealed in a glass flask at 1 atmosphere pressure and 20 °C. The flask can withstand up to a pressure of 2 atmosphere. Find the temperature to which the gas can be heated so that the flask doesn't break.
(a) 513 °C (b) 413 °C
(c) 313 °C (d) 213 °C

28. An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12 °C. To what volume does it grow when it reaches the surface, which is at a temperature of 35 °C?
(a) 3.28×10^{-6} m^3 (b) 5.28×10^{-6} m^3
(c) 0.23×10^{-6} m^3 (d) 1.15×10^{-6} m^3

29. A certain gas is taken to the five states represented by dots in the graph. The plotted lines are isotherms. Order of the most probable speed v_p of the molecules at these five states is



(a) v_p at 3 > v_p at 1 = v_p at 2 > v_p at 4 = v_p at 5
(b) v_p at 1 > v_p at 2 = v_p at 3 > v_p at 4 = v_p at 5
(c) v_p at 3 > v_p at 2 = v_p at 4 > v_p at 1 = v_p at 5
(d) insufficient information to predict the result.

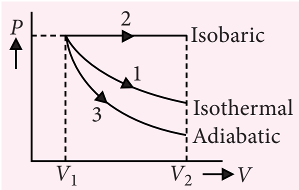
30. The average kinetic energy of the molecules of a low density gas at 27 °C is
(Boltzmann constant = 1.38×10^{-23} J K^{-1})
(a) 3.1×10^{-20} J (b) 3.5×10^{-21} J
(c) 5.3×10^{-18} J (d) 6.21×10^{-21} J

MPP-7 CLASS XII ANSWER KEY

1. (b)	2. (d)	3. (a)	4. (b)	5. (b)
6. (a)	7. (a)	8. (d)	9. (c)	10. (c)
11. (d)	12. (a)	13. (b)	14. (a)	15. (a)
16. (b)	17. (b)	18. (d)	19. (c)	20. (b,d)
21. (b,c,d)	22. (a,c)	23. (b,c)	24. (2)	25. (6)
26. (3)	27. (b)	28. (b)	29. (b)	30. (b)

SOLUTIONS

1. (a): Three given processes are shown in the P-V graphs.



As work done by the gas = area under the P-V graph

$\because$ $(\text{Area})_2 > (\text{Area})_1 > (\text{Area})_3$

$\therefore$ $W_2 > W_1 > W_3$

2. (c) : In the given diagram T is constant and $P_1 > P_2$. Curve (iii) represents $P_1 > P_2$ and straight line graph, parallel to pressure axis indicates constant T.

3. (b): Given; $V \propto T^2$

$V = kT^2$ where k is constant.

Differentiating both sides, of the above equation, we have, $dV = 2kTdT$

Also, from equation of state $V = \frac{nRT}{P} = kT^2$

or $P = \frac{nR}{kT}$

$dW = PdV = \frac{nR}{kT} 2kTdT = 2nRdT$

$\Rightarrow \frac{dW}{ndT} = 2R$

The molar heat capacity of the gas for the process will be

$$C = C_v + \frac{dW}{ndT} = \frac{R}{\gamma-1} + 2R = \left(\frac{2\gamma-1}{\gamma-1}\right)R$$

4. (c) : For an adiabatic process, PV^{γ} = constant

$\therefore$ $P_i V_i^{\gamma} = P_f V_f^{\gamma}$

or $P_f = P_i\left(\frac{V_i}{V_f}\right)^{\gamma} = P\left(\frac{V}{V/8}\right)^{4/3} = P(8)^{4/3} = 16P$

5. (c) : Work done = area of ΔABC

$$= \frac{AC \times BC}{2} = \frac{(P_2 - P_1)(V_2 - V_1)}{2}$$

6. (d): As initial and final states in the two processes are same. Therefore, $\Delta U_1 = \Delta U_2$.

As area under curve a > area under curve b, therefore, $\Delta W_a > \Delta W_b$

As $\Delta Q = \Delta U + \Delta W$ $\therefore$ $\Delta Q_1 > \Delta Q_2$

7. (a) : Coefficient of performance $= \frac{T_2}{T_1 - T_2}$

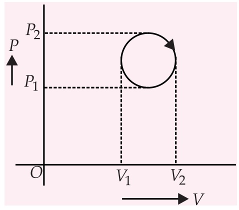
$$= \frac{273-23}{(273+77)-(273-23)} = \frac{250}{100} = 2\cdot5$$

As coefficient of performance $= \frac{Q_2}{W}$

or $2.5 = \frac{1000 \times 80 \times 4.2}{W}$ or $W = \frac{1000 \times 80 \times 4.2}{2.5}$

= 134400 J

8. (c) : For a cyclic process, work done = area enclosed by the cycle.



Treat the circle as an ellipse of semi–major axis $= \frac{V_2 - V_1}{2}$

and semi-minor axis $= \frac{P_2 - P_1}{2}$

$\therefore$ Area $= \pi\left(\frac{V_2 - V_1}{2}\right)\left(\frac{P_2 - P_1}{2}\right)$

$\therefore$ Work done $= \frac{\pi}{4}(P_2 - P_1)(V_2 - V_1)$

9. (b): The amount of heat required at constant pressure is

$\Delta Q = nC_P\Delta T$

$\therefore$ $C_P = \frac{\Delta Q}{n\Delta T} = \frac{70}{2(35-30)} = 7$ cal

As $C_P - C_V = R$

$\therefore$ $C_V = C_P - R = 7 - 2 = 5$ cal mol^{-1} K^{-1}

The amount of heat required at constant volume is

$\Delta Q = nC_V\Delta T = 2 \times 5 \times (35 - 30) = 50$ cal

10. (d): From $\eta = 1 - \frac{T_2}{T_1}$ or $\frac{T_2}{T_1} = 1 - \eta = 1 - \frac{40}{100} = \frac{3}{5}$

$\therefore$ $T_2 = \frac{3}{5}T_1 = \frac{3}{5} \times 500 = 300$ K

Again $\frac{T_2}{T_1} = 1 - \eta$ or $\frac{300}{T_1'} = 1 - \frac{50}{100} = \frac{1}{2}$

or $T_1' = 600$ K

11. (a)

12. (b): Work done during an adiabatic change is

$$W = \frac{\mu R(T_1 - T_2)}{\gamma - 1}$$

As $\gamma = \frac{C_P}{C_V}$ $\therefore$ $\gamma - 1 = \frac{C_P - C_V}{C_V} = \frac{R}{C_V}$ $[\because C_P - C_V = R]$

$\therefore$ $W = \frac{\mu R(T_1 - T_2)C_V}{R} = \mu(T_1 - T_2)C_V$

13. (a) : According to the first law of thermodynamics

$\Delta Q = \Delta U + \Delta W$

In a adiabatic process, $\Delta Q = 0$

$\therefore$ $\Delta U = -\Delta W = -(-146 \text{ kJ}) = 146$ kJ

As $\Delta U = nC_V\Delta T$

$$\therefore\; C_V = \frac{\Delta U}{n\Delta T} = \frac{146\times10^3}{1\times10^3\times7} \quad (\because\; \Delta T = 7°C = 7\text{ K})$$

$= 20.8\text{ J mol}^{-1}\text{ K}^{-1}$

For diatomic gas, $C_V = \frac{5}{2}R = \frac{5}{2}\times8.3$

$= 20.8\text{ J mol}^{-1}\text{ K}^{-1}$

Hence, the gas is diatomic.

14. (a): Here, T_2 = –3°C = 270 K, T_1 = 27°C = 300 K

Efficiency of perfect engine,

$$\eta_{en} = 1 - \frac{T_2}{T_1} = 1 - \frac{270}{300} = 1 - \frac{9}{10} = \frac{1}{10} = 0.1$$

Efficiency of refrigerator, η_{ref} = 50% of η_{en} = 0.05

If Q_1 is the heat transferred per second at higher temperature by doing work W, then

$$\eta_{ref} = \frac{W}{Q_1} \Rightarrow Q_1 = \frac{W}{\eta_{ref}} = \frac{1\text{ kW}\times1\text{ s}}{0.05} \Rightarrow Q_1 = 20\text{ kJ}$$

Heat taken out of the refrigerator per second is

$Q_2 = Q_1 - \eta_{ref}\, Q_1 = 20 - 0.05 \times 20 = 19$ kJ

15. (b): The given process is represented by

$$P = \frac{P_0}{1+\left(\frac{V_0}{V}\right)^2}\;;\; \text{At } V = V_0,\; P = \frac{P_0}{1+1} = \frac{P_0}{2}$$

$$\therefore\; T_i = \frac{PV}{nR} = \frac{\left(\frac{P_0}{2}\right)V_0}{1\times R} = \frac{P_0V_0}{2R}$$

$$\text{At } V = 2V_0,\; P = \frac{P_0}{1+1/4} = \frac{4P_0}{5}$$

$$\therefore\; T_f = \frac{PV}{nR} = \frac{\left(\frac{4P_0}{5}\right)(2V_0)}{1\times R} = \frac{8P_0V_0}{5R}$$

$$\Delta T = T_f - T_i = \frac{8P_0V_0}{5R} - \frac{P_0V_0}{2R} = \frac{11P_0V_0}{10R}$$

16. (b): The average speed of molecules of an ideal gas is given by

$$\langle v\rangle = \sqrt{\frac{8RT}{M}} \; i.e., \langle v\rangle \propto \sqrt{T} \text{ for same gas.}$$

Since, temperature of A and C are same, average speed of O_2 molecules will be equal in A and C *i.e.*, v_1.

17. (b): According to ideal gas equation, $PV = nRT$

For 32 g of O_2, $n = 1$ $\therefore$ $P = RT/V$

For 4 g of H_2 (*i.e.* $n = 2$) at temperature $2T$,

$$P' = \frac{2R\times2T}{V} = \frac{4RT}{V} \quad\text{or}\quad P' = 4P$$

18. (d): Here, v = 150 km h^{-1}, N = 10

$V = 20 \times 20 \times 1.5$ km^3.

Diameter of plane, $d = 2R = 2 \times 10 = 20$ m

$= 20 \times 10^{-3}$ km

$$n = \frac{N}{V} = \frac{10}{20\times20\times1.5} = 0.0167\text{ km}^{-3}$$

Mean free path of a plane

$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 n}$$

Time elapse before collision of two planes randomly,

$$t = \frac{\lambda}{v} = \frac{1}{\sqrt{2}\,\pi d^2 nv}$$

$$= \frac{1}{1.414\times3.14\times(20)^2\times10^{-6}\times(0.0167)\times(150)}$$

$$= \frac{10^6}{4449.5} = 224.74\text{ h} \approx 225\text{ h}$$

19. (d): Kinetic energy of vessel $= \frac{1}{2}mv^2$

Increase in internal energy, $\Delta U = nC_V\Delta T$

where n is the number of moles of the gas in vessel. As the vessel is stopped suddenly, its kinetic energy is used to increase the temperature of the gas.

$$\therefore\; \frac{1}{2}mv^2 = \Delta U = nC_V\Delta T$$

$$= \frac{m}{M}C_V\Delta T \quad \left(\because\; n = \frac{m}{M}\right)$$

$$\text{or}\quad \Delta T = \frac{Mv^2}{2C_V} = \frac{Mv^2(\gamma-1)}{2R}\text{ K} \quad \left(\because\; C_V = \frac{R}{(\gamma-1)}\right)$$

20. (d): Molecules of an ideal gas move randomly with different speeds.

21. (b): As $P = nk_BT$, where n = number of molecules per unit volume

$$\therefore\; n = \frac{P}{k_BT} = \frac{4\times10^{-10}}{1.38\times10^{-23}\times300} \cong 10^{11}\text{ per m}^3$$

$= 10^5$ per cm^3

Solution Senders of Physics Musing

SET-51

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22. (c) : As $\gamma = 1 + \frac{2}{f}$, $\therefore\ f = \frac{2}{\gamma - 1}$

23. (d): According to gas equation

$$PV = nRT \quad \text{or} \quad \frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1} \quad \text{or} \quad \frac{P_2}{P_1} = \frac{V_1}{V_2} \cdot \frac{T_2}{T_1}$$

Here, $T_2 = 3000$ K, $T_1 = 300$ K

Since H_2 splits into hydrogen atoms, therefore volume become half, *i.e.*, $V_2 = \frac{1}{2}V_1$

$$\therefore \quad \frac{P_2}{P_1} = \frac{V_1}{\frac{1}{2}V_1} \times \frac{3000}{300} \quad \text{or} \quad \frac{P_2}{P_1} = 20$$

24. (a) : Mean free path, $\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$ where,

n = Number of molecules per unit volume

d = Diameter of the molecule

As $PV = k_BNT$ $\therefore\ n = \frac{N}{V} = \frac{P}{k_BT}$ $\therefore\ \lambda = \frac{k_BT}{\sqrt{2}\pi d^2 P}$

25. (a) : Internal energy of a gas, $U = nC_VT$ where n is the number of the moles of a gas and C_V is the molar specific heat at constant volume and T is the temperature of the gas.

As hydrogen is a diatomic gas,

$$\therefore \quad (C_V)_{H_2} = \frac{5}{2}R$$

As helium is a monatomic gas,

$$\therefore \quad (C_V)_{He} = \frac{3}{2}R$$

According to the question, $U_{H_2} = U_{He}$

$$\therefore\ n_{H_2} \times (C_V)_{H_2} \times T_{H_2} = n_{He} \times (C_V)_{He} \times T_{He}$$

$$\text{or} \quad n_{H_2} \times \frac{5}{2}R \times T = n_{He} \times \frac{3}{2}R \times 2T \quad \text{or} \quad \frac{n_{H_2}}{n_{He}} = \frac{6}{5}$$

26. (a) : According to ideal gas equation

$$PV = nRT \quad \text{or} \quad \frac{V}{T} = \frac{nR}{P}$$

At constant pressure, $\frac{V}{T}$ = constant

Hence, graph (a) is correct.

27. (c) : Here, P_1 = 1 atmosphere, P_2 = 2 atmosphere, $T_1 = 20°C = 293$ K, T_2 = ?

As volume remains constant,

$$\therefore \quad \frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \text{or} \quad \frac{1}{293} = \frac{2}{T_2}$$

or $T_2 = 586$ K $= 586 - 273 = 313°C$

28. When the air bubble is at depth of 40 m,

Volume of air bubble $V_1 = 1.0\ cm^3 = 1.0 \times 10^{-6}\ m^3$

Pressure of air bubble $P_1 = 1\ atm + h_1\rho g$

$= 1.013 \times 10^5 + 40 \times 10^3 \times 9.8 = 4.93 \times 10^5$ Pa

Temperature of air bubble $T_1 = 12 + 273 = 285$ K

When the air bubbles reaches at the surface of lake, pressure of air bubbles,

P_2 = 1 atmosphere = 1.013×10^5 Pa

Temperature of air bubble $T_2 = 35°C = (35 + 273)$ K = 308 K

Volume of air bubbles = ?

$$\text{Now,} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_2 = \frac{P_1V_1T_2}{T_1P_2} = \frac{4.93 \times 10^5 \times 1.0 \times 10^{-6} \times 308}{285 \times 1.013 \times 10^5}$$

$= 5.275 \times 10^{-6}\ m^3$

29. (a) : States 1 and 2 are at the same temperature. Also states 4 and 5 are at same temperature.

As v_p is more at higher temperature and same at all states at equal temperature.

$$\therefore \quad v_p \text{ at } 3 > v_p \text{ at } 1 = v_p \text{ at } 2 > v_p \text{ at } 4 = v_p \text{ at } 5$$

30. (d): Kinetic energy of an ideal gas,

$$E_K = \frac{3}{2}RT = \frac{3}{2}\ (1.38) \times 10^{-23})\ (300)$$

$= 6.21 \times 10^{-21}$ J

◆◆

Exam Update

JEE (Advanced) 2018

The Joint Entrance Examination (Advanced) 2018 will be conducted by the IITs . The performance of a candidate in this examination will form the basis for admission to the Bachelor's, Integrated Master's and Dual Degree programs (entry at the 10+2 level) in all the IITs. The decisions of the JAB 2018 will be final in all matters related to JEE (Advanced) 2018 and admission to IITs.

Candidates should be among the top 2,24,000 (including all categories) by scoring positive marks in Paper-1 of JEE (Main)-2018.

Examination will be held on May 20, 2018. The entire JEE (Advanced) 2018 Examination will be conducted in fully computer based test mode. The exam consists of two papers, Paper 1 and Paper 2, each of three hours' duration, and will be held in two sessions. Both the papers are compulsory.

For more information visit www.jeeadv.ac.in

ACE YOUR WAY CBSE XI

Series 5

CHAPTERWISE PRACTICE PAPER

MECHANICAL PROPERTIES OF FLUIDS | THERMAL PROPERTIES OF MATTER

Time Allowed : 3 hours | Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION - A

1. Is viscosity a vector?
2. Is the bulb of a thermometer made of diathermic or adiabatic wall?
3. What is the effect on the viscosity of a gas if the temperature is decreased?
4. Coffee runs up into a lump of sugar when one corner is held in the liquid. Why?
5. If we put on wet clothes, the temperature of the body is lowered considerably. Why?

SECTION - B

6. Two bodies of specific heats s_1 and s_2 having same heat capacities are combined to form a single composite body. What is the specific heat of the composite body?
7. Why two holes are made to empty an oil tin?
8. What do you understand by hydrostatic paradox?
9. Explain why vacuum is created between two walls of thermos flask (Dewar flask).
10. Water on a clean glass plate spreads into a thin film but a small amount of water placed on a flat surface of wax forms into drops. Why?

OR

Bernoulli's theorem holds for incompressible, non-viscous fluids. How this relationship changed when the viscosity of fluid is not negligible?

SECTION - C

11. A tank filled with fresh water has hole in its bottom and water is flowing out of it.
    (a) If the size of the hole is increased, what will be the change in :
    (i) volume of water flowing out per second?
    (ii) velocity of the outcoming water?
    (b) If in the above tank, the fresh water is replaced by sea water, will the velocity of outcoming water change?
12. A horizontal pipe line carries water in a streamline flow. At a point along the pipe where the cross-sectional area is 10 cm^2, the water velocity is 1 m s^{-1} and the pressure is 2000 Pa. What is the

pressure at another point where the cross-sectional area is 5 cm^2 ?

13. Derive Newton's law of cooling from Stefan's law.

14. The sap in trees, which consists mainly of water in summer, rises in a system of capillaries of radius, $r = 2.5 \times 10^{-5}$ m. The surface tension of sap is $S = 7.28 \times 10^{-2}$ N m^{-1} and the angle of contact is 0°. Does surface tension alone account for the supply of water to the top of all trees?

15. The difference between the lengths of a certain brass rod and that of a steel rod is claimed to be constant at all the temperatures. Is this possible?

16. Define the terms molecular range, sphere of influence and surface film.

17. What is thermal conduction? Briefly explain the molecular mechanism of thermal conduction.

18. A lead bullet penetrates into a solid object and melts. Assuming that 50% of its kinetic energy was used to heat it, calculate the initial speed of the bullet. The initial temperature of the bullet is 27 °C and its melting point is 327 °C. Latent heat of fusion of lead = $2{\cdot}5 \times 10^4$ J kg^{-1} and specific heat capacity of lead = 125 J kg^{-1} K^{-1}.

OR

There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if **(a)** they are heated to the same temperature? **(b)** same amount of heat is given to each of them?

19. The cylindrical tube of a spray pump has a cross-section of 8.0 cm^2, one end of which has 40 fine holes, each of diameter 1.0 mm. If the liquid flow inside the tube is 1.5 m per minute, what is the speed of ejection of the liquid through the holes?

20. Derive equation of continuity for steady, irrotational flow of a perfectly mobile and incompressible fluid. What conclusion do you draw from it?

21. A body takes 5 min to cool from 80 °C to 65 °C and 10 min to cool to 55 °C. Find the temperature of the surroundings.

22. An air bubble of diameter 2 mm rises steadily through a solution of density 1750 kg m^{-3} at the rate of 0·35 cm s^{-1}. Calculate the coefficient of viscosity of the solution. The density of air is negligible.

SECTION - D

23. In one of the demonstration experiments, one day the Physics teacher in class XI took a slab of ice and supported it on two wooden blocks. He took a metallic wire and attached two heavy weights at its ends. He placed the wire over the slab. It was seen that the wire gradually cut its way through the ice without cutting it into two pieces. The students were very happy to see this. The teacher asked his students can any one explain what is happening. Rakesh stood up and explained that just below the wire, ice melts at a lower temperature due to the increase in pressure. When the wire has passed, water above the wire freezes again. Thus the wire passes through the ice slab without cutting it into two pieces.

(a) What are the values being displayed by Rakesh in his actions?

(b) Name the phenomenon involved in the above demonstration and briefly explain this phenomenon.

(c) What will happen if one use plastic wire in place of metallic wire to conduct the stated experiment?

SECTION - E

24. An aluminium container of mass 100 g contains 200 g of ice at –20°C. Heat is added to the system at a rate of 100 cal s^{-1}. What is the temperature of the system after 4 minutes? Draw a rough sketch showing the variation in the temperature of the system as a function of time. Specific heat capacity of ice = 0.5 cal g^{-1} $°C^{-1}$, specific heat capacity of aluminium = 0·2 cal g^{-1} $°C^{-1}$, specific heat capacity of water = 1 cal g^{-1} $°C^{-1}$ and latent heat of fusion of ice = 80 cal g^{-1}.

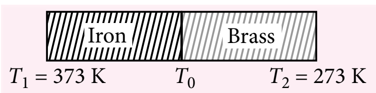
OR

An iron bar (L_1 = 0.1 m, A_1 = 0.02 m^2, K_1 = 79 W m^{-1} K^{-1}) and a brass bar (L_2 = 0.1 m, A_2 = 0.02 m^2, K_2 = 109 W m^{-1} K^{-1}) are soldered end to end as shown in figure. The free ends of the iron bar and brass bar are maintained at 373 K and 273 K respectively. Obtain expression and hence compute :

(a) the temperature of the junction of the two bars

(b) the equivalent conductivity of the compound bar

(c) the heat current through the compound bar.



25. (a) Explain the principle and working of hydraulic lift with the help of a schematic diagram.

(b) Why should we blow over the piece of paper and not below it to keep it horizontal?

OR

Define heat current and thermal resistance. Deduce the mathematical expressions for them in terms of thermal conductivity K. Also, write its units and dimensions.

26. Surface tension is exhibited by liquids due to force of attraction between molecules of the liquid. The surface tension decreases with increase in temperature and vanishes at boiling point.
Given that the latent heat of vaporisation for water L_v = 540 kcal kg^{-1}, the mechanical equivalent of heat J = 4.2 J cal^{-1}, density of water $\rho_w = 10^3$ kg m^{-3}, Avogadro's number $N_A = 6.0 \times 10^{26}$ kmole $^{-1}$ and the molecular weight of water M_A = 18 kg for 1 kmole.

(a) Estimate the energy required for one molecule of water to evaporate.

(b) Show that the inter–molecular distance for water is $d = \left[\frac{M_A}{N_A} \times \frac{1}{\rho_w}\right]^{1/3}$ and find its value.

(c) 1 g of water in the vapour state at 1 atm occupies 1601cm^3. Estimate the intermolecular distance at boiling point, in the vapour state.

OR

Derive an expression for the excess pressure inside a soap bubble.

SOLUTIONS

1. No, viscosity is a scalar quantity.

2. The bulb of a thermometer is made up of diathermic wall because it receives heat from the body to measure the temperature of the body.

3. If the temperature is decreased, there is a decrease in the viscosity.

4. The lump of sugar has fine pores in it which act like small capillaries. We know that whenever a capillary is dipped into such a liquid which wets it, the liquid rises into it. Since coffee wets sugar, it will rise up into the lump of sugar through these capillaries in it.

5. If we put on wet clothes, the temperature of the body is lowered due to the high value of specific heat of water.

6. Let m_1 and m_2 are the masses of two bodies.
Heat capacity of the composite body

$(m_1 + m_2)s = m_1s_1 + m_2s_2$

$\therefore \quad (m_1 + m_2)s = m_1s_1 + m_1s_1 = 2m_1s_1$

($\because$ Two bodies have same heat capacity)

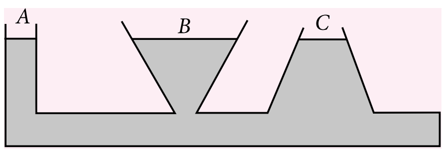
or $\quad s = \dfrac{2m_1s_1}{m_1 + m_2}$

But $m_2s_2 = m_1s_1$ or $\quad m_2 = m_1\dfrac{s_1}{s_2}$

$$\therefore \quad s = \frac{2m_1s_1}{m_1 + m_1\frac{s_1}{s_2}} = \frac{2s_1s_2}{s_1 + s_2}$$

7. When oil comes out through a tin with one hole, the pressure inside the tin becomes less than the atmospheric pressure, soon the oil stops flowing out. When two holes are made in the tin, air keeps on entering the tin through the other hole and maintains pressure inside.

8. Consider three vessels A, B and C of different shapes as shown in figure. They are connected at the bottom by a horizontal pipe. On filling with water the level in the three vessels is the same though they hold different amounts of water. This phenomenon is called hydrostatic paradox.



9. Thermos flask is a container which maintains the temperature of liquid kept in it. Since, both conduction and convection require a material medium. Vacuum is created between the two walls of the flask so that the heat does not flow in or out of the flask and hence, liquid in the flask remains hot or cold for a long time.

10. The force of adhesion between the molecules of water and glass is more than the force of cohesion between the water molecules. Thus, when water is dropped on the glass plate, it spreads on it. On the other hand, the force of cohesion between water molecules is more than the force of adhesion between water and wax molecules. Thus, on account of this higher force of cohesion, water instead of spreading on wax forms into drops.

OR

The effect of viscosity, which is internal friction in case of fluids, is to dissipate mechanical energy of the fluid into heat. Thus, in case the viscosity of a fluid is not negligible, the quantity $P + \rho gh + (1/2)\rho v^2$ goes on decreasing along the direction of flow of the fluid.

11. (a) (i) The volume of water flowing out per second will increase as its volume depends directly on the size of hole.

(ii) The velocity of outflow of water remains unchanged because it depends upon the height of water level and is independent of the size of the hole.

(b) No, though the density of sea water is more than that of fresh water, but the velocity of outflow of water is independent of the density of water.

12. According to the equation of continuity,

$a_1v_1 = a_2v_2$ or $10\text{ cm}^2 \times 1\text{ m s}^{-1} = 5\text{ cm}^2 \times v_2$

$\therefore\ v_2 = 2\text{ m s}^{-1}$

Using Bernoulli's theorem for horizontal flow,

$$P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2$$

or $P_2 + \frac{1}{2} \times 10^3 \times 2^2 = 2000 + \frac{1}{2} \times 10^3 \times 1^2$

or $P_2 = 2000 + 500 - 2000 = 500\text{ Pa}$

13. Suppose a body of surface area A at an absolute temperature T is kept in an enclosure at lower temperature T_0. According to Stefan-Boltzmann law, the net rate of loss of heat from the body due to radiation is

$$H = e\sigma A(T^4 - T_0^4) = e\sigma A(T - T_0)(T + T_0)(T^2 + T_0^2)$$
$$= e\sigma A(T - T_0)(T^3 + T^2T_0 + TT_0^2 + T_0^3)$$

As $(T - T_0)$ is small, T can be taken approximately equal to T_0.

$$\therefore\ H = e\sigma A(T - T_0)(T_0^3 + T_0^3 + T_0^3 + T_0^3)$$
$$= 4e\sigma AT_0^3(T - T_0)$$

If s is the specific heat of the body and m is its mass, then the rate of fall of temperature will be

$$-\frac{dT}{dt} = \frac{H}{ms} = \frac{4e\sigma AT_0^3}{ms}(T - T_0)$$

$$\left(\text{where, } K = \frac{4e\sigma AT_0^3}{ms}\right)$$

or $\frac{dT}{dt} = -K(T - T_0)$

This prove the Newton's law of cooling.

14. Here, $r = 2.5 \times 10^{-5}$ m, $S = 7.28 \times 10^{-2}\text{ N m}^{-1}$, $\rho = 10^3\text{ kg m}^{-3}$, $\theta = 0°$

If h is the height to which sap rises in trees, then

$$h = \frac{2S\cos 0°}{r\rho g}$$
$$= \frac{2(7.28\times10^{-2}\text{ N m}^{-1})}{(2.5\times10^{-5}\text{ m})(10^3\text{ kg m}^{-3})(10\text{ m s}^{-2})} = 0.58\text{ m}$$

No, as many trees have heights much greater than 0.58 m. Water reaches all parts of the trees through transpiration pull.

15. Let L_1, L_2 = lengths of brass and steel rods respectively, α_1, α_2 = their corresponding coefficients of linear expansion.

If the temperature of both the rods is changed from T to $(T + \Delta T)$,

change in length of brass rod = $\alpha_1L_1\Delta T$

and change in length of steel rod = $\alpha_2L_2\Delta T$

For the difference between their lengths to be constant,

$\alpha_1L_1\Delta T = \alpha_2L_2\Delta T$ or $\alpha_1L_1 = \alpha_2L_2$ or $\frac{L_1}{L_2} = \frac{\alpha_2}{\alpha_1}$

Since the lengths of rods are in the inverse ratio of the coefficient of linear expansion of their materials, therefore they must have a constant difference in length of rods of brass and steel.

16. Molecular range : It is the maximum distance upto which a molecule can exert some appreciable force of attraction on other molecules. It is of the order of 10^{-9} m in solids and liquids.

Sphere of influence : A sphere drawn around a molecule as centre and with a radius equal to the molecular range is called the sphere of influence of the molecule. The molecule at the centre attracts all the molecules lying in its sphere of influence.

While studying the behaviour of a molecule under the influence of cohesive forces, we need to consider only the molecules lying in its sphere of influence.

Surface film : A thin film of liquid near its surface having thickness equal to the molecular range for that liquid is called surface film.

MPP-7 CLASS XI ANSWER KEY

1. (d)	2. (d)	3. (c)	4. (d)	5. (d)
6. (a)	7. (b)	8. (b)	9. (d)	10. (c)
11. (c)	12. (c)	13. (c)	14. (d)	15. (c)
16. (a)	17. (b)	18. (b)	19. (c)	20. (a,c)
21. (b,d)	22. (a,d)	23. (b,d)	24. (6)	25. (5)
26. (3)	27. (b)	28. (a)	29. (b)	30. (d)

17. Thermal conduction : It is a process in which heat is transmitted from one part of a body to another at a lower temperature through molecular collisions, without any actual flow of matter.

Molecular mechanism of thermal conduction : Solids are heated through conduction. When one end of a metal rod is heated, the molecules at the hot end vibrate with greater amplitude. So they have greater average kinetic energy. As these molecules collide with the neighbouring molecules of lesser kinetic energy, the energy is shared between them. The kinetic energy of the neighbouring molecules increases. This energy transfer takes place from one layer to the next, without the molecules leaving their average location. This way, heat is passed to the colder end of the rod.

18. Let the mass of the bullet = m.

Heat required to take the bullet from 27 °C to 327 °C

$= m \times (125 \text{ J kg}^{-1} \text{ K}^{-1})(300 \text{ K})$

$= m \times (3{\cdot}75 \times 10^4 \text{ J kg}^{-1})$.

Heat required to melt the bullet

$= m \times (2{\cdot}5 \times 10^4 \text{ J kg}^{-1})$.

If the initial speed be v, the kinetic energy is $\frac{1}{2}mv^2$

and hence the heat developed is $\frac{1}{2}\left(\frac{1}{2}mv^2\right) = \frac{1}{4}mv^2$.

Thus, $\frac{1}{4}mv^2 = m(3{\cdot}75 + 2{\cdot}5) \times 10^4 \text{ J kg}^{-1}$

or $v = 500 \text{ m s}^{-1}$.

OR

(a) As thermal expansion of isotropic solids is similar to true photographic enlargement, the expansion of a cavity is same as if it were a solid body of the same material *i.e.*, $\Delta V = \gamma V \Delta T$. As here V, γ and ΔT are same for both solid and hollow spheres, so the expansions of both will be equal.

(b) If same amount of heat is given to the two spheres, then due to lesser mass, rise in temperature of hollow sphere will be more (as $\Delta T = Q/Ms$) and hence, the expansion will be more as $\Delta V = \gamma V \Delta T$.

19. Cross-sectional area of the pump,

$A_1 = 8.0 \text{ cm}^2 = 8.0 \times 10^{-4} \text{ m}^2$

Cross-sectional area of the hole in the pump,

$$A_2 = \frac{\pi(1.0 \times 10^{-3})^2}{4}$$

Velocity of flow in the pump,

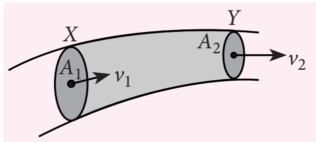
$$v_1 = \frac{1.5}{60} \text{ m s}^{-1} = 0.025 \text{ m s}^{-1}$$

If v_2 is the velocity of ejection of the liquid through each one of the 40 holes,

$$A_1 v_1 = 40(A_2 v_2) \text{ or } v_2 = \frac{A_1 v_1}{40 A_2}$$

or $$v_2 = \left[\frac{(8.0 \times 10^{-4})0.025}{40 \times \pi(1.0 \times 10^{-3})^2/4}\right] \text{m s}^{-1} = 0.64 \text{ m s}^{-1}$$

20. Consider a liquid flowing through a tube XY of variable area of cross-section as shown in figure. Let A_1, A_2 be the cross-section areas of the tube at X and Y and v_1, v_2 be the velocities of flow of liquid at X and Y respectively.



Volume of the liquid flowing per second through $X = A_1 v_1$

Mass of liquid flowing per second through $X = A_1 v_1 \rho_1$

where ρ_1 is the density of the liquid at X.

Similarly, mass of the liquid flowing per second through $Y = A_2 v_2 \rho_2$

where ρ_2 is the density of the liquid at Y.

According to the law of conservation of mass, mass of liquid entering A per second = mass of liquid leaving B per second

or $A_1 v_1 \rho_1 = A_2 v_2 \rho_2$

In case the liquid is incompressible, $\rho_1 = \rho_2$

Hence, $A_1 v_1 = A_2 v_2$ or Av = constant

This is known as the equation of continuity of flow of an incompressible liquid and is a statement of conservation of mass.

It can be concluded that if $A_2 < A_1$, then $v_2 > v_1$.

Thus, the velocity of flow increases as the cross-sectional area decreases and vice-versa.

21. In the first case, $T_1 = 80$ °C, $T_2 = 65$ °C, $t = 5$ min

Let T_0 be the temperature of the surroundings.

From $\ln\frac{(T_1 - T_0)}{(T_2 - T_0)} = Kt$, we get

$$\ln\frac{(80 - T_0)}{(65 - T_0)} = 5K \quad \text{...(i)}$$

In the second case,

$T_1 = 65$ °C, $T_2 = 55$ °C, $t = 10$ min – 5 min = 5 min

From $\ln \frac{(T_1 - T_0)}{(T_2 - T_0)} = Kt$, we get

$$\ln \frac{(65 - T_0)}{(55 - T_0)} = 5K \quad \text{...(ii)}$$

From eqns. (i) and (ii),

$$\ln \frac{(80 - T_0)}{(65 - T_0)} = \ln \frac{(65 - T_0)}{(55 - T_0)}$$

or $\frac{(80 - T_0)}{(65 - T_0)} = \frac{(65 - T_0)}{(55 - T_0)}$

or $(80 - T_0)(55 - T_0) = (65 - T_0)^2$

or $4400 - 55T_0 - 80T_0 + T_0^2 = 4225 - 130T_0 + T_0^2$

or $5T_0 = 175$ or $T_0 = 35$ °C

22. The force of buoyancy B is equal to the weight of the displaced liquid. Thus, $B = \frac{4}{3}\pi r^3 \sigma g$.

This force is upward. The viscous force acting downward is $F = 6\pi\eta r v$.

The weight of the air bubble may be neglected as the density of air small. For uniform velocity,

$F = B$

or, $6\pi\eta r v = \frac{4}{3}\pi r^3 \sigma g$

or, $\eta = \frac{2r^2 \sigma g}{9v}$

or, $\eta = \frac{2 \times (1 \times 10^{-3}\text{ m})^2 \times (1750\text{ kg m}^{-3})(9.8\text{ m s}^{-2})}{9 \times (0.35 \times 10^{-2}\text{ m s}^{-1})}$

≈ 11 poise.

This appears to be highly viscous liquid.

23. **(a)** Attentive, intelligent and ability to explain things scientifically.

(b) This phenomenon is called regelation in which ice melts when pressure is increased and again freezes when pressure is removed

(c) The phenomenon of regelation will be observed as it is independent of the material of wire used.

24. Total heat supplied to the system in 4 minutes is $Q = 100 \times 240 = 2.4 \times 10^4$ cal.

Heat required to take the system from –20°C to 0°C

$= (100) \times (0.2) \times (20) + (200) \times (0.5 \times (20)$

$= 400 + 2000 = 2400$ cal

The time taken in this process $= \frac{2400}{100} = 24\text{ s}$

The heat required to melt the ice at 0 °C

$= (200)(80) = 16000$ cal

The time taken in this process $= \frac{16000}{100}\text{ s} = 160\text{ s}$

If the final temperature is T, the heat required to take the system to the final temperature

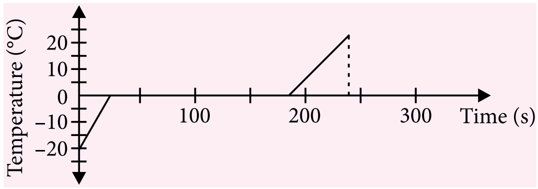
$= (100) \times (0.2)T + (200)(1)T = 220\,T$.

Thus,

$2.4 \times 10^4 = 2400 + 16000 + (220)\,T$

or, $T = \frac{5600}{220} = 25.5$ °C.

The variation in the temperature as a function of time is sketched in figure.



OR

(a) Let T_0 be temperature at junction of two bars.

In steady state, $\frac{dQ}{dt} = \frac{K_1 A_1 (T_1 - T_0)}{L_1} = \frac{K_2 A_2 (T_0 - T_2)}{L_2}$

whence, $K_1(T_1 - T_0) = K_2(T_0 - T_2)$

(as $A_1 = A_2$ and $L_1 = L_2$)

or $T_0 = \frac{K_1 T_1 + K_2 T_2}{K_1 + K_2} = \frac{79 \times 373 + 109 \times 273}{79 + 109} = 315$ K

(b) Also, $\frac{dQ}{dt} = \frac{K_1 A}{L_1}(T_1 - T_0) = \frac{K_1 A}{L}\left(T_1 - \frac{K_1 T_1 + K_2 T_2}{K_1 + K_2}\right)$

$\left(\because\ L_1 = L \text{ and } T_0 = \frac{K_1 T_1 + K_2 T_2}{K_1 + K_2}\right)$

$$\frac{dQ}{dt} = \frac{K_1 K_2}{(K_1 + K_2)} \frac{A}{L}(T_1 - T_2) \quad \text{...(i)}$$

If K is the equivalent conductivity of the compound bar, then

$$\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{2L} \quad \text{...(ii)}$$

From eqns. (i) and (ii),

$$\frac{KA(T_1 - T_2)}{2L} = \frac{K_1 K_2}{(K_1 + K_2)} \frac{A}{L}(T_1 - T_2)$$

or $K = \frac{2K_1 K_2}{K_1 + K_2} = \frac{2 \times 79 \times 109}{79 + 109}$ W m^{-1} K^{-1}

$= 91.6$ W m^{-1} K^{-1}

(c) Heat current through compound bar, *i.e.*,

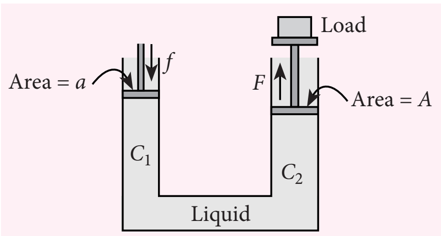
$$\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{2L}$$

$$= \frac{(91.6 \text{ W m}^{-1} \text{ K}^{-1})(0.02 \text{ m}^2)(373 - 273)\text{K}}{2 \times 0.1 \text{m}} = 916 \text{ W}$$

25. (a) Hydraulic lift : Hydraulic lift is an application of Pascal's law. It is used to lift heavy objects. It is a force multiplier.

It consists of two cylinders C_1 and C_2 of different cross-sectional areas connected to each other by a pipe. The cylinders are fitted with water-tight frictionless pistons. The cylinders and the pipe contain a liquid. Suppose a force f is applied on the smaller piston of cross-sectional area a. Then

Pressure exerted on the liquid, $P = \frac{f}{a}$



According to Pascal's law, the same pressure P is also transmitted to the larger piston of cross-sectional area A.

∴ Force on larger piston is

$$F = P \times A = \frac{f}{a} \times A = \frac{A}{a} \times f$$

As $A > a$, therefore, $F > f$.

Hence, by making the ratio A/a large, very heavy loads (like cars and trucks) can be lifted by the application of a small force. However, there is no gain of work. The work done by force f is equal to the work done by F. The piston P_1 has to be moved down by a larger distance compared to the distance moved up by piston P_2.

(b) If we blow over a piece of paper, velocity of air above the paper becomes more than that below it. As kinetic energy of air above the paper increases, so in accordance with Bernoulli's theorem ($P + \frac{1}{2}\rho v^2$ = constant), its pressure energy and hence its pressure decreases. Due to greater value of pressure below the piece of paper (*i.e.*, atmospheric pressure), it remains horizontal and does not fall.

On the other hand if we blow under the paper, the pressure on the lower side decreases. The atmospheric pressure above the paper will therefore bend the paper downwards. So the paper will not remain horizontal.

OR

Heat current and thermal resistance: Heat flows in a conductor due to temperature difference between its two points. The flow of heat per unit time is called heat current, denoted by H. Thus

$$H = \frac{Q}{t}$$

Its SI unit is J s^{-1} or watt (W).

From Ohm's law, electric resistance is given by

$$R = \frac{V}{I}$$

That is electric resistance is the ratio of the potential difference and the electric current. Similarly, the ratio of the temperature difference between the ends of a conductors to the heat current through it is called thermal resistance, denoted by R_H. Thus

$$R_H = \frac{\Delta T}{H}$$

As $Q = KA\frac{\Delta T}{\Delta x}t$

$$\therefore \quad H = \frac{Q}{t} = KA\frac{\Delta T}{\Delta x}$$

and $R_H = \frac{\Delta T}{H} = \frac{\Delta x}{KA}$

Hence greater the coefficient of thermal conductivity of a material, smaller is the thermal resistance of rod of that material.

As $R_H = \Delta T/H$,

so SI unit of $R_H = \frac{\text{K}}{\text{J s}^{-1}} = \frac{\text{K}}{\text{W}} = \text{K W}^{-1}$.

Dimensions of $R_H = \frac{[\text{K}]}{[\text{ML}^2\text{T}^{-2}]\cdot[\text{T}^{-1}]}$

$= [\text{M}^{-1} \text{L}^{-2} \text{T}^3 \text{K}]$

26. (a) For the evaporation of 1 kg of water,

energy required = 540 kcal = $540 \times 10^3 \times 4.2$ J

$= 2268 \times 10^3$ J

For the evaporation of 1 kmol (18 kg) of water,

energy required = $(2268 \times 10^3 \text{ J})(18)$

$= 40824 \times 10^3 \text{ J} = 4.0824 \times 10^7$ J

Since 1 kmol of water contains 6×10^{26} molecules, energy required for 1 molecule to evaporate,

$$u = \frac{4.0824 \times 10^7}{6 \times 10^{26}} \text{J} = 0.68 \times 10^{-19} \text{J}$$

$= 6.8 \times 10^{-20}$ J

(b) If we consider water molecules to be placed at a distance d from each other,

volume around one molecule = d^3

Also, volume around one molecule

$$= \frac{\text{volume of 1 kmol}}{\text{number of molecules / kmol}}$$

$$= \frac{M_A/\rho_w}{N_A} = \frac{M_A}{N_A\rho_w}$$

Thus, $d^3 = \dfrac{M_A}{N_A\rho_w}$

Hence, $d = \left(\dfrac{M_A}{N_A\rho_w}\right)^{1/3}$

$$= \left[\frac{18 \text{ kg kmol}^{-1}}{(6 \times 10^{26} \text{kmol}^{-1})(10^3 \text{kg m}^{-3})}\right]^{1/3}$$

$= (30 \times 10^{-30})^{1/3}$

$\approx 3.1 \times 10^{-10}$ m

(c) Volume occupied by 1 kmol (18 kg) of water molecules

$$= 1601 \times 10^{-6} \text{m}^3 \text{ g}^{-1}(18 \times 10^3 \text{g})$$

$= 28818 \times 10^{-3}$ m^3

Since 1 kmol contains 6×10^{26} molecules, volume occupied by 1 molecule

$$= \frac{28818 \times 10^{-3} \text{m}^3}{6 \times 10^{26}} = 48030 \times 10^{-30} \text{m}^3$$

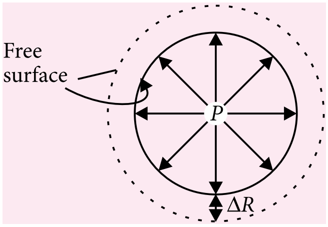
If d' is the intermolecular distance, then

$(d')^3 = 48030 \times 10^{-30}$ m^3

Hence, $d' = 36.3 \times 10^{-10}$ m

OR

Let us consider a liquid bubble (say a soap bubble) of radius R and let S be the surface tension of the liquid. We know that there is an excess pressure (p) inside the bubble and it acts normally outwards. Let the excess pressure increase the radius of the bubble from R to $(R + \Delta R)$ as shown.



Work done by the excess pressure, *i.e.*,

W = force × distance

= excess pressure × surface area × distance

$= P \times 4\pi R^2 \times \Delta R$

i.e., $W = 4\pi PR^2 \times \Delta R$ ...(i)

The soap bubble has two free surfaces, one inside the bubble the other outside it.

Total increase in the surface area of the bubble, *i.e.*,

$\Delta A = 2[4\pi(R + \Delta R)^2 - 4\pi R^2]$

$= 8\pi[R^2 + (\Delta R)^2 + 2R \times \Delta R - R^2]$

$= 16\pi R\Delta R$ ...(ii)

[neglecting $(\Delta R)^2$ as it is very small]

We know that $S = \dfrac{W}{\Delta A}$ ...(iii)

From eqns. (i), (ii) and (iii),

$$S = \frac{4\pi PR^2 \Delta R}{16\pi R\Delta R} = \frac{PR}{4} \text{ or } P = \frac{4S}{R}$$

Since the excess pressure is inversely proportional to the radius of the bubble, it means smaller the bubble, greater the excess pressure inside it.

MPP-7 MONTHLY Practice Problems

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Oscillations

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

1. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force, $f(t) \propto \cos \omega t$ $(\omega \neq \omega_0)$ is applied to the oscillator. How does the displacement (y) of oscillator vary?

(a) $y \propto m(\omega_0^2 - \omega^2)$ (b) $y \propto (\omega_0^2 - \omega^2)/m$

(c) $y \propto \dfrac{m}{\omega_0^2 - \omega^2}$ (d) $y \propto \dfrac{1}{m(\omega_0^2 - \omega^2)}$

2. A mass attached to a spring is free to oscillate, with angular velocity ω, in a horizontal plane without friction or damping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time $t = 0$. The amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0 will be

(a) $\sqrt{\dfrac{\omega^2}{v_0^2} + x_0}$ (b) $\sqrt{\dfrac{x_0^2}{v_0^2} + \omega}$

(c) $\sqrt{\dfrac{\omega}{v_0} + x_0}$ (d) $\sqrt{\dfrac{v_0^2}{\omega^2} + x_0^2}$

3. A particle of mass 0.2 kg executes simple harmonic motion along a path of length 0.2 m at the rate of 600 oscillations per minute. Assume at $t = 0$, the particle starts SHM in positive direction. The kinetic and potential energies in joules when the displacement is $x = A/2$, where A stands for the amplitude, are

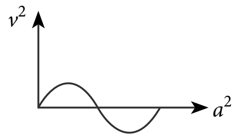
(a) $\dfrac{\pi^2}{10}, \dfrac{5\pi^2}{10}$ (b) $\dfrac{\pi^2}{15}, \dfrac{3\pi^2}{10}$

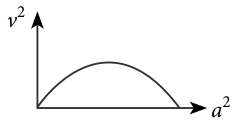
(c) $\dfrac{3\pi^2}{10}, \dfrac{\pi^2}{10}$ (d) $\dfrac{3\pi^2}{10}, \dfrac{5\pi^2}{15}$

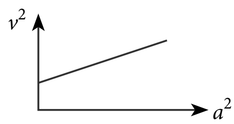
4. Time period of a particle executing SHM is 8 s. At $t = 0$, it is at the mean position. The ratio of the displacement of the particle in the 1st second to the 2nd second is

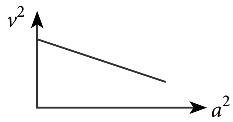
(a) $\dfrac{1}{\sqrt{2}+1}$ (b) $\sqrt{2}$ (c) $\dfrac{1}{\sqrt{2}}$ (d) $\sqrt{2} + 1$

5. A graph of the square of the velocity against the square of the acceleration of a given simple harmonic motion is

(a) 

(b) 

(c) 

(d) 

6. Starting from origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy?

(a) $\dfrac{1}{6}$ s (b) $\dfrac{1}{4}$ s (c) $\dfrac{1}{3}$ s (d) $\dfrac{1}{12}$ s

7. If a spring has time period T and is cut into n equal parts, then the time period of each part will be

(a) $T\sqrt{n}$ (b) $\dfrac{T}{\sqrt{n}}$ (c) nT (d) T

8. When the displacement is half of the amplitude, then what fraction of the total energy of a simple harmonic oscillator is kinetic?

(a) (2/7)th (b) (3/4)th
(c) (2/9)th (d) (5/7)th

9. Maximum speed of a particle in SHM is v_{max}. Then average speed of a particle in SHM is equal to

(a) $\frac{v_{max}}{2}$ (b) $\frac{\pi v_{max}}{2}$ (c) $\frac{v_{max}}{2\pi}$ (d) $\frac{2v_{max}}{\pi}$

10. Masses m and $3m$ are attached to the two ends of a spring of constant k. If the system vibrates freely, the period of oscillation will be

(a) $\pi\sqrt{\frac{m}{k}}$ (b) $2\pi\sqrt{\frac{m}{k}}$ (c) $\pi\sqrt{\frac{3m}{k}}$ (d) $2\pi\sqrt{\frac{3m}{k}}$

11. A mass M suspended from a spring of negligible mass. The spring is pulled a little and then released, so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $5T/3$. The ratio of m/M is

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$ (c) $\frac{16}{9}$ (d) $\frac{25}{9}$

12. If the displacement (x) and velocity (v) of a particle executing SHM are related through the expression $4v^2 = 25 - x^2$, then its time period is

(a) π (b) 2π (c) 4π (d) 6π

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. Assertion : Resonance is a special case of forced vibration in which the natural frequency of vibration of the body is the same as the impressed frequency and the amplitude of forced vibration, is maximum.

Reason : The amplitude of forced vibrations of a body increases with increase in the frequency of the externally impressed periodic force.

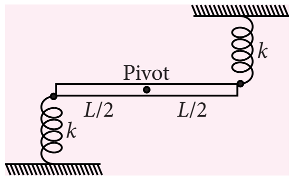
14. Assertion : The graph between velocity and displacement for a harmonic oscillator is a parabola.
Reason : Velocity changes uniformly with displacement.

15. Assertion : The maximum velocity in SHM is v_m. The average velocity during motion from one extreme position to other extreme position will be $2v_m/\pi$.
Reason : Average velocity is the mean of the maximum and minimum velocity of particle in SHM.

JEE MAIN / JEE ADVANCED

Only One Option Correct Type

16. The uniform stick in figure has mass m and length L and is pivoted at its center. In the equilibrium position shown, the identical light springs have their natural length. Stick is turned through small angle θ_0 from the position shown and released. How fast will the tip of the stick be moving when the stick passes through the horizontal?



(a) $L\theta_0\sqrt{\frac{3k}{2m}}$ (b) $L\theta_0\sqrt{\frac{k}{m}}$

(c) $L\theta_0\sqrt{\frac{2k}{3m}}$ (d) $L\theta_0\sqrt{\frac{5k}{m}}$

17. $x = A \sin \omega t$, represents the equation of a SHM. If displacements of the particle are x_1 and x_2 and velocities are v_1 and v_2, respectively, then amplitude of SHM is

(a) $\left[\frac{(v_2 - v_1)(x_2 - x_1)}{v_2^2 - v_1^2}\right]^{\frac{1}{2}}$ (b) $\left[\frac{(v_2x_1)^2 - (v_1x_2)^2}{v_2^2 - v_1^2}\right]^{\frac{1}{2}}$

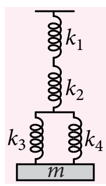
(c) $\frac{v_1x_2}{(v_2 - v_1)^2}$ (d) $\left[\frac{v_1x_2 - v_2x_1}{v_1^2 - v_2^2}\right]^{\frac{1}{2}}$

18. A tunnel is made across the earth of radius R, passing through its centre. A ball is dropped from a height h in the tunnel. The motion will be periodic with time period

(a) $2\pi\sqrt{\frac{R}{g}} + 4\sqrt{\frac{h}{g}}$ (b) $2\pi\sqrt{\frac{R}{g}} + 4\sqrt{\frac{2h}{g}}$

(c) $2\pi\sqrt{\frac{R}{g}} + \sqrt{\frac{h}{g}}$ (d) $2\pi\sqrt{\frac{R}{g}} + \sqrt{\frac{2h}{g}}$

19. A system of springs with their spring constants are as shown in figure. The frequency of oscillations of the mass m will be (assuming the springs to be massless)



(a) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_1k_2(k_3+k_4)}{[(k_1+k_2)+(k_3+k_4)+k_1k_4]m}}$

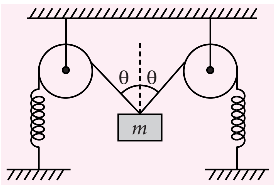
(b) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_1k_2(k_3+k_4)}{[(k_1+k_2)+(k_3+k_4)+k_1k_2]m}}$

(c) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_1k_2(k_3+k_4)}{[(k_1+k_2)\times(k_3+k_4)+k_1k_2]m}}$

(d) $\dfrac{1}{2\pi}\sqrt{\dfrac{(k_1+k_2)(k_3+k_4)+k_1k_2}{k_1k_2(k_3+k_4)m}}$

More than One Options Correct Type

20. In figure, the block of mass m is in equilibrium initially. Now the block is pushed down by a slight distance and released (springs are identical and massless having spring constant k). Then



(a) Initial elongation of the spring is $\dfrac{mg}{2k\cos\theta}$

(b) Initial elongation of the spring is $\dfrac{mg}{2k\cos^2\theta}$

(c) Time period of oscillation of the block is $2\pi\sqrt{\dfrac{m}{2k\cos^2\theta}}$

(d) Time period of oscillation of the block is $2\pi\sqrt{\dfrac{m}{2k}}$.

21. A particle moves along the x-axis as per the equation $x = 4 + 3\sin(2\pi t)$. Here, x is in cm and t in seconds. Select the correct alternative(s).

(a) The motion of the particle is simple harmonic with mean position at $x = 0$.

(b) The motion of the particle is simple harmonic with mean position at $x = 4$ cm.

(c) The motion of the particle is simple harmonic with mean position at $x = -4$ cm.

(d) Amplitude of oscillation is 3 cm.

22. A simple pendulum is oscillating between extreme positions P and Q about the mean position O. Which of the following statements are true about the motion of pendulum?

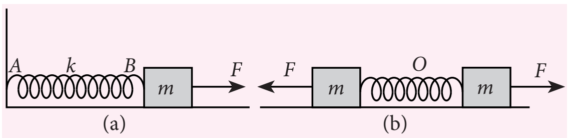
(a) At point O, the acceleration of the bob is different from zero.

(b) The acceleration of the bob is constant throughout the oscillation.

(c) The tension in the string is constant throughout the oscillation.

(d) The tension is maximum at O and minimum at P or Q.

23. The given figure (a) shows a spring of force constant k fixed at one end and carrying a mass m at the other end placed on a horizontal frictionless surface. The spring is stretched by a force F. Figure (b) shows the same spring with both ends free and a mass m fixed at each free end. Each of the spring is stretched by the same force F. The mass in case (a) and the masses in case (b) are then released. Which of the following statements are true?



(a) While oscillating, the maximum extension of the spring is more in case (a) than in case (b).

(b) The maximum extension of the spring is same in both cases.

(c) The time period of oscillation is the same in both cases.

(d) The time period of oscillation in case (a) is $\sqrt{2}$ times that in case (b).

Integer Answer Type

24. The minimum phase difference between two SHMs

$$y_1 = \sin\frac{\pi}{6}\sin\omega t + \sin\frac{\pi}{3}\cos\omega t$$

$$y_2 = \cos\frac{\pi}{6}\sin\omega t + \cos\frac{\pi}{3}\cos\omega t$$

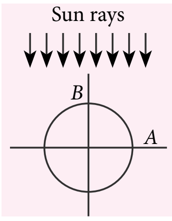
is π/n. What is the value of n?

25. A block is performing SHM along a vertical line with amplitude of 40 cm on a horizontal plank. When the plank is momentarily at rest, the block just lose the contact with plank. The period of oscillations of block is $\dfrac{2\pi}{n}$ s. What is the value of n? [Use $g = 10$ m s^{-2}]

26. The equation of a particle executing SHM is given by $x = 3\cos\left(\frac{\pi}{2}\right)t$ cm, where t is in second. The distance travelled by the particle in the first 8.5 s is $\left(24+\frac{n}{\sqrt{2}}\right)$ then the value of n is

Comprehension Type

A particle B is moving in a vertical circle of radius a in anticlockwise direction with constant time period T. Another particle A is undergoing SHM on its horizontal diameter with amplitude a and same time period T. A is initially at its right extreme and B is at topmost point as shown. Sun is directly overhead.



27. At what time does shadow of B first fall on A?

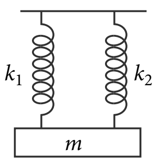
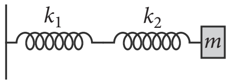
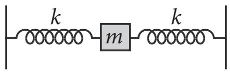
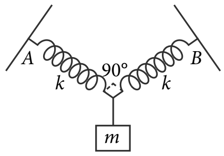
(a) $\frac{T}{8}$ (b) $\frac{7T}{8}$ (c) $\frac{5T}{8}$ (d) $\frac{3T}{8}$

28. At what time does the acceleration of B first point directly towards A?

(a) $\frac{T}{4}$ (b) $\frac{T}{2}$ (c) $\frac{3T}{4}$ (d) T

Matrix Match Type

29. Column I has different combinations of springs and column II shows their time period in different order. Match column I and column II and select the correct option.

Column I	Column II
A. 	P. $2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$
B. 	Q. $2\pi\sqrt{\frac{m}{(k_1+k_2)}}$
C. 	R. $2\pi\sqrt{\frac{m}{k}}$
D. 	S. $2\pi\sqrt{\frac{m}{2k}}$

	A	B	C	D
(a)	P	Q	R	S
(b)	Q	P	S	R
(c)	Q	S	P	R
(d)	P	Q	S	R

30. Two particles A and B start SHM at $t = 0$. Their positions as function of time are given by

$x_A = A\sin\omega t$

$x_B = A\sin(\omega t + \pi/3)$

Column I	Column II
A. Minimum time when x is same	P. $\frac{5\pi}{6\omega}$
B. Minimum time when velocity is same	Q. $\frac{\pi}{3\omega}$
C. Minimum time after which $v_A < 0$ and $v_B < 0$	R. $\frac{\pi}{\omega}$
D. Minimum time after which $x_A < 0$ and $x_B < 0$	S. $\frac{\pi}{2\omega}$

	A	B	C	D
(a)	P	S	R	Q
(b)	S	P	Q	R
(c)	Q	R	P	S
(d)	Q	P	S	R

◈◈

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......

No. of questions correct ......

Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

NEET | JEE ESSENTIALS

Class XII

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Unit 5 OPTICS

Optics is the branch of physics that studies the phenomena and laws associated with the generation, propagation and interaction of light with substances. Here, we will deal with five important properties of light namely reflection, refraction, interference, diffraction and polarisation.

RAY OPTICS

Ray optics treats the law of propagation of light in a transparent media on the basis of notions of light as a combination of light-rays-lines along which energy of electromagnetic waves propagates.

Reflection

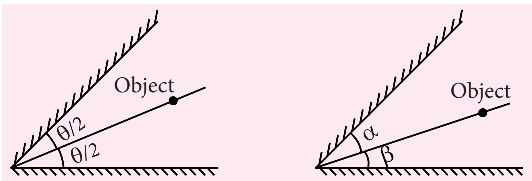
If the incident light after interacting with a boundary separating the two media comes back in the same medium, the phenomenon is called reflection and the boundary is known as reflector.

Laws of Reflection

- Angle of incidence ($\angle i$) is equal to the angle of reflection ($\angle r$).
- The incident ray, the reflected ray and the normal, all lie in the same plane.

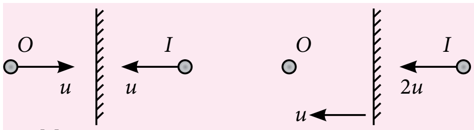
Reflection through Plane Mirrors

- After reflection, velocity, wavelength and frequency of light remains same but intensity decreases.
- **Number of images by two plane mirrors inclined at an angle θ**
  - $n = \left(\frac{360°}{\theta} - 1\right)$; if $\frac{360°}{\theta}$ = even integer
  - If $360°/\theta$ = odd integer, then there are two possibilities.



Object is placed symmetrically $n = \left(\frac{360°}{\theta} - 1\right)$ | Object is placed asymmetrically $n = \frac{360°}{\theta}$

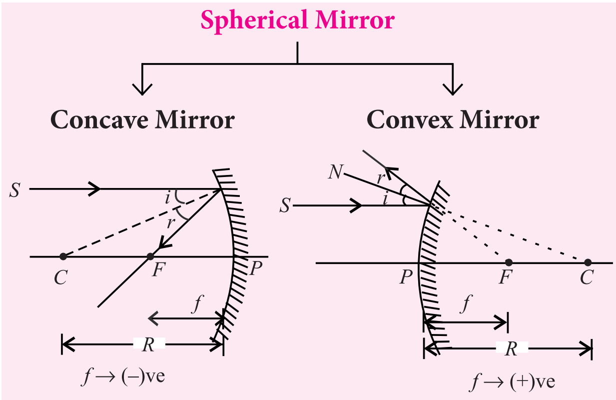
- When the object moves with speed u towards (or away) from the plane mirror then image also moves towards (or away) with speed u. But relative speed of image with respect to object is $2u$.
- When mirror moves towards the stationary object with speed u, the image will move with speed $2u$ in same direction as that of mirror.



Mirror at rest | Mirror is moving

- A man of height h requires a mirror of length at least equal to $h/2$, to see his own complete image.
- To see complete wall behind himself, a person requires a plane mirror of at least one third the height of wall. It should be noted that person is standing in the middle of the room.

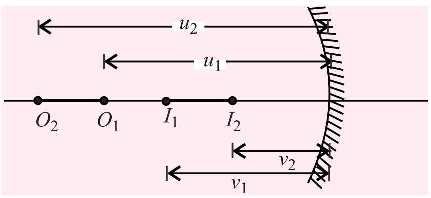
Reflection through Spherical Surfaces



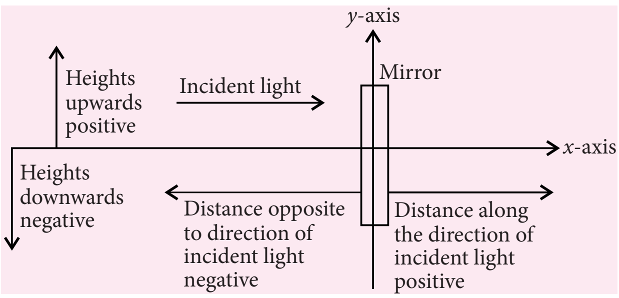
- Focal length $f = R/2$
- Mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$
- Lateral magnification,

$$m = \frac{h_i}{h_0} = \frac{-v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$$

- Longitudinal Magnification
  - For large object (O_1O_2) : $m_l = \frac{|v_1| - |v_2|}{|u_2| - |u_1|}$
  - For very small object : $m_l = \frac{dv}{du} = \frac{-v^2}{u^2} = -m^2$



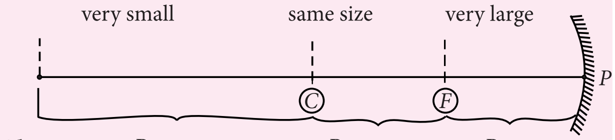
- Sign convention



Characteristic of Image

- **Concave mirror**

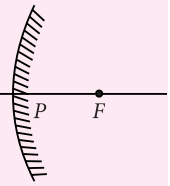
Object :	$u = \infty$	$u = R = 2f$	$u = f$
Image :	Image at F	Image at C	Image at ∞
Nature :	Real, inverted very small	Real, inverted same size	Real, inverted very large



Object :	Between ∞ and C $u > R$	Between F and C $R < u < f$	Between F and P $u < f$
Image :	Between F and C	Between C and ∞	Behind mirror
Nature :	Real, inverted, small	Real, inverted, enlarged	Virtual, upright, enlarged

- **Convex mirror**

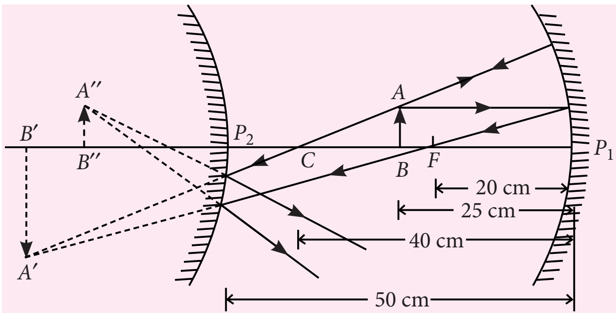
Object : $u = \infty$
Image : Image at F
Nature : Virtual, erect, very small



Object : For all position (P to ∞)
Image : Between F and P
Nature : Virtual, erect, small

Illustration 1 : An object is placed exactly midway between a concave mirror of radius of curvature 40 cm and a convex mirror of radius of curvature 30 cm. The mirrors face each other and are 50 cm apart. Determine the nature and position of the image formed by successive reflections first at the concave mirror and then at the convex mirror.

Soln.: The image formation is shown in figure.



(i) For concave mirror : $u_1 = -25$ cm, $f_1 = -20$ cm

$$\frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1} = -\frac{1}{20} + \frac{1}{25} = -\frac{1}{100} \quad \therefore v_1 = -100 \text{ cm}$$

As v_1 is negative, the image $A'B'$ is real and is formed in front of concave mirror such that $P_1B' = 100$ cm.

(ii) For convex mirror : The image $A'B'$ acts as virtual object

$\therefore \quad u_2 = +(100 - 50) = 50$ cm, $f_2 = +15$ cm

Hence, $\frac{1}{v_2} = \frac{1}{f_2} - \frac{1}{u_2} = \frac{1}{15} - \frac{1}{50} = \frac{7}{150}$

or $\quad v_2 = +21.43$ cm

As v_2 is positive, the final image $A''B''$ is virtual and is formed behind the convex mirror such that $A''B'' = 21.43$ cm.

Refraction

When light passes obliquely from one transparent medium to another, the direction of its path may change at the interface of the two media. This phenomenon is known as refraction of light.

Laws of refraction

- The incident ray, the normal to the interface at the point of incidence and the refracted ray, all lie in the same plane.
- **Snell's law :** The ratio of the sine of the angle of incidence to the sine of the angle of refraction is always a constant (a different constant for a different set of media).

  i.e., $\frac{\sin i}{\sin r} = \text{constant} = \mu_{21}$

- The refractive index of a medium for a light of given wavelength may be defined as the ratio of the speed of light in vacuum to its speed in that medium.

  $$\mu = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}} = \frac{c}{v}$$

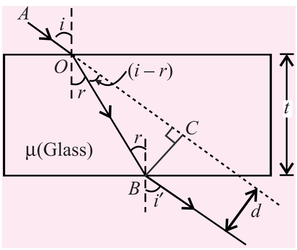
Refraction through a Glass Slab

- Lateral shift in rectangular glass slab,

  $d = BC = OB \sin(i - r)$

  $= \frac{t}{\cos r} \sin(i - r)$

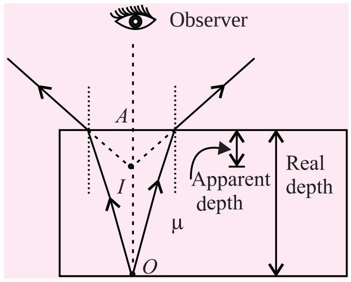
  $\therefore \quad d = t\frac{\sin(i-r)}{\cos r}$



- Apparent depth

  $= \frac{\text{Real depth}}{\text{Refractive index}\,(\mu)}$

  $AI = \frac{AO}{\mu}$ or $AO = \mu \times AI$

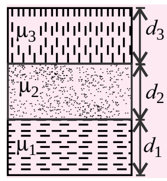


- If there is an ink spot at the bottom of the glass slab of thickness t, it appears to be raised by a distance, d = thickness – apparent depth

  $= t - \frac{t}{\mu} = t\left(1 - \frac{1}{\mu}\right)$

- Apparent depth of the container shown in the figure is

  $= \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \frac{d_3}{\mu_3}.$



Total Internal Reflection (TIR)

- Conditions for total internal reflection to take place.
  - Light must travels from denser medium to rarer medium ($\mu_1 > \mu_2$)
  - Angle of incidence at the interface must be greater than the critical angle for the pair of media ($i > \theta_c$).
  - Using Snell's law,

    $\mu_1 \sin\theta_c = \mu_2 \sin 90° = \mu_2$

    $\theta_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$

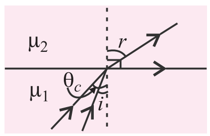
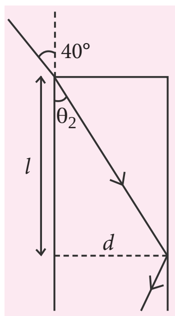


Illustration 2 : The optical fiber in figure is 2 m long and has a diameter of 20 μm. If a ray of light is incident on one end of the fiber at an angle $\theta_1 = 40°$, how many reflections does it make before emerging from the other end? (The index of refraction of the fiber is 1.30.)



Soln.: The condition for total internal reflection is fulfilled. Then by Snell's law,

$$\sin\theta_2 = \frac{\mu_1}{\mu_2}\sin\theta_1 = \frac{\sin 40°}{1.3} = 0.495 \text{ and } \theta_2 = 29.7°$$

The length along fiber to the first reflection is

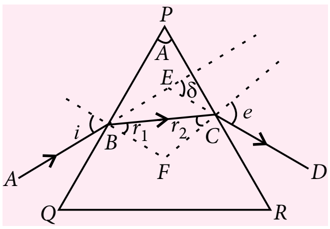
$$l = \frac{d}{\tan\theta_2} = \frac{2\times10^{-5}}{0.570} = 3.51 \times 10^{-5}\ \text{m}$$

$\therefore$ By the law of reflection, l also represents the length along the fiber between successive reflections, the number of reflections is

$$\frac{L}{l} = \frac{2\ \text{m}}{3.51\times10^{-5}\ \text{m}} \approx 57000$$

Refraction through a Glass Prism

- Angle of deviation, $\delta = (i - r_1) + (e - r_2) = i + e - A$
- Angle of minimum deviation occurs at $i = e$. Also, $r_1 = r_2 = A/2$
  $\therefore\ \delta_m = 2i - A$



- Refractive index of prism,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

- If A and δ_m are small, then $\delta_m = (\mu - 1)A$

Dispersion : Splitting of Light

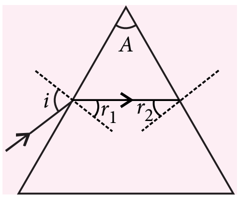
- Refractive index of a material depends slightly on the wavelength of light, $\mu = \mu_0 + \frac{A}{\lambda^2}$; This is Cauchy's formula.
- Deviation of violet, yellow and red light are $\delta_v = (\mu_v - 1)A$; $\delta_y = (\mu_y - 1)A$, $\delta_r = (\mu_r - 1)A$
  - δ_y is also known as average deviation.
- Angular dispersion, $\theta = \delta_v - \delta_r = (\mu_v - \mu_r)A$
- Dispersive power of the medium, $\omega = \frac{\theta}{\delta_y} = \frac{\mu_v - \mu_r}{\mu_y - 1}$
  - It depends only on prism material.

Illustration 3 : What is the required condition, if the light incident on one face of a prism, does not emerge from the other face?

Soln.: For no emergence, $r_2 > \theta_c$

$A - r_1 > \theta_c$

$\sin(A - r_1) > \sin\theta_c$



$$\sin A \cos r_1 - \cos A \sin r_1 > \frac{1}{\mu}$$

$$\sin A\,[\cos r_1] - \cos A\left[\frac{\sin i}{\mu}\right] > \frac{1}{\mu}$$

$$\Rightarrow\quad \mu \sin A\sqrt{1-\sin^2 r_1} - \cos A \sin i > 1$$

$$\Rightarrow\quad \mu \sin A\sqrt{\left(1-\frac{\sin^2 i}{\mu^2}\right)} > 1 + \cos A \sin i$$

$$\Rightarrow\quad \sin A \times \sqrt{\mu^2 - \sin^2 i} > (1 + \cos A \sin i)$$

Squaring both sides,

$\sin^2 A\,(\mu^2 - \sin^2 i) > (1 + \cos A \sin i)^2$

$(\mu^2 \sin^2 A - \sin^2 A \sin^2 i) > 1 + \cos^2 A \sin^2 i + 2\cos A \sin i$

$\mu^2 \sin^2 A > 1 + (\cos^2 A + \sin^2 A)\sin^2 i + 2\cos A \sin i$

$\mu^2 \sin^2 A > 1 + \sin^2 i + 2\cos A \sin i$

The greatest value of $\sin i = 1$

$\Rightarrow\quad \mu^2 \sin^2 A > 1 + 1 + 2\cos A$

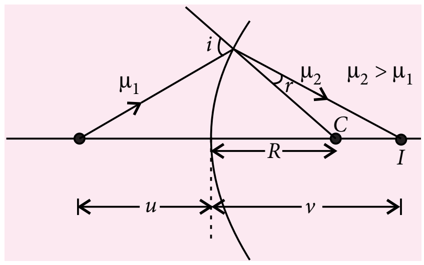
$$\mu^2\left(2^2 \sin^2\frac{A}{2}\cos^2\frac{A}{2}\right) > 2\,(1 + \cos A)$$

$$4\mu^2 \sin^2\frac{A}{2}\cos^2\frac{A}{2} > 4\cos^2\frac{A}{2}$$

$$\Rightarrow\quad \mu^2 > \frac{1}{\sin^2\frac{A}{2}} \Rightarrow \mu > \operatorname{cosec}\left(\frac{A}{2}\right)$$

Refraction at Transparent Spherical Surfaces

- Standard result : $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$
- This formula is valid for any curved spherical surface.



- If refracting surface is plane then $R = \infty$.
- $\frac{\mu_2}{v} - \frac{\mu_1}{u} = 0$ or $\frac{u}{v} = \frac{\mu_1}{\mu_2}$ or $\frac{\text{Actual distance}}{\text{Apparent distance}} = \frac{\mu_1}{\mu_2}$
- Lens maker's formula, $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
- Thin lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$;

  Power of a lens, $P = \frac{1}{f(\text{m})} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

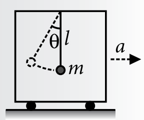
BRAIN MAP

CLASS XI

WORK AND ENERGY

Pendulum Suspended in an Accelerating Trolley

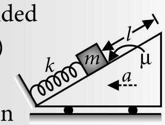
- For a pendulum suspended from the ceiling of a trolley moving with acceleration a, the maximum deflection θ of the pendulum from the vertical is $\theta = 2\tan^{-1}\left(\frac{a}{g}\right)$



Work Energy Theorem for Non-inertial Frames

For a block of mass m welded with light spring (relaxed) When the wedge fitted moves with an acceleration a, block slides through maximum distance l relative to wedge,

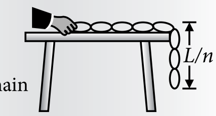
$$l = \frac{2m}{k}[a(\cos\theta - \mu\sin\theta) - g(\sin\theta + \mu\cos\theta)]$$



Work Done in Pulling the Chain

$$W = \frac{MgL}{2n^2}$$

{M = Mass of chain
L = Length
n = Fraction of chain hanged}



Different cases explained using work energy theorem

Work Energy Theorem

Work done by a force acting on a body is equal to the change in the kinetic energy of the body. It is valid for a system in presence of all types of forces.

$W_{total} = \Delta K$

Work

Work done by a force (F) is equal to the scalar product of the force and the displacement (S) of the body.

$W = \vec{F} \cdot \vec{S}$ (θ is the angle between F and S)

$W = FS\cos\theta$

Energy

The energy of a body is defined as its capacity for doing work. Energy is a scalar quantity.

Unit and dimensions for both energy and work are same

Dimensions : $[ML^2T^{-2}]$

S.I. Unit : joule (J)

Nature of Work Done

- Positive work ($0° \le \theta < 90°$)
  Component of force is parallel to displacement
- Negative work ($90° < \theta \le 180°$)
  Component of force is opposite to displacement
- Zero work ($\theta = 90°$)
  Force is perpendicular to displacement

Work Depends on Frame of Reference

With change of the frame of reference (inertial), force does not change while displacement may change. So the work done by a force will vary in different frames.

Work Done by Friction

- Work done by static friction is always zero.
- Work done by kinetic friction on the system is always negative.

Work Done by a Spring Force

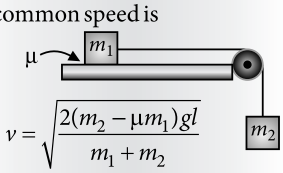
- Work done for a displacement from x_i to x_f

$$W_s = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

Motion of Blocks Connected with Pulley

- Two blocks connected by a string, as shown. If they are released from rest. After they have moved a distance l, their common speed is

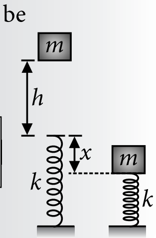
$$v = \sqrt{\frac{2(m_2 - \mu m_1)gl}{m_1 + m_2}}$$



An Application of Conservation of Energy

- A block of mass m, falling from height h, on a mass less spring of stiffness k.
  - The maximum compression in the spring will be

$$x = \frac{mg}{k}\left[1 + \sqrt{1 + \frac{2kh}{mg}}\right]$$



- If block is released slowly ($h = 0$), maximum compression, $x = \frac{2mg}{k}$
- Work done in bringing the block to stable equilibrium, $W_{ext} = -\frac{m^2g^2}{2k}$

Potential Energy

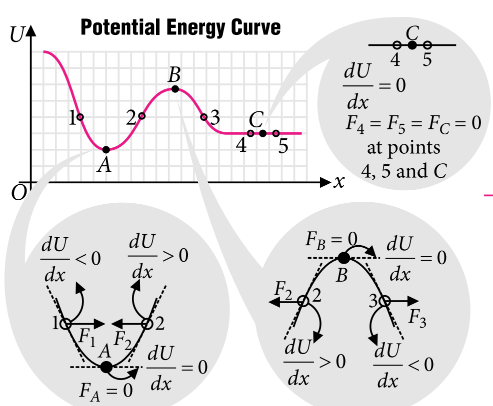
It is the ability of doing work by a conservative force. It arises from the configuration of the system or position of the particles in the system.

Relation between Conservative Force and Potential Energy

Negative gradient of the potential energy gives force.

$$F = -\frac{dU}{dr}$$

Potential Energy Curve



BIOT-SAVART LAW

BRAIN MAP
CLASS XII

APPLICATIONS OF BIOT-SAVART LAW

BIOT-SAVART LAW

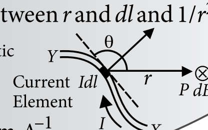
Whenever current is passed through a conductor, a magnetic field is set up around it. Magnetic field at any point was first explained by **Jean Biot (1774- 1862)** and **Felix Savart (1791-1841)**.

If dB is the magnetic field at a point P, at a distance r from element length dl, then according to this law, dB is proportional to current I, dl, sine of angle between r and dl and $1/r^2$.

$$dB = k\frac{Idl\sin\theta}{r^2}$$

μ_0 is called magnetic permeability of the medium.

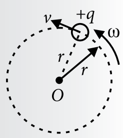
$$k = \frac{\mu_0}{4\pi} = 10^{-7}\ \text{T m A}^{-1}$$



Magnetic Field due to a Circulating Charge at Centre

$$B_0 = \frac{\mu_0}{4\pi}\frac{q\omega}{r}$$

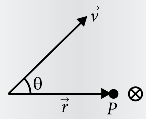
(ω is the angular velocity)



Magnetic Field due to a Moving Charge

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$$

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$$



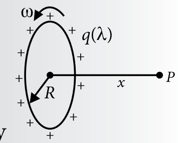
Magnetic Field on the Axis of a Spinning Charged Ring

$$B = \frac{\mu_0\omega\lambda}{2}\frac{R^3}{(R^2+x^2)^{3/2}}$$

At centre of the ring,

$x = 0$; $B = \dfrac{\mu_0\omega\lambda}{2}$

λ = linear charge density



Magnetic field at a distance R from the centre of the strip at a right angle to the strip

$$B = \frac{\mu_0 I}{2\pi a}\tan^{-1}\left(\frac{a}{R}\right)$$

Magnetic Field on the Axis of a Spinning Charged Non-conducting thin Disc

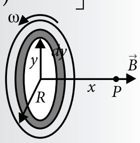
$$B = \frac{\mu_0\sigma\omega}{2}\left[\frac{(R^2+2x^2)}{\sqrt{(R^2+x^2)}} - 2(x)\right]$$

At the centre,

$x = 0$

$$B_{centre} = \frac{\mu_0\sigma\omega R}{2}$$

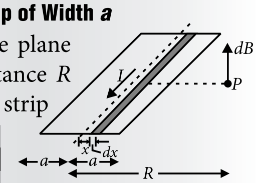
σ = surface charge density



Field due to a Current Carrying Flat Strip of Width *a*

Magnetic field in the plane of the strip at a distance R from the centre of the strip

$$B = \frac{\mu_0 I}{4\pi a}\ln\left[\frac{R+a}{R-a}\right]$$



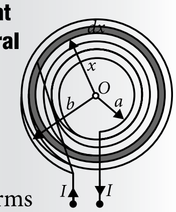
Magnetic Field at the Centre of Spiral

$$B = \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

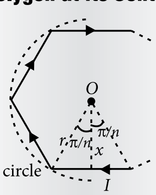
a = inner radius

b = outer radius

N = number of terms



Field due to Regular Polygon at its Centre



$$B_{net} = \frac{\mu_0}{2\pi}\frac{nI}{r}\tan\frac{\pi}{n}$$

(n = no. of sides of polygon)

Magnetic Field due to Current Carrying Solenoid

$$B = \frac{\mu_0 nI}{2}[\sin\alpha + \sin\beta]$$

Inside a long solenoid,

$\alpha = \beta = 90°$; $B = \mu_0 nI$.

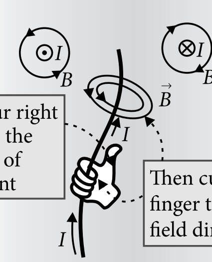
At one end of a long solenoid,

$\alpha = 0°, \beta = 90°$; $B = \mu_0 nI/2$.

Direction of magnetic field

Right Hand Thumb Rule

Direction of magnetic field because of current carrying wire may be obtained in following way.

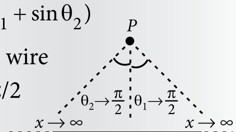


Point your right thumb in the direction of the current

Then curl your finger to get the field direction.

Field due to a Current Carrying Straight Wire

$$B = \frac{\mu_0 I}{4\pi R}(\sin\theta_1 + \sin\theta_2)$$



- For infinite wire

  $\theta_1 = \theta_2 = \pi/2$

  $$B = \frac{\mu_0 I}{2\pi R}$$

- At the end of an infinite wire

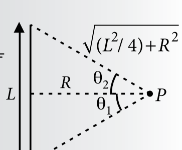
  For $\theta_1 = 0$ and $\theta_2 = \pi/2$; $B = \dfrac{\mu_0 I}{4\pi R}$

- Field at the perpendicular bisector of a straight wire

  $$B = \frac{\mu_0 I}{4\pi R}\frac{2L}{\sqrt{L^2+4R^2}}$$

  In this case

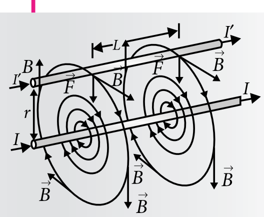
  $\theta_1 = \theta_2$



Force between Current Carrying Parallel Wires

Force per unit length

$$\frac{F_{21}}{L} = \frac{F_{12}}{L} = \frac{\mu_0 II'}{2\pi r}$$



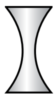
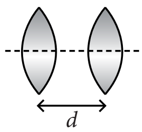
Lens		Focal Length
Biconvex lens $R_1 = R$ $R_2 = -R$		$f = \dfrac{R}{2(\mu - 1)}$
Plano-convex lens $R_1 = \infty$ $R_2 = -R$		$f = \dfrac{R}{(\mu - 1)}$
Biconcave lens $R_1 = -R$ $R_2 = +R$		$f = -\dfrac{R}{2(\mu - 1)}$
Plano-concave lens $R_1 = \infty$ $R_2 = R$		$f = \dfrac{-R}{(\mu - 1)}$
Two lenses separated by small distance d		$\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{d}{f_1 f_2}$ $P = P_1 + P_2 - dP_1P_2$
Plano convex lens with silvered plane surface		$\lvert f \rvert = \dfrac{\lvert f_L \rvert}{2} = \dfrac{R}{2(\mu - 1)}$ $P = \dfrac{2(\mu - 1)}{R}$
Plano convex lens with silvered curved surface		$\lvert f \rvert = \dfrac{R}{2\mu}$; $P = \dfrac{2\mu}{R}$
Double convex lens with one side silvered		$f = \dfrac{R}{2(2\mu - 1)}$; $P = \dfrac{2(2\mu - 1)}{R}$

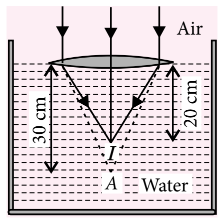
Illustration 4 : An equiconvex lens of refractive index (3/2) and focal length 10 cm is held with its axis vertical and its lower surface immersed in water (μ = 4/3), the upper surface being in air. At what distance from the lens, will a vertical beam of parallel light incident on the lens be focussed?

Soln.: According to lens-maker's formula,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Here, f = 10 cm ; μ = (3/2) ; $R_1 = R$ and $R_2 = -R$

So, $\dfrac{1}{10} = \left[\dfrac{3}{2} - 1\right]\left[\dfrac{1}{R} - \dfrac{1}{-R}\right]$ *i.e.*, R = 10 cm



Now in case of refraction from curved surface,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

So for air-glass boundary,

$$\frac{(3/2)}{v_A} - \frac{1}{-\infty} = \frac{(3/2) - 1}{+10} \Rightarrow v_A = 30 \text{ cm.}$$

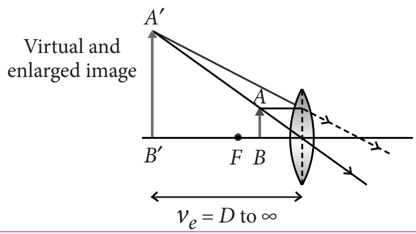
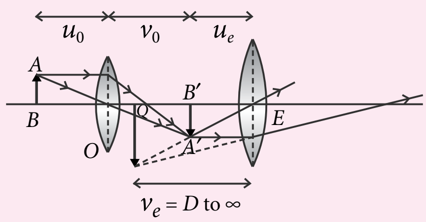
i.e., the air-glass surface forms image A at a distance of 30 cm from it as shown in figure. This image will act as an object for glass-water interface which is curved, so that

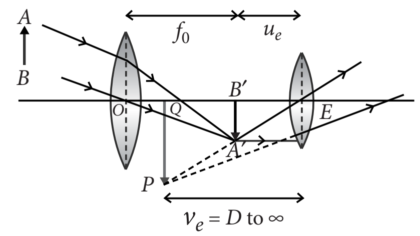
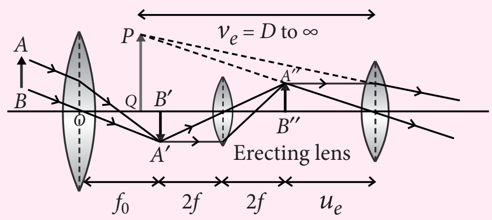
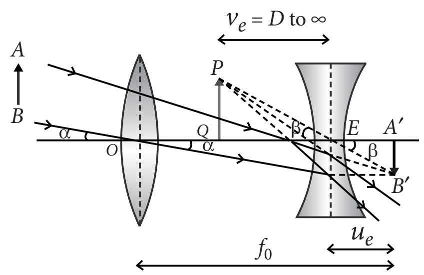
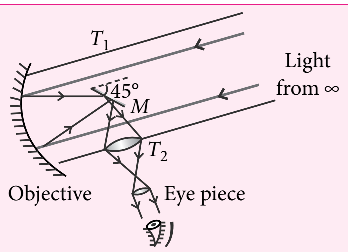
$$\frac{(4/3)}{v} - \frac{(3/2)}{+30} = \frac{(4/3) - (3/2)}{-10}$$

i.e., $\dfrac{4}{3v} = \dfrac{1}{60} + \dfrac{1}{20}$ or $v = 20$ cm

So the incident beam will converge at a distance of 20 cm from the lens inside the water.

Optical Instruments

<table>
<tr><th>Optical Instruments</th><th>Schematic Representation</th><th>Magnification</th></tr>
<tr><td>Simple Microscope</td><td>
</td><td>(i) for final image at D
$m_D = \left(1 + \dfrac{D}{f}\right)$
(ii) for final image at ∞
$m_\infty = \dfrac{D}{f}$</td></tr>
<tr><td>Compound Microscope</td><td>
</td><td>(i) for final image at D
$m_D = -\dfrac{L}{f_0}\left(1 + \dfrac{D}{f_e}\right)$
(ii) for final image at ∞
$m_\infty = -\dfrac{v_0}{u_0} \times \left(\dfrac{D}{f_e}\right)$</td></tr>
</table>

Astronomical Telescope (Refracting type)		(i) for final image at D $m_D = -\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$ $L_D = f_0 + u_e$ (ii) for final image at ∞ $m_\infty = -\frac{f_0}{f_e}$; $L_\infty = f_0 + f_e$
Terrestrial Telescope		(i) for final image at D $m_D = \frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$ $L_D = f_0 + 4f + u_e$ (ii) for final image at ∞ $m_\infty = \left(\frac{f_0}{f_e}\right)$ $L_\infty = f_0 + 4f + f_e$
Gallilean Telescope		(i) for final image at D $m_D = \frac{f_0}{f_e}\left(1-\frac{f_e}{D}\right)$ $L_D = f_0 - u_e$; $L_\infty = f_0 - f_e$ (ii) for final image at ∞ $m_\infty = \frac{f_0}{f_e}$
Reflecting Telescope		$m_D = \frac{f_0}{f_e} = \left(\frac{R/2}{f_e}\right)$ R = radius of curvature of concave mirror

WAVE OPTICS

Wavefront

Wavefront is a locus of points which vibrate in same phase. A ray of light is perpendicular to the wavefront.

Wavefronts

- Spherical Wavefront (Point source)
- Cylindrical Wavefront (Thin rod source)
- Plane Wavefront (Parallel beam of light)

Huygen's Principle

Each point on a wavefront acts as fresh source of new disturbance called secondary waves or wavelets. The new wavefront at any later time is given by the forward envelope (tangential surface) in the forward direction of the secondary wavelets at that time. The secondary wavelets spread out in all directions with the speed of light in the given medium.

- **Coherent Sources**
  - Two sources of light which continuously emit light waves of same frequency (or wavelength) with a zero or constant phase difference between them, are called coherent sources.
  - Conditions for obtaining two coherent sources of light.
    - The two sources of light must be obtained from a single source by some method.

- The two sources must give monochromatic light.
- The path difference between the waves arriving on the screen from the two sources must not be large.

Interference of Light

If two light waves of the same frequency and having zero or constant phase difference travelling in the same direction then their superposition gets redistributed becoming maximum at some points and minimum at others. This phenomenon is called interference of light.

Addition of Coherent Waves

If A_1, A_2 are the amplitudes of interfering waves due to two coherent sources and ϕ is constant phase difference between the two waves at any point P, then the resultant amplitude at P will be

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

Resultant intensity at P is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

- When $\phi = 2n\pi$, where $n = 0, 1, 2, ...$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

- When $\phi = (2n - 1)\pi$, where $n = 1, 2, 3, ...$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

- If the amplitudes of the two waves are equal $A_1 = A_2 = A_0$, then resultant amplitude

$$A = \sqrt{2A_0^2 + 2A_0^2\cos\phi} = 2A_0\cos\left(\frac{\phi}{2}\right)$$

Resultant intensity, $I = A^2 = 4I_0\cos^2\left(\frac{\phi}{2}\right)$

In this case, $A_{max} = 2A_0$, $I_{max} = 4I_0$

$A_{min} = 0$, $I_{min} = 0$

Note : If the sources are incoherent, $I = I_1 + I_2$.

Young's Double Slit Experiment

The phenomenon of interference of light was first observed by a British Physicist Thomas Young. Using two slits illuminated by a monochromatic light source, he obtained alternately bright and dark band on the screen. These bands are called interference fringes or interference bands.

- **Constructive interference :** (*i.e.*, formation of bright fringes)
  - For n^{th} bright fringe,

    Path difference = $n\lambda$ or $d\sin\theta = \frac{dy}{D} = n\lambda$

    d = distance between the two slits

    D = distance of slits from the screen
  - The position of n^{th} bright fringe from the centre of the screen is given by $y_n = n\lambda\frac{D}{d}$
- **Destructive interference :** (*i.e.*, formation of dark fringes)
  - For n^{th} dark fringe,

    path difference = $(2n-1)\frac{\lambda}{2}$
  - The position of n^{th} dark fringe from the centre of the screen is given by

$$y'_n = (2n-1)\frac{\lambda}{2}\frac{D}{d}$$

- The distance between any two consecutive bright or dark fringes is called fringe width.

  Fringe width, $\beta = \frac{\lambda D}{d}$
  - Angular fringe width, $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$
- Intensity and width ratio, $\frac{W_1}{W_2} = \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2}$
- Fringe visibility, $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$
- When entire apparatus of Young's double slit experiment is immersed in a medium of refractive index μ, then fringe width becomes

$$\beta' = \frac{\lambda' D}{d} = \frac{\lambda D}{\mu d} = \frac{\beta}{\mu}$$

- When a thin transparent plate of thickness, t and refractive index, μ is placed in the path of one of the interfering waves, fringe width remains unaffected but the entire pattern shifts by

$$\Delta y = (\mu - 1)\,t\,\frac{D}{d} = (\mu - 1)\,t\,\frac{\beta}{\lambda}$$

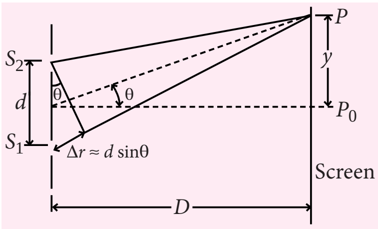
This shifting is towards the side in which transparent plate is introduced.

  - Number of shifted fringes, $N = \frac{\Delta y}{\beta} = \frac{(\mu - 1)t}{\lambda}$

Illustration 5 : In a Young's double-slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelengths, λ = 750 nm and λ' = 900 nm. At what minimum distance from the common central bright fringe on a screen 2 m from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other?

Soln.: The m^{th} bright fringe of the λ pattern and the m'^{th} bright fringe of the λ' pattern are located at

$$y_m = \frac{mD\lambda}{d} \quad \text{and} \quad y'_m = \frac{m'D\lambda'}{d}$$



Equating these distances gives

$$\frac{m}{m'} = \frac{\lambda'}{\lambda} = \frac{900}{750} = \frac{6}{5}$$

Hence, the first position at which overlapping occurs is

$$y_6 = y'_5 = \frac{(6)(2)(750\times10^{-9})}{2\times10^{-3}} = 4.5 \text{ mm}$$

Interference due to Thin Films

- Interference due to reflected light (reflected system of light)
  - For a bright fringe,

    $$2\mu t\cos r = (2n+1)\frac{\lambda}{2}\text{ ; where, } n = 0, 1, 2, 3, \ldots$$
  - For a dark fringe

    $2\mu t\cos r = n\lambda$; where, n = 0, 1, 2, 3 …
- Interference due to transmitted light (transmitted system of light)
  - For a bright-fringe,

    $2\mu t\cos r = n\lambda$; where n = 0, 1, 2, 3, …
  - For a dark fringe,

    $$2\mu t\cos r = (2n+1)\frac{\lambda}{2}\text{ ; where } n = 0, 1, 2, 3, \ldots$$
- The conditions for maxima and minima in the reflected system are just opposite to those for the transmitted system. Thus, the reflected and transmitted systems are complimentary *i.e.*, a film which appears bright by reflected light will appear dark by transmitted light and vice-versa.

Diffraction of Light

The phenomenon of bending of light around the corners of an obstacle or aperture is called diffraction of light.

- In ray optics, we ignore diffraction and assume that light travels in straight lines. This assumption is reasonable because under ordinary conditions, diffraction (bending) of light is negligible.
- The smaller the size of the obstacle or aperture, the greater is the bending (or diffraction) of light around the corners of the obstacle or aperture and vice-versa.
- The diffraction phenomenon is generally divided into the following two classes :
  - **Fresnel diffraction :** In this case, either the source or the screen or both are at finite distances from the aperture or obstacle causing diffraction.
  - **Fraunhofer diffraction :** In this case, the source and the screen on which the pattern is observed are at infinite distances from the aperture or the obstacle causing diffraction.

Single Slit Diffraction

The diffraction pattern produced by a single slit of width a consists of a central maximum bright band with alternating bright and dark bands of decreasing intensity on both sides of the central maximum.

- Condition for n^{th} secondary maximum is,

  $$\text{Path difference } = a\sin\theta_n = (2n+1)\frac{\lambda}{2}$$

  where n = 1, 2, 3, ...
- Condition for n^{th} secondary minimum is,

  Path difference = $a\sin\theta_n = n\lambda$

  where n = 1, 2, 3,.......
- Width of secondary maxima or minima

  $$\beta = \frac{\lambda D}{a} = \frac{\lambda f}{a} \quad \text{where, } a = \text{width of slit}$$

  D = distance of screen from the slit

  f = focal length of lens for diffracted light
- Width of central maximum $= \dfrac{2\lambda D}{a} = \dfrac{2f\lambda}{a}$
- Angular fringe width of central maximum $= \dfrac{2\lambda}{a}$.
- Angular fringe width of secondary maxima or minima $= \dfrac{\lambda}{a}$

Illustration 6 : Angular width of central maxima in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, the angular width decreases by 30%. What will be the wavelength of this light?

Soln.: Angular width, $\theta = \frac{2\lambda}{a}$

$$\Rightarrow \frac{\theta_1}{\theta_2} = \frac{\lambda_1}{\lambda_2} \Rightarrow \frac{\theta}{\frac{70}{100}\theta} = \frac{6000}{\lambda_2} \Rightarrow \lambda_2 = 4200 \text{ Å}$$

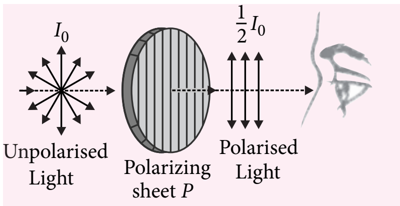
Resolving Power of Optical Instruments

Resolving power is the ability of the instrument to resolve or to see as separate, the images of two close objects.

- Resolving power $(R) = \frac{1}{\text{Resolving limit}}$
  - Resolving power of microscope, $R = \frac{1}{\Delta d} = \frac{2\mu \sin\theta}{\lambda}$
  - Resolving power of telescope, $R = \frac{1}{\Delta\theta} = \frac{a}{1.22\lambda}$
    a = diameter of objective lens of the telescope.

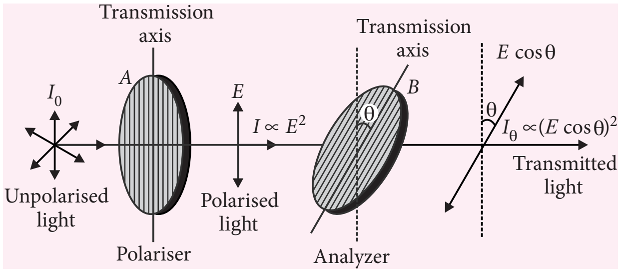
Polarisation of Light

The phenomenon of restricting the vibrations of light (electric vector) in a particular direction, perpendicular to direction of wave motion is called polarisation of light.

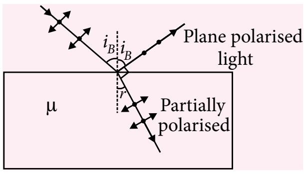


- Polarisation of light confirms the transverse nature of light. The plane in which vibrations of polarised light are confined is called plane of vibration. A plane which is perpendicular to the plane of vibration is called plane of polarisation.
- Plane polarised light can be produced by the following methods
  - By reflection
  - By scattering
  - By refraction
  - By dichroism
  - By double refraction

- **Malus's Law :** Intensity of transmitted light by analyser depends on the angle between the polariser and analyser.
  $I_\theta \propto \cos^2\theta \Rightarrow I_\theta = I\cos^2\theta$
  I = intensity of polarised light passing through the polariser.



- **Polarisation by reflection (Brewster's law)**
  - When unpolarised light is reflected from a plane boundary between two transparent media, the reflected light is partially polarised. The degree of polarisation depends on the angle of incidence and the ratio of wave speeds in the two media.
  - At polarising angle (Brewster's angle), the reflected and refracted beams are mutually perpendicular to each other.



$\sin i_B = \mu\sin(r)$
$\sin i_B = \mu\sin(90 - i_B)$
$\Rightarrow \mu = \tan i_B$
This is the Brewster's law.

Illustration 7 : Two polaroids are oriented with their planes perpendicular to incident light and transmission axis making an angle of 30° with each other. What fraction of incident unpolarised light is transmitted?

Soln.: Using Malus' law, $I_{transmitted} = I_{incident} \times \cos^2\theta$

On the first polaroid, $I_1 = I_0 (\cos^2\theta)_{average}$

For unpolarized light $(\cos^2\theta)_{average} = \frac{1}{2}$

$\Rightarrow I_1 = \frac{I_0}{2}$

For the second polaroid, $\theta = 30°$

$$\Rightarrow I_2 = I_1\cos^2 30° = \left(\frac{I_0}{2}\right)\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8}I_0 \text{ or } \frac{I_2}{I_0} = \frac{3}{8}$$

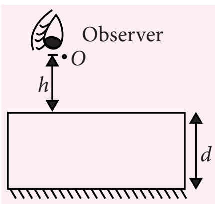
SPEED PRACTICE

1. Convex lens made up of glass (refractive index of glass = 1.5) and has radius of curvature R is dipped into water. Its focal length will be (refractive index of water = 1.33)
(a) $4R$ (b) $2R$ (c) R (d) $R/2$

2. A convex lens is made of a material having refractive index 1.2. If it is dipped in water (μ = 1.33), it will behave like a
(a) convergent lens (b) divergent lens
(c) a rectangular slab (d) a prism

3. Path difference for the first secondary maximum in the diffraction pattern of a single slit is given by (a is the width of the slit)
(a) $a \sin\theta = \frac{\lambda}{2}$ (b) $a \cos\theta = \frac{3\lambda}{2}$
(c) $a \sin\theta = \lambda$ (d) $a \sin\theta = \frac{3\lambda}{2}$

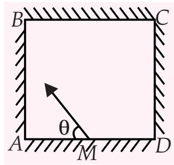
4. A point luminous object (O) is at a distance h from front face of a glass slab of width d and of refractive index μ. A reflecting plane mirror is fixed on the back face of slab. An observer sees the image of object in mirror as shown in figure. Distance of image from front face as seen by observer will be



(a) $h + \frac{2d}{\mu}$ (b) $h + 2d$
(c) $h + d$ (d) $2h + \frac{d}{\mu}$

5. A point source of light is placed at a depth h below the surface of water of refractive index μ. A floating opaque disc is placed on the surface of water so that light from the source is not visible from the surface. The minimum diameter of the disc is
(a) $\frac{2h}{(\mu^2 - 1)^{1/2}}$ (b) $2h(\mu^2 - 1)^{1/2}$
(c) $\frac{h}{2(\mu^2 - 1)^{1/2}}$ (d) $h(\mu^2 - 1)^{1/2}$

6. Four identical mirrors are made to stand vertically to form a square arrangement as shown in a top view. A ray starts from the midpoint M of mirror AD and after two reflections reaches corner D. Then, angle θ must be



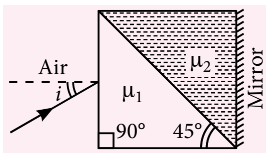
(a) $\tan^{-1}(0.75)$ (b) $\cot^{-1}(0.75)$
(c) $\sin^{-1}(0.75)$ (d) $\cos^{-1}(0.75)$

7. An eye specialist prescribes spectacles having a combination of a convex lens of focal length 40 cm in contact with a concave lens of focal length 25 cm. The power of this lens combination is
(a) + 1.5 D (b) – 1.5 D
(c) + 6.67 D (d) – 6.67 D

8. A thin convergent glass lens (μ_g = 1.5) has a power of +5.0 D. When this lens is immersed in a liquid of refractive index μ_l, it acts as a divergent lens of focal length 100 cm. The value of μ_l is
(a) 4/3 (b) 5/3 (c) 5/4 (d) 6/5

9. When a thin transparent sheet of refractive index $\mu = \frac{7}{5}$ is placed near one of the slits in Young's double slits experiment, the intensity at the centre of the screen reduces to half of the maximum intensity. The minimum thickness of the sheet should be
(a) $\frac{\lambda}{4}$ (b) $\frac{5\lambda}{8}$ (c) $\frac{3\lambda}{2}$ (d) $\frac{\lambda}{3}$

10. In the given figure, for what value of i, the incidence ray will retrace its initial path?



(a) $\cos^{-1}\left[\mu_1 \sin\left(\frac{\pi}{4} - \sin^{-1}\frac{\mu_2}{\sqrt{2}\mu_1}\right)\right]$

(b) $\sin^{-1}\left[\mu_1 \sin\left(\frac{\pi}{4} - \sin^{-1}\frac{\mu_2}{\sqrt{2}\mu_1}\right)\right]$

(c) $\sin^{-1}\left[\mu_2 \sin\left(\frac{\pi}{4} - \sin^{-1}\frac{\mu_1}{\sqrt{2}\mu_2}\right)\right]$

(d) $\cos^{-1}\left[\mu_2 \sin\left(\frac{\pi}{4} - \sin^{-1}\frac{\mu_1}{\sqrt{2}\mu_2}\right)\right]$

11. A boat has green light of wavelength 500 nm on the mast. What wavelength would be measured and what colour would be observed for this light as seen by a diver submerged in water by the side of the boat? (Refractive index of water = $\frac{4}{3}$)

(a) Green of wavelength 375 nm
(b) Red of wavelength 665 nm
(c) Green of wavelength 500 nm
(d) Blue of wavelength 375 nm

12. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is

(a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$
(c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$

13. The ratio of resolving powers of an optical microscope for two wavelengths λ_1 = 4000 Å and λ_2 = 6000 Å is

(a) 9 : 4 (b) 3 : 2 (c) 16 : 81 (d) 8 : 27

[NEET 2017]

14. Young's double slit experiment is first performed in air and then in a medium other than air. It is found that 8^{th} bright fringe in the medium lies where 5^{th} dark fringe lies in air. The refractive index of the medium is nearly

(a) 1.59 (b) 1.69 (c) 1.78 (d) 1.25

[NEET 2017]

15. A beam of light from a source L is incident normally on a plane mirror fixed at a certain distance x from the source. The beam is reflected back as a spot on a scale placed just above the source L. When the mirror is rotated through a small angle θ, the spot of the light is found to move through a distance y on the scale. The angle θ is given by

(a) $\frac{y}{x}$ (b) $\frac{x}{2y}$ (c) $\frac{x}{y}$ (d) $\frac{y}{2x}$

[NEET 2017]

16. A thin prism having refracting angle 10° is made of glass of refractive index 1.42. This prism is combined with another thin prism of glass of refractive index 1.7. This combination produces dispersion without deviation. The refracting angle of second prism should be

(a) 6° (b) 8°
(c) 10° (d) 4° **[NEET 2017]**

17. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is

(a) real and at a distance of 40 cm from convergent lens
(b) virtual and at a distance of 40 cm from convergent lens
(c) real and at a distance of 40 cm from the divergent lens
(d) real and at a distance of 6 cm from the convergent lens. **[JEE Main Offline 2017]**

18. In a Young's double slit experiment, slits are separated by 0.5 mm and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is

(a) 1.56 mm (b) 7.8 mm
(c) 9.75 mm (d) 15.6 mm

[JEE Main Offline 2017]

19. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer ? (speed of light = 3×10^8 m s^{-1})

(a) 10.1 GHz (b) 12.1 GHz
(c) 17.3 GHz (d) 15.3 GHz

[JEE Main Offline 2017]

20. Let the refractive index of a denser medium with respect to a rarer medium be μ_{12} and its critical angle θ_C. At an angle of incidence A when light is travelling from denser medium to rarer medium, a part of the light is reflected and the rest is refracted and the angle between reflected and refracted rays is 90°. Angle A is given by

(a) $\frac{1}{\tan^{-1}(\sin\theta_C)}$ (b) $\frac{1}{\cos^{-1}(\sin\theta_C)}$
(c) $\tan^{-1}(\sin\theta_C)$ (d) $\cos^{-1}(\sin\theta_C)$

[JEE Main Online 2017]

SOLUTIONS

1. **(a) :** $\frac{1}{f_w}=\left(\frac{\mu_g}{\mu_w}-1\right)\left(\frac{1}{R}-\frac{1}{(-R)}\right)=\left(\frac{(3/2)}{(4/3)}-1\right)\left(\frac{2}{R}\right)$

or $f_w = 4R$

2. **(b) :** As per lens maker's formula,

$$\frac{1}{f}=(\mu_l-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ...(i)$$

For given convex lens, $\mu_l = 1.2$ and $f > 0$.

When it is dipped in water,

$$\frac{1}{f'}=\left(\frac{\mu_l}{\mu_w}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ...(ii)$$

From eqns. (i) and (ii),

$$\frac{f'}{f}=\frac{(\mu_l-1)}{\left(\frac{\mu_l}{\mu_w}-1\right)}=\frac{(1.2-1)}{\left(\frac{1.2}{1.33}-1\right)} \quad (\because\ \mu_w = 1.33)$$

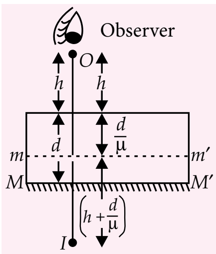
or $f' = -2.05f$; $\because f > 0$,

$\therefore$ $f' < 0$, *i.e.*, it will behave like a divergent lens.

3. **(d) :** For n^{th} secondary maximum, path difference

$a \sin\theta = (2n+1)\frac{\lambda}{2}$; For $n = 1$, $a\sin\theta = \frac{3\lambda}{2}$

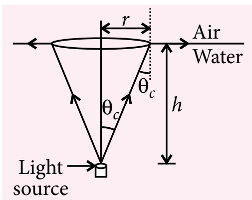
4. **(a) :** As shown in figure, glass slab will form the image of bottom *i.e.*, mirror MM' at a depth (d/μ) from its front face. So the distance of object O from virtual mirror mm' will be $h + (d/\mu)$.



Now as a plane mirror forms image behind the mirror at the same distance as the object is in front of it, the distance of image I from mm' will be $h + (d/\mu)$ and as the distance of virtual mirror from the front face of slab is (d/μ), the distance of image I from front face as seen by observer

$$=\left[h+\frac{d}{\mu}\right]+\frac{d}{\mu}=h+\frac{2d}{\mu}.$$

5. **(a) :** The figure shows incidence of ray from water at critical angle θ_c,



Now, $\sin\theta_c = \frac{1}{\mu}$

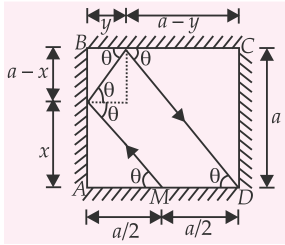
so that $\tan\theta_c = \frac{1}{(\mu^2-1)^{1/2}}$

From figure, $\tan\theta_c = \frac{r}{h}$

where r is the radius of the disc.

$\therefore$ Diameter of the disc $= 2r = 2h\tan\theta_c = \frac{2h}{(\mu^2-1)^{1/2}}$

6. **(b) :** The ray starting from midpoint M at an angle θ reaches the corner D at the right along a parallel path. Let a be the length of the side.



From figure,

$$\tan\theta=\frac{x}{(a/2)} \quad ...(i); \qquad \tan\theta=\frac{a-x}{y} \quad ...(ii)$$

$$\tan\theta=\frac{a}{a-y} \quad ...(iii)$$

From eqns. (i) and (ii), we get

$$\frac{2x}{a}=\frac{a-x}{y} \text{ or } 2xy = a^2 - xa \quad ...(iv)$$

From eqns. (ii) and (iii), we get

$$\frac{a-x}{y}=\frac{a}{a-y} \Rightarrow 3xy = 2ay \quad \text{(Using (iv))}$$

$$x=\frac{2a}{3}$$

Substituting this value of x in eqns. (i), we get

$$\tan\theta=\frac{(2a/3)}{(a/2)}=\frac{4}{3} \quad \therefore\ \cot\theta=\frac{1}{\tan\theta}=\frac{3}{4}$$

or $\theta = \cot^{-1}(0.75)$

7. **(b) :** Power of the lens combination is

$$P = P_1 + P_2 = \frac{1}{f_1(\text{in m})}+\frac{1}{f_2(\text{in m})}$$

$$=\frac{1}{+0.40\,\text{m}}+\frac{1}{-0.25\,\text{m}} = -1.5\ \text{m}^{-1} = -1.5\ \text{D}$$

8. **(b) :** Given, power = +5 D; $\therefore f = 20$ cm

$$\frac{1}{f}=\left(\frac{\mu_g-\mu_a}{\mu_a}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{20}=\left(\frac{1.5-1}{1}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ...(i)$$

$$\text{and } \frac{1}{-100}=\left(\frac{1.5-\mu_l}{\mu_l}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad ...(ii)$$

Dividing eqn. (i) by (ii), we get $\mu_l = \frac{5}{3}$

9. **(b) :** $I = 4I_0\cos^2\frac{\phi}{2} \Rightarrow 2I_0 = 4I_0\cos^2\frac{\phi}{2}$

$$\therefore \quad \cos\frac{\phi}{2}=\frac{1}{\sqrt{2}} \Rightarrow \phi = 2\times\frac{\pi}{4}=\frac{\pi}{2}$$

Path difference, $\Delta x = \frac{\lambda}{2\pi}\times\phi = \frac{\lambda}{2\pi}\frac{\pi}{2} = \frac{\lambda}{4}$

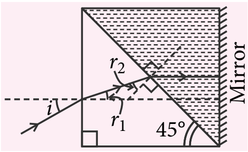
$\Rightarrow \quad (\mu - 1)t = \frac{\lambda}{4}$ or $0.4t = \frac{\lambda}{4} \Rightarrow t = \frac{5\lambda}{8}$

10. (b): Since incident ray retraces its path, it must strike the plane mirror perpendicularly.

From Snell's law, $\quad \sin i = \mu_1 \sin r_1 \quad$ ...(i)

and $\mu_1 \sin r_2 = \mu_2 \sin 45° \Rightarrow \mu_1 \sin r_2 = \frac{\mu_2}{\sqrt{2}} \quad$ ...(ii)

$\Rightarrow \quad r_2 = \sin^{-1}\left(\frac{\mu_2}{\sqrt{2}\mu_1}\right)$



Also, $r_1 + r_2 = \frac{\pi}{4}$

$\therefore \quad r_1 = \frac{\pi}{4} - \sin^{-1}\left(\frac{\mu_2}{\sqrt{2}\mu_1}\right) \quad$ (Using (ii))

From eqn. (i),

$\therefore \quad i = \sin^{-1}\left[\mu_1 \sin\left(\frac{\pi}{4} - \sin^{-1}\frac{\mu_2}{\sqrt{2}\mu_1}\right)\right]$

11. (a)

12. (b): As $I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

or, $\quad \frac{I_{max}}{2} = I_{max}\cos^2\left(\frac{\phi}{2}\right) \Rightarrow \frac{1}{2} = \cos^2\frac{\phi}{2}$

$\Rightarrow \quad$ Phase difference, $\phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

or $\quad \phi = (2n+1)\frac{\pi}{2}$ where $n = 0, 1, 2, \ldots$

$\therefore \quad$ Path difference $= \frac{\lambda}{2\pi}\times$ phase difference

$= \frac{\lambda}{2\pi}\times(2n+1)\frac{\pi}{2} = (2n+1)\frac{\lambda}{4}$

13. (b): The resolving power of an optical microscope,

$\text{RP} = \frac{2\mu\sin\theta}{\lambda}; \therefore \frac{\text{RP}_1}{\text{RP}_2} = \frac{6000}{4000} = \frac{3}{2}$

14. (c) : Position of 8th bright fringe in medium,

$x = \frac{8\lambda_m D}{d}$

Position of 5th dark fringe in air,

$x' = \frac{(5 - 1/2)\lambda_{air} D}{d} = \frac{4.5\lambda_{air} D}{d}$

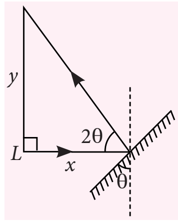
Given $x = x'$

$\therefore \quad \frac{8\lambda_m D}{d} = \frac{4.5\lambda_{air} D}{d}; \ \mu_m = \frac{\lambda_{air}}{\lambda_m} = \frac{8}{4.5} \approx 1.78$

15. (d): When mirror is rotated by θ, reflected ray will be rotated by 2θ.

For small angle θ,

$\tan 2\theta \approx 2\theta = \frac{y}{x} \quad \therefore \ \theta = \frac{y}{2x}$



16. (a) : The condition for dispersion without deviation is given as $(\mu - 1)A = (\mu' - 1)A'$

Given $\mu = 1.42$, $A = 10°$, $\mu' = 1.7$, $A' = ?$

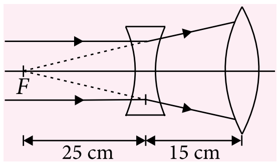
$\therefore (1.42 - 1) \times 10 = (1.7 - 1)A'; (0.42) \times 10 = 0.7 \times A'$

or $A' = \frac{0.42\times 10}{0.7} = 6°$

17. (a) : Given : focal length of concave lens, $f = -25$ cm

Focal length of convex lens, $f' = 20$ cm

The formation of image is shown in the figure.



The image for diverging lens will form at F, *i.e.*, at focal length of concave lens.

Now, this image will serve as the object for convex lens. It is at twice the focal length of the convex lens (*i.e.*, $2f'$). Hence, the final image will also form at $2f'$, which is at a distance of 40 cm from the convergent lens. Also, the image formed is real.

18. (b): Let y be the distance from the central maximum to the point where the bright fringes due to both the wavelengths coincides.

Now, for λ_1, $y = \frac{m\lambda_1 D}{d}$; for λ_2, $y = \frac{n\lambda_2 D}{d}$

$\therefore \quad m\lambda_1 = n\lambda_2; \ \frac{m}{n} = \frac{\lambda_2}{\lambda_1} = \frac{520}{650} = \frac{4}{5}$

i.e., with respect to central maximum, 4th bright fringe of λ_1 coincides with 5th bright fringe of λ_2.

Now, $y = \frac{4\times 650\times 10^{-9}\times 1.5}{0.5\times 10^{-3}}$ m $= 7.8$ mm

19. (c) : Frequency of the microwave measured by the observer will be given by Doppler's effect of light.

$\frac{\upsilon'}{\upsilon} = \sqrt{\frac{1+\beta}{1-\beta}} \quad \left(\text{Here } \beta = \frac{v}{c}\right)$

$\Rightarrow \quad \frac{\upsilon'}{\upsilon} = \sqrt{\frac{1+v/c}{1-v/c}} = \sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{c+c/2}{c-c/2}} = \sqrt{3}$

$= 10\sqrt{3}$ GHz $= 17.3$ GHz

20. (c)

◈◈

EXAM PREP 2018

CLASS XII

Useful for Medical/Engg. Entrance Exams

CHAPTERWISE MCQS FOR PRACTICE

ELECTROMAGNETIC WAVES

1. Light ranges in wavelength roughly from violet at 390 nm to red at 780 nm. Its speed in vacuum is about 3×10^8 m s^{-1}, as is the case for all electromagnetic waves. The corresponding frequency range is

(a) 450-900 THz (b) 380-770 THz
(c) 560-780 THz (d) 680-980 THz

2. Given a harmonic plane electromagnetic wave whose electric field has the form

$E_z(y,t) = E_{0z}\sin\left[\omega\left(t - \frac{y}{c}\right) + \varepsilon\right]$ (where ε is a constant phase angle). The corresponding magnetic field will be

(a) $\frac{1}{c}E_z(y,t)$ (b) $cE_z(y, t)$
(c) $c^2E_z(y, t)$ (d) $\frac{1}{c^2}E_z(y,t)$

3. Imagine an electromagnetic plane wave in vacuum whose electric field (in SI units) is given by
$E_x = 10^2 \sin \pi(3 \times 10^6 z - 9 \times 10^{14} t)$; $E_y = 0$; $E_z = 0$.
The frequency and wavelength will be

(a) 6.2×10^{14} Hz and 530 nm
(b) 3.2×10^{15} Hz and 630 nm
(c) 4.5×10^{14} Hz and 666 nm
(d) 4.5×10^{14} Hz and 450 nm

4. A plane electromagnetic wave moving through free space has an electric field (also referred to as the optical field) given by $E_x = 0$, $E_y = 0$ and

$$E_z = 100\sin\left[8\pi\times10^{14}\left(t - \frac{x}{3\times10^8}\right)\right] \text{ V m}^{-1}.$$

The corresponding flux density is

(a) 13.3 W m^{-2} (b) 11.4 W m^{-2}
(c) 25.0 W m^{-2} (d) 20.4 W m^{-2}

5. The charge on a parallel plate capacitor is varying as $q = q_0 \sin 2\pi \cdot \upsilon t$. The plates are very large and close together. Neglecting the edge effects, the displacement current through the capacitor is

(a) $\frac{q}{\varepsilon_0 \upsilon}$ (b) $\frac{q_0}{\varepsilon_0}\sin 2\pi\upsilon t$
(c) $2\pi\upsilon q_0 \cos 2\pi\upsilon t$ (d) $\frac{2\pi\upsilon q_0}{\varepsilon_0}\cos 2\pi\upsilon t$

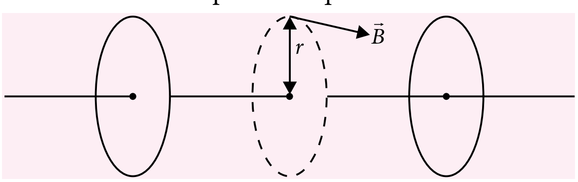
6. A plane electromagnetic wave
$E_z = 10 \cos (6 \times 10^8 t + 4x)$ V m^{-1} is propagating in a medium of refractive index μ. Find μ.

(a) 1.2 (b) 1.5 (c) 2.0 (d) 2.4

7. In a region of free space during the propagation of electromagnetic wave, the electric field at some instant of time is $\vec{E} = (90\,\hat{i} + 40\,\hat{j} - 70\,\hat{k})$ N C^{-1} and the magnetic field is $\vec{B} = (0\cdot18\hat{i} + 0\cdot08\hat{j} + 0\cdot30\hat{k})\,\mu$T. The poynting vector for these fields is

(a) $(14\cdot0\hat{i} - 3\cdot148\,\hat{j})$ (b) $(14\cdot0\hat{i} - 31\cdot48\,\hat{j})$
(c) $(1\cdot4\,\hat{i} + 3\cdot148\,\hat{j})$ (d) $(14\cdot0\,\hat{i} + 31\cdot48\,\hat{j})$

8. The magnetic field strength B at the point between the capacitor plates is indicated in figure. B in terms of the rate of change of the electric field strength, *i.e.*, dE/dt between the plates is equal to



(a) $\dfrac{\mu_0}{2\pi r}\dfrac{dE}{dt}$ (b) $\dfrac{\varepsilon_0\mu_0 r}{2}\dfrac{dE}{dt}$

(c) zero (d) $\dfrac{\mu_0}{2r}\dfrac{dE}{dt}$

9. A plane electromagnetic wave in the visible region is moving along the z-direction. The frequency of the wave is 0.5×10^{15} Hz and the electric field at any point is varying sinusoidally with time with an amplitude of 1 V m^{-1}. The average values of the energy densities of the electric and the magnetic fields respectively are

(a) 2.21×10^{-12} J m^{-3}, 2.21×10^{-12} J m^{-3}
(b) 1.22×10^{-11} J m^{-3}, 1.22×10^{-11} J m^{-3}
(c) 1.22×10^{-12} J m^{-3}, 2.21×10^{-12} J m^{-3}
(d) 3.11×10^{-12} J m^{-3}, 1.22×10^{-11} J m^{-3}

10. A laser beam has intensity 2.5×10^{14} W m^{-2}. The amplitudes of electric and magnetic fields in the beam respectively are

(a) 1.2×10^7 V m^{-1}, 1.44 T
(b) 4.3×10^8 V m^{-1}, 1.44 T
(c) 5.6×10^9 V m^{-1}, 1.44 T
(d) 9.2×10^8 V m^{-1}, 3.12 T

11. A light beam travelling in the x-direction is described by the electric field :

$E_y = 270 \sin \omega\left(t - \dfrac{x}{c}\right)$ V m^{-1}. An electron is constrained to move along y-direction with a speed of 2.0×10^7 m s^{-1}. The maximum electric force and maximum magnetic force on the electron are

(a) 4.20×10^{-15} N, 3.82×10^{-16} N
(b) 3.62×10^{-19} N, 1.63×10^{-18} N
(c) 4.32×10^{-17} N, 2.88×10^{-18} N
(d) 5.31×10^{-18} N, 5.62×10^{-18} N

12. Light with an energy flux of 18 W cm^{-2} falls on a non-reflecting surface at normal incidence. If the surface has an area of 20 cm^2, then the average force exerted on the surface during a 30 min time span is

(a) 3.4×10^{-6} N (b) 4.5×10^{-7} N
(c) 1.2×10^{-6} N (d) 1.2×10^{-7} N

13. Electromagnetic waves travel in a medium at a speed of 2.0×10^8 m s^{-1}. The relative permeability of the medium is 1.0. Then, the relative permittivity will be

(a) 1.25 (b) 5.52 (c) 3.26 (d) 2.25

14. The electric field of an electromagnetic wave is given by $E = (50 \text{ N C}^{-1}) \sin \omega(t - x/c)$.

The energy contained in a cylinder of cross section 10 cm^2 and length l dm along the x-axis is 5.5×10^{-12} J. The value of l is

(a) 3 (b) 9 (c) 4 (d) 5

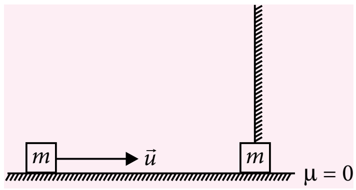
15. A beam of light travelling along x-axis is described by the magnetic field, $B_z = 5.2 \times 10^{-9}$ T $\sin \omega(t - x/c)$. The maximum electric force on alpha particle moving along y-axis with a speed 3×10^7 m s^{-1} is $n \times 10^{-19}$ N. The value of n is

(Charge on electron = 1.6×10^{-19} C)

(a) 5 (b) 3 (c) 9 (d) 4

RAY OPTICS AND OPTICAL INSTRUMENTS

16. An object of mass m is moving with velocity $\vec{u}$ towards a plane mirror kept on a stand as shown in figure. The mass of the mirror and stand system is m. A head-on elastic collision takes place between the object and the mirror stand, the velocity of image before and after the collision is



(a) $\vec{u}, 2\vec{u}$ (b) $-\vec{u}, -2\vec{u}$
(c) $-\vec{u}, 2\vec{u}$ (d) $\vec{u}, -2\vec{u}$

17. A plano-convex lens has a focal length of 30 cm and an index of refraction of 1.50. Find the radius of the convex surface.

(a) 12 cm (b) 20 cm (c) 25 cm (d) 15 cm

18. Two symmetric double convex lenses, A and B, have the same focal length, but the radii of curvature differ so that $R_A = 0.9R_B$. If $\mu_A = 1.63$, then μ_B will be

(a) 1.33 (b) 1.70 (c) 2.10 (d) 1.60

19. A 2.0 cm tall object is placed at 15 cm in front of a concave mirror of focal length 10 cm. What is the size and nature of the image?

(a) 4 cm, real (b) 4 cm, virtual
(c) 1.0 cm, real (d) none of these

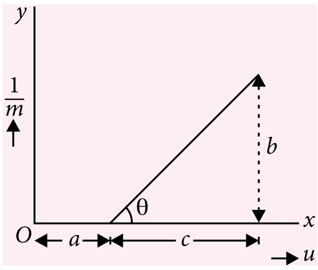
20. A glass convex lens has a power of + 10 D. When this lens is totally immersed in a liquid, it acts as a concave lens of focal length 50 cm. Calculate the refractive index of the liquid. Given ${}^a\mu_g = 1.5$.

(a) 1.33 (b) 1.67 (c) 1.50 (d) 2.32

21. In a thin prism of glass ($^a\mu_g$ = 1.5), which of the following relations between the angle of minimum deviation δ_m and angle of refraction r will be correct?
(a) $\delta_m = r$ (b) $\delta_m = 1.5\ r$
(c) $\delta_m = 2r$ (d) $\delta_m = 0.5\ r$

22. A ray is incident at an angle of incidence i on one surface of a prism of small angle A and emerges normally from opposite surface. If the refractive index of the material of prism is μ, the angle of incidence i is nearly equal to
(a) A/μ (b) $A/2\mu$ (c) μA (d) $\mu A/2$

23. The graph in figure shows how the inverse of magnification $(1/m)$ produced by a thin convex lens varies with object distance u. The focal length of the lens used is



(a) $\frac{b}{ac}$ (b) $\frac{c}{ba}$ (c) $\frac{b}{c}$ (d) $\frac{c}{b}$

24. A point source of light is kept at a depth of h in water of refractive index 4/3. The radius of the circle at the surface of water through which light emits is
(a) $\frac{3}{\sqrt{7}}h$ (b) $\frac{\sqrt{7}}{3}h$ (c) $\frac{\sqrt{3}}{7}h$ (d) $\frac{7}{\sqrt{3}}h$

25. A microscope is having objective of focal length 1 cm and eyepiece of focal length 6 cm. If tube length is 30 cm and image is formed at the least distance of distinct vision, what is the magnification produced by the microscope ? Take D = 25 cm.
(a) 25 (b) 6 (c) 125 (d) 150

26. Which of the following statements is correct?
(a) Objective of microscope is of large focal length and of small aperture.
(b) Eyepiece of telescope is of small focal length and of large aperture.
(c) Microscope produces linear magnification.
(d) In telescope, the image is farther to the eye but the size does not increase.

27. In an astronomical telescope in normal adjustment, a straight black line of length L is drawn on the objective lens. The eyepiece forms a real image of this line. The length of this image is l. The magnification of the telescope is
(a) $\frac{L}{l}$ (b) $\frac{L}{l}+1$ (c) $\frac{L}{l}-1$ (d) $\frac{L+l}{L-l}$

28. A combination is made of two lenses of focal lengths f and f' in contact. The dispersive power of the materials of the lenses are ω and ω'. The combination is achromatic, when
(a) $\omega = \omega_0$, $\omega' = 2\ \omega_0$, $f' = 2f$
(b) $\omega = \omega_0$, $\omega' = 2\ \omega_0$, $f' = f/2$
(c) $\omega = \omega_0$, $\omega' = 2\ \omega_0$, $f' = -f/2$
(d) $\omega = \omega_0$, $\omega' = 2\ \omega_0$, $f' = -2f$

29. In a laboratory, four convex lenses L_1, L_2, L_3 and L_4 of focal lengths 2, 4, 6 and 8 cm respectively are available. Two of these lenses form a telescope of length 10 cm and magnifying power 4. The objective and eye lenses are respectively
(a) L_2, L_3 (b) L_1, L_4 (c) L_1, L_2 (d) L_4, L_1

30. An object and a screen are fixed at a distance d apart. When a lens of focal length f is moved between the object and the screen, sharp images of the object are formed on the screen for two positions of the lens. If the magnifications produced at these two positions are M_1 and M_2, then
(a) $\frac{M_1}{M_2}=1$ (b) $|M_1| + |M_2| = 1$
(c) $M_1 M_2 = 1$ (d) $|M_1| - |M_2| = 1$

SOLUTIONS

1. **(b):** Since $v = \upsilon\lambda$,

$$\upsilon_{vio} = \frac{3\times10^8 \text{ m s}^{-1}}{390\times10^{-9} \text{ m}} = 7.7\times10^{14} \text{ Hz and}$$

$$\upsilon_{red} = \frac{3\times10^8 \text{ m s}^{-1}}{780\times10^{-9} \text{ m}} = 3.8\times10^{14} \text{ Hz}$$

The frequency range is then from 380 to 770 THz (1 THz = 10^{12} Hz).

2. **(a):** From Maxwell's equation for electromagnetism,

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t}$$

Since $E_x = E_y = 0$,

$$\frac{\partial E_z}{\partial y} = -\frac{\partial B_x}{\partial t} \text{ or } \frac{\partial B_x}{\partial t} = \frac{\omega}{c}E_{0z}\cos\left[\omega\left(t-\frac{y}{c}\right)+\varepsilon\right]$$

Integrating both sides with respect to t leads to

$$B_x(y,t) = \frac{1}{c}E_{0z}\sin\left[\omega\left(t-\frac{y}{c}\right)+\varepsilon\right] = \frac{1}{c}E_z(y,t)$$

(We neglect the "constant" of integration, as it represents at most an additive, steady magnetic field.)

3. **(c) :** The wave function has the basic form $E_x(z, t) = E_{0x} \sin k(z - vt)$.
Consequently, it can be reformulated as
$E_x = 10^2 \sin [3 \times 10^6\pi(z - 3 \times 10^8 t)]$,
where $k = 3 \times 10^6\pi$ m^{-1} and $v = 3 \times 10^8$ m s^{-1}.

Since $k = 2\pi/\lambda \Rightarrow 3 \times 10^6\pi = \dfrac{2\pi}{\lambda}$ or

$\lambda = \dfrac{2}{3}\times 10^{-6}$ m ≈ 666 nm,

Furthermore, $\upsilon = \dfrac{v}{\lambda} = \dfrac{3\times 10^8}{\frac{2}{3}\times 10^{-6}} = 4.5\times 10^{14}$ Hz

4. **(a) :** Electromagnetic flux density is given by $I = (c\varepsilon_0/2)E_0^2$.
Since $\varepsilon_0 = 8.85\times 10^{-12}$ C^2 N^{-1} m^{-2} and $E_0 = 100$ V m^{-1},

$$I = \frac{(3\times 10^8)(8.85\times 10^{-12})\times 100^2}{2} = 13.3 \text{ W m}^{-2}$$

5. **(c) :** Displacement current,

$$I_D = \frac{dq}{dt} = \frac{d}{dt}(q_0 \sin 2\pi\upsilon t) = 2\pi\upsilon q_0 \cos 2\pi\upsilon t$$

6. **(c) :** Comparing the given equation with the equation of plane electromagnetic wave,

$$E = E_0 \cos\left(\frac{2\pi}{\lambda}vt + \frac{2\pi}{\lambda}x\right),$$

we get $\dfrac{2\pi v}{\lambda} = 6\times 10^8$ and $\dfrac{2\pi}{\lambda} = 4$

or $4 \times v = 6 \times 10^8$ or $v = \dfrac{6}{4}\times 10^8 = \dfrac{3}{2}\times 10^8$ m s^{-1}

Refractive index, $\mu = \dfrac{c}{v} = \dfrac{3\times 10^8}{(3/2)\times 10^8} = 2$

7. **(b):** Poynting vector, $\vec{S} = \dfrac{1}{\mu_0}(\vec{E}\times\vec{B})$

$$= \frac{1}{4\pi\times 10^{-7}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 90 & 40 & -70 \\ 0\cdot 18 & 0\cdot 08 & 0\cdot 30 \end{vmatrix}\times 10^{-6}$$

$$\approx (14\cdot 0\,\hat{i} - 31\cdot 48\,\hat{j})$$

8. **(b):** $B = \dfrac{\mu_0}{4\pi}\dfrac{2i_D}{r} = \dfrac{\mu_0}{4\pi}\dfrac{2}{r}\times\varepsilon_0\dfrac{d\phi_E}{dt}$

$$= \frac{\mu_0}{2\pi r}\varepsilon_0\frac{d}{dt}(E\pi r^2) = \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt}$$

9. **(a) :** Average energy density of the electric field is

$$u_E = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{4}\times 8.85\times 10^{-12}\times 1^2$$

$= 2.21 \times 10^{-12}$ J m^{-3}

Average energy density of the magnetic field is

$$u_B = \frac{B_0^2}{4\mu_0} = \frac{1}{4}\frac{E_0^2}{\mu_0 c^2} \qquad \left[\because B_0 = \frac{E_0}{c}\right]$$

$$= \frac{1}{4}\times\frac{1^2}{4\pi\times 10^{-7}}\times\frac{1}{(3\times 10^8)^2}$$

$= 2.21 \times 10^{-12}$ J m^{-3}

10. **(b):** The intensity of a plane electromagnetic wave,

$$I = u_{av}\cdot c = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$\therefore$ Amplitude of electric field,

$$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2\times 2.5\times 10^{14}}{8.85\times 10^{-12}\times 3\times 10^8}}$$

$= 4.3 \times 10^8$ V m^{-1}

Amplitude of magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{4.3\times 10^8}{3\times 10^8} = 1.44 \text{ T}$$

11. **(c) :** Maximum electric field, $E_0 = 270$ V m^{-1}
Maximum magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{270}{3\times 10^8} = 9\times 10^{-7} \text{ T},$$

directed along z-direction.
Maximum electric force on the electron,
$F_e = qE_0 = 1.6 \times 10^{-19} \times 270 = 4.32 \times 10^{-17}$ N
Maximum magnetic force on the electron
$F_m = qvB_0 = 1.6 \times 10^{-19} \times 2.0 \times 10^7 \times 9 \times 10^{-7}$
$= 2.88 \times 10^{-18}$ N

12. **(c) :** Energy flux = 18 W cm^{-2} = 18 J s^{-1} cm^{-2}
Area = 20 cm^2
Time = 30 min = 1800 s
Total energy falling on the surface,
U = energy flux × time × area
= (18 J s^{-1} cm^{-2}) × 1800 s × 20 cm^2
= 6.48 × 10^5 J
The total momentum delivered to the surface,

$$p = \frac{U}{c} = \frac{6.48\times 10^5 \text{ J}}{3\times 10^8 \text{ m s}^{-1}} = 2.16\times 10^{-3} \text{ kg m s}^{-1}$$

The average force exerted on the surface,

$$F = \frac{p}{t} = \frac{2.16\times 10^{-3}\text{J}}{1800} = 1.2\times 10^{-6}\text{N}$$

13. (d): Speed of an electromagnetic wave in a medium is given by

$$v=\frac{1}{\sqrt{\mu\varepsilon}}=\frac{1}{\sqrt{\mu_r\mu_0\varepsilon_r\varepsilon_0}}=\frac{1}{\sqrt{\mu_0\varepsilon_0}}\times\frac{1}{\sqrt{\mu_r\varepsilon_r}}=\frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

$$\therefore\ v^2=\frac{c^2}{\mu_r\varepsilon_r}$$

Hence relative permittivity,

$$\varepsilon_r=\frac{c^2}{\mu_r v^2}=\frac{(3\times10^8)^2}{1.0\times(2\times10^8)^2}=2.25$$

14. (d): Average value of energy density, *i.e.*, (energy/volume) of electromagnetic wave,

$$u_{av}=\frac{1}{2}\varepsilon_0E_0^2$$

Total volume of the cylinder, $V = AL$

Total energy contained in the cylinder,

$$U=u_{av}\times V=\frac{1}{2}\varepsilon_0E_0^2\times AL$$

Here, $E_0 = 50\ \text{N C}^{-1}$, $A = 10\ \text{cm}^2 = 10\times10^{-4}\ \text{m}^2$, $L = (l/10)$ m ; $U = 5.5\times10^{-12}$ J

$$\therefore\quad 5.5\times10^{-12}=\frac{1}{2}\times(8.86\times10^{-12})\,(50)^2\times(10\times10^{-4})\times(l/10)$$

On solving, $l \approx 5$

15. (a): Here, $B_0 = 5{\cdot}2\times10^{-9}$ T,

Charge on alpha particle,

$q = 2e = 2\times1.6\times10^{-19}$ C

Maximum electric field,

$E_0 = cB_0 = (3\times10^8)\times(5.2\times10^{-9}) = 1.56\ \text{V m}^{-1}$

Maximum force on alpha particle due to electric field, $F_{max} = qE_0 = (2\times1.6\times10^{-19})\times1.56$
$\approx 5\times10^{-19}$ N

According to question, $n\times10^{-19} = 5\times10^{-19}$ or $n = 5$

16. (c): Before collision, $\vec{v}_{image}=-\vec{v}_{object}=-\vec{u}$

After collision, $\vec{v}_{mirror}=\vec{u}$

$\vec{v}_{image}=2\vec{v}_{mirror}=2\vec{u}$

17. (d): Using the lens maker's formula with $R_1 = \infty$,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Rightarrow\frac{1}{30}=(1.50-1)\left(\frac{1}{\infty}-\frac{1}{R_2}\right)$$

$$\Rightarrow\quad\frac{1}{30}=\frac{0.50}{-R_2}\quad\text{or}\quad R_2=-15\ \text{cm}$$

The negative sign means that surface is convex to the right.

18. (b): Since the focal lengths are the same,

$(\mu_A - 1)(2/R_A) = (\mu_B - 1)(2/R_B)$

or $\quad(\mu_B - 1) = (R_B/R_A)\,(\mu_A - 1) = 0.63/0.9 = 0.70$

or $\quad\mu_B = 1.70$

19. (a): According to new cartesian sign convention, $u = -15$ cm, $f = -10$ cm and $h_O = 2.0$ cm

According to mirror formula, $\dfrac{1}{v}+\dfrac{1}{u}=\dfrac{1}{f}$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-10}-\frac{1}{-15}\quad\text{or}\quad v=-30\ \text{cm}$$

Magnification of the mirror, $m=\dfrac{-v}{u}=\dfrac{h_I}{h_O}$

$$\Rightarrow\quad\frac{-(-30)}{-15}=\frac{h_I}{2}\ \Rightarrow\ h_I=-4\ \text{cm}$$

Negative sign shows that image is real and inverted.

20. (b): Here $P = +10$ D

$$f_a=\frac{1}{P}=\frac{1}{10}\text{m}=10\ \text{cm}$$

$$\text{But}\quad\frac{1}{f_a}=({}^a\mu_g-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\text{or}\quad\frac{1}{10}=(1.5-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\text{or}\quad\frac{1}{R_1}-\frac{1}{R_2}=\frac{1}{10\times0.5}=\frac{1}{5}$$

When the lens is immersed in the liquid,

$$\frac{1}{f_l}=\left(\frac{{}^a\mu_g}{{}^a\mu_l}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\text{or}\quad\frac{1}{-50}=\left(\frac{1.5}{{}^a\mu_l}-1\right)\times\frac{1}{5}$$

$$\text{or}\quad\frac{1.5}{{}^a\mu_l}=1-0.1=0.9\quad\text{or}\quad{}^a\mu_l=\frac{1.5}{0.9}=1.67.$$

21. (a): $\delta = i + e - (r_1 + r_2)$

when $\delta = \delta_m$, $i = e$; $r_1 = r_2 = r$

$\therefore\quad\delta_m = 2i - 2r = 2\,{}^a\mu_g\,r - 2r = 2r\,({}^a\mu_g - 1)$

(Using Snells' law)

$$\delta_m=2r(1.5-1)\quad\text{or}\quad\delta_m=r$$

22. (c): As refracted ray emerges normally from opposite surface, $r_2 = 0$.

As $\quad A = r_1 + r_2 \qquad \therefore\ r_1 = A$

$$\text{Now,}\ \mu=\frac{\sin i_1}{\sin r_1}\approx\frac{i_1}{r_1}=\frac{i}{A}\ ;\ i=\mu A$$

23. (d): For a thin convex lens

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{u+f}{uf}$$

$$v=\frac{uf}{u+f} \quad \text{or} \quad \frac{v}{u}=\frac{f}{u+f}$$

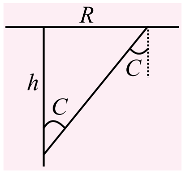
$$\therefore \quad \frac{1}{m}=\frac{u+f}{f}=\frac{1}{f}(u)+1$$

This is the equation of a straight line.

$\therefore$ For graph between $(1/m)$ and u,

Slope of the line = $\tan\theta = \frac{1}{f}=\frac{b}{c} \quad \therefore \quad f=\frac{c}{b}$

24. (a) : $\sin C=\frac{1}{\mu} \Rightarrow \frac{R}{\sqrt{R^2+h^2}}=\frac{3}{4}$



Squaring, $16R^2 = 9R^2 + 9h^2$

$$7R^2=9h^2 \Rightarrow R=\frac{3}{\sqrt{7}}h$$

25. (d): Here, f_o = 1 cm, f_e = 6 cm, D = 25 cm, L = 30 cm
When the image is formed at the least distance of distinct vision, the magnification is

$$m=\frac{L}{f_o}\left(1+\frac{D}{f_e}\right)=\frac{30}{1}\left(1+\frac{25}{6}\right) = 155 \approx 150$$

26. (c) : Objective of microscope is of small focal length and of small aperture and eyepiece of telescope is of small focal length and of small aperture.
Telescope produces image nearer to the eye but the size of image does not increases.

27. (a) : Let f_o and f_e be the focal lengths of the objective and eyepiece respectively.
For normal adjustment, distance from the objective to the eyepiece (tube length) = $f_o + f_e$.
Treating the line on the objective as the object, and the eyepiece as the lens, $u = -(f_o + f_e)$ and $f = f_e$.

$$\therefore \quad \frac{1}{v}=\frac{1}{f_e}-\frac{1}{f_o+f_e}=\frac{f_o}{(f_o+f_e)f_e}$$

$$\text{or} \quad v=\frac{(f_o+f_e)f_e}{f_o}$$

Magnification = $\left|\frac{v}{u}\right|=\frac{f_e}{f_o}=\frac{\text{image size}}{\text{object size}}=\frac{I}{O}$.

$\therefore \quad \frac{f_o}{f_e}=\frac{L}{l}$ = magnification of telescope in normal adjustment.

28. (d): For achromatic combination, the necessary condition is

$\frac{\omega}{\omega'}=-\frac{f}{f'}$ which is satisfied by option (d).

29. (d): $L=f_o+f_e=10$; $M=\frac{f_o}{f_e}=4 \;\therefore\; f_o=4f_e$

$L = 4f_e + f_e = 10$

$$\text{or} \quad f_e=\frac{10}{5}=2\text{ cm} \;;\; f_o=4\times 2=8\text{ cm}$$

$\therefore$ The objective lens is L_4 and eye lens is L_1.

30. (c) : From displacement method for the determination of focal length of a lens, $d > 4f$ and $M_1 M_2 = 1$.

ACE YOUR WAY CBSE XII

Series 6

CHAPTERWISE PRACTICE PAPER
DUAL NATURE OF MATTER AND RADIATION | ATOMS AND NUCLEI

Time Allowed : 3 hours

Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION - A

1. Which colour photon is more energetic : red or violet?

2. In hydrogen atom, if the electron is replaced by a particle which is 200 times heavier but has the same charge, how would its radius change ?

3. Does each incident photon essentially eject a photoelectron?

4. Is a free neutron a stable particle?

5. Why is it found experimentally difficult to detect neutrinos in nuclear β-decay?

SECTION - B

6. Explain the term stopping potential and threshold frequency.

7. You are given two nuclides $^{7}_{3}X$ and $^{4}_{3}Y$.
(i) Are they the isotopes of the same element? Why?
(ii) Which one of the two is likely to be more stable? Give reason.

8. A piece of wood from the ruins of an ancient building was found to have a ^{14}C activity of 12 disintegrations per minute per gram of its carbon content. The ^{14}C activity of the living wood is 16 disintegrations per minute per gram. How long ago did the tree, from which the given wooden sample die? (Given half-life of ^{14}C is 5760 yrs).

OR

Name two radioactive elements which are not found in observable quantities in nature. Why it is so? Why is it said that nuclear forces are saturated forces?

9. An α-particle and a proton are accelerated through the same potential difference. Calculate the ratio of linear momenta acquired by the two.

10. Explain why the spectrum of hydrogen atom has many lines, although a hydrogen atom contains only one electron.

SECTION - C

11. A star converts all its hydrogen into helium achieving 100% helium composition. It then converts helium to carbon via the reaction.

$$^{4}_{2}He + ^{4}_{2}He + ^{4}_{2}He \rightarrow ^{12}_{6}C + 7.27\ MeV$$

The mass of the star is 5.0×10^{32} kg and it generates energy at the rate of 5×10^{30} W. How long will it take to convert all the helium to carbon at this rate?

12. The deuteron is bound by nuclear forces just as H-atom is made up of p and e bound by electrostatic forces. If we consider the force between neutron and proton in deuteron as given in the form of a Coulomb potential but with an effective charge e' :
$F = \frac{1}{4\pi\varepsilon_0} \frac{e'^2}{r}$. Estimate the value of (e'/e) given that the binding energy of a deuteron is 2.2 MeV.

13. **(a)** Using the Bohr's model, calculate the speed of the electron in a hydrogen atom in the $n = 1, 2$ and 3 levels.
(b) Calculate the orbital periods in each of these levels.

14. Two particles A and B of de Broglie wavelengths λ_1 and λ_2 combine to form a particle C. The process conserves momentum. Find the de Broglie wavelength of the particle C. (The motion is one dimensional.)

15. $_{86}Rn^{222}$ is converted into $_{84}Po^{218}$ and $_{93}Np^{239}$ is converted into $_{94}Pu^{239}$. Name the particles emitted in each case and write down the corresponding equations.

16. Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion? To what temperature must the gases be heated to initiate the reaction?

17. What is the minimum energy that must be given to a H-atom in ground state so that it can emit an H_γ line in Balmer series? If the angular momentum of the system is conserved, what would be the angular momentum of such H_γ photon?

18. Sketch the graphs, showing the variation of stopping potential V_s with frequency υ of the incident radiations for two photosensitive materials A and B having threshold frequencies $\upsilon_0 > \upsilon'_0$ respectively.
(a) Which of the two metals A or B has higher work function?
(b) What information do you get from the slope of the graphs?
(c) What does the value of the intercept of graph A on the potential axis represent?

19. Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.
(a) Is the probability of backward scattering (*i.e.*, scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?
(b) Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?
(c) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

20. Prove that the instantaneous rate of change of activity of a radioactive substance is inversely proportional to the square of its half-life.

OR

A source contains two phosphorus radio-nuclides $^{32}_{15}P$ ($T_{1/2}$ = 14.3 days) and $^{33}_{15}P$ ($T_{1/2}$ = 25.3 days). Initially, 10% of the decays come from $^{33}_{15}P$. How long one must wait until 90% do so?

21. In the Auger process, an atom makes a transition to a lower state without emitting a photon. The excess energy is transferred to an outer electron which may be ejected by the atom. (This is called an Auger electron). Assuming the nucleus to be massive, calculate the kinetic energy of a $n = 4$ Auger electron emitted by Chromium by absorbing the energy from $n = 2$ to $n = 1$ transition. (For Cr, take $Z = 24$)

22. Why can an electron not exist inside the nucleus?

SECTION - D

23. Ramaswami, a resident of Kundakulam, was all set to leave everything and shift to another place in view of the decision of Government to start nuclear thermal power plant at Kundakulam. His granddaughter Manika, a science student, was really upset on the ignorant decision of her grandfather. She could finally convince him not to shift, since adequate safety measures to avoid any nuclear mishappening have already been taken by the Government before starting nuclear thermal plants.

(a) What is the value displayed by Manika in convincing her grandfather?
(b) Name the working principle of a nuclear reactor.
(c) Name the main components of a nuclear reactor.
(d) Why is heavy water used as a moderator?

SECTION - E

24. Derive the expression for the de-Broglie wavelength of an electron moving under a potential difference of V.
Describe Davisson and Germer experiment to establish the wave nature of electrons. Draw a labelled diagram of the apparatus used.

OR

(a) State Bohr's postulates for hydrogen atom.
(b) Draw the energy level diagram showing the ground state, and the next few excited states for hydrogen (H) atom. Mark the transition which corresponds to the emission of spectral lines for the Balmer series.

25. Draw a labelled diagram for α-particle scattering experiment. Give Rutherford's observation and discuss the significance of this experiment. Obtain the expression which helps us to get an idea of the size of a nucleus, using these observations.

OR

What is photoelectric effect? Explain experimentally the variation of photoelectric current with
(i) intensity of light
(ii) the potential difference between the plates, and
(iii) the frequency of incident light and hence state the laws of photoelectric emission.

26. On the basis of photon theory, obtain Einstein's photoelectric equation. Use this equation to show that there must exist a threshold frequency for each photosensitive surface.
Radiations of frequencies υ_1 and υ_2 are made to fall in turn, on a photosensitive surface. The stopping potentials required for stopping the most energetic emitted photo electrons in the two cases are V_1 and V_2 respectively. Obtain a formula for calculating Planck's constant and the threshold frequency in terms of these parameters.

OR

(a) Draw the plot of binding energy per nucleon (BE/A) as a function of mass number A. Write two important conclusions that can be drawn regarding the nature of nuclear force.
(b) Use this graph to explain the release of energy in both the processes of nuclear fusion and fission.
(c) Write the basic nuclear process of neutron undergoing β-decay.

SOLUTIONS

1. Since, energy of photon is given by, $E = h\upsilon$ and $\upsilon_{violet} > \upsilon_{red}$, $E_{violet} > E_{red}$.

2. Radius of n^{th} orbit of H-atom with electron is

$$r = \frac{n^2h^2\varepsilon_0}{\pi me^2} \quad \text{or} \quad r \propto \frac{1}{m}$$

So, if electron is replaced with a charged particle of same charge but of mass $200m$, then radius of n^{th} orbit of H-atom will become

$$\frac{r'}{r} = \frac{m}{200m} = \frac{1}{200} \quad \text{or} \quad r' = \frac{r}{200}$$

i.e., will reduce to $\frac{1}{200}$ times that with electron.

3. No, it may be absorbed in some other way. If the frequency of the incident photon is less than the threshold frequency, there will be no emission of photoelectrons at all.

4. No, a free neutron is not a stable particle. It decays spontaneously into a proton, an electron and antineutrino.
$_0n^1 \rightarrow {}_1H^1 + {}_{-1}e^0 + \bar{\upsilon}$

5. Neutrinos are neutral (chargeless), almost massless particles that hardly interact with matter.
The neutrinos can penetrate large quantity of matter without any interaction. Hence it is difficult to detect neutrinos experimentally.

6. Stopping potential : It is the minimum negative potential given to the anode in a photocell for which the photoelectric current becomes zero. If V_0 is the stopping potential, then maximum kinetic energy of emitted photoelectron is
$K_{max} = eV_0 = h\upsilon - \phi_0$

$$\therefore \quad V_0 = \frac{h\upsilon}{e} - \frac{\phi_0}{e}$$

Threshold frequency : It is the minimum frequency of the incident radiation for which emission of photoelectrons just take place from a metal surface without any kinetic energy. If υ_0 is the threshold frequency, then using Einstein's photoelectric equation,

$$0 = h\upsilon_0 - \phi_0 \quad \text{or} \quad \upsilon_0 = \frac{\phi_0}{h}$$

7. **(i)** Yes, they are the isotopes of the same element because they have same atomic number ($Z = 3$).
(ii) The isotope ${}^7_3X$ has 3 protons and 4 neutrons while the isotope ${}^4_3Y$ has 3 protons and 1 neutron. Due to the presence of a greater number of neutrons in ${}^7_3X$, the strong attractive nuclear force dominates over the electrostatic repulsion between the protons. So ${}^7_3X$ is more stable than ${}^4_3Y$.

8. Here, half-life of ${}^{14}C$, $T_{1/2} = 5760$ yrs, $t = ?$
$R_0 = 16$ disintegrations per minute per gram
$R = 12$ disintegrations per minute per gram
From law of activity,

$$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right) = \frac{T_{1/2}}{0.693}\times 2.303\log\left(\frac{R_0}{R}\right)$$

$$\left(\because \lambda = \frac{\ln 2}{T} = \frac{0.693}{T}\right)$$

$$= \frac{5760}{0.693}\times 2.303\log\left(\frac{16}{12}\right)$$

$$t = \frac{5760\times 2.303\times 0.125}{0.693}\text{ yrs} = 2392.7\text{ yrs}$$

OR

Tritium and plutonium are two radioactive elements which are not found in observable quantities in nature. This is because their half-lives are short as compared to the age of the universe.
A nucleon interacts only with its nearest neighbouring nucleon. It does not interact with those nucleons which are not in direct contact with it. That is why we say that nuclear forces show saturated effect. This is supported by the fact that binding energy per nucleon is same over a wide range of mass numbers.

9. As $\frac{1}{2}mv^2 = qV$, $v = \sqrt{\frac{2qV}{m}}$

Momentum, $p = mv = m\sqrt{\frac{2qV}{m}} = \sqrt{2mqV}$

Since potential difference (V) is the same for both the particles,
$p \propto \sqrt{mq}$

or $\frac{p_\alpha}{p_p} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)}$

or $\frac{p_r}{p_p} = \sqrt{4\times 2} = 2\sqrt{2}:1 \left(\text{As } \frac{m_\alpha}{m_p} = 4 \text{ and } \frac{q_\alpha}{q_p} = 2\right)$

10. A source of hydrogen spectrum has billions of hydrogen atoms. Each hydrogen atom has many stationary states. All possible transitions can occur from any higher level to any lower level. This gives rise to a large number of spectral lines.

11. As 4×10^{-3} kg of He consists of 6.023×10^{23} He atoms, so 5×10^{32} kg of He consists of

$$\frac{6.023\times 10^{23}\times 5\times 10^{32}}{4\times 10^{-3}} = 7.52875\times 10^{58}\text{ atoms}$$

Now, 3 atoms of He produce energy
$= 7.27\times 10^6\times 1.6\times 10^{-19}$ J
So, all He atoms in star produce

$$\text{total energy} = \frac{7.27\times 1.6\times 10^{-13}}{3}\times 7.52875\times 10^{58}\text{ J}$$

$= 29.2\times 10^{45}$ J
As power generated is $P = 5\times 10^{30}$ W
∴ Time taken to convert all the atoms into carbon

$$= \frac{29.2\times 10^{45}}{5\times 10^{30}} = 5.84\times 10^{15}\text{ s}$$

$$= \frac{5.84\times 10^{15}}{365\times 24\times 60\times 60} = 1.85\times 10^{8}\text{ yrs}$$

12. The binding energy in ground state of hydrogen atom is $E = \frac{me^4}{8\varepsilon_0^2 h^2} = 13.6\text{ eV}$ ...(i)

If proton and neutron (in a deuteron) had charge e' each, and were governed by the same electrostatic force, then in (i), we replace e by e' and m by m', the reduced mass of neutron-proton, where

$$m' = \frac{M\times M}{M+M} = \frac{M}{2} = \frac{1836\,m}{2} = 918\,m$$

Here, M represents mass of a neutron/proton.

∴ Binding energy, $E' = \frac{918\,m\,e'^4}{8\varepsilon_0^2 h^2} = 2.2\text{ MeV}$ ...(ii)

Dividing (ii) by (i), we get

$$918\left(\frac{e'}{e}\right)^4 = \frac{2.2\text{ MeV}}{13.6\text{ eV}} = \frac{2.2\times 10^6}{13.6}$$

$$\left(\frac{e'}{e}\right)^4 = \frac{2.2\times10^6}{13.6\times918} = 176.21$$

$$\frac{e'}{e} = (176.21)^{1/4} = 3.64$$

13. **(a)** Speed of the electron in Bohr *n*th orbit, *i.e.*,

$v_n = \frac{c}{n}\alpha$; where α (fine struture constant) = $\frac{1}{137}$

Thus, $v_1 = \frac{c}{1}\alpha = \frac{1}{137}c = \frac{1}{137}(3\times10^8 \text{ m s}^{-1})$

$= 2.18 \times 10^6 \text{ m s}^{-1}$

$$v_2 = \frac{c}{2}\alpha = \frac{v_1}{2} = 1.09\times10^6 \text{ m s}^{-1}$$

$$v_3 = \frac{c}{3}\alpha = \frac{v_1}{3} = 7.27\times10^5 \text{ m s}^{-1}$$

(b) $T_1 = \frac{2\pi r_1}{v_1} = \frac{2\times3.14\times(0.53\times10^{-10})}{2.18\times10^6}$

$= 1.52 \times 10^{-16}$ s (as $r_1 = 0.53$ Å)

Further, as $T_n = n^3 T_1$,

$T_2 = (2)^3(1.52 \times 10^{-16}\text{s}) = 1.22 \times 10^{-15}\text{s}$

$T_3 = (3)^3(1.52 \times 10^{-16}\text{s}) = 4.10 \times 10^{-15}\text{s}$

14. $|\vec{p}_C| = |\vec{p}_A| + |\vec{p}_B|$

$\Rightarrow \frac{h}{\lambda_C} = \frac{h}{\lambda_A} + \frac{h}{\lambda_B}$ ...(i)

Following cases may arise,

Case (i) : $p_A > 0$, $p_B > 0$, *i.e.*, both p_A and p_B are positive.

From (i) : $\frac{h}{\lambda_C} = \frac{h(\lambda_A+\lambda_B)}{\lambda_A\lambda_B}$ or $\lambda_C = \frac{\lambda_A\lambda_B}{\lambda_A+\lambda_B}$

Case (ii) : $p_A < 0$, $p_B < 0$, *i.e.*, both p_A and p_B are negative.

From (i), $\frac{h}{\lambda_C} = -\frac{h}{\lambda_A} - \frac{h}{\lambda_B} = -\frac{h(\lambda_A+\lambda_B)}{\lambda_A\lambda_B}$

or $\lambda_C = -\frac{\lambda_A\lambda_B}{(\lambda_A+\lambda_B)}$

Case (iii) : $p_A > 0$, $p_B < 0$, *i.e.*, p_A is positive and p_B is negative.

From (i), $\frac{h}{\lambda_C} = \frac{h}{\lambda_A} - \frac{h}{\lambda_B} = \frac{(\lambda_B-\lambda_A)h}{\lambda_A\lambda_B}$

or $\lambda_C = \frac{\lambda_A\lambda_B}{\lambda_B-\lambda_A}$

Case (iv) $p_A < 0$, $p_B > 0$, *i.e.*, p_A is negative and p_B is positive.

From (i), $\frac{h}{\lambda_C} = -\frac{h}{\lambda_A} + \frac{h}{\lambda_B} = \frac{(\lambda_A-\lambda_B)h}{\lambda_A\lambda_B}$

or $\lambda_C = \frac{\lambda_A\lambda_B}{\lambda_A-\lambda_B}$

15. Let the particle emitted in each case be represented as ${}_ZX^A$. Therefore,

(i) ${}_{86}\text{Rn}^{222} \to {}_{84}\text{Po}^{218} + {}_ZX^A$

Using the law of conservation of mass number and charge number, we get

$222 = 218 + A$

$\therefore A = 222 - 218 = 4$

$86 = 84 + Z$

$\therefore Z = 86 - 84 = 2$

Now, $A = 4$ and $Z = 2$ correspond to an alpha particle ${}_2\text{He}^4$. Therefore, emitted particle is an alpha particle, and the equation is

${}_{86}\text{Rn}^{222} \to {}_{84}\text{Po}^{218} + {}_2\text{He}^4$

(ii) ${}_{93}\text{Np}^{239} \to {}_{94}\text{Pu}^{239} + {}_ZX^A$

Using the law of conservation of mass number and charge number, we get

$239 = 239 + A$, $\therefore$ $A = 239 - 239 = 0$

$93 = 94 + Z$, $\therefore$ $Z = 93 - 94 = -1$.

Now, $A = 0$ and $Z = -1$ correspond to electron (${}_{-1}e^0$). Therefore, emitted particle is a beta particle, and the equation is

${}_{93}\text{Np}^{239} \to {}_{94}\text{Pu}^{239} + {}_{-1}\text{e}^0 + \bar{\nu}$

16. 2 × kinetic energy of each nucleus = Potential energy of system

$\Rightarrow 2\times K = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{2R}$

$K = \frac{1}{4\pi\varepsilon_0}\frac{e\times e}{4R} = 9\times10^9 \times \frac{(1.6\times10^{-19})^2}{4\times2\times10^{-15}}$

or $K = \frac{2.88\times10^{-14}}{1.6\times10^{-19}}$ eV = 1.8×10^5 eV = 180 keV

Temperature of gas must be heated so that kinetic energy of each molecule $\frac{3}{2}kT$ is equal to 180 keV, *i.e.*,

$\frac{3}{2}kT = 180 \text{ keV} = 180 \times 10^3 \times 1.6 \times 10^{-19}$ J

$= 2.88 \times 10^{-14}$ J

$$T = \frac{2\times2.88\times10^{-14}}{3\times1.38\times10^{-23}} = 1.39\times10^9 \text{ K}$$

17. H_γ line in the Balmer series corresponds to transition from state $n_i = 5$ to state $n_f = 2$.

As $E_n = -\frac{13.6}{n^2}\text{eV}, E_5 = -\frac{13.6}{(5)^2}\text{eV} = -0.54\text{ eV}$

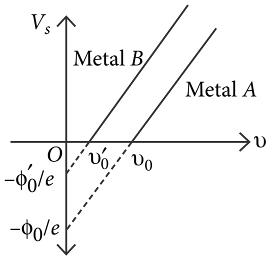
Minimum energy that must be given to the electron in the ground state ($n = 1$) to raise it to state $n = 5$, *i.e.*, $E_5 - E_1 = -0.54\text{ eV} - (-13.6\text{ eV}) = 13.06\text{ eV}$

If the angular momentum of system (electron + emitted photon) is conserved, change in angular momentum of photon = change in angular momentum of electron

$$= 5\left(\frac{h}{2\pi}\right) - 2\left(\frac{h}{2\pi}\right) = 3\left(\frac{h}{2\pi}\right)$$

$$= \frac{3(6.63\times10^{-34}\text{ J s})}{2\times3.14} = 3.17\times10^{-34}\text{ J s}$$

18. The variation of stopping potential (V_s) with frequency (υ) of the incident radiations for two photosensitive materials is shown in figure.



(a) Metal A has higher work function, as

$\phi_0 = h\upsilon_0$ and $\phi_0' = h\upsilon_0'$, since $\upsilon_0 > \upsilon_0'$.

$\therefore$ $\phi_0 > \phi_0'$

(b) Slope of $V_s - \upsilon$ graph $= h/e$ = constant

Slope of curve does not depend on type of material.

(c) Intercept of graph A on the potential axis

$$= -\frac{\phi_0}{e} = -\frac{h\upsilon_0}{e}.$$

19. (a) Much less, because there is no such massive core (nucleus) in Thomson's model as in Rutherford's model.

(b) This suggests that scattering is predominantly due to a single collision, because the chance of single collision increases linearly with the number of the target atoms, and hence linearly with the thickness of the foil.

(c) In Thomson model, positive charge is distributed uniformly in the atom. So single collision causes very little deflection. The observed average scattering angle can be explained only by considering multiple scattering. Hence it is wrong to ignore multiple scattering in Thomson's model.

20. Instantaneous activity, $R = \lambda N$

Thus, $\frac{dR}{dt} = \frac{d}{dt}(R) = \frac{d}{dt}(\lambda N) = \lambda\frac{dN}{dt}$

Since, $\frac{dN}{dt} = \lambda N$, (neglecting negative sign)

$\Rightarrow \frac{dR}{dt} = \lambda(\lambda N) = \lambda^2 N$

As, $\lambda = \frac{\ln 2}{T_{1/2}}, \frac{dR}{dt} = \left(\frac{\ln 2}{T_{1/2}}\right)^2 N$ or $\frac{dR}{dt} \propto \frac{1}{(T_{1/2})^2}$

OR

In the mixture of P-32 and P-33, initially 10% decay came from P-33.

Hence initially 90% of the mixture is P-32 and 10% of the mixture is P-33.

Let after time t the mixture is left with 10% of P-32 and 90% of P-33.

Half life of both P-32 and P-33 are given as 14.3 days and 25.3 days respectively.

Let x be total mass undecayed initially and y be total mass undecayed finally.

Let initial number of P-32 nuclides = 0.9 x

Final number of P-32 nuclides = 0.1 y

Similarly, initial number of P-33 nuclides = 0.1x

Final number of P-33 nuclides = 0.9 y

For isotope P-32,

$N = N_0\, 2^{-t/T_{1/2}}$ or $0.1\, y = 0.9\, x\, 2^{-t/14.3}$ ...(i)

For isotope P-33,

$N = N_0\, 2^{-t/T_{1/2}}$ or $0.9\, y = 0.1\, x\, 2^{-t/25.3}$ ...(ii)

On dividing, we get, $\frac{1}{9} = 9\frac{2^{-t/14.3}}{2^{-t/25.3}}$

or $\frac{1}{81} = 2^{\left(-\frac{t}{14.3}+\frac{t}{25.3}\right)} \Rightarrow \frac{1}{81} = 2^{-t\left[\frac{11}{14.3\times25.3}\right]}$

or $81 = 2^{t\left[\frac{11}{14.3\times25.3}\right]}$

Taking log, $\log_e 81 = t\left(\frac{11}{14.3\times25.3}\right)\log_e 2$

or $1.9082 = \frac{11t}{25.3\times14.3}\times0.3010$

$t = 208.5$ days $\simeq 209$ days

21. As the nucleus is massive, recoil momentum of the atom can be ignored. We can assume that the entire energy of transition is transferred to the Auger electron.

As there is a single valence electron in chromium (Z = 24), the energy states may be thought of as given by Bohr model. The energy of the n^{th} state is

$E_n = -\frac{RhcZ^2}{n^2}$ where R is Rydberg constant.

In the transition from $n = 2$ to $n = 1$, energy released,

$$\Delta E = -RhcZ^2\left(\frac{1}{4}-1\right) = \frac{3}{4}RhcZ^2$$

The energy required to eject a n = 4 electron

$$= RhcZ^2\left(\frac{1}{4}\right)^2 = \frac{RhcZ^2}{16}$$

$$\therefore \text{ KE of Auger electron} = \frac{3RhcZ^2}{4} - \frac{RhcZ^2}{16}$$

$$\text{KE} = RhcZ^2\left(\frac{3}{4}-\frac{1}{16}\right) = \frac{11}{16}RhcZ^2$$

$$= \frac{11}{16}(13.6\text{ eV})\times 24\times 24 = 5385.6\text{ eV}$$

[$\because Rhc = 13.6$ eV]

22. If an electron is to exist inside a nucleus, its de Broglie wavelength (λ) should be less than the nuclear diameter which is of the order of 10^{-14} m. According to de Broglie hypothesis,

$$p = \frac{h}{\lambda} = \frac{6.63\times 10^{-34}\text{ J s}^{-1}}{10^{-14}\text{ m}}$$

or $p = 6.63 \times 10^{-20}$ kg m s^{-1}

Further, for energetic electrons,

$E = pc = (6.63 \times 10^{-20}$ kg m s$^{-1})(3 \times 10^8$ m s$^{-1})$

$= 2 \times 10^{-11}$ J = 125 MeV

(as 1 MeV = 1.6×10^{-13} J)

Thus, for an electron to exist inside the nucleus, it should have at least an energy of 125 MeV. This energy is much larger than the binding energy which Coulomb force is capable of providing within the nucleus. Moreover, it is experimentally found that the energy of β-rays (electrons) emitted by radioactive nuclei is of the order of 2 to 3 MeV. Thus, electrons cannot reside inside the nucleus.

23. **(a)** Awareness and social responsibility.

(b) Controlled chain reaction.

(c) Nuclear fuel, moderator, control rods, coolant and outer shielding.

(d) Heavy water contains protons. Fast moving neutrons undergo elastic collisions with these slow moving protons and thus get slowed down.

24. Refer to point 7.4(4, 6), page no. 492, 493 (MTG Excel in Physics)

OR

Refer to point 8.1(4), 8.2(4)(b), 8.2(7)(ii), page no. 526, 527, 528 & 529 (MTG Excel in Physics)

25. Refer to point 8.1(2), page no. 525, 526 (MTG Excel in Physics)

OR

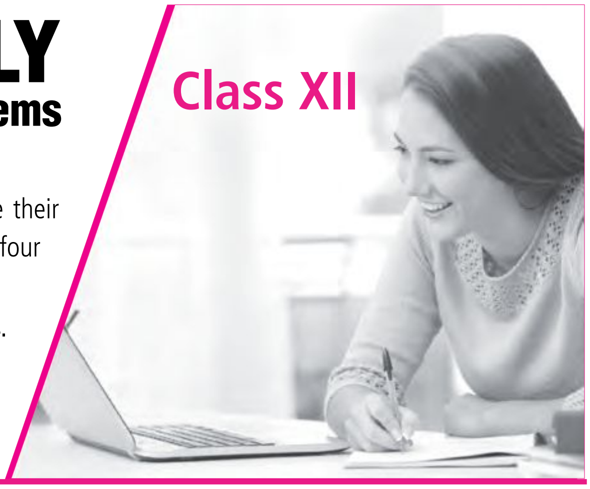
Refer to point 7.2, page no. 489, 490 (MTG Excel in Physics)

26. Refer to point 7.3, page no. 491-492 (MTG Excel in Physics)

OR

Refer to point 8.3(15), 8.4(5), page no. 534 and 535 (MTG Excel in Physics)

MPP-7 MONTHLY Practice Problems



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

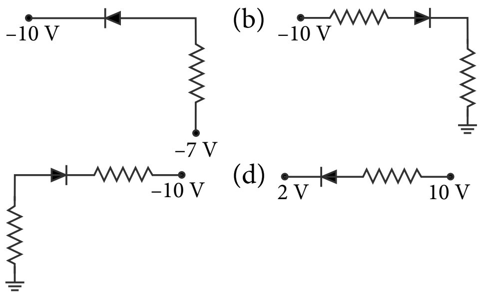
Semiconductor Electronics and Communication Systems

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

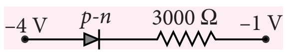
1. In which of the following figure, the junction diode is reverse biased?



2. For a junction diode, when I_0 is the reverse saturation current then the forward current is given by

(a) $I_0 e^{-eV/kT}$ (b) $I_0 e^{eV/kT}$
(c) $I_0(e^{-eV/kT} - 1)$ (d) $I_0(e^{eV/kT} - 1)$.

3. Consider that the junction diode is ideal.



The value of current in the diode is

(a) 0 (b) 10^{-1} A (c) 10^{-2} A (d) 10^{-3} A

4. In a transistor connected in a common emitter mode, $R_o = 4\ \text{k}\Omega$, $R_i = 1\ \text{k}\Omega$, $I_c = 1$ mA and $I_b = 20\ \mu$A. The voltage gain is

(a) 100 (b) 200 (c) 300 (d) 400

5. If α and β are the current gain in the CB and CE configurations respectively of the transistor circuit, then $\frac{\beta - \alpha}{\alpha\beta} =$

(a) zero (b) 1 (c) 2 (d) 0.5

6. The following table provides the set of values of V and I obtained for a given diode:

	V	I
Forward biasing	2.0 V 2.4 V	60 mA 80 mA
Reverse biasing	0 V −2 V	0 μA −0.25 μA

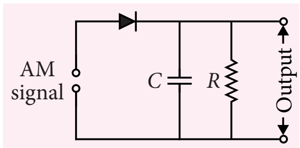
Assuming the characteristics to be nearly linear, over this range, the forward and reverse bias resistance of the given diode respectively are

(a) 20 Ω, 8×10^6 Ω (b) 40 Ω, 4×10^6 Ω
(c) 10 Ω, 8×10^6 Ω (d) 20 Ω, 4×10^6 Ω

7. If the highest modulating frequency of the wave is 5 kHz, the number of stations that can be accommodated in a 150 kHz bandwidth is

(a) 15 (b) 10
(c) 5 (d) none of these

8. Given the circuit diagram of an AM demodulator. For a good demodulation of AM signal of carrier frequency f, the value of RC should be



(a) RC = 1/f (b) RC < 1/f
(b) RC ≥ 1/f (d) RC >> 1/f

9. If carrier and modulating signals are given by $V_c = 20 \sin 10^6 \pi t$ and $V_m = 12 \sin 500 \pi t$ then the modulation index is

(a) 40% (b) 50% (c) 60% (d) 64%

10. A TV transmission tower at a particular station has a height of 80 m. By how much the height of tower be increased to double its coverage range?
(a) 160 m (b) 240 m
(c) 320 m (d) 120 m

11. In optical communication system operating at 1200 nm, only 2% of the source frequency is available for TV transmission having a bandwidth of 5 MHz. The number of TV channel that can be transmitted is
(a) 2 million (b) 10 million
(c) 0.1 million (d) 1 million

12. Sinusoidal carrier voltage of frequency 1.5 MHz and amplitude 50 V is amplitude modulated by sinusoidal voltage of frequency 10 kHz producing 50% modulation. The lower and upper side-band frequencies in kHz are, respectively
(a) 1490, 1510 (b) 1510, 1490
(c) $\frac{1}{1490}, \frac{1}{1510}$ (d) $\frac{1}{1510}, \frac{1}{1490}$

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. **Assertion :** Thickness of depletion layer is not fixed in all semiconductor devices.
Reason : No free charge carriers are available in depletion layer.

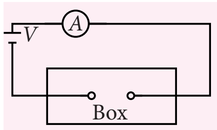
14. **Assertion :** A silicon optical fibre with a core diameter large enough has a core refractive index of 1.50 and a cladding refractive index 1.47. The acceptance angle in air for this fibre = $\sin^{-1}(0.30)$.
Reason : Acceptance angle $\theta_a = \sin^{-1}$ (NA) where NA = numerical aperture = $(\mu_1^2 - \mu_2^2)^{1/2}$.

15. **Assertion :** Many channels get allowed when transmission frequency is high.
Reason : At low frequencies, band width is too small.

JEE MAIN / PETS

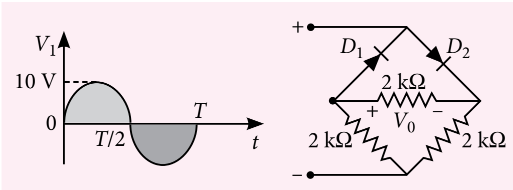
Only One Option Correct Type

16. A semiconductor diode and a resistor of constant resistance are connected in some way inside a box having two external terminals. When a potential difference V of 1 V is applied as shown, current I = 25 mA flows through box. If potential difference is reversed, I = 50 mA. Forward resistance and diode resistance are



(a) 40 Ω, 20 Ω (b) 40 Ω, 40 Ω
(c) 0 Ω, ∞ (d) 6 Ω, 12 Ω

17. In the circuit shown in figure the maximum output voltage V_0 is



(a) 0.2 V (b) 5 V
(c) 10 V (d) $\frac{5}{\sqrt{2}}$ V

18. A potential barrier of 0.3 V exists across a p-n junction. The depletion region is 1 μm wide. An electron with speed 400 km s^{-1} approached this p-n junction from n-side. Its speed on entering the p-side will be
(a) 1.3×10^4 m s^{-1} (b) 2.3×10^4 m s^{-1}
(c) 1.3×10^5 m s^{-1} (d) 2.3×10^5 m s^{-1}

19. For light of wavelength 1.3 μm, how many phone calls could be carried at maximum bit rate? Band width of optical fibre = 2 GHz.
(a) 2.9×10^7 (b) 1.5×10^6
(c) 2.3×10^5 (d) 1.7×10^4

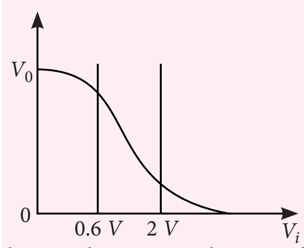
More than One Options Correct Type

20. In amplitude modulation, the modulation index m, is kept less than or equal to 1 because
(a) $m > 1$, will result in interference between carrier frequency and message frequency, resulting into distortion.
(b) $m > 1$, will result in overlapping of both side bands resulting into loss of information.

(c) $m > 1$, will result in change in phase between carrier signal and message signal.

(d) $m > 1$, indicates amplitude of message signal greater than amplitude of carrier signal resulting into distortion.

21. Figure shows the transfer characteristics of a base biased CE transistor. Which of the following statements are true?



(a) At $V_i = 0.4$ V, transistor is in active state.

(b) At $V_i = 1$ V, it can be used as an amplifier.

(c) At $V_i = 0.5$ V, it can be used as a switch turned off.

(d) At $V_i = 2.5$ V, it can be used as a switch turned on.

22. When an electric field is applied across a semiconductor,

(a) electrons move from lower energy level to higher energy level in the conduction band

(b) electrons move from higher energy level to lower energy level in the conduction band

(c) holes in the valence band move from higher energy level to lower energy level

(d) holes in the valence band move from lower energy level to higher energy level

23. Consider an *n-p-n* transistor with its base-emitter junction forward biased and collector base junction reverse biased. Which of the following statements are true?

(a) Electrons crossover from emitter to collector.

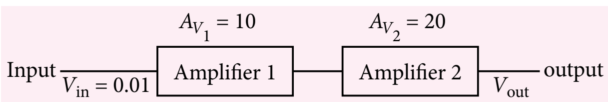
(b) Holes move from base to collector.

(c) Electrons move from emitter to base.

(d) Electrons from emitter move out of base without going to the collector.

Integer Answer Type

24. Two amplifiers are connected one after the other in series (cascaded). The first amplifier has a voltage gain of 10 and the second has a voltage gain of 20. If the input signal is 0.01 V, calculate the output ac signal (in V).

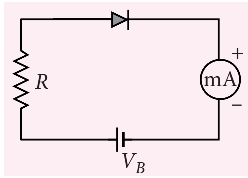


25. A change of 0.2 mA in the base current causes a change of 5 mA in the collector current for a common emitter amplifier. If the input resistance is 2 kΩ and its voltage gain is 75, calculate the load resistance (in kΩ) used in the circuit.

26. An AM transmitter records an antenna current of 10.5 A. The antenna current drops to 10 A when only carrier is carried. If the percentage modulation is $n \times 15.1\%$, then find the value of integer n.

Comprehension Type

A silicon *p-n* junction diode is connected to a resistor R and a battery of voltage V_B through milliammeter (mA) as shown in figure. The knee voltage for this junction diode is $V_N = 0.7$ V.



The *p-n* junction diode requires a minimum current of 1 mA to attain a value higher than the knee point on the *I-V* characteristics of this junction diode. Assuming that the voltage V across the junction is independent of the current above the knee point.

27. If $V_B = 5$ V, the maximum value of R so that the voltage V is above the knee point voltage is

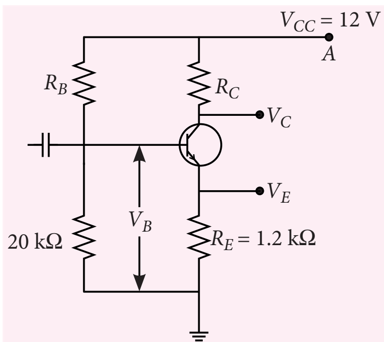
(a) 4.0 kΩ (b) 4.3 kΩ
(c) 5.0 kΩ (d) 5.7 kΩ

28. If $V_B = 5$ V, the value of R in order to establish a current of 6 mA flows in the circuit is

(a) 833 Ω (b) 717 Ω
(c) 950 Ω (d) 733 Ω

Matrix Match Type

29. For the transistor circuit as shown in figure,



Given; $I_c = 1$ mA, $V_{CE} = 3$ V, $V_{BE} = 0.5$ V and $V_{CC} = 12$ V, $\beta = 100$.

Match the column I with column II

	Column I		Column II
(A)	V_E (in V)	(P)	1.7
(B)	V_B (in V)	(Q)	1.2
(C)	R_C (in kΩ)	(R)	108
(D)	R_B (in kΩ)	(S)	7.8

	A	B	C	D
(a)	P	S	Q	R
(b)	Q	P	S	R
(c)	P	Q	S	R
(d)	P	S	R	Q

30. In column I, we have frequency range of em wave and in column II the main purpose of use of those em waves.

	Column I		Column II
(A)	54 – 72 MHz	(P)	FM broadcasting
(B)	840 – 935 MHz	(Q)	Television (VHF)
(C)	88 – 108 MHz	(R)	Cellular Mobiles Radio
(D)	3.7 – 6.4 GHz	(S)	Satellite communication

	A	B	C	D
(a)	R	P	S	Q
(b)	Q	R	P	S
(c)	Q	P	R	S
(d)	R	S	P	Q

◈◈

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......

No. of questions correct ......

Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / NEET / AIIMS / JIPMER with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / NEET. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

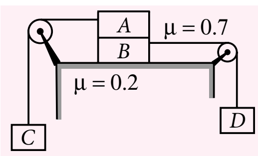
PROBLEM Set 52

MORE THAN ONE OPTION CORRECT TYPE

1. The current velocity of a river grows in proportion to the distance from its bank and reaches a maximum value v_0 in the middle. Near the banks the velocity is zero. A boat moves on the river with speed u relative to river its direction is constant and always perpendicular to the river flow. Width of river is d.
   (a) The distance through which the boat will be carried away or drift is $v_0d/2u$.
   (b) The distance through which the boat will be carried away or drift is $v_0d/4u$.
   (c) The path of boat as seen by observer on ground will be parabolic for each half of journey.
   (d) The path of boat as seen by observer on ground will be curved but not parabolic.

2. A block of mass 1 kg falls from a height h = 0.4 m on a massless spring of stiffness constant k = 300 N m^{-1}. If g = 10 m s^{-2}, then
   (a) when compression in the spring is 1/30 m, the acceleration of the block is zero
   (b) maximum compression in the spring is 0.2 m
   (c) maximum acceleration of the block is 50 m s^{-2}
   (d) maximum extension in the spring is 0.2 m

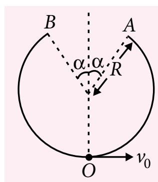
3. An arrangement of masses and pulleys is shown in the figure. Strings connecting masses A and B with the pulleys are horizontal and all the pulleys and the strings are light. Friction coefficient between the surface and the block B is 0.2 and that between blocks A and B is 0.7. The system is released from rest.



   (Given : mass of blocks A, B, C and D are 6 kg, 3 kg, 6 kg and 1 kg respectively.)
   (a) The magnitude of acceleration of the system is 2 m s^{-2} and there is no slipping between block A and block B.
   (b) The magnitude of friction between block A and block B is 42 N
   (c) Acceleration of block C is 1 m s^{-2} downwards.
   (d) Tension in the string connecting blocks B and D is 12 N.

4. A thin rod of length l is kept vertically (along y-axis) with end A being at rest on a smooth horizontal plane. A slight disturbance makes the end A slip on the plane (along x-axis) and the rod falls. Then,
   (a) the path described by a point P at a distance of $\frac{l}{4}$ from A is a quadrant of a circle
   (b) the centre of mass of the rod moves along a straight line, parallel to y-axis.
   (c) the acceleration of the centre of mass as the rod becomes horizontal is $\frac{3g}{4}$
   (d) the greatest angular velocity of the rod about the instantaneous centre of rotation is $\sqrt{\frac{3g}{l}}$

5. A wire is bent in the shape of an arc as shown. A bead can move on the wire without friction. Initially the bead is at point O. It is given a velocity v_0 so that it just reaches point B on the wire and moves along the wire again.



By Akhil Tewari, Author Foundation of Physics for JEE Main & Advanced, Professor, IITians PACE, Mumbai.

(a) $v_0 = \sqrt{gR\left(2+2\cos\alpha+\dfrac{1}{\cos\alpha}\right)}$

(b) $v_0 = \sqrt{gR\left(1+2\cos\alpha+\dfrac{1}{\cos\alpha}\right)}$

(c) During the motion of bead on wire the normal reaction on bead may reverse direction twice.

(d) The normal reaction on the bead is always radially away from the center.

6. A ball of mass 1 kg collides with a wall at a speed 10 m s^{-1} and rebounds on the same line with the same speed. If the mass of the wall is taken as infinite, then

(a) the work done by the ball on the wall is zero

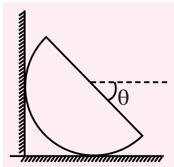
(b) the magnitude of the impulse of force exerted by the ball on the wall is 20 kg m s^{-1}

(c) both the kinetic energy and momentum of the ball remains unchanged

(d) the kinetic energy of the ball remains unchanged while the momentum changes

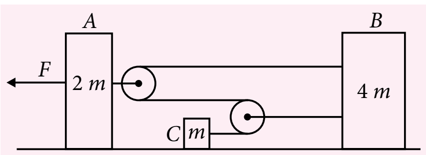
INTEGER TYPE

7. A semicircular disc rests in the vertical plane with its curved surface on a vertical and horizontal rough surface as shown in figure.



If coefficient of friction is μ for all surfaces then the largest value of θ is $\sin^{-1}\left(\dfrac{3\pi}{n}\left(\dfrac{\mu+\mu^2}{1+\mu^2}\right)\right)$, find n.

8. Consider three blocks A, B and C placed on a horizontal surface as shown in figure and a force F is applied on A. Assuming all surfaces to be smooth, the ratio of acceleration of block B when C is fixed to when all blocks are free to move is $3x/17$. Find x.

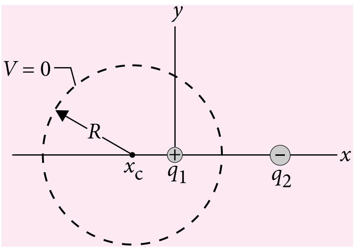


9. An adiabatic piston of mass m equally divides a diathermic container of volume V_0 and length l. A light spring connects the piston to the right wall. In equilibrium, pressure on each side of the piston is P_0. The container starts moving with acceleration a towards right. The stretch x of the spring when the acceleration of the piston equals the acceleration of the container is given as $n\left[k+\dfrac{n\rho_0\gamma A}{l}\right]^{-1}$. Find n. (Assume that $x << l$, the gas in the container has the adiabatic exponent, ratio of C_p and C_v is γ, m = 2 kg, a = 2 m s^{-2}).

10. A particle is projected from a point O with initial speed of 30 m s^{-1} to pass through a point which is 40 m from O horizontally and 10 m above O. There are two angles of projection for which this is possible. If these angles are α and β then find the value of $\tan[-(\alpha+\beta)]$.

JEE WORK OUTS

1. A point charge $q_1 = +\,6e$ is fixed at the origin of a rectangular coordinate system, and a second point charge $q_2 = -10e$ is fixed at $x = 9.60$ nm, $y = 0$. With $V = 0$ at infinity, the locus of all points in the xy plane with $V = 0$ is a circle centered on the x axis, as shown in figure. Find (a) the location x_c of the centre of the circle and (b) the radius R of the the circle.

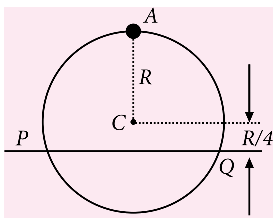


2. A room is maintained at 20 °C by a heater of resistance of 20 ohms connected to 200 V mains. The temperature is uniform throughout the room and the heat is transmitted through a glass window of area 1 m^2 and thickness 0.2 cm. Calculate the temperature (in °C) outside. Thermal conductivity of glass is 0.2 cal $m^{-1}s^{-1}°C^{-1}$ and mechanical equivalent of heat is 4.2 J cal^{-1}.

3. A very small circular loop of area 5×10^{-4} m^2, resistance 2 Ω and negligible inductance is initially coplanar and concentric with a larger fixed circular loop of radius 0.1 m. A constant current of 1 A is passed in the bigger loop and the smaller loop is rotated with angular velocity 100 rad s^{-1} about a diameter. Find the maximum emf induced in the smaller loop.

4. Two swimmers leave point A on one bank of the river to reach point B lying right across on the other bank. One of them crosses the river along the straight line AB while the other swims at right angles to the stream and then walks the distance that he has been carried away by the stream to get to point B. What was the velocity u (in km h^{-1}) of her walking if both swimmers reached the destination simultaneously? The stream velocity is 2 km h^{-1} and the velocity of each swimmer with respect to still water is 2.5 km h^{-1}.

5. A uniform circular disc has radius R and mass m. A particle also of mass m, is fixed at a point A on the edge of the disc as shown in figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance $R/4$ from the centre C of the disc. The line AC is perpendicular to PQ. Initially, the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle as it reaches its lowest position.



6. Distance between the centre of two stars is 10 a. The masses of these stars are M and $16M$ and their radii are a and $2a$ respectively. A body of mass m is fired straight from the surface of the large star towards the smaller star. What should be its minimum speed to reach the surface of the smaller star ?

7. A piece of copper wire has a resistance per unit length of 5.9×10^{-3} Ω m^{-1}. The wire is then wound into a thin flat coil of many turns that has a radius of 0.05 m. The ends of the wire are connected to a 12 V battery. Determine the magnetic field at the centre of coil.

8. A small ideally absorbing plate of mass 10 mg is suspended from a practically weightless quartz fibre 2 cm long. A light flash from a laser falls on its surface perpendicular to it, causing the fibre with the plate to deflect from the vertical by an angle of 0.6°. Calculate the energy of the laser flash. (in joule)

9. A lead bullet just melts when stopped by an obstacle. Assuming that 25 percent of the heat is absorbed by the obstacle, find the velocity of the bullet. Its initial temperature is 27 °C.
(Melting point of lead = 327 °C, specific heat of lead = 0.03 cal g^{-1} °C^{-1}, latent heat of fusion of lead = 6 cal g^{-1} °C^{-1}, J = 4.2 joule cal^{-1}).

10. A disabled tanker leaks kerosene (μ = 1.20) into the Persian Gulf, creating a large slick on top of the water (μ = 1.33). If you are looking straight down from an airplane into a region of the slick where its thickness is 460 nm, for which wavelength(s) of visible light is the reflection the greatest?

SOLUTIONS

1. (a) The potential at any point will be the sum of the contribution from each charge,

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}+\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2},$$

where r_1 is the distance between x_c and q_1 and r_2 is the distance between x_c and q_2.

$$r_1=\sqrt{x^2+y^2}$$

Since q_2 is at (d, 0), where d = 9.60 nm,

$$r_2=\sqrt{(x-d)^2+y^2}$$

$$S=\frac{q_1}{r_1}+\frac{q_2}{r_2} \text{ (where, } S=4\pi\varepsilon_0 V)$$

If S = 0, we can rearrange and square this expression,

$$\frac{q_1}{r_1}=-\frac{q_2}{r_2} \Rightarrow \frac{r_1^2}{q_1^2}=\frac{r_2^2}{q_2^2},$$

$$\frac{x^2+y^2}{q_1^2}=\frac{(x-d)^2+y^2}{q_2^2},$$

Let $\alpha = q_2/q_1$, then we can write

$$\alpha^2(x^2+y^2)=(x-d)^2+y^2$$

or $\alpha^2x^2+\alpha^2y^2=x^2-2xd+d^2+y^2$

or $(\alpha^2-1)x^2+2xd+(\alpha^2-1)y^2=d^2$

By adding $d^2/(\alpha^2-1)$ to both sides of the equation. Then

$$(\alpha^2-1)\left[\left(x+\frac{d}{\alpha^2-1}\right)^2+y^2\right]=d^2\left(1+\frac{1}{\alpha^2-1}\right)$$

$$\left(x+\frac{d}{\alpha^2-1}\right)^2+y^2=\frac{d^2}{(\alpha^2-1)}\left(1+\frac{1}{\alpha^2-1}\right)$$

By comparing the equation of circle, we get

$$x=-\frac{d}{\alpha^2-1}=\frac{-(9.60\,\text{nm})}{(-10/6)^2-1}=-5.4\text{ nm};\ y=0$$

The centre of the circle is at (–5.4 nm, 0).

(b) The radius of the circle is

$$\sqrt{d^2\frac{\left(1+\frac{1}{\alpha^2-1}\right)}{\alpha^2-1}},$$

which can be simplified to

$$d\frac{\alpha}{\alpha^2-1}=(9.6\,\text{nm})\frac{|(-10/6)|}{(-10/6)^2-1}=9.00\,\text{nm}$$

2. Heat entering per second to the room,

$$\frac{\Delta H}{t}=\frac{V^2}{R}=\frac{200\times200}{20}=2000\,\text{J}$$

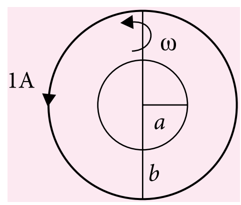
Also, $\frac{\Delta H}{t}=\frac{KA(\Delta T)}{l}$

$$\therefore\ 2000=\frac{0.2\times4.2\times1\times(20-T)}{0.2\times10^{-2}}$$

$$\Rightarrow\ 20-T=\frac{2000\times0.2\times10^{-2}}{0.2\times4.2}=4.76$$

$$\Rightarrow\ T=15.2°\text{C}$$

3. The situation is shown in the diagram. The field at the centre of larger loop,



$$B_1=\frac{\mu_0}{4\pi}\frac{2\pi i}{R}=2\pi\times10^{-6}\,\text{T}$$

This field is initially along the normal to the area of smaller loop. Now as the smaller loop (and hence normal to the plane) is rotating at angular velocity ω, so in time t, it will turn by an angle, θ = ωt, with respect to B and hence the flux linked with smaller loop at time t,

$$\phi=B_1S_2\cos\theta=(2\pi\times10^{-6})\times5\times10^{-4}\cos\omega t$$

or $\phi=\pi\times10^{-9}\cos\omega t$

emf induced in the smaller loop,

$$\varepsilon=-\frac{d\phi}{dt}=-\frac{d}{dt}(\pi\times10^{-9}\cos\,\omega t)$$

$\varepsilon=\pi\times10^{-9}\,\omega\sin\omega t$

Maximum induced emf = $\pi\times10^{-9}\,\omega$

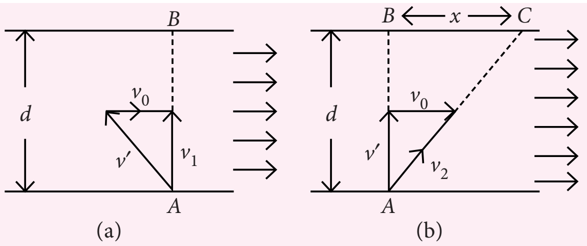
$= 3.14\times10^{-9}\times100=3.14\times10^{-7}$ V

4. Let one of the swimmer crosses the river along AB, which is obviously the shortest path. Time taken to cross the river by the swimmer 1,

$$t_1 = \frac{d}{\sqrt{v'^2 - v_0^2}} = \frac{d}{\sqrt{2.5^2 - 2^2}} = \frac{d}{1.5} \quad ...(i)$$

For the other swimmer which follows the quickest path, the time taken to cross the river,

$$t_2 = \frac{d}{v'} \quad ...(ii)$$



In the time t_2, drifting of the swimmer 2, becomes

$$x = v_0 t_2 = \frac{v_0}{v'} d \text{ or } t_2 = \frac{d}{v'} = \frac{d}{2.5} \quad ...(iii)$$

(using eqn. (ii))

Let t_3 be the time for swimmer 2 to walk the distance x to come from C to B (figure (b)), then

$$t_3 = \frac{x}{u} = \frac{v_0 d}{v'u} = \frac{2d}{2.5u} \quad ...(iv)$$

According to the problem, $t_1 = t_2 + t_3$

$$\frac{d}{1.5} = \frac{d}{2.5} + \frac{2d}{2.5u} \Rightarrow u = 3 \text{ km h}^{-1}$$

5. Initially the particle of mass m is fixed at point A on a disc. When the disc is held vertical, the point A is at its highest position.

When the disc rotates about PQ, the point A, along with the particle of mass m, reaches its lowest position at A'.

Let BD be taken as reference level for potential energy.

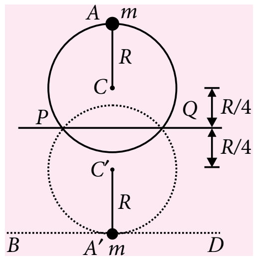
$$\therefore AA' = AC + CC' + C'A' = R + \left(\frac{R}{4} + \frac{R}{4}\right) + R$$

$$= \frac{5R}{2}$$

$$\therefore CC' = \frac{R}{4} + \frac{R}{4} = \frac{R}{2}$$

Decrease in potential energy of particle

$$= mg \times (AA') = \frac{5mgR}{2}$$



Decrease in potential energy of disc

$$= mg \times (CC') = \frac{mgR}{2}$$

Total decrease in potential energy of system

$$\frac{5mgR}{2} + \frac{mgR}{2}$$

$\therefore$ Decrease in potential energy of system $= 3mgR$ ...(i)

Gain in kinetic energy of system $= \frac{1}{2} I\omega^2$

I = Moment of inertia of system about PQ

or $I = I_{disc} + I_{particle}$

$$\text{or } I = \frac{mR^2}{4} + \frac{mR^2}{16} + \frac{25mR^2}{16} = \frac{30mR^2}{16} = \frac{15mR^2}{8}$$

$\therefore$ Gain in kinetic energy of system

$$= \frac{1}{2} \times \left(\frac{15mR^2}{8}\right)\omega^2 \quad ...(ii)$$

Equating the energies of the system,

$$\therefore \frac{15\omega R^2\omega^2}{16} = 3\, mgR, \text{ From (i) and (ii)}$$

$$\text{or } \omega^2 = \frac{16g}{5R} \quad \text{or} \quad \omega = \sqrt{\frac{16g}{5R}}$$

($\because$ Angular speed is about PQ)

The particle rotates in a circle of radius

$$\left(\frac{AA'}{2}\right) = \text{radius} \left(\frac{5R}{4}\right)$$

$\therefore$ Linear speed of particle at position $A' = v$

$$\therefore v = \frac{5R\omega}{4} = \left(\frac{5R}{4}\right)\sqrt{\frac{16g}{5R}} = \sqrt{5gR}$$

6. Let A denote the point along O_1O_2 where gravitational field strength is zero.

$\therefore$ Field strength due to S_1 = Field strength due to S_2

$$\text{or } \frac{GM}{r_1^2} = \frac{G(16M)}{r_2^2}$$

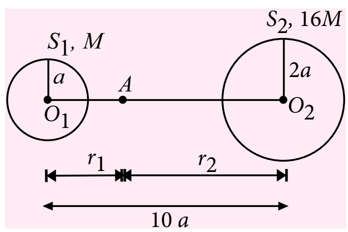
or $r_2 = 4r_1$

But $r_1 + r_2 = 10a$

$\therefore r_1 + 4r_1 = 10a$

or $r_1 = 2a$ .....(i)

and $r_2 = 8a$ .....(ii)



When the body of mass m is fired straight from the surface of S_2 towards S_1 along O_2O_1, it travels under the gravitational forces exerted by S_2 and S_1.

From O_2 to A, it is attracted by S_2.
At A, there is a region of no attraction.
From A to O_1, it is attracted by S_1.
Let v denote velocity with which the body of mass m is fired so that it just crosses A along AO_1.
Using the law of conservation of energy,

$$\therefore \frac{1}{2}mv^2 = \left[-\frac{GmM}{r_1} - \frac{16GmM}{r_2}\right] - \left[-\frac{GmM}{(10a-2a)} - \frac{16GmM}{2a}\right]$$

$$\text{or } \frac{1}{2}mv^2 = -GmM\left[\frac{1}{2a} + \frac{16}{8a} + \left(\frac{-1}{8a} - \frac{16}{2a}\right)\right]$$

$$\text{or } \frac{1}{2}mv^2 = \frac{45GmM}{8a} \Rightarrow v^2 = \frac{45GM}{4a}$$

$$\text{or } v = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

7. Let l be the length of copper wire and it is wound into a coil of n turns of radius 0.05 m.

$\therefore\ l = n \times 2\pi\ (0.05)$

$$I = \frac{V}{R} = \frac{12}{5.9\times10^{-3}\,l}$$

$$B = \frac{\mu_0 nI}{2(0.05)} = \frac{\mu_0 \times 12}{0.1\times5.9\times10^{-3}\times2\pi\times0.05} = 0.081\text{ T}$$

8. Let E = energy of laser flash

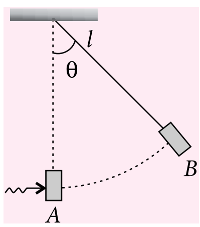
Then its momentum is $\frac{E}{c}$

$\therefore$ momentum transfer to the plate $= \frac{E}{c}$

If v be the velocity acquired by the plate, then

$$mv = \frac{E}{c}$$

By conservation of energy of plate between the points A and B.



$$\frac{1}{2}mv^2 = mgl(1-\cos\theta)$$

$$\Rightarrow v^2 = 2\,gl\,(1 - \cos\theta)$$

$$\Rightarrow \frac{E^2}{m^2c^2} = 2gl\times2\sin^2\frac{\theta}{2}$$

$$\Rightarrow \frac{E}{mc} = 2\sin(\theta/2)\sqrt{gl}$$

(since $\sin\frac{\theta}{2} \approx \frac{\theta}{2}$, for deflection being small)

$$\therefore\quad E = 2mc\frac{\theta}{2}\sqrt{gl}$$

$$= 10\times10^{-6}\times3\times10^8\times(0.6\,\pi/180)\ \sqrt{9.8\times0.02}$$

$$= 13.9\text{ J} \simeq 14\text{ J}$$

9. Kinetic energy of bullet is converted into heat.
25% of heat is absorbed by obstacle.
75% of heat is absorbed by lead. This heat raises temperature of lead to 327 °C and then melts it.

$$75\%\times\left(\frac{1}{2}Mv^2\right) = Ms\Delta T + ML$$

$$\text{or } \frac{75}{100}\times\frac{Mv^2}{2} = M(s\Delta T + L)$$

$$s = \frac{0.03\text{ cal}}{\text{g °C}} = \frac{0.03\times4.2\text{ J}}{(10^{-3}\text{kg})\text{°C}} = \frac{0.03\times4.2\times1000\text{ J}}{\text{kg °C}}$$

$$L = \frac{6\text{ cal}}{\text{g}} = \frac{6\times4.2}{10^{-3}\text{kg}} = 6\times4.2\times10^3\text{ J kg}^{-1}$$

$$\therefore \frac{3}{4}\times\frac{v^2}{2} = 126\times(327-27) + 25200$$

$$= 37800 + 25200$$

$$\text{or } v^2 = \frac{8}{3}\times63000 = 8\times21000 = 168000$$

or $v = 409.8$ m s^{-1}.

10. Light from above the oil slick can be reflected back up from the top of the oil layer or from the bottom of the oil layer, For both reflections, the light is reflecting off a substance with a higher refractive index so both reflected rays pick up a phase change of π. Therefore, the equation for a maxima is

$$2d + \frac{1}{2}\lambda_n + \frac{1}{2}\lambda_n = m\lambda_n.$$

that $\lambda_n = \lambda/\mu$, where μ is the refractive index of the thin film. Then $2\mu d = (m - 1)\lambda$
is the condition for a maxima. We know $\mu = 1.20$ and $d = 460$ nm. We need to find the wavelength in the visible range (400 nm to 700 nm) which has an integer m. If $\lambda = 700$ nm, then m is

$$m = \frac{2\mu d}{\lambda} + 1 = \frac{2(1.20)(460\text{ nm})}{(700\text{ nm})} + 1 = 2.58$$

But m needs to be an integer. If we increase m to 3, then

$$\lambda = \frac{2(1.20)(460\text{ nm})}{(700\text{ nm})} = 552\text{ nm}$$

which is clearly in the visible range. Trying $m = 2$ will result in a value in the infrared range (1100 nm), while $m = 4$ will yield a value in the ultraviolet range (368 nm). So the oil slick will appear green.

◆◆

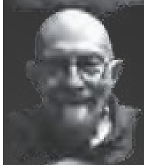
Live Physics

TRIO RIDES GRAVITATIONAL WAVES TO NOBEL HONOUR

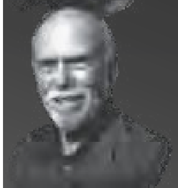
The Physics Nobel this year recognises the detection of gravitational waves - a discovery that can revolutionise astrophysics and help us analyse the beginning of the universe. The three scientists who have bagged the award are all associated with the LIGO project through which they detected the waves that Albert Einstein had predicted 100 years ago...

THE WINNERS

Rainer Weiss | 85
Massachusetts Institute of Technology



Kip Thorne | 77
California Institute of Technology



Barry Barish | 81
California Institute of Technology

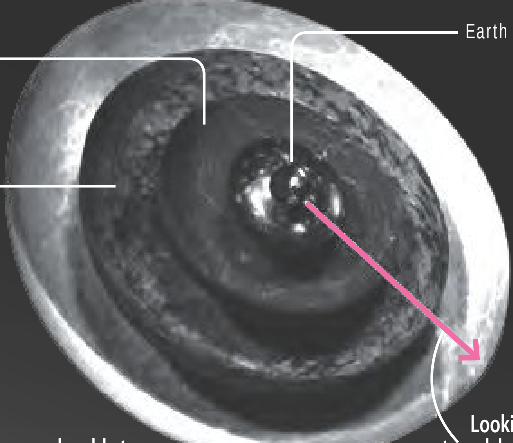
One half of the $1.1m prize for Weiss and the other half jointly for Barish and Thorne

The trio were the architects and founders of LIGO

What are gravitational waves

Gravitational waves pass through boundaries that light cannot and can give us "ears" into every corner of the universe

They can transport information about what happens inside black holes, which even light cannot escape, and pass through cosmic microwave background (CMB) radiation, which prevents us from seeing the universe before it turned 380,000 years old



4.25bn
light years away
The farthest distance at which LIGO can pick up signals

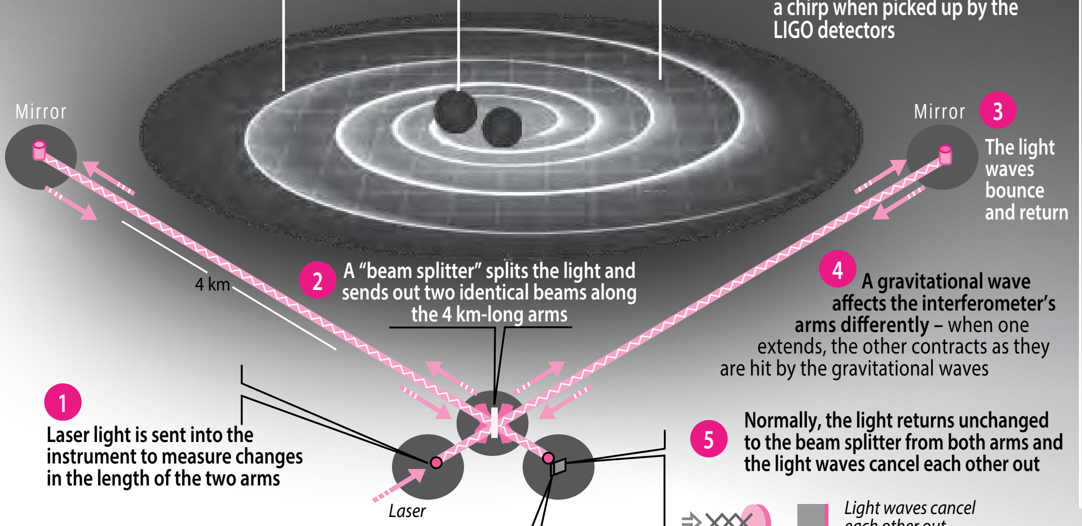
300mn
years after big bang
First stars form

3,80,000
years back
Farthest distance we will ever see with light

Less than a second
after big bang
Future gravitational-wave detectors may be able to directly measure fluctuations in the earliest universe

How LIGO detected the waves

The waves came from a collision between two black holes and took 1.3 bn years to reach Earth. They sounded like a chirp when picked up by the LIGO detectors



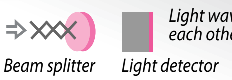
1 Laser light is sent into the instrument to measure changes in the length of the two arms

2 A "beam splitter" splits the light and sends out two identical beams along the 4 km-long arms

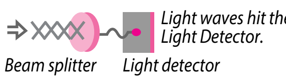
3 The light waves bounce and return

4 A gravitational wave affects the interferometer's arms differently – when one extends, the other contracts as they are hit by the gravitational waves

5 Normally, the light returns unchanged to the beam splitter from both arms and the light waves cancel each other out



6 If the arms are disturbed by a gravitational wave, the **light waves will have travelled different distances.** Light then escapes through the splitter and hits the detector



LIGO is a pair of L-shaped antennas that detected the waves. The antennas are located in US in Hanford, Washington, and Livingston, Louisiana, 3,000 km apart

LIGO: Laser Interferometer Gravitational-Wave Observatory, a project with over 1,000 researchers from 20+ countries

Source: **Nobel.org**
Graphic: **Sunil Singh**

Courtsey : The Times of India

PHYSICS MUSING

SOLUTION SET-51

1. **(b) :** Initial energy of electron = 2 eV
Energy of electron in 1st excited state

$$(i.e., n=2) = \frac{-13.6}{2^2} = -3.4 \text{ eV}$$

Photon of energy = 2 – (–3.4) = 5.4 eV will be emitted.

$$\Rightarrow KE_{max} = E_{photon} - W_0 = \left(5.4 - \frac{12400}{4600}\right)\text{eV} = 2.7\text{eV}.$$

$$= 5.4 - 2.7 = 2.7 \text{ eV}$$

2. **(c):** $I_R = 4I_0 \cos^2\left(\frac{\Delta\theta}{2}\right)$

As given I_R at centre is $2I_0$, then,

$$2I_0 = 4I_0 \cos^2\left(\frac{\Delta\theta}{2}\right)$$

$$\Rightarrow \left(\frac{\Delta\theta}{2}\right) = \frac{\pi}{4}$$

$$\therefore \quad \Delta\theta = \frac{\pi}{2} \text{ and } \Delta x = \Delta\theta \times \frac{\lambda}{2\pi} = \frac{\lambda}{4}$$

For slab of thickness t path difference $\Delta x = (\mu - 1)t$

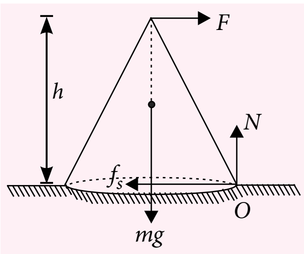
$$\left(\frac{3}{2} - 1\right)t = \frac{\lambda}{4} \text{ or } t = \frac{\lambda}{2}$$

3. **(c):** torque (τ) $\tau = r \times F$
where, r = distance of force from the axial point of the rotation. For toppling,

$\tau_O = 0$

$Fh = mgr$

$$F = \frac{mgr}{h}$$



For slipping,

$$F = f_{s_{max}} = \mu mg$$

For slipping before toppling,

$$\frac{mgr}{h} \geq \mu mg \Rightarrow \mu \leq \frac{r}{h}$$

$$\therefore \quad \mu_{max} = \frac{r}{h}$$

4. **(c):** Given that 1 MSD = 1 mm
9 MSD = 10 VSD
∵ Least count,

$$\text{LC} = 1 \text{ MSD} - 1 \text{ VSD} = 1\,\text{mm} - \frac{9}{10}\,\text{mm} = \frac{1}{10}\,\text{mm}$$

Measuring reading of edge = MSR + VSR × (LC)

$$= 10 + 1 \times \frac{1}{10} = 10.1\,\text{mm}$$

Volume of cube $V = (1.01)^3 = 1.03$ cm^3

$$\because \quad \text{Density of cube} = \frac{2.736}{1.03} = 2.6563 \text{ g cm}^{-3}$$

$$= 2.66 \text{ g cm}^{-3}$$

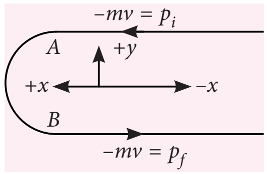
5. **(a) :** From figure, $OP = 2r \sin\theta = vt$

$$\sin\theta = \frac{vt}{2r} \quad \Rightarrow \quad \cos\theta \frac{d\theta}{dt} = \frac{v}{2r}$$

$$\text{or} \quad \frac{d\theta}{dt} = \frac{v}{2r\sqrt{1-\sin^2\theta}}$$

$$= \frac{v}{2r\sqrt{1-\frac{v^2t^2}{4r^2}}} = \frac{v}{\sqrt{4r^2 - v^2t^2}}$$

6. **(b) :** Choosing the x–y areas as shown in the figure,



The initial momentum of bead at A is $p_i = + mv$
The final momentum of bead at B is $p_f = - mv$
Now, Δp at A and B is

$$\Delta p = p_f - p_i = -2mv$$

Now, time taken by the bead on moving through AB is

$$\Delta t = \frac{\pi.d/2}{v} = \frac{\pi d}{2v}$$

Average force exerted by bead on the wire is

$$|F_{av}| = \frac{\Delta p}{\Delta t} = \left(\frac{2mv}{\pi d/2v}\right) = \frac{4mv^2}{\pi d}$$

7. **(b, d) :** Binding energy of two nuclei between $1 < A < 50$ = 2 × Binding energy of each nucleus = 2 (2A)
Binding energy of fused nucleus
mass number (2< A < 100) = (2 × 2A) = 4A
⇒ No energy will be released.
Binding energy of two nuclei between $51 < A < 100$
= 2 × Binding energy of each nucleus
= 2 × (2A) = 4A
and binding energy of fused nucleus
mass number $100 < A < 200$ = (8 × 2A) = 16A
⇒ Energy will be released.

Binding energy of nucleus (mass number $100 < A < 200$) $= 8A$

Binding energy of two nuclei (mass number $50 < \frac{A}{2} < 100$) after fission $= 2\left(2 \times \frac{A}{2}\right) = 2A$

$\Rightarrow$ No energy will be released

Binding energy of nucleus (mass number $200 < A < 260$) $= 4A$

Binding energy of two nuclei (mass number $100 < \frac{A}{2} < 130$) on fusion $= 2\left(8 \times \frac{A}{2}\right) = 8A$

$\Rightarrow$ Energy will be released

Hence option (b) and (d) are correct only.

8. (a, b) : Given, $g = \frac{2h}{t^2}$

For relative error, $\frac{\Delta g}{g} = \frac{\Delta h}{h} + \frac{2\Delta t}{t}$

Mean value should be calculated using, $g = \frac{2h}{t^2}$

For 1st experiment, $g_1 = \frac{2 \times 3}{(0.781)^2} = 9.8366 = 9.84$

$$\Delta g_1 = g_1\left(\frac{\Delta h}{h} + \frac{2\Delta t}{t}\right)$$

$$= 9.84\left(\frac{0.003}{3} + \frac{2 \times 0.002}{0.781}\right) = 0.06 \text{ m s}^{-2}$$

$\therefore \quad g_1 = (9.84 \pm 0.06)$ m s^{-2}

Repeating similar calculation for second experiment, we get

$g_2 = (9.88 \pm 0.09)$ m s^{-2}

9. (a,b,c,d) : If block moves the distance x and let say x_2 and x_1 be compressions in S_2 and S_1 respectively.

$x_1 + x_2 = x$, $k_1x_1 = k_2x_2$

$\therefore \quad x_1 = \frac{k_2 x}{k_2 + k_1}$ and $x_2 = \frac{k_1 x}{k_1 + k_2}$

(a) Total work done by block on springs is equal to work done by S_2 on block and that is equal to total potential energy in S_1 and S_2.

$$\Delta U = \frac{k_1}{2}\left(\frac{k_2 x}{k_1 + k_2}\right)^2 + \frac{k_2}{2}\left(\frac{k_1 x}{k_1 + k_2}\right)^2 = \frac{1}{2}\frac{k_1 k_2}{k_1 + k_2}x^2$$

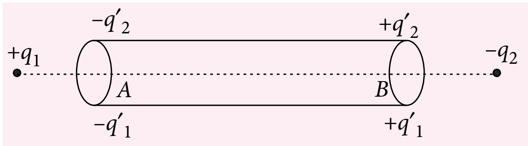
$$W_{S_2} \text{ on block} = -\Delta U = -\frac{1}{2}\frac{k_1 k_2}{k_1 + k_2}x^2$$

(b) W_{S_2} on $S_1 = \Delta U_{S_1} = \frac{1}{2}k_1 x_1^2 = \frac{k_1 k_2^2 x^2}{2(k_1 + k_2)^2}$

(c) W_F on block $= \Delta U_{S_2 \text{ and } S_1} = \frac{1}{2}\left(\frac{k_1 k_2}{k_1 + k_2}\right)x^2$

(d) $W_{S_{1,wall}} = 0$

10. (a, b, c, d) : Due to induction effect, the situation is shown in figure.



Due to $+q_1$, net induced charges is $-q'_1$ at end A and q'_1 at end B while due to $-q_2$ induced charges are $-q'_2$ and q'_2 at ends A and B respectively. Thus, the end A acquires negatively charged and B acquires positive charge. Electric force experienced by q_1 or $-q_2$ has to be computed by using principle of superposition.

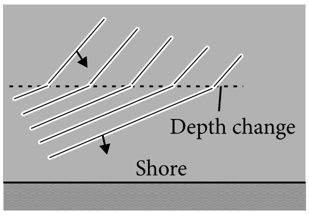
YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

Q1. Waves can approach a beach from many directions, depending on the wind and the locations of distant storms. Why do the waves generally turn so as to be parallel to the beach figure shown.



– Jaspreet Singh, Ludhiana

Ans. This tendency to turn, is due to the decrease in speed of the waves as water depth decreases. As a particular wave crest crosses from deeper water to shallower water, the portion of the crest to cross first slows and then lags behind the rest of the crest. This lagging puts a kink in the crest.

The slower portion in the shallower water is now travelling more directly toward shore than the portion still in the deeper water. Eventually all of the crest will have passed into the shallower water and be headed more directly toward shore.

The extent of the slowing, and thus also the turning, depends on wavelength, and so the individual waves slow and turn by different amounts.

Q2. How does a long, heavy bar help a tight rope walker maintain balance, especially if the performance is outdoors and in a moderately gusty wind?

– Divyansh Prasad, Kolkata

Ans. Balance is maintained if the center of mass is kept, on the average, over the rope. When the performer leans too far in one direction, the body must bend back in the opposite direction. A heavy bar helps, if the performer leans, say, to the left, the bar is shoved to the right so that the combined center of mass of the performer and bar is kept over the rope. The procedure must be executed quickly before the performer leans too far. A light bar is of little help because with little mass, it would have to be shifted too far to be practical.

Q3. While we put an audio cassette player in its play mode without a cassette in place (or with a blank cassette) and turn the volume control to maximum. Then bring a strong magnet up to the play head. Why does the motion produce a fizzing sound on the player?

– Akanksha Chokker, Delhi

Ans. The play head in the cassette player is ferromagnetic and consists of many magnetic domains, or regions in which the magnetic properties are uniform and produce a magnetic field in a certain direction. However, the field direction differs from domain to domain. As you bring the magnet near the play head, the domains shift abruptly to bring their magnetic fields into alignment with the magnet's field. As these domain fields change, a varying current is produced in a coil wrapped around the play head. Those changes in current are amplified and fed to a speaker, where a fizzing sound is heard — the coming and going of current as the magnetic field of the domains shift.

4. People waiting near the tracks of an electrically driven train have sometimes noticed a tingling if they touch a conducting object, such as a pipe, that is connected to ground. What causes the tingling?

– Meenakshi Pandey, Patna

Ans. If the train is driven electrically by an overhead line, it probably carries alternating current (AC). Because such current continually changes direction and strength, the magnetic field it produces in the surrounding region also changes direction and strength. In conductors this variation in magnetic field produces currents, but the currents are probably too small for a person to notice. However, if the person touches a larger conducting object, such as a metallic sign, the currents can be larger and noticeable.

◈◈

Rural IIT Aspirants Hit as Entrance Test Goes Online

Mastering Pythagoras and Bayes' Theorem may come in handy for a teenager preparing for a berth at the nation's most prestigious engineering colleges, but means nothing if he doesn't have a working knowledge of the computer.

Of the 2.2 lakh students appearing for the IIT Joint Entrance Examination (Advanced) in 2018, an estimated 50-60% are from rural and semi-urban areas. Many of these students have never used a computer, said experts and coaching institutes. And these students have just few months to get comfortable with the computer before they take online one of the toughest engineering entrance exams in the world.

The Joint Admission Board (JAB) announced that the entrance test to Indian Institutes of Technology will completely go online from 2018. The last written IIT JEE-Advanced was held in May this year.

Last year's JEE-Mains, a preliminary exam to the JEE-Advanced, was conducted both offline and online. According to expert estimates, just 5-10% students opted then to take the test online.

The IIT JEE is the qualifying examination for candidates seeking admission to undergraduate engineering programmes at IITs and other centrally funded technical institutions. The tests are conducted in two parts, JEE-Main and JEE-Advanced (the sole admission test for the IITs).

Coaching institutes are gearing up to make their students comfortable taking the tests online. Several of them already have their digital platforms and will start familiarising the students to it.

"All coaching institutes have online testing platforms — at least for JEE-Mains, which they will transfer for JEE-Advanced,". However, the students who have taken admission in April-June are at the peak of their preparation for the tests and do not have much time to familiarise themselves with the new system.

JEE Advanced Online from 2018

CHALLENGES

- 50-60% of the 2.2 lakh students to take the test are from rural and semi-urban areas, of whom many have never used a computer
- Students get only 10 months to familiarise themselves with the computer
- Logistical challenges of holding the exam for a large number of students on one day

ADVANTAGES

- Going online can bring down errors
- Results can be declared faster
- A step forward for India as most countries are already conducting online examinations

Another challenge could be managing over 2 lakh students taking the test online on one day. However, Bhaskar Ramamurthi, director of IIT Madras and chairman of JAB 2017, ruled out the possibility of conducting multiple exams.

"The reason we had not started it earlier is because we felt it is not possible to manage the over 2 lakh students together online but now we know it is feasible," he said, adding that online tests are being already done in other exams like GATE. "We will release more information on how students can familiarise themselves and sample papers, etc.," he added.

However, unlike IIT JEE that is for Class 12 students, GATE is conducted for admissions to postgraduate courses in engineering/ technology/ architecture/science, where there is more scope of an earlier exposure and familiarity with computer. Similarly, the Common Admission Test (CAT), an entrance examination to the premier management and business schools like IIMs, is also a computer-based test. However, CAT applicants are at least graduates who are more likely to have basic knowledge of the computer than the young 18-year-olds from rural hinterlands.

But there are advantages too of conducting the test online. Going online can bring down the errors that had embarrassed the IITs in the recent past with the Supreme Court issuing a stay order on the admission process at all IITs. Also, results can be declared faster and it could be logistically easier.

Taking the entrance online is a step forward for India as most countries are already conducting online examinations.

Last year's paper had huge translational and printing errors. Going online will certainly help reduce this.

However, when computer is not even a compulsory subject in CBSE Class 12 examination, some experts said conducting a crucial examination like the JEE-Advanced online with just few months for the students to get comfortable with the machine could prove to be a tough decision.

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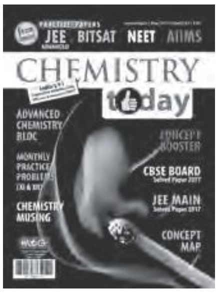

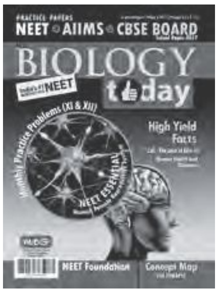

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