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NEET | JEE ESSENTIALS

Class XI

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Unit 4

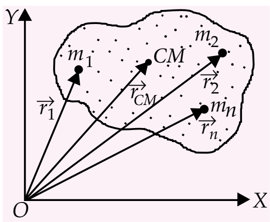
SYSTEM OF PARTICLES AND ROTATIONAL MOTION

CENTRE OF MASS

- For a system of particles or a body, centre of mass is an imaginary point at which its total mass is supposed to be concentrated.
- The position of centre of mass of a rigid body depends on two factors :
  - The geometrical shape of the body.
  - The distribution of mass in the body.
- Position of centre of mass of n point masses m_1, m_2, ..., m_n whose position vectors from origin O are $\vec{r}_1, \vec{r}_2, ..., \vec{r}_n$ is given by,

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n};$$

$$\vec{r}_{CM} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i}$$



$$\vec{r}_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i$$

where $M\left(= \sum_{i=1}^{n} m_i\right)$ is the total mass of the system.

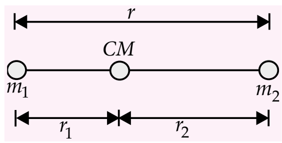
Since $\vec{r}_{CM} = x_{CM}\hat{i} + y_{CM}\hat{j} + z_{CM}\hat{k}$, therefore,

$$x_{CM} = \left(\frac{m_1x_1 + m_2x_2 + ...}{m_1 + m_2 + ...}\right) = \frac{1}{M}\sum m_ix_i,$$

$$y_{CM} = \left(\frac{m_1y_1 + m_2y_2 + ...}{m_1 + m_2 + ...}\right) = \frac{1}{M}\sum m_iy_i,$$

$$z_{CM} = \left(\frac{m_1z_1 + m_2z_2 + ...}{m_1 + m_2 + ...}\right) = \frac{1}{M}\sum m_iz_i$$

- **Position of centre of mass of two particles :** The distance of centre of mass (r) from any of the particle is inversely proportional to the mass of the particle (m)



i.e., $r \propto \frac{1}{m}$

or $\frac{r_1}{r_2} = \frac{m_2}{m_1}$

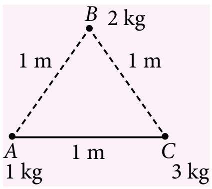
or $m_1r_1 = m_2r_2$

or $r_1 = \left(\frac{m_2}{m_2 + m_1}\right)r$ and $r_2 = \left(\frac{m_1}{m_1 + m_2}\right)r$

Here, r_1 = distance of centre of mass from m_1 and r_2 = distance of centre of mass from m_2

  - From this discussion, we see that $r_1 = r_2 = \frac{1}{2}r$ if $m_1 = m_2$, *i.e.*, centre of mass lies midway between the two particles of equal masses.

Illustration 1 : Consider a system of 3 particles of masses 1 kg, 2 kg and 3 kg are kept at the vertices of an equilateral triangle of side 1 m as shown in figure. Find the coordinates of the centre of mass.



Soln.: Taking particle A as origin and x-axis along AC, we write down the coordinates of the particles.

Particles	x-coordinate	y-coordinate
A	0 cm	0 cm
B	$\frac{1}{2}$ cm	$\frac{\sqrt{3}}{2}$ cm
C	1 cm	0 cm

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C}$$

$$= \frac{1\,\text{kg}(0\,\text{m}) + 2\,\text{kg}\left(\frac{1}{2}\,\text{m}\right) + 3\,\text{kg}(1\,\text{m})}{1\,\text{kg} + 2\,\text{kg} + 3\,\text{kg}} = \frac{2}{3}\,\text{m}$$

$$y_{CM} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C}$$

$$= \frac{(1\,\text{kg})(0\,\text{m}) + (2\,\text{kg})\cdot\left(\frac{\sqrt{3}}{2}\,\text{m}\right) + 3\,\text{kg}\,(0\,\text{m})}{1\,\text{kg} + 2\,\text{kg} + 3\,\text{kg}}$$

$$= \frac{\sqrt{3}}{6}\,\text{m}$$

Hence the coordinates of centre of mass for the choice of axes is $\left(\frac{2}{3}\,\text{m}, \frac{\sqrt{3}}{6}\,\text{m}\right)$.

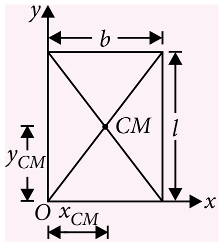
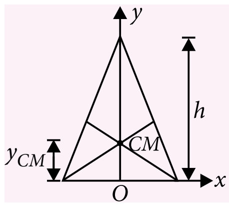
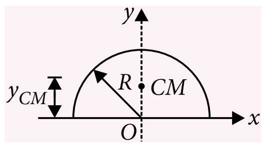
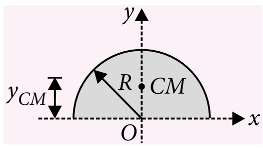
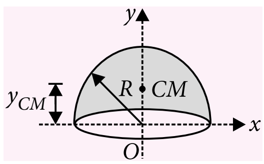
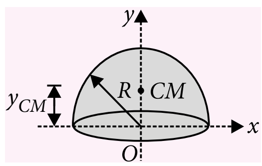
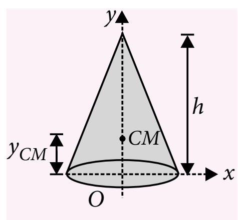
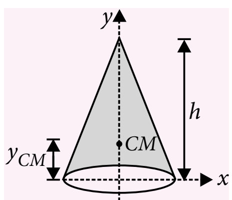
- **Centre of mass of a continuous mass distribution:** Consider a small mass element dm at position $\vec{r}$ as a point mass and replacing the summation by integration.

  $$\vec{R}_{CM} = \frac{1}{M}\int \vec{r}\, dm$$

  Consequently, $x_{CM} = \frac{1}{M}\int x\, dm$, $y_{CM} = \frac{1}{M}\int y\, dm$

  and $z_{CM} = \frac{1}{M}\int z\, dm$

- **Centre of mass of some well known rigid bodies :**

Different Body	Position of Centre of Mass
	Rectangular plate $x_{CM} = \frac{b}{2}$, $y_{CM} = \frac{l}{2}$
	Triangular plate At the centroid, $y_{CM} = \frac{h}{3}$, $x_{CM} = 0$
	Semi-circular ring $y_{CM} = \frac{2R}{\pi}$, $x_{CM} = 0$
	Semi-circular disc $y_{CM} = \frac{4R}{3\pi}$, $x_{CM} = 0$
	Hemispherical shell $y_{CM} = \frac{R}{2}$, $x_{CM} = 0$
	Solid hemisphere $y_{CM} = \frac{3R}{8}$, $x_{CM} = 0$
	Circular cone (solid) $y_{CM} = \frac{h}{4}$, $x_{CM} = 0$
	Circular cone (hollow) $y_{CM} = \frac{h}{3}$, $x_{CM} = 0$

- For a lamina type (two-dimensional) body with uniform negligible thickness, the formulae for finding the position of centre of mass are as follows:

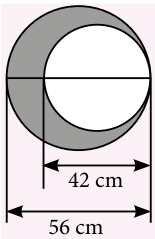
$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...}{m_1 + m_2 + ...}$$
$$= \frac{\rho A_1 t\vec{r}_1 + \rho A_2 t\vec{r}_2 + ...}{\rho A_1 t + \rho A_2 t + ...} \qquad (\because m = \rho A t)$$
$$= \frac{A_1\vec{r}_1 + A_2\vec{r}_2 + ...}{A_1 + A_2 + ...}$$

Here, A stands for area and ρ for density.

- If some mass is removed from a rigid body, then the position of centre of mass of the remaining portion is obtained using.
  - $\vec{r}_{CM} = \frac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 - m_2} = \frac{A_1\vec{r}_1 - A_2\vec{r}_2}{A_1 - A_2}$
  - $x_{CM} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$
  - $y_{CM} = \frac{m_1y_1 - m_2y_2}{m_1 - m_2} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$
  - $z_{CM} = \frac{m_1z_1 - m_2z_2}{m_1 - m_2} = \frac{A_1z_1 - A_2z_2}{A_1 - A_2}$

  Here, m_1, $\vec{r}_1$, x_1, y_1 and z_1 are the values for the whole mass while m_2, A_2, $\vec{r}_2$, x_2, y_2 and z_2 are the values for the mass which has been removed.

Illustration 2 : A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in the figure. Find the position of the centre of mass of the remaining portion.



Soln.: Let us assume the centre of mass of the circular plate be origin. Let r_1 be the distance of the centre of mass of remaining portion from centre of the bigger circle. Remaining area,

$$A_1 = \frac{\pi[(56)^2 - (42)^2]}{4}$$

$\therefore \quad A_1r_1 = A_2r_2 \quad$ or $\quad r_1 = \left(\frac{A_2}{A_1}\right)r_2$

Here r_2 is the distance of the centre of mass of removed portion from centre of the bigger circle and A_2 is the area of removed portion.

or $\quad r_1 = \frac{\pi(42)^2 \times 4}{\pi[(56)^2 - (42)^2] \times 4} \times \frac{7}{1}$

or $\quad r_1 = \frac{42 \times 42 \times 7}{98 \times 14} = 9$ cm

$\therefore \quad r_1 = 9$ cm = Required distance of centre of mass of remaining part.

Motion of Centre of Mass

- For a system of particles, position of centre of mass is

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...}$$

- Velocity of centre of mass

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ...}{m_1 + m_2 + ...} \qquad \left(\because \frac{d\vec{r}}{dt} = \vec{v}\right)$$

- Acceleration of centre of mass

$$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...} \qquad \left(\because \vec{a} = \frac{d\vec{v}}{dt}\right)$$

- Linear momentum of a system of particles is equal to the product of mass of the system with the velocity of its centre of mass.

PURE ROTATIONAL MOTION

- A body is said to be in pure rotational motion if the perpendicular distance of each particle remains constant from a fixed line or point and do not move parallel to the line, and that line is known as axis of rotation.
- Angular displacement, $\theta = \frac{s}{r}$

  where s = length of arc traced by the particle.
  r = distance of particle from the axis of rotation.
- Angular velocity, $\omega = \frac{d\theta}{dt}$
- Angular acceleration, $\alpha = \frac{d\omega}{dt}$
- All the parameters θ, ω and α are same for all the particles. Axis of rotation is perpendicular to the plane of rotation of particles.
- If α = constant, then

  $\omega = \omega_0 + \alpha t \qquad$ where ω_0 = initial angular speed
  t = time interval

  $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

  $\omega^2 = \omega_0^2 + 2\alpha\theta$

  These equations are known as equations of rotational motion.

Illustration 3 : A wheel rotates with an angular acceleration given by $\alpha = 4at^3 - 3bt^2$, where t is the time and a and b are constants. If the wheel has initial angular speed ω_0, find the equations for the (i) angular speed, and (ii) angular displacement.

Soln.: (i) As $\alpha = \dfrac{d\omega}{dt} \Rightarrow d\omega = \alpha dt$

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha\, dt = \int_0^t (4at^3 - 3bt^2)\, dt$$

$$\omega = \omega_0 + at^4 - bt^3$$

(ii) Further, $\omega = \dfrac{d\theta}{dt} \;\Rightarrow\; d\theta = \omega\, dt$

$$\int_0^{\theta} d\theta = \int_0^t \omega\, dt = \int_0^t (\omega_0 + at^4 - bt^3)\, dt$$

$$\theta = \omega_0 t + \frac{at^5}{5} - \frac{bt^4}{4}$$

COMBINED ROTATION AND TRANSLATION

- If a body rotates about an axis with an angular frequency ω with respect to the axis of rotation, linear velocity of any particle in the body at a distance r from the axis of rotation is equal to

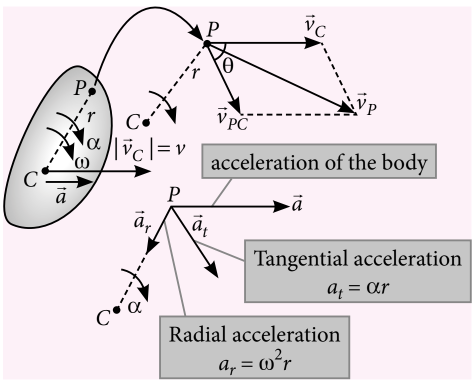
$$\vec{v} = \vec{\omega} \times \vec{r}$$

If the axis of rotation also moves with velocity $\vec{v}_0$, then the net velocity of the particle relative to stationary frame will be

$$\vec{v} = \vec{\omega} \times \vec{r} + \vec{v}_0$$

- Take a body which is moving with velocity $\vec{v}$ and also rotating about centre of mass with angular velocity ω. Let us analyse a point P on the body.
  - ➢ The velocity of point P in the body is

$$|\vec{v}_P = |\vec{v}_{PC} + \vec{v}_C| = \sqrt{v_{PC}^2 + v_C^2 + 2v_{PC} \cdot v_C \cdot \cos\theta}$$



We have $v_{PC} = r\,\omega$ and $v_C = v$. Thus,

$$v_P = \sqrt{(r\omega)^2 + v^2 + 2(r\omega)v \cdot \cos\theta}$$

- In same way we can write the acceleration of point P

$$\vec{a}_P = \vec{a}_{PC} + \vec{a}_C$$

$\vec{a}_{PC}$ has both tangential as well as radial acceleration components. Hence we can express $\vec{a}_{PC}$ as

$$\vec{a}_{PC} = (\vec{a}_{PC})_{\text{tangential}} + (\vec{a}_{PC})_{\text{radial}} = \vec{a}_t + \vec{a}_r$$

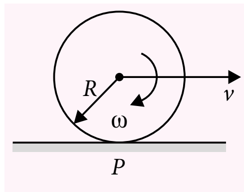
$\vec{a}_t = r\,\alpha$ (acts perpendicular to line CP)

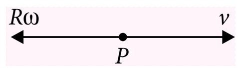
$\vec{a}_r = \omega^2 r$ (acts along the line PC)

Hence net acceleration of point P can be express as

$$\vec{a}_P = (\vec{a}_t + \vec{a}_r) + \vec{a} \qquad (\because \vec{a}_C = a)$$

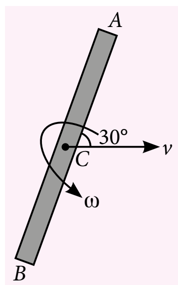
- **Rolling :** A body in combined motion said to be rolling over a surface if the surfaces in contact do not slide relative to each other. It means that the relative velocity between the points of contact is zero.
- Condition of pure rolling is $v = R\omega$. In this case bottommost point of the spherical body is at rest. It has no slipping with its contact point on ground. Because ground point is also at rest.





  - ➢ If $v > R\omega$, then net velocity of point P is in the direction of v. This is called forward slipping.
  - ➢ If $v < R\omega$, then net velocity of point P is in opposite direction of v. This is called backward slipping.

Illustration 4 : A uniform rod of length l is spinning with an angular velocity $\omega = 2v/l$ while its centre of mass moves with a velocity v as shown in figure. Find the velocity of the end A of the rod.



Soln.: Velocity of end A,

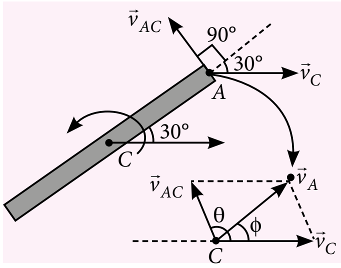
$$\vec{v}_A = \vec{v}_{AC} + \vec{v}_C$$

Hence the velocity of A is

$$v_A = \sqrt{v_{AC}^2 + v_C^2 + 2v_{AC}v_C\cos\theta}$$

We know, $v_C = v$, $v_{AC} = \dfrac{l}{2}\,\omega$ and $\theta = 90° + 30° = 120°$

$$\text{Hence, } v_A = \sqrt{v^2 + \frac{l^2\omega^2}{4} + lv\,\omega\cos 120°}$$



But, $\omega = \frac{2v}{l}, v_{AC} = \frac{l}{2}\omega = \frac{l}{2}\left(\frac{2v}{l}\right) = v$

$\therefore \quad v_A = v$

Let $\vec{v}_A$ make angle ϕ with the direction of $\vec{v}_C$. Then

$$\phi = \tan^{-1}\left(\frac{v\sin\theta}{v_{AC}\cos\theta + v_C}\right)$$

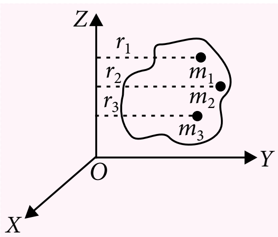
We get, $\phi = \tan^{-1}\left(\frac{v\sin 120°}{v\cos 120° + v}\right) = \frac{\pi}{3}$

MOMENT OF INERTIA

- The moment of inertia of a rigid body about a given axis is the sum of the product of the masses of its constituent particles and the square of their respective distances from the axis of rotation.

$$I = \sum_{i=1}^{n} m_i r_i^2$$

- The radius of gyration k of a body about an axis of rotation is defined as the root mean square distances of the particles from the axis of rotation and its square when multiplied with the mass of the body gives moment of inertia of the body about the axis.



$$k = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}}$$

= root mean square distance of a particles from axis OZ.

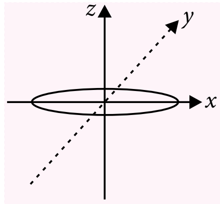
- Two important theorems on moment of inertia :
  - **Perpendicular axes theorem** (Only applicable to plane lamina *i.e.*, 2-D objects only):

    $I_z = I_x + I_y$ (Object is in x-y plane)

    where I_z = moment of inertia of the body about z-axis.

    I_x = moment of inertia of the body about x-axis.

    I_y = moment of inertia of the body about y-axis.



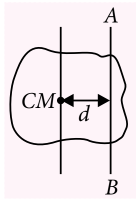
Body is in x-y plane

    $I_y = I_x + I_z$ (Object is in x-z plane)

    $I_x = I_y + I_z$ (Object is in y-z plane)

  - **Parallel axes theorem** (Applicable to any type of object) :

    $I_{AB} = I_{CM} + Md^2$



    where,

    I_{CM} = Moment of inertia of the object about an axis passing through centre of mass and parallel to axis AB

    I_{AB} = Moment of inertia of the object about axis AB

    M = Total mass of object

    d = Perpendicular distance between axis about which moment of inertia is to be calculated and the one passing through the centre of mass.

- **Some important points about rolling motion :**
  - Kinetic energy of a rolling body = translational kinetic energy (K_T) + rotational kinetic energy (K_R)

    $$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2\left[1 + \frac{k^2}{R^2}\right]$$

  - When a body rolls down an inclined plane of inclination θ without slipping its velocity at the bottom of incline is given by

    $$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$$

    where h is the height of the incline.

  - When a body rolls down on an inclined plane without slipping, its acceleration down the inclined plane is given by

    $$a = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}}$$

  - When a body rolls down on an inclined plane without slipping, time taken by the body to reach the bottom is given by

$$t = \sqrt{\frac{2l\left(1+\frac{k^2}{R^2}\right)}{g\sin\theta}}$$

where l is the length of the inclined plane.

- **Moment of inertia of some important cases :**

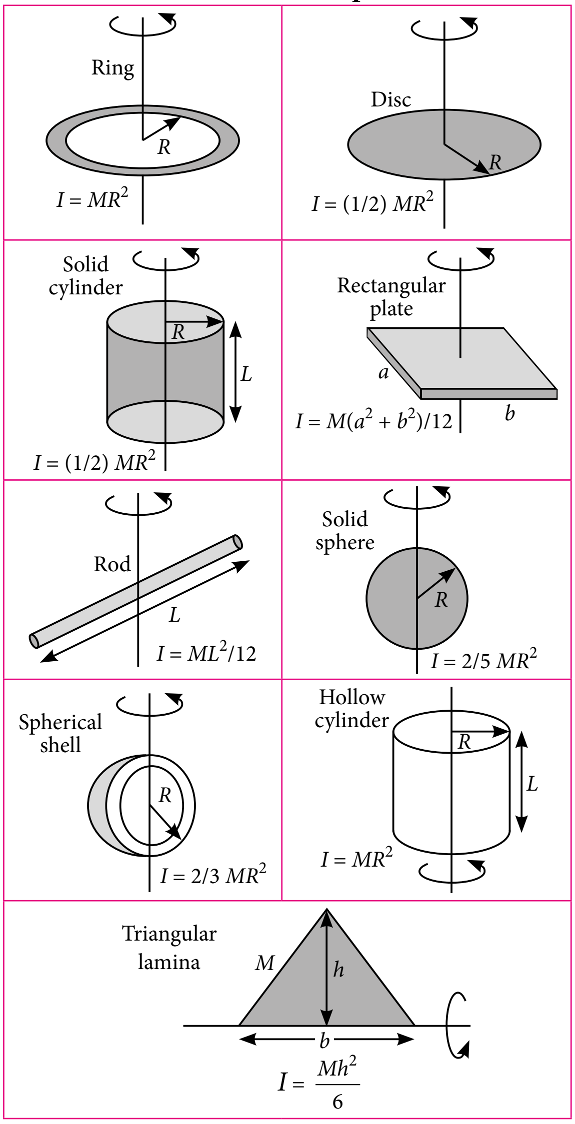


Illustration 5 : A circular disc A of radius r is made from an iron plate of thickness t and another circular disc B of radius $4r$ is made from an iron plate thickness $t/4$. What is the relation between the moments of inertia I_A and I_B?

Soln.: $I_{Disc} = \frac{1}{2}MR^2$

Let ρ be the density of iron, then $I_A = \frac{1}{2}(\rho)[\pi r^2 t](r^2)$ as radius of plate A is r and thickness is t.

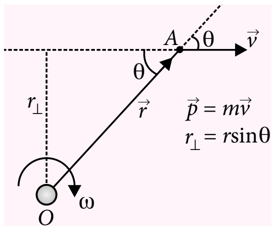
$\therefore \quad I_A = \frac{\rho\pi r^4 t}{2}$.

Now, $I_B = \frac{1}{2}(\rho)\left(\pi(4r)^2\frac{t}{4}\right)(4r)^2$ as radius of plate B is $4r$ and thickness is $t/4$.

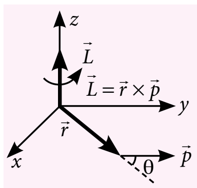
$\Rightarrow \quad I_B = \left[\frac{\rho\pi r^4 t}{2}\right]\left(\frac{16\times16}{4}\right)$ or $I_B = 64\, I_A \;\therefore\; I_A < I_B$.

ANGULAR MOMENTUM

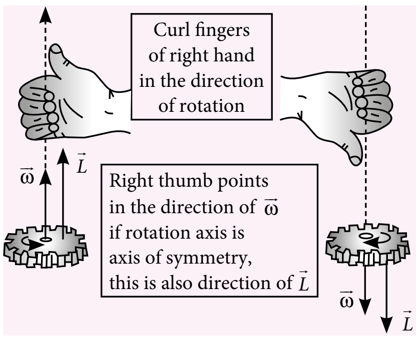
- Angular momentum of particle A about point O will be, $\vec{L} = \vec{r}\times\vec{p} = \vec{r}\times(m\vec{v}) = m(\vec{r}\times\vec{v})$
- Magnitude of $\vec{L}$ is $L = mvr\sin\theta = mvr_\perp$ where θ is the angle between $\vec{r}$ and $\vec{p}$.



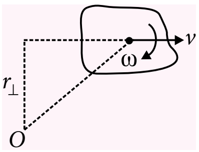
- Direction of $\vec{L}$ will be given by right hand screw rule. For the given figure, direction of $\vec{L}$ is perpendicular to paper inwards.
- If a body is rotating about a fixed axis, we can write angular momentum of the body $L = I\omega$
  I = Moment of inertia, about fixed axis,
  ω = angular velocity of rotation
- Direction of angular momentum
  - We define angular momentum even if the particle is not moving in a circular path. Even a particle moving in a straight line has angular momentum about any axis displaced from the path of the particle.



  - For rotation about an axis of symmetry, $\vec{\omega}$ and $\vec{L}$ are parallel and along the axis, respectively. The directions of both vectors are given by the right hand rule.



- Angular momentum of a rigid body in rotation plus translation about a general axis :

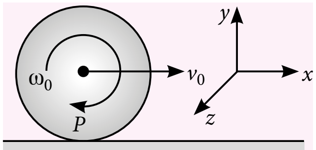


- There will be two terms :
  (a) $I_{CM}\,\omega$ (b) $mv_{CM}r_\perp = mvr_\perp$
- From right hand screw rule, we can see that $I_{CM}\,\omega$ and $mvr_\perp$ both terms are perpendicular to the paper in inward direction. Hence, they are added or $L_{Total} = I_{CM}\,\omega + mvr_\perp$

Illustration 6 : A sphere of mass M and radius R rolls without slipping on a rough surface with centre of mass has constant speed v_0. Find the angular momentum of the sphere about the point of contact.

Soln.: Since $\vec{L}_P = \vec{L}_{CM} + \vec{r} \times \vec{p}_{CM}$

$= I_{CM}\omega_0(-\hat{k}) + Mv_0R(-\hat{k})$



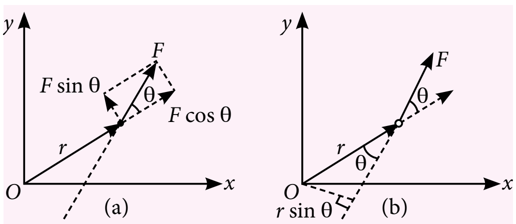
Since sphere is in pure rolling motion, hence

$\omega_0 = v_0/R$

$$\Rightarrow \vec{L}_P = \left[\frac{2}{5}MR^2\left(\frac{v_0}{R}\right)(-\hat{k}) + Mv_0R(-\hat{k})\right] = \frac{7}{5}Mv_0R(-\hat{k})$$

TORQUE

- The turning ability of a force about an axis is called its torque about that axis.
- Consider force $\vec{F}$ which acts through a point whose position vector is $\vec{r}$, as shown in figure.



- Its torque about the origin is defined as $\vec{\tau} = \vec{r} \times \vec{F}$.
- The direction of the torque can be obtained from the right hand rule and its magnitude is given by $\tau = rF\sin\theta$, where θ is the angle between the vector $\vec{r}$ and $\vec{F}$.

  $\tau = r(F\sin\theta) = rF_\perp$

  where $F_\perp$ is the component of F perpendicular to r.

  $\tau = (r\sin\theta)\,F = r_\perp F$

  where $r_\perp$ is the perpendicular distance from the origin to the line of action of the force. It is also called the lever arm.

- Torque is also the rate of change of angular momentum

$$\vec{\tau} = \frac{d\vec{L}}{dt} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha}$$

- If a torque τ applied on a body rotates it through an angle $\Delta\theta$, the work done by the torque is

  $\Delta W = \tau\Delta\theta$

  or work done = torque × angular displacement

  Power of a torque is given as

$$P = \frac{\Delta W}{\Delta t} = \frac{\tau\Delta\theta}{\Delta t} = \tau\omega$$

  i.e., Power of a torque = torque × angular velocity

- **Law of conservation of angular momentum :** If no external torque acts on a system, total angular momentum of the system remains unchanged. In the absence of any external torque,

  $L = I\omega$ = constant

  or $I_1\omega_1 = I_2\omega_2$ or $I_1 \cdot \frac{2\pi}{T_1} = I_2 \cdot \frac{2\pi}{T_2}$

Illustration 7 : A turntable turns about a fixed vertical axis, making one revolution in 10 s. The moment of inertia of the turntable about the axis is 1200 kg m^2. A man of 80 kg, initially standing at the centre of the turntable, runs out along the radius. What is the angular velocity of the turntable when the man is 2 m from the centre?

Soln.: Let I_0 be the initial moment of inertia of the system (man + table)

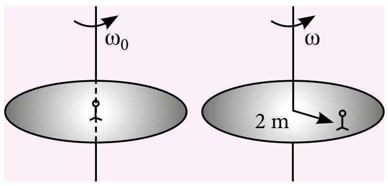
$I_0 = I_{man} + I_{table} = 0 + 1200 = 1200$ kg m^2

($I_{man} = 0$ as the man is at the axis)

I = final moment of inertia of the system

$= I_{man} + I_{table} = mr^2 + 1200$

$= 80(2)^2 + 1200 = 1520$ kg m^2



As there is no external torque about the axis, we can conserve angular momentum.

By conservation of angular momentum : $I_0\omega_0 = I\omega$

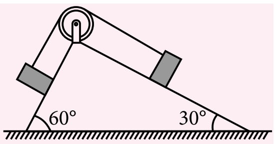
Now $\omega_0 = 2\pi/T_0 = 2\pi/10 = \pi/5$ rad s^{-1}

$$\Rightarrow \quad \omega = \frac{I_0\omega_0}{I} = \frac{1200\times\pi}{1520\times 5} = 0.49 \text{ rad s}^{-1}$$

SPEED PRACTICE

1. A man of mass 80 kg is riding on a small cart of mass 40 kg which is moving along a level floor at a speed of 2 m s^{-1}. He is running on the cart, so that his velocity relative to the cart is 3 m s^{-1} in the direction opposite to the motion of cart. What is the speed of the centre of mass of the system?
(a) 1.5 m s^{-1} (b) 1 m s^{-1}
(c) 3 m s^{-1} (d) zero

2. Two blocks of equal mass are tied with a light string, which passes over a massless pulley as shown in figure. The magnitude of acceleration of centre of mass of both the blocks is (neglect friction everywhere)



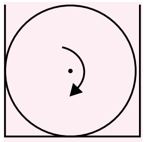
(a) $\left(\frac{\sqrt{3}-1}{4\sqrt{2}}\right)g$ (b) $(\sqrt{3}-1)g$
(c) $\frac{g}{2}$ (d) $\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)g$

3. The position of a particle is given by $\vec{r} = \hat{i} + 2\hat{j} - \hat{k}$ and its linear momentum is given by $\vec{p} = 3\hat{i} + 4\hat{j} - 2\hat{k}$. Then its angular momentum, about the origin is perpendicular to
(a) *yz*-plane (b) *z*-axis
(c) *y*-axis (d) *x*-axis

4. A uniform rod of mass m and length l makes a constant angle θ with an axis of rotation which passes through one end of the rod. Its moment of inertia about this axis is
(a) $\frac{ml^2}{3}$ (b) $\frac{ml^2}{3}\sin\theta$
(c) $\frac{ml^2}{3}\sin^2\theta$ (d) $\frac{ml^2}{3}\cos^2\theta$

5. A ring of radius R is rotating with an angular speed ω_0 about a horizontal axis. It is placed on a rough horizontal table. The coefficient of kinetic friction is μ_k. The time after which it starts rolling is
(a) $\frac{\omega_0 \mu_k R}{2g}$ (b) $\frac{\omega_0 g}{2\mu_k R}$
(c) $\frac{2\omega_0 R}{\mu_k g}$ (d) $\frac{\omega_0 R}{2\mu_k g}$

6. A solid sphere of mass 5 kg and radius 1 m after rotating with angular speed $\omega_0 = 40$ rad s^{-1} is placed between two smooth walls on a rough ground. Distance between the walls is slightly greater than the diameter of the sphere. Coefficient of friction between the sphere and the ground is $\mu = 0.1$. Sphere will stop rotating after



(a) 8 s (b) 12 s (c) 20 s (d) 16 s

7. A uniform rod AB of mass m and length l is at rest on a smooth horizontal surface. An impulse J is applied to the end B perpendicular to the rod in horizontal direction. Speed of particle P at a distance $l/6$ from the center towards A of the rod after time $t = \frac{\pi ml}{12J}$ is
(a) $2\frac{J}{m}$ (b) $\frac{J}{\sqrt{2}m}$
(c) $\frac{J}{m}$ (d) $\sqrt{2}\frac{J}{m}$

8. A billiard ball is hit by a cue at a height h above that center. It acquires a linear velocity v_0. Mass of the ball is m and radius is r. The angular velocity ω_0 acquired by the ball is
(a) $\frac{2v_0h}{5r^2}$ (b) $\frac{5v_0h}{2r^2}$
(c) $\frac{2v_0r^2}{5h}$ (d) $\frac{5v_0r^2}{2h}$

9. A child is sitting at one end of a long trolley moving with a uniform speed v on a smooth horizontal track. If the child starts running towards the other end of the trolley with a speed u (w.r.t. trolley), the speed of the centre of mass of the system will
(a) $u + v$ (b) $v - u$
(c) v (d) none

10. A body of mass m slides down an inclined plane and reaches the bottom with a velocity v. If the same mass were in the form of a ring which rolls

down this incline, the velocity of the ring at the bottom would be

(a) v (b) $v/\sqrt{2}$

(c) $\sqrt{2}\,v$ (d) $\sqrt{2/5}\,v$

11. A circular platform is mounted on a frictionless vertical axle. Its radius R = 2 m and its moment of inertia about the axle is 200 kg m^2. It is initially at rest. A 50 kg man stands on the edge of the platform and begins to walk along the edge at the speed of 1 m s^{-1} relative to the ground. Time taken by the man to complete one revolution is

(a) π s (b) $\frac{3\pi}{2}$ s (c) 2π s (d) $\frac{\pi}{2}$ s

12. In a gravity free space, a man of mass M standing at a height h above the floor, throws a ball of mass m straight down with a speed u. When the ball reaches the floor, the distance of the man above the floor will be

(a) $h\,(1 + m/M)$

(b) $h\,(2 - m/M)$

(c) $2h$

(d) a function of m, M, h and u

13. A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N?

(a) 0.25 rad s^{-2} (b) 25 rad s^{-2}

(c) 5 m s^{-2} (d) 25 m s^{-2}

[NEET 2017]

14. Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities ω_1 and ω_2. They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is

(a) $\frac{1}{4}I(\omega_1-\omega_2)^2$ (b) $I(\omega_1-\omega_2)^2$

(c) $\frac{1}{8}I(\omega_1-\omega_2)^2$ (d) $\frac{1}{2}I(\omega_1+\omega_2)^2$

[NEET 2017]

15. Two rotating bodies A and B of masses m and $2m$ with moments of inertia I_A and I_B ($I_B > I_A$) have equal kinetic energy of rotation. If L_A and L_B be their angular momenta respectively, then

(a) $L_A = \frac{L_B}{2}$ (b) $L_A = 2L_B$

(c) $L_B > L_A$ (d) $L_A > L_B$

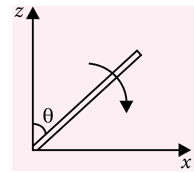
[NEET Phase-II 2016]

16. A solid sphere of mass m and radius R is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with an angular speed twice that of the sphere. The ratio of their kinetic energies of rotation (E_{sphere} / $E_{cylinder}$) will be

(a) 2 : 3 (b) 1 : 5 (c) 1 : 4 (d) 3 : 1

[NEET Phase-II 2016]

17. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is



(a) $\frac{3g}{2l}\sin\theta$ (b) $\frac{2g}{3l}\sin\theta$

(c) $\frac{3g}{2l}\cos\theta$ (d) $\frac{2g}{3l}\cos\theta$

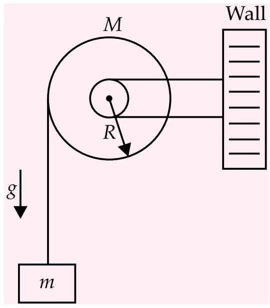
[JEE Main 2017]

18. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum ?

(a) $\sqrt{\frac{3}{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) $\frac{3}{\sqrt{2}}$

[JEE Main 2017]

19. A uniform disc of radius R and mass M is free to rotate only about its axis. A string is wrapped over its rim and a body of mass m is tied to the free end of the string as shown in the figure. The body is released from rest. Then the acceleration of the body is



(a) $\frac{2mg}{2m+M}$ (b) $\frac{2Mg}{2m+M}$

(c) $\frac{2Mg}{2M+m}$ (d) $\frac{2mg}{2M+m}$

[JEE Main Online 2017]

20. In a physical balance working on the principle of moments, when 5 mg weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct?
(a) Left arm is shorter than the right arm
(b) Left arm is longer than the right arm
(c) Every object that is weighed using this balance appears lighter than its actual weight
(d) Both the arms are of same length
[JEE Main Online 2017]

SOLUTIONS

1. (d): Velocity of man with respect to ground is 1 m s^{-1} in opposite direction of motion of the car.

Hence, $v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{40\times 2 - 80\times 1}{40+80} = 0$

2. (a): Acceleration of system,

$a = \frac{mg\sin 60° - mg\sin 30°}{2m}$ or $a = \left(\frac{\sqrt{3}-1}{4}\right)g$

Here, m = mass of each block

Now, $\vec{a}_{CM} = \frac{m\vec{a}_1 + m\vec{a}_2}{2m}$

Here, $\vec{a}_1$ and $\vec{a}_2$ are $\left(\frac{\sqrt{3}-1}{4}\right)g$ at right angles.

Hence, $|\vec{a}_{CM}| = \frac{\sqrt{2}}{2}a = \left(\frac{\sqrt{3}-1}{4\sqrt{2}}\right)g$

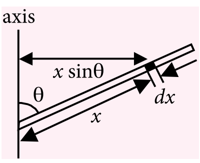
3. (d): $\vec{L} = \vec{r}\times\vec{p} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -1\\ 3 & 4 & -2\end{vmatrix}$

$= \hat{i}(-4+4) - \hat{j}(-2+3) + \hat{k}(4-6) = 0\hat{i} - 1\hat{j} - 2\hat{k}$

$\vec{L}$ has components along $-y$ axis and $-z$ axis but it has no component along the x-axis. The angular momentum is in yz plane, *i.e.*, perpendicular to x-axis.

4. (c): Mass of the element $= \left(\frac{m}{l}\right)dx$

Moment of inertia of the element about the axis



$= \left(\frac{m}{l}dx\right)(x\sin\theta)^2 = \frac{m}{l}\sin^2\theta x^2 dx$

$\therefore\ I = \frac{m}{l}\sin^2\theta\int_0^l x^2dx = \frac{ml^2}{3}\sin^2\theta$

5. (d): Acceleration produced in the centre of mass due to friction,

$a = \frac{f}{M} = \frac{\mu_k Mg}{M} = \mu_k g$ ... (i)

where M is the mass of the ring

Angular retardation produced by the torque due to friction,

$\alpha = -\frac{\tau}{I} = -\frac{fR}{I} = -\frac{\mu_k MgR}{I}$ ... (ii)

As $v = u + at$ $(\because u = 0)$

$\therefore$ $v = 0 + \mu_k gt$ (Using (i))

As $\omega = \omega_0 + \alpha t$

$\therefore$ $\omega = \omega_0 - \frac{\mu_k MgR}{I}t$ (Using (ii))

For rolling without slipping

$v = R\omega$

$\therefore$ $\frac{v}{R} = \omega_0 - \frac{\mu_k MgR}{I}t$

$\frac{\mu_k gt}{R} = \omega_0 - \frac{\mu_k MgR}{I}t \Rightarrow \frac{\mu_k gt}{R}\left[1 + \frac{MR^2}{I}\right] = \omega_0$

$\frac{\mu_k gt}{R} = \frac{\omega_0}{1+\frac{MR^2}{I}} \Rightarrow t = \frac{R\omega_0}{\mu_k g\left(1+\frac{MR^2}{I}\right)}$

For ring, $I = MR^2$

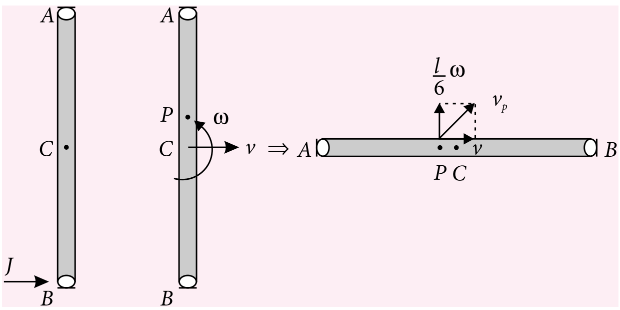
$\therefore$ $t = \frac{R\omega_0}{\mu_k g\left(1+\frac{MR^2}{MR^2}\right)} = \frac{R\omega_0}{2\mu_k g}$

6. (d): Angular retardation, $\alpha = -\frac{\tau}{I} = -\frac{(\mu mgR)}{\frac{2}{5}mR^2}$

$\therefore$ $\alpha = -\frac{5\mu g}{2R} = -\frac{5\times 0.1\times 10}{2\times 1} = -2.5\ \text{rad s}^{-2}$

Now, $0 = \omega_0 + \alpha t \Rightarrow t = \frac{40}{2.5} = 16$ s

7. (d): Let v and ω be the linear and angular speeds of the rod after applying an impulse J at B.



$\because$ Impulse = Change in momentum

We have, $mv = J$ or $v = \frac{J}{m}$ ...(i)

Due to impulse at end B, the rod will also rotate,

$\therefore \quad I\omega = J \cdot \frac{l}{2}$

$\frac{ml^2}{12} \cdot \omega = J \cdot \frac{l}{2}$ or $\omega = \frac{6J}{ml}$ ...(ii)

After the given time $t = \frac{\pi ml}{12J}$, the rod will rotate an angle

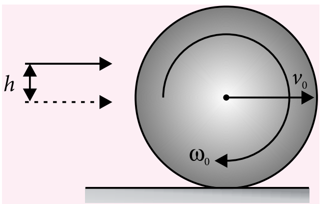
$\theta = \omega t = \left(\frac{6J}{ml}\right)\left(\frac{\pi ml}{12J}\right) = \frac{\pi}{2}$ (from eqn. (ii))

As, shown in figure, the particle at P will have velocity $\vec{v}_P$ due to both rotational and translational motion of rod. Due to rotation,

$\frac{l}{6} \cdot \omega = \left(\frac{l}{6}\right)\left(\frac{6J}{ml}\right) = \frac{J}{m} = v$ (using eqn. (i))

$\therefore \quad |\vec{v}_P| = \sqrt{v^2 + v^2} = \sqrt{2}v = \sqrt{2}\frac{J}{m}$

8. (b) : Let J be linear impulse imparted to the ball. Applying; Impulse = Change in momentum



We have $J = mv_0$ ...(i)

$J \cdot h = I\omega_0 = \frac{2}{5}mr^2\omega_0$ ...(ii)

From equations (i) and (ii), we get $\omega_0 = \frac{5}{2}\frac{v_0 h}{r^2}$

9. (c) : Velocity of centre of mass will remain unaffected as no external force acts on system.

10. (b) : Kinetic energy of translation of mass m sliding down the incline with velocity, $v = \frac{1}{2}mv^2.$

For pure rolling, $v_{CM} = r\omega$

Total kinetic energy of the ring rolling down the incline with velocity v_{CM}

$= \frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_{CM}^2 + \frac{1}{2}(mr^2)\omega^2 = mv_{CM}^2$

Since both the bodies have mass m, they will gain same amount of kinetic energy.

Thus, $mv_{CM}^2 = \frac{1}{2}mv^2$ or $v_{CM} = v/\sqrt{2}$

11. (c) : As the system is initially at rest, therefore, initial angular momentum, $L_i = 0$.

According to the principle of conservation of angular momentum,

final angular momentum, $L_f = 0$.

$\therefore$ Angular momentum of platform = Angular momentum of man in opposite direction.

i.e., $mvR = I\omega$

or $\omega = \frac{mvR}{I} = \frac{50 \times 1 \times 2}{200} = \frac{1}{2}$ rad s^{-1}

Angular velocity of man relative to platform is

$\omega_r = \omega + \frac{v}{R} = \frac{1}{2} + \frac{1}{2} = 1$ rad s^{-1}

Time taken by the man to complete one revolution is $T = \frac{2\pi}{\omega_r} = \frac{2\pi}{1} = 2\pi$ s

12. (a) : In gravity free space, displacement of centre of mass of man and ball system should not move. If displacement of the ball be h then the displacement of man in upward direction.

$mh = Mh' \Rightarrow h' = \frac{mh}{M}$ ...(i)

Hence, the position of man from ground

$H = h + h' = h\left[1 + \frac{m}{M}\right]$

13. (b) : $m = 3$ kg, $r = 40$ cm $= 40 \times 10^{-2}$ m $= 0.4$ m, $F = 30$ N

Moment of inertia of hollow cylinder about its axis

$= mr^2 = 3 \text{ kg} \times (0.4)^2 \text{ m}^2 = 0.48 \text{ kg m}^2$

The torque is given by, $\tau = I\alpha$

where, I = moment of inertia,

α = angular acceleration

In the given case, $\tau = rF$, as the force is acting perpendicularly to the radial vector.

$\therefore \quad \alpha = \frac{\tau}{I} = \frac{Fr}{mr^2} = \frac{F}{mr} = \frac{30}{3 \times 40 \times 10^{-2}} = \frac{30 \times 100}{3 \times 40}$

$\alpha = 25$ rad s^{-2}

14. (a) : Initial angular momentum $= I\omega_1 + I\omega_2$

Let ω be angular speed of the combined system.

Final angular momentum $= 2I\omega$

$\therefore$ According to conservation of angular momentum

$I\omega_1 + I\omega_2 = 2I\omega$ or $\omega = \frac{\omega_1 + \omega_2}{2}$

Initial rotational kinetic energy

$E_i = \frac{1}{2}I(\omega_1^2 + \omega_2^2)$

Final rotational kinetic energy

$$E_f = \frac{1}{2}(2I)\omega^2 = \frac{1}{2}(2I)\left(\frac{\omega_1+\omega_2}{2}\right)^2 = \frac{1}{4}I(\omega_1+\omega_2)^2$$

$\therefore$ Loss of energy $\Delta E = E_i - E_f$

$$= \frac{I}{2}(\omega_1^2+\omega_2^2) - \frac{I}{4}(\omega_1^2+\omega_2^2+2\omega_1\omega_2)$$

$$= \frac{I}{4}\left[\omega_1^2+\omega_2^2-2\omega_1\omega_2\right] = \frac{I}{4}(\omega_1-\omega_2)^2$$

15. (c) : Here, $m_A = m$, $m_B = 2\,m$

Both bodies A and B have equal kinetic energy of rotation

$$K_A = K_B \Rightarrow \frac{1}{2}I_A\omega_A^2 = \frac{1}{2}\,I_B\omega_B^2$$

$$\Rightarrow \frac{\omega_A^2}{\omega_B^2} = \frac{I_B}{I_A} \quad \text{...(i)}$$

Ratio of angular momenta,

$$\frac{L_A}{L_B} = \frac{I_A\omega_A}{I_B\omega_B} = \frac{I_A}{I_B}\times\sqrt{\frac{I_B}{I_A}} \quad \text{[Using eqn. (i)]}$$

$$= \sqrt{\frac{I_A}{I_B}} < 1 \quad (\because I_B > I_A)$$

$\therefore \quad L_B > L_A$

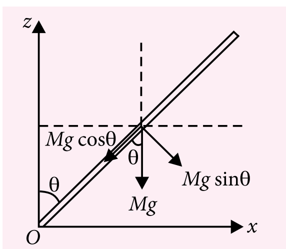
16. (b) : $$\frac{E_{\text{sphere}}}{E_{\text{cylinder}}} = \frac{\frac{1}{2}I_s\omega_s^2}{\frac{1}{2}I_c\omega_c^2} = \frac{I_s\omega_s^2}{I_c\omega_c^2}$$

Here, $I_s = \frac{2}{5}mR^2$, $I_c = \frac{1}{2}mR^2$ and $\omega_c = 2\omega_s$

$$\therefore \frac{E_{\text{sphere}}}{E_{\text{cylinder}}} = \frac{\frac{2}{5}mR^2\times\omega_s^2}{\frac{1}{2}mR^2\times(2\omega_s)^2} = \frac{4}{5}\times\frac{1}{4} = \frac{1}{5}$$

17. (a) : The torque of the weight Mg of the rod about the pivot O is given by

$$\tau = Mg\sin\theta\times\left(\frac{l}{2}\right) \quad \text{...(i)}$$



(Mg cosθ is passing through the pivot O. Hence, its contribution to the torque will be zero)

Also,

$$\tau = I\alpha \quad \text{...(ii)}$$

$$\therefore \quad I\alpha = Mg\sin\theta\times\left(\frac{l}{2}\right) \quad \text{(Using (i) and (ii))}$$

Now, moment of inertia of the rod about the pivot O is $I = \frac{1}{3}Ml^2$

$$\therefore \quad \frac{1}{3}Ml^2\alpha = Mg\sin\theta\left(\frac{l}{2}\right) \Rightarrow \alpha = \frac{3}{2}\frac{g}{l}\sin\theta$$

18. (a) : Moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is given by

$$I = \frac{1}{12}ml^2 + \frac{mR^2}{4}$$

$$\text{or} \quad I = \frac{m}{4}\left(\frac{1}{3}l^2 + R^2\right) \quad \text{... (i)}$$

Also, $m = \rho V = \rho\pi R^2 l$ or $R^2 = \frac{m}{\rho\pi l}$

Substitute R^2 in eqn. (i), we get

$$I = \frac{m}{4}\left(\frac{l^2}{3} + \frac{m}{\rho\pi l}\right)$$

For moment of inertia to be maximum or minimum,

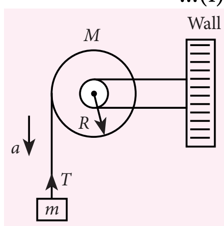
$$\frac{dI}{dl} = 0 \Rightarrow \frac{m}{4}\left(\frac{2l}{3} - \frac{m}{\rho\pi l^2}\right) = 0$$

$$\Rightarrow \frac{2l}{3} - \frac{R^2}{l} = 0 \quad \left(\text{Using } \frac{R^2}{l} = \frac{m}{\rho\pi l^2}\right)$$

$$\Rightarrow \frac{2l}{3} = \frac{R^2}{l} \Rightarrow \frac{l}{R} = \sqrt{\frac{3}{2}}$$

19. (a) : From figure, we conclude

$mg - T = ma$ ...(i)



Moment of inertia of a uniform disc, $I = \frac{MR^2}{2}$

and an acceleration is,

$a = \alpha R$

$\because \quad RT = I\alpha$

$$\therefore \quad RT = \frac{MR^2}{2}\times\frac{a}{R} \Rightarrow T = \frac{Ma}{2}$$

Putting this value in equation (i)

$$mg - \frac{Ma}{2} = ma \text{ or } mg = a\left(m + \frac{M}{2}\right) \Rightarrow a = \frac{2mg}{M+2m}$$

20. (a)

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EXAM PREP 2018

CLASS XI

Useful for Medical/Engg. Entrance Exams

CHAPTERWISE MCQs FOR PRACTICE

MECHANICAL PROPERTIES OF FLUIDS

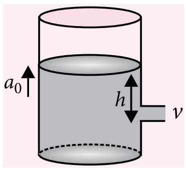
1. Eight identical droplets of mercury (surface tension 0.55 N m^{-1}) of radius 1 mm. All the droplets merge into one bigger drop. Find the amount of energy evolved.

(a) 24.62×10^{-5} J (b) 27.65×10^{-6} J
(c) 24.62×10^{-6} J (d) 27.65×10^{-5} J

2. A spherical soap bubble of radius 2 cm is attached to a spherical bubble of radius 4 cm. Find the radius of the curvature of the common surface.

(a) 4 cm (b) 3 cm (c) 2 cm (d) 0.4 cm

3. For the area a of the hole is much lesser than the area of the base of a vessel of liquid, velocity of efflux v of the liquid in an accelerating vessel as shown in figure is



(a_0 = vertical acceleration)

(a) $\sqrt{2gh}$ (b) $\sqrt{2(g-a_0)h}$
(c) $\sqrt{2(g+a_0)h}$ (d) none of these

4. Water from a tap emerges vertically downwards with an initial velocity v_0. Assume pressure is constant throughout the stream of water and the flow is steady, find the distance from the tap at which cross-sectional area of stream is half of the cross-sectional area of stream at the tap.

(a) $\frac{v_0^2}{2g}$ (b) $\frac{3v_0^2}{2g}$ (c) $\frac{2v_0^2}{g}$ (d) $\frac{5v_0^2}{2g}$

5. The work done in increasing the size of a rectangular soap film with dimensions 8 cm × 3.75 cm to 10 cm × 6 cm is 2×10^{-4} J. The surface tension of the film in N m^{-1} is

(a) 1.65×10^{-2} (b) 3.3×10^{-2}
(c) 6.6×10^{-2} (d) 8.25×10^{-2}

6. Water from a tap emerges vertically downward with an initial speed of 1.0 m s^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. What is the cross-sectional area (in m^2) of the stream 0.15 m below the tap?

(a) 2×10^{-4} (b) 5×10^{-4}
(c) 5×10^{-5} (d) 2×10^{-5}

7. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, what is the value of R?

(a) $\frac{L}{\sqrt{2\pi}}$ (b) $\frac{L}{\sqrt{\pi}}$ (c) $\frac{L}{2}$ (d) $\frac{L}{4}$

8. The narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U shaped tube open at both ends. If U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water is 7.3×10^{-2} N m^{-1}. Take the angle of contact to be zero, and density of water to be 1.0×10^3 kg m^{-3} and $g = 9.8$ m s^{-2}.

(a) 0.5×10^{-2} m (b) 0.4×10^{-2} m
(c) 0.3×10^{-2} m (d) 0.1×10^{-2} m

9. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density ρ_2 ($\rho_2 < \rho_1$). Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, *i.e.*, $F_{viscous} = -kv^2$ ($k > 0$). The terminal speed of the ball is

(a) $\frac{Vg(\rho_1 - \rho_2)}{k}$ (b) $\sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$

(c) $\frac{Vg\rho_1}{k}$ (d) $\sqrt{\frac{Vg\rho_1}{k}}$

10. The drop of liquid of density ρ is floating with $\frac{1}{4}$th inside the liquid A of density ρ_1 and remaining in the liquid B of density ρ_2. Then

(a) Upthrust in liquid $A = \frac{\delta_1}{4\delta_2}$ times upthrust in liquid B.

(b) $4\delta = \delta_1 + 3\delta_2$

(c) $3\delta = 4\delta_1 + \delta_2$

(d) Upthrust in liquid $B = \frac{\delta_2}{3\delta_1}$ times upthrust in liquid A.

11. Three capillaries of internal radii $2r$, $3r$ and $4r$, all of the same length, are joined end to end. A liquid passes through the combination and the pressure difference across this combination is 20.2 cm of mercury. The pressure difference across the capillary of internal radius $2r$ is

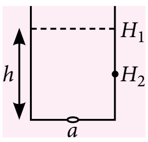
(a) 2 cm of Hg (b) 4 cm of Hg

(c) 8 cm of Hg (d) 16 cm of Hg

12. A frame made of metallic wire enclosing a surface area A is covered with a soap film. If the area of the frame of metallic wire is reduced by 50%, the energy of the soap film will be changed by

(a) 100% (b) 75% (c) 50% (d) 25%

13. A vessel having area of cross-section A contains a liquid upto a height h. At the bottom of the vessel, there is a small hole having area of cross-section a. Then the time taken for the liquid level to fall from height H_1 to H_2 is given by



(a) $\sqrt{2g(H_1 - H_2)}$ (b) $\frac{A}{a}\sqrt{\frac{2}{g}}(\sqrt{H_1} - \sqrt{H_2})$

(c) $\frac{A}{a}\sqrt{\frac{g}{2}}(\sqrt{H_1} - \sqrt{H_2})$ (d) $\sqrt{2gH}$

14. Two soap bubbles of radii a and b combine to form a single bubble of radius c. If P is the external pressure, then the surface tension of the soap solution is

(a) $\frac{P(c^3 + a^3 + b^3)}{4(a^2 + b^2 - c^2)}$ (b) $\frac{P(c^3 - a^3 - b^3)}{4(a^2 + b^2 - c^2)}$

(c) $Pc^3 - 4a^2 - 4b^2$ (d) $Pc^2 - 2a^2 - 3b^2$

15. Water rises in a capillary tube to a height of 2.0 cm. In another capillary tube whose radius is one-third of it, how much the water will rise?

(a) 5 cm (b) 3 cm (c) 6 cm (d) 9 cm

THERMAL PROPERTIES OF MATTER

16. A body cools from 50°C to 40° C in 5 minutes. The temperature of surroundings is 20°C. Find the temperature of body after another 5 minutes.

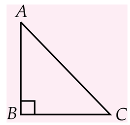
(a) 33.3°C (b) 43.3°C

(c) 35.3°C (d) 30.3°C

17. Three rods of length L of identical cross-sectional area made from the same metal form the sides of an isosceles triangle ABC right angled at B. The points A and B are maintained at temperatures T and $\sqrt{2}T$ respectively in the steady state. Assuming that only heat conduction takes place, temperature of point C will be

(a) $\frac{3T}{\sqrt{2}+1}$ (b) $\frac{T}{\sqrt{2}+1}$

(c) $\frac{T}{\sqrt{3}(\sqrt{2}-1)}$ (d) $\frac{T}{\sqrt{2}-1}$

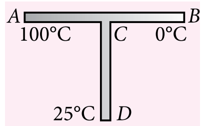


18. An icebox made of 1.5 cm thick styrofoam has dimensions 60 cm × 60 cm × 30 cm. It contains ice at 0 °C and is kept in a room at 40 °C. Find the rate at which the ice is melting. Latent heat of fusion of ice = 3.36×10^5 J kg^{-1} and thermal conductivity of styrofoam = 0.04 W m^{-1}°C^{-1}.

(a) 0.5 g s^{-1} (b) 0.56 g s^{-1}

(c) 0.46 g s^{-1} (d) 0.48 g s^{-1}

19. A rod CD of thermal resistance 5.0 K W^{-1} is joined at the middle of an identical rod AB as shown in figure. The ends A, B and D are maintained at 100°C, 0°C and 25°C respectively. Find the heat current in CD.



(a) 4 W (b) 5 W

(c) 6 W (d) 7 W

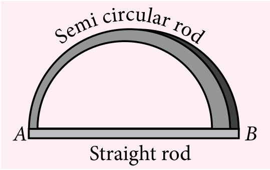
20. An electric heater emits 1000 W of thermal radiation. The coil has a surface area of 0.020 m^2. Assuming that the coil radiates like a blackbody, find its temperature.

(Stefan's constant, $\sigma = 6.00 \times 10^{-8}$ W m^{-2} K^{-4}.)

(a) 980 K (b) 970 K

(c) 950 K (d) 955 K

21. Two rods (one semi-circular and other straight) of same material and of same cross-sectional area are joined as shown in the figure. The points A and B are maintained at different temperatures. Find the ratio of the heat transferred through a cross-section of semi-circular rod to the heat transferred through a cross-section of the straight rod in a given time.



(a) $\frac{2}{\pi}$ (b) $\frac{1}{\pi}$ (c) π (d) $\frac{\pi}{2}$

22. An ice cube of mass 0.1 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat s of the container varies with temperature T according to the empirical relation $s = A + BT$, where A = 100 cal kg^{-1} K^{-1} and $B = 2\times 10^{-2}$ cal kg^{-1} K^{-2}. If the final temperature of the container is 27°C, find the mass of the container.
(Latent heat of fusion of water = 8×10^4 cal kg^{-1} specific heat of water = 10^3 cal kg^{-1} K^{-1})
(a) 0.495 kg (b) 0.224 kg
(c) 0.336 kg (d) 0.621 kg

23. The two opposite faces of a cubical piece of iron (thermal conductivity = 0.2 cal cm^{-1} s^{-1} °C^{-1}) are at 100 °C and 0 °C in ice. If the area of surface is 4 cm^2, then the mass of ice melted in 10 minutes will be
(a) 30 g (b) 300 g (c) 5 g (d) 50 g

24. The sun's surface temperature is about 6000 K. The sun's radiation is maximum at a wavelength of 0.5 μm. A certain light bulb filament emits radiation with a maximum at 2 μm. If both the surface of the sun and of the filament have the same emissive characteristics, what is the temperature of the filament?
(a) 1500 K (b) 1600 K
(c) 1000 K (d) 1200 K

25. Three discs A, B and C having radii 2 m, 4 m, and 6 m, respectively are coated with carbon black on their outer surfaces. The wavelengths corresponding to maximum intensity are 3 m, 4 m and 5 m respectively. The power radiated by them are P_A, P_B and P_C respectively. Then
(a) P_A is minimum (b) P_B is minimum
(c) P_C is minimum (d) $P_A = P_B = P_C$

26. A hot liquid is filled in a container and kept in a room of temperature of 25 °C. The liquid emits heat at the rate of 200 J s^{-1} when its temperature is 75 °C. When the temperature of the liquid becomes 40 °C, the rate of heat loss in J s^{-1} is
(a) 160 (b) 140 (c) 80 (d) 60

27. If an anisotropic solid has coefficients of linear expansion α_x, α_y, and α_z for three mutually perpendicular directions in the solid, its coefficient of volume expansion will be
(a) $(\alpha_x\alpha_y\alpha_z)^{1/3}$ (b) $\alpha_x + \alpha_y + \alpha_z$
(c) $(\alpha_x^2+\alpha_y^2+\alpha_z^2)^{1/2}$ (d) $(\sqrt{\alpha_x}+\sqrt{\alpha_y}+\sqrt{\alpha_z})^2$

28. The surface temperature of the stars is determined using
(a) Planck's law
(b) Wien's displacement law
(c) Rayleigh Jeans law
(d) Kirchhoff's law

29. A sphere of 3 cm radius acts like a blackbody. It absorbs 30 kW of power radiated to it from the surroundings. What is the temperature of the sphere? (Stefan's constant, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^4)
(a) 2600 K (b) 2500 K
(c) 2400 K (d) 2200 K

30. A refrigerator door is 150 cm high, 80 cm wide, and 6 cm thick. If the coefficient of conductivity is 0.0005 cal cm^{-1} s $^{-1}$ °C^{-1}, and the inner and outer surfaces are at 0 °C and 30 °C, respectively, what is the heat loss per minute through the door, in cal?
(a) 2000 (b) 1500 (c) 1800 (d) 2200

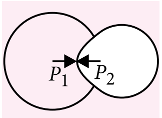
SOLUTIONS

1. **(b):** Surface area of 8 droplets = $8 \times 4\pi r^2$
$= 32\pi \times 10^{-6}$ m^2
∴ Surface energy of 8 droplets = $32\pi \times 10^{-6} \times 0.55$ J
Let R be the radius of the big drop formed.
The volume of the big drop $= \frac{4\pi}{3}R^3$
Since volume remains constant in the process of combination
$$8\times\frac{4\pi}{3}\times(1\times10^{-3})^3 = \frac{4\pi}{3}\times R^3 \text{ or } R = 2\times 10^{-3}\text{ m}$$
Surface energy of big drop formed
$= 4\pi(2 \times 10^{-3})^2 \times 0.55 = 16\pi \times 10^{-6} \times 0.55$ J
∴ Energy evolved
$= 32\pi \times 10^{-6} \times 0.55 - 16\pi \times 10^{-6} \times 0.55$
$= 16\pi \times 0.55 \times 10^{-6} = 27.65 \times 0^{-6}$ J

2. **(a):** P_1(excess pressure in the spherical bubble) $= \frac{4S}{r_1}$

P_2(excess pressure in the spherical soap bubble) $= \frac{4S}{r_2}$

$\therefore \quad P_2 - P_1 = 4S\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{4S}{r}$



Where r = radius of curvature of the common surface

$\therefore \quad \frac{1}{r} = \frac{1}{r_2} - \frac{1}{r_1} = \frac{1}{2} - \frac{1}{4} = \frac{2-1}{4} = \frac{1}{4}$

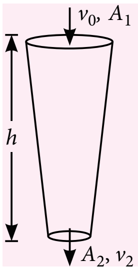
$\therefore \quad r = 4$ cm

3. **(c) :** Effective value of acceleration due to gravity becomes $(g + a_0)$.

Required velocity of efflux, $v = \sqrt{2(g + a_0)h}$

4. **(b):** In the shown figure, $v_2^2 = v_0^2 + 2gh$

and $A_1v_0 = A_2v_2$



Solving, $\frac{A_2}{A_1} = \frac{v_0}{\sqrt{v_0^2 + 2gh}}$

$\frac{A_2}{A_1} = \frac{A_2}{2A_2} = \frac{v_0}{\sqrt{v_0^2 + 2gh}}$

$4v_0^2 = v_0^2 + 2gh \quad \Rightarrow \quad h = \frac{3v_0^2}{2g}$

5. **(b):** Since the soap film has two surfaces,

$\therefore$ Increase in surface area,

$\Delta A = 2[10 \times 6 - 8 \times 3.75] \times 10^{-4}$ m^2

Surface tension, $S = \frac{W}{\Delta A} = \frac{2 \times 10^{-4}}{2 \times [60 - 30] \times 10^{-4}}$

$= 3.3 \times 10^{-2}$ N m^{-1}

6. **(c) :** Here : $v_1 = 1.0$ m s^{-1}, $A_1 = 10^{-4}$ m^2,

$h_1 - h_2 = 0.15$ m, $v_2 = ?$ $A_2 = ?$

According to Bernoulli's theorem,

$$P + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$[\because P_1 = P_2 = P]$

or $\quad \frac{1}{2}v_1^2 + gh_1 = \frac{1}{2}v_2^2 + gh_2$

or $\quad v_2^2 = v_1^2 + 2g(h_1 - h_2) = (1.0)^2 + 2 \times 10 \times 0.15 = 4$

or $\quad v = 2$ m s^{-1}

By equation of continuity, $A_1v_1 = A_2v_2$

$\therefore \quad A_2 = \frac{A_1v_1}{v_2} = \frac{10^{-4} \times 1}{2} = 5 \times 10^{-5}$ m^2

7. **(a):** Equating the rate of flow, $v_1A_1 = v_2A_2$

But $\quad v_1 = \sqrt{2gy},\ A_1 = L^2, v_2 = \sqrt{2g \times 4y},\ A_2 = \pi R^2$

$\therefore \quad \sqrt{2gy} \times L^2 = \sqrt{2g \times 4y} \times \pi R^2$

or $\quad L^2 = 2\pi R^2 \quad$ or $\quad R = \frac{L}{\sqrt{2\pi}}$

8. **(a) :** Here : $r_1 = \frac{3.0}{2} = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m,

$r_2 = \frac{6.0}{2} = 3.0 \text{ mm} = 3.0 \times 10^{-3}$ m

$S = 7.3 \times 10^{-2}$ N m^{-1}, $\theta = 0°$

$\rho = 1.0 \times 10^3$ kg m^{-3}, $g = 9.8$ m s^{-2}

Let h_1 and h_2 be the heights to which water rises in the two tubes. Then

$h_1 = \frac{2S\cos\theta}{r_1\rho g}$ and $h_2 = \frac{2S\cos\theta}{r_2\rho g}$

Therefore,

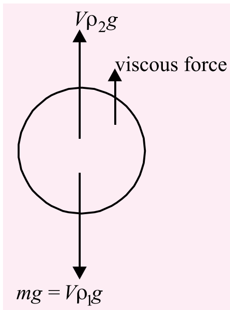
$$h_1 - h_2 = \frac{2S\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2 \times 7.3 \times 10^{-2}\cos 0°}{10^3 \times 9.8}\left[\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right]$$

$$= \frac{14.6 \times 10^{-2}}{10^3 \times 9.8 \times 10^{-3}}\left[\frac{1}{1.5} - \frac{1}{3}\right]$$

$$= \frac{14.6 \times 10^{-2}}{9.8} \times \frac{1}{3} = 0.5 \times 10^{-2} \text{ m}$$

9. **(b):** The forces acting on the solid ball when it is falling through a liquid are mg downwards, upthrust upwards and the viscous force also acting upwards. The viscous force rapidly increases with velocity, attaining a maximum when the ball reaches the terminal velocity.



Then the acceleration is zero. $mg - V\rho_2 g - kv^2 = ma$ where V is volume, v is the terminal velocity.

When the ball is moving with terminal velocity, $a = 0$

$\therefore \quad V\rho_1 g - V\rho_2 g - kv^2 = 0$

or $\quad v^2 = \frac{Vg(\rho_1 - \rho_2)}{k} \quad$ or $\quad v = \sqrt{\frac{Vg(\rho_1 - \rho_2)}{k}}$

10. **(b) :** Upthrust in liquid A, $U_1 = \frac{1}{4}V\rho_1 g$

Upthrust in liquid B, $U_2 = \frac{3}{4}V\rho_2 g$

$\therefore \quad \frac{U_1}{U_2} = \frac{\rho_1}{3\rho_2}$

For floatation,

$\frac{1}{4}V\rho_1 g + \frac{3}{4}V\rho_2 g = V\rho g \quad \Rightarrow \quad \rho_1 + 3\rho_2 = 4\rho$

11. (d): $V=\dfrac{\pi P r^4}{8\eta l}$

As capillaries of same length are joined end to end, the volume rate V across each of the capillaries is same.

Here, $P_1\cdot r_1^4 = P_2\cdot r_2^4 = P_3\cdot r_3^4$

or $P_1(2r)^4 = P_2(3r)^4 = P_3(4r)^4 \Rightarrow 16P_1 = 81P_2 = 256P_3$

$\Rightarrow\ P_2=\dfrac{16}{81}P_1$ and $P_3=\dfrac{P_1}{16}$

(Given) : $(P_1+P_2+P_3) = 20.2$ cm of Hg

$\Rightarrow\ P_1\left(1+\dfrac{16}{81}+\dfrac{1}{16}\right)=20.2 \Rightarrow P_1 = 16$ cm of Hg.

12. (c) : Surface energy = surface tension × surface area

$E = S\times 2A$

($\because$ Soap film has two surfaces)

As $A'=\left[A-\dfrac{50A}{100}\right]=\dfrac{A}{2}\quad\therefore\ E'=S\times 2\left(\dfrac{A}{2}\right)=SA$

Percentage decrease in surface energy

$=\dfrac{E-E'}{E}\times 100\%=\dfrac{2SA-SA}{2SA}\times 100\%$

$=\dfrac{1}{2}\times 100\% = 50\%$

13. (b): The velocity of efflux from the hole, $v=\sqrt{2gh}$

Using equation of continuity, $A_1v_1 = A_2v_2$

$A\left(-\dfrac{dh}{dt}\right)=a(\sqrt{2gh})\ \Rightarrow\ \dfrac{-dh}{\sqrt{h}}=\dfrac{a}{A}\sqrt{2g}\,dt$

$-\displaystyle\int_{H_1}^{H_2} h^{-\frac{1}{2}}dh=\dfrac{a}{A}\sqrt{2g}\int_0^t dt$ or $-\left[2h^{\frac{1}{2}}\right]_{H_1}^{H_2}=\dfrac{a}{A}\sqrt{2g}(t)$

or $2(\sqrt{H_1}-\sqrt{H_2})=\dfrac{a}{A}\sqrt{2\cdot g}\ t$

$\Rightarrow\ t=\dfrac{A}{a}\cdot\sqrt{\dfrac{2}{g}}(\sqrt{H_1}-\sqrt{H_2})$

14. (b): As the total mass of the air inside the bubble and the temperature remains constant.

$\therefore\ P_aV_a + P_bV_b = P_cV_c$

As pressure inside the soap bubble is $\dfrac{4S}{r}$ more than the external pressure, (here S is the surface tension)

$\left(P+\dfrac{4S}{a}\right)\cdot\left(\dfrac{4}{3}\pi a^3\right)+\left(P+\dfrac{4S}{b}\right)\cdot\left(\dfrac{4}{3}\pi b^3\right)$

$=\left(P+\dfrac{4S}{c}\right)\cdot\left(\dfrac{4}{3}\pi c^3\right)$

$\therefore\ S=\dfrac{P(c^3-a^3-b^3)}{4(a^2+b^2-c^2)}$

15. (c) : According to ascent formula

$h=\dfrac{2S\cos\theta}{r\rho g}$

For a given liquid, $hr=\dfrac{2S\cos\theta}{\rho g}=$ constant

$\therefore\ h_1r_1 = h_2r_2$

or $h_2=\dfrac{h_1r_1}{r_2}=\dfrac{(2.0)(r_1)}{(r_1/3)}=(2.0)(3)=6.0$ cm

16. (a) : Rate of fall of temperature $=\dfrac{50-40}{5}$

$=2°$C per minute

Average excess of temperature $=\dfrac{40+50}{2}-20$

$=45-20=25$

According to law of cooling,

rate of cooling $\propto$ excess of temperature

$\therefore\ 2=k\cdot 25$ or $k=\dfrac{2}{25}$

Let T = temperature after another 5 minutes

Then rate of fall of temperature $=\dfrac{40-T}{5}$

and average excess of temperature $=\dfrac{T+40}{2}-20=\dfrac{T}{2}$

$\therefore\ \dfrac{40-T}{5}=k\dfrac{T}{2}$

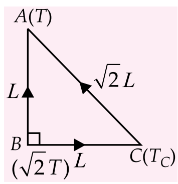
or $\dfrac{40-T}{5}=\dfrac{2}{25}\cdot\dfrac{T}{2}$ or $40-T=\dfrac{T}{5}$ or $6T=200$

or $T=33.33°$C

17. (a) : As $T_B > T_A$, heat flows from B to A through both paths BA and BCA.

Rate of heat flow in BC = Rate of heat flow in CA

$\therefore\ \dfrac{KA(\sqrt{2}T-T_C)}{L}=\dfrac{KA(T_C-T)}{\sqrt{2}\,L}$



Solving this, we get

$T_C=\dfrac{3T}{\sqrt{2}+1}$

18. (c) : The total surface area of the walls

= 2(60 cm × 60 cm + 60 cm × 30 cm + 60 cm × 30 cm)

= 1.44 m^2.

The thickness of the walls = 1.5 cm = 0.015 m.

The rate of heat flow into the box is

$\dfrac{\Delta Q}{\Delta t}=\dfrac{KA(T_1-T_2)}{x}$

$=\dfrac{(0.04\ \text{W m}^{-1}°\text{C}^{-1})(1.44\ \text{m}^2)(40°\text{C})}{0.015\ \text{m}}=154$ W

The rate at which the ice melts is

$$= \frac{154 \text{ W}}{3.36\times10^5 \text{ J kg}^{-1}} = 0.46 \text{ g s}^{-1}$$

19. (a): The thermal resistance of AC is equal to that of CB and is equal to 2.5 K W^{-1}. Suppose, the temperature at C is T. The heat currents through AC, CB and CD are

$$\frac{\Delta Q_1}{\Delta t} = \frac{100°\text{C} - T}{2.5 \text{ K W}^{-1}}, \quad \frac{\Delta Q_2}{\Delta t} = \frac{T - 0°\text{C}}{2.5 \text{ K W}^{-1}}$$

and $\frac{\Delta Q_3}{\Delta t} = \frac{T - 25°\text{C}}{5.0 \text{ K W}^{-1}}$

We also have

$$\frac{\Delta Q_1}{\Delta t} = \frac{\Delta Q_2}{\Delta t} + \frac{\Delta Q_3}{\Delta t}$$

or $\frac{100°\text{C} - T}{2.5} = \frac{T - 0°\text{C}}{2.5} + \frac{T - 25°\text{C}}{5}$

or $225°\text{C} = 5\,T$

or $T = 45°\text{C}$

Thus, $\frac{\Delta Q_3}{\Delta t} = \frac{45°\text{C} - 25°\text{C}}{5.0 \text{ K W}^{-1}} = \frac{20 \text{ K}}{5.0 \text{ K W}^{-1}} = 4.0$ W.

20. (d): Let the temperature of the coil be T. The coil will emit radiation at a rate $A\sigma T^4$. Thus,

1000 W = (0.020 m^2) × (6.0 × 10^{-8} W m^{-2} K^{-4}) × T^4

or $T^4 = \frac{1000}{0.020\times 6.00\times10^{-8}}$ K^4 = 8.33 × 10^{11} K^4

or T = 955 K

21. (a): $\frac{dQ}{dt} = \frac{KA\Delta T}{l}$, for both rods K, A and ΔT are same

$$\Rightarrow \frac{dQ}{dt} \propto \frac{1}{l}$$

so $\frac{(dQ/dt)_{\text{semicircular}}}{(dQ/dt)_{\text{straight}}} = \frac{l_{\text{straight}}}{l_{\text{semicircular}}} = \frac{2r}{\pi r} = \frac{2}{\pi}$

22. (a): Heat received by ice is

$Q_1 = mL + ms\Delta T$

= (0.1) (8 × 10^4) + (0.1) × 10^3 × 27 = 10700 cal

Heat lost by the container

$$Q_2 = \int_{300}^{500} m(A + BT)dT = m\left[AT + \frac{BT^2}{2}\right]_{300}^{500}$$

= 21600 m

By principle of calorimetry, $Q_1 = Q_2$

$\Rightarrow m = 0.495$ kg

23. (b): $Q = mL = KA\frac{(T_1 - T_2)}{l}t$

$$\Rightarrow m = \frac{1}{L}\times KA\frac{(T_1 - T_2)}{l}\times t$$

$$= \frac{1}{80}\times 0.2\times 4\times\frac{(100-0)}{\sqrt{4}}\times 10\times 60$$

$(\because\ l^2 = 4 \Rightarrow l = \sqrt{4})$

$$= \frac{0.2\times4\times100\times600}{80\times2} = 300g$$

24. (a): Use Wien's displacement law,

$\lambda_m T$ = constant

$\therefore\ \lambda_1 T_1 = \lambda_2 T_2$

$\Rightarrow 0.5\times10^{-6}\times6000 = 2\times10^{-6}\times T_2$

$\therefore\ T_2 = \frac{0.5\times6000}{2} = 1500$ K

25. (a): Stefan's law, $P \propto AT^4$

Wien's law, $\lambda_m T$ = constant

$$\therefore\ P \propto \frac{A}{(\lambda_m)^4} \propto \frac{r^2}{\lambda_m^4}$$

$$P_A : P_B : P_C = \frac{2^2}{3^4} : \frac{4^2}{5^4} : \frac{6^2}{5^4} = \frac{4}{81} : \frac{1}{16} : \frac{36}{625}$$

= 0.05 : 0.0625 : 0.0576

Hence P_A is minimum.

26. (d): Applying Newton's law of cooling,

$$\frac{ms(75-25)}{dt} = 200 \text{ J s}^{-1}$$

and $\frac{ms(40-25)}{dt} = x \quad \therefore\ x = \frac{200}{50}\times15 = 60 \text{ J s}^{-1}$

27. (b)

28. (b): The surface temperature of the stars is determined by Wien's displacement law.

29. (a): The power absorbed by a blackbody is $P = \sigma AT^4$

or (30 × 10^3) = (5.67 × 10^{-8}) 4π(0.03)$^2T^4$.

T^4 = 4.68 × 10^{13}

T = temperature of sphere = 2600 K

30. (c): $Q = \frac{KA\,\Delta T(t)}{d}$

$$= \frac{0.0005(150\times80)(30°-0°)(60)}{6} = 1800 \text{ cal}$$

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ACE YOUR WAY CBSE XI

Series 4

CHAPTERWISE PRACTICE PAPER

GRAVITATION | MECHANICAL PROPERTIES OF SOLIDS

Time Allowed : 3 hours | Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION-A

1. Why does a tennis ball bounce higher on a hill than on plains?

2. What is the difference between inertial mass and gravitational mass of a body?

3. The radius of a wire is doubled. How does the couple per unit angle of twist change?

4. Identical springs of steel and copper are equally stretched. On which, more work will have to be done?

5. The ratio stress/strain remains constant for small deformation. What will be the effect on this ratio when the deformation made is very large?

SECTION-B

6. Where on the Earth's surface is the centrifugal acceleration due to its rotation the greatest? Where is it the least? Why?

7. The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

8. Out of aphelion and perihelion, where is the speed of the earth more and why?

9. Explain why one can jump higher on the surface of the Moon than on the Earth.

OR

To what depth must a rubber ball be taken in deep sea so that its volume is decreased by 0.1%. (The bulk modulus of rubber is 9.8×10^8 N m^{-2} and the density of sea water is 10^3 kg m^{-3}.)

10. The length of a metal wire is l_1 when the tension in it is T_1 and l_2 when the tension in it is T_2. Find the original length of the wire.

SECTION-C

11. A uniform solid sphere of mass M and radius a is surrounded symmetrically by a uniform thin spherical shell of equal mass and radius $2a$. Find the gravitational field at a distance

(a) $\frac{3}{2}a$ from the centre, (b) $\frac{5}{2}a$ from the centre.

12. Determine the gravitational force of attraction between a uniform sphere of mass M and a uniform rod of length L and of mass m, placed such that, r, is the distance between the centre of the sphere and near end of rod.

13. Two identical heavy spheres are separated by a distance 10 times their radius. Will an object placed at the mid point of the line joining their centres be in stable equilibrium or unstable equilibrium? Give reason for your answer.

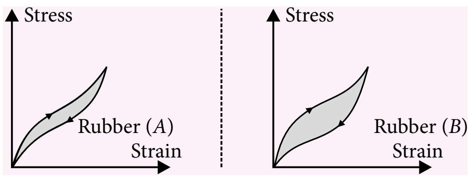
14. One end of a wire 2 m long and 0.2 cm^2 in cross section is fixed in a ceiling and a load of 4.8 kg is attached to the free end. Find the extension of the wire and energy stored in it. Young modulus of steel $= 2.0 \times 10^{11}$ N m^{-2}. Take $g = 10$ m s^{-2}.

15. A box-shaped piece of gelatin dessert has a top area of 15 cm^2 and a height of 3 cm. When a shearing force of 0.50 N is applied to the upper surface, the upper surface displaces 4 mm relative to the bottom surface. What are the shearing stress, shearing strain and the shear modulus for the gelatin?

16. Describe stress-strain relationship for a loaded steel wire and hence explain the terms elastic limit, permanent set, yield point, and tensile strength.

OR

Two different types of rubbers are found to have the stress-strain curves as shown in figure.



(a) In which significant ways do these curves differ from the stress-strain curve of a metal?

(b) A heavy machine is to be installed in a factory. To absorb vibrations of the machine, a block of rubber is placed between the machinery and the floor. Which of the two rubbers, A and B would you prefer to use for this purpose? Why ?

(c) Which of the two rubber materials would you choose for a car tyre ?

17. A square lead slab of side 50 cm and thickness 10 cm is subjected to a shearing force (on its narrow face) of 9.0×10^4 N. The lower edge is riveted to the floor. How much will the upper edge be displaced? If modulus of rigidity (G) is 5.6×10^9 N m^{-2}.

18. A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.5 rps. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (mass of the sun $= 2 \times 10^{30}$ kg)

19. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force f, its length increases by l. Another wire of the same material of length $2L$ and radius $2r$, is pulled by a force $2f$. Find the increase in length of this wire.

20. How can the knowledge of elasticity be used to estimate the maximum height of a mountain on earth?

21. A rocket is launched vertically from the surface of the earth with an initial velocity of 10 km s^{-1}. How far above the surface of the earth would it go ?

Radius of the earth = 6400 km and $g = 9.8$ m s^{-2}.

22. What is the percentage increase in the length of a wire of diameter 2.5 mm stretched by a force of 100 kg wt ? Young's modulus of elasticity of the wire is 12.5×10^{11} dyne cm^{-2}.

SECTION-D

23. Mythili was a student of Class XI. She was sitting in a garden along with her grandmother, who was retired Physics teacher. Suddenly she saw an orange falling from the tree. Immediately she asked her grandmother that both orange and earth experience equal and opposite forces of gravitation, then why it is ;the orange that falls towards the earth and not the earth towards the orange. Her grandmother explained her the reason in a simple way.

(a) What are the values being displayed by Mythili?

(b) What in your opinion may be the reason for this observation?

SECTION-E

24. State Kepler's laws of planetary motion. Deduce Newton's law of gravitation from Kepler's law.

OR

Define the term elasticity. Give an explanation of the elastic properties of materials in terms of interatomic forces ?

25. A steel rod of length $2l$, cross sectional area A and mass M is set rotating in a horizontal plane about an axis passing through the centre. If Y is the Young's modulus for steel, find the extension in the length of the rod. (Assume the rod is uniform.)

OR

Derive an expression for the elastic potential energy stored in a stretched wire under stress. Define the terms elastic after effect and elastic fatigue.

26. Earth's orbit is an ellipse with eccentricity 0.0167. Thus, earth's distance from the sun and speed as it moves around the sun varies from day to day. This means that the length of the solar day is not constant throughout the year. Assume that earth's spin axis is normal to its orbital plane and find out the length of the shortest and the longest day. A day should be taken from noon to noon. Does this explain variation of length of the day during the year?

OR

What are geostationary satellites ? Calculate the height of the orbit above the surface of the earth in which a satellite, if placed, will appear stationary. State the necessary conditions for a satellite to be geostationary.

SOLUTIONS

1. The value of g is less on hills because they are comparatively at a greater distance from the centre of the earth. Therefore, the gravitational pull on the tennis ball is less on hill tops and so it bounces higher on hills than on plains.

2. The property of an object responsible for the gravitational force it exerts on another object is called its gravitational mass. On the other hand, the property of an object that measures its resistance to acceleration is called its inertial mass.

3. The couple per unit angle for twist is given by $\tau = \pi\eta r^4/2L$. On doubling the radius(r) of the wire, τ becomes $(2)^4$, *i.e.*, 16 times.

4. Work done to stretch a spring is given by

$$W = \frac{YAl^2}{2L}$$

For two identical spring A and L are same and both are equally stretch by same length l.

$\therefore \quad W \propto Y$

$$\frac{W_{steel}}{W_{Cu}} = \frac{Y_{steel}}{Y_{Cu}} < 1$$

i.e., $W_{steel} < W_{Cu}$

Hence, more work will be done in case of spring made of copper.

5. When the deforming force is applied beyond elastic limit, the strain produced is more than that has been observed within elastic limit. Due to which the ratio stress/strain will decrease.

6. The centrifugal acceleration acting on a body of mass m placed on the surface of the Earth is given by $mr\omega^2$, where r is the radius of the circle described by the body while rotating with the Earth with angular speed ω. At the equator, r is maximum and as such centrifugal acceleration is also maximum. But at the poles, $r = 0$ and as such the centrifugal acceleration is zero *i.e.*, minimum.

7. Using law of conservation of energy

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 - \frac{1}{2}mv_e^2$$

Here, v_0 = speed of projectile when far away from the earth,

v = velocity of projection of the body,

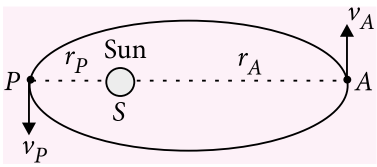
v_e = escape velocity

$$\therefore \quad v_0 = \sqrt{v^2 - v_e^2} = \sqrt{(3v_e)^2 - v_e^2} = \sqrt{8}\,v_e$$

[Given; $v = 3v_e$]

$$= \sqrt{8} \times 11.2 = 31.68 \text{ km s}^{-1}.$$

8. In the figure, point A represents aphelion and point P perihelion.



According to Kepler's second law, areal velocity is constant. So, $r_P \times v_P = r_A \times v_A$

$$\Rightarrow \frac{r_A}{r_P} = \frac{v_P}{v_A}$$

$\because \quad r_A > r_P$ so, $\quad v_P > v_A$.

Hence, speed of the earth at the perihelion is more than that at the aphelion.

9. The maximum height to which a man can jump is given by $h = v_0^2 \sin 2\theta / 2g$. For a given value of v_0 and θ, $h \propto 1/g$.
Since on the surface of the Moon, acceleration due to gravity (g) is less than that on the Earth's surface, therefore h is more on the surface of the Moon than that on Earth's surface. Therefore, One can jump higher on the surface of the Moon than on the Earth.

OR

Bulk modulus of rubber (B) = 9.8×10^8 N m^{-2}
Density of sea water (ρ) = 10^3 kg m^{-3}
Percentage decrease in volume,

$$\left(\frac{\Delta V}{V} \times 100\right) = 0.1 \text{ or } \frac{\Delta V}{V} = \frac{0.1}{100} = \frac{1}{1000}$$

Let the rubber ball be taken up to depth h.

$\therefore$ Change in pressure (p) = $h\rho g$

$$\therefore \text{ Bulk modulus } (B) = \frac{p}{(\Delta V / V)} = \frac{h\rho g}{(\Delta V / V)}$$

$$\text{or} \quad h = \frac{B \times (\Delta V / V)}{\rho g} = \frac{9.8 \times 10^8 \times \frac{1}{1000}}{10^3 \times 9.8} = 100 \text{ m}$$

10. Let l be the original length and A the area of cross-section of the wire.
Change in length in first case = $l_1 - l$
Change in length in second case = $l_2 - l$
Young's modulus of the wire will be,

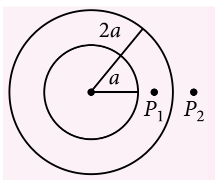
$$Y = \frac{T_1}{A} \cdot \frac{l}{l_1 - l} = \frac{T_2}{A} \cdot \frac{l}{l_2 - l}$$

or $T_1 l_2 - T_1 l = T_2 l_1 - T_2 l$

or $l(T_2 - T_1) = T_2 l_1 - T_1 l_2$

$$\text{or} \quad l = \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$$

11. Let the point P_1 is at a distance $\frac{3}{2}a$ from the centre and P_2 is at a distance $\frac{5}{2}a$ from the centre as shown in figure.



(a) As P_1 is inside the cavity of the thin spherical shell, the field here due to the shell is zero. The field due to the solid sphere is

$$E = \frac{GM}{\left(\frac{3}{2}a\right)^2} = \frac{4GM}{9a^2}$$

This is also the resultant field.
The direction is towards the centre.

(b) The point P_2 is outside the sphere as well as the shell. Both may be replaced by single particles of the same mass at the centre. The field due to each of them is

$$E' = \frac{GM}{\left(\frac{5}{2}a\right)^2} = \frac{4GM}{25a^2}$$

The resultant field is $E = 2E' = \frac{8GM}{25a^2}$ towards the centre.

12. Mass per unit length of rod, $= m/L$. Consider a small element of the rod of thickness dx at a distance x from the centre of sphere.

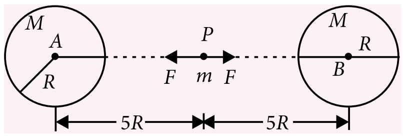
Mass of this element, $dm = \frac{m}{L} dx$.

Gravitational attraction between element and the sphere is, $dF = \frac{GM\,dm}{x^2} = \frac{GM\,m}{L} \frac{dx}{x^2}$

Total gravitational attraction acting on the rod can be obtained by integrating it within the limits $x = r$ to $x = r + L$. We get,

$$F = \frac{GM\,m}{L} \int_r^{r+L} \frac{dx}{x^2} = \frac{GM\,m}{L}\left[-\frac{1}{x}\right]_r^{r+L} = \frac{GM\,m}{r(r+L)}$$

13. Situation is shown in the figure,



P is the mid point of AB. Magnitude of force applied by each sphere on the mass m, is given by

$$F = \frac{GMm}{(5R)^2}$$

The direction of forces are opposite, hence the resultant force is zero.
If mass m is displaced towards right by small distance r, then the force on the mass m, due to object B is

$$F_B = \frac{GMm}{(5R - r)^2}$$

Force on mass m due to object A is $F_A = \frac{GMm}{(5R + r)^2}$
At new point, $F_B > F_A$.
Therefore a resultant force ($F_B - F_A$) will act on particle towards right. The particle will move towards B.
So, object of mass m will be in unstable condition.

14. Here; mass = 4.8 kg, Area = 0.2 cm^2 = 0.2 × 10^{-4} m^2

length = 2 m

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{l/L}$$

with symbols having their usual meanings. The extension is $l = \frac{TL}{AY}$

As the load is in equilibrium after the extension, the tension in the wire is equal to the weight of the load = 4.8 kg × 10 m s^{-2} = 48 N

$$\text{Thus, } l = \frac{(48\text{ N})(2\text{ m})}{(0.2\times10^{-4}\text{ m}^2)\times(2.0\times10^{11}\text{ N m}^{-2})}$$

$= 2.4 \times 10^{-5}$ m.

Energy stored in wire,

$$U = \frac{1}{2}\times\text{stress}\times\text{strain}\times\text{volume}$$

$$= \frac{1}{2}\times\frac{T}{A}\times\frac{l}{L}\times AL = \frac{1}{2}Tl$$

$$= \frac{1}{2}\times 48\times 2.4\times10^{-5} = 5.76\times10^{-4}\text{ J}$$

15. Here, $A = 15 \times 10^{-4}$ m^2, $l = 3 \times 10^{-2}$ m, $F = 0.50$ N, $\Delta x = 4 \times 10^{-3}$ m

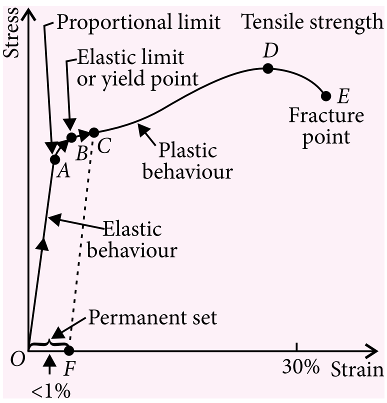
$$\text{Stress} = \frac{F}{A} = \frac{0.50}{15\times10^{-4}}\text{ N m}^{-2} = 333.3\text{ N m}^{-2}$$

$$\text{Strain} = \frac{\Delta x}{l} = \frac{4\times10^{-3}}{3\times10^{-2}} = 0.133$$

$$\text{Shear modulus, } G = \frac{\text{stress}}{\text{strain}} = \frac{F}{A}\times\frac{l}{\Delta x}$$

$$= \frac{0.50}{15\times10^{-4}}\times\frac{3\times10^{-2}}{4\times10^{-3}}\text{ N m}^{-2} = 2500\text{ N m}^{-2}$$

16. Figure shows a stress-strain curve for a steel wire which is gradually being loaded.



(i) The initial part OA of the graph is a straight line indicating that stress is proportional to strain. Upto the point A, Hooke's law is obeyed. The point A is called the proportional limit. In this region, the wire is perfectly elastic.

(ii) After the point A, the stress is not proportional to strain and a curved portion AB is obtained. However, if the load is removed at any point between O and B, the curve is retraced along BAO and the wire attains its original length. The portion OB of the graph is called elastic region and the point B is called elastic limit or yield point. The stress corresponding to the yield point is called yield strength (σ_y).

(iii) Beyond the point B, the strain increases more rapidly than stress. If the load is removed at any point C, the wire does not come back to its original length but traces dashed line CF. Even on reducing the stress to zero, a residual strain equal to OF is left in the wire. The material is said to have acquired a permanent set.

(iv) If the load is increased beyond the point C, there is large increase in the strain or the length of the wire. The stress corresponding to the point D is called ultimate strength or tensile strength of the material. Beyond this point, additional strain is produced even by a reduced applied force and fracture occurs at point E.

OR

(a) (i) Hooke's law is not obeyed at all.

(ii) There is no permanent set even for large stresses.

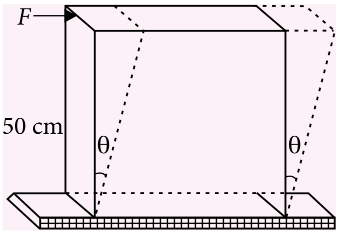
In fact, each curve forms a loop, called the elastic hysteresis loop and the area of this loop represents the energy dissipated by the material while undergoing stretching and then recovering its original configuration.

(b) To absorb vibrations of the machinery, the material with larger hysteresis loop area is preferred. The material with smaller loop area would not provide stability to the machinery installed on it. Therefore, we prefer rubber B.

(c) In case of a car tyre, energy wasted (which ultimately appears as heat) is to be avoided as it would heat up the tyre. Obviously, rubber A with lesser energy loss is to be preferred.

17. The lead slab is fixed and force is applied parallel to narrow face as shown in figure. The area of the face parallel to which is applied is

A = 50 cm × 10 cm = 0.5 m × 0.1 m = 0.5 m^2



Therefore, the stress applied $= \frac{9.0\times10^4\,\text{N}}{0.05\,\text{m}^2}$

$= 1.80 \times 10^6\ \text{N m}^{-2}$

We know that shearing strain $= \frac{\Delta x}{L} = \frac{\text{Stress}}{G}$

Therefore, the displacement, $\Delta x = \frac{\text{Stress}\times L}{G}$

$$\Delta x = \frac{1.8\times10^6\,\text{N m}^{-2}\times0.5\text{m}}{5.6\times10^9\,\text{N m}^{-2}} = 1.6\times10^{-4}\ \text{m}$$

$\Delta x = 0.16$ mm

18. The object will remain stuck to the surface of star due to gravity, if the acceleration due to gravity is more than the centrifugal acceleration due to its rotation.

Acceleration due to gravity,

$$g = \frac{GM}{R^2} = \frac{6.67\times10^{-11}\times2.5\times2\times10^{30}}{(12000)^2}$$

$= 2.3 \times 10^{12}\ \text{m s}^{-2}$

Centrifugal acceleration $= r\omega^2 = r(2\pi\upsilon)^2$

$= 12000(2\pi \times 1.5)^2 = 1.1 \times 10^6\ \text{m s}^{-2}$

Since $g > r\omega^2$, therefore the body will remain stuck with the surface of star.

19. First case, length of wire $= L$, radius of wire $= r$

Force applied $= f$, increase in length $= l$

$$Y_1 = \frac{\frac{f}{\pi r^2}}{l/L} = \frac{fL}{\pi r^2 l} \qquad ...(i)$$

In second case, length of wire $= 2L$,

radius of wire $= 2r$

Force applied $= 2f$, increase in length $= x$ (say)

$$Y_2 = \frac{\frac{2f}{\pi(2r)^2}}{x/2L} = \frac{fL}{\pi r^2 x} \qquad ...(ii)$$

Both the wires are of same material,

So, $Y_1 = Y_2$

Hence, $x = l$ (Using eqn. (i) and (ii))

20. The maximum height of mountain on earth depends upon shear modulus of rock. At the base of the mountain, the stress due to all the rock on the top should be less than the critical shear stress at which the rock begins to flow. Suppose the height of the mountain is h and the density of its rock is ρ. The force per unit area (due to the weight of the mountain) at the base $= h\rho g$. The material at the base experiences this force per unit area in the vertical direction, but sides of the mountain are free. Hence, there is a shear component of the order of $h\rho g$. The elastic limit for a typical rock is about $3 \times 10^8\ \text{N m}^{-2}$ and its density is $3 \times 10^3\ \text{kg m}^{-3}$.

Hence

$h_{max}\,\rho g = 3 \times 10^8$

or $h_{max} = \frac{3\times10^8}{\rho g} = \frac{3\times10^8}{3\times10^3\times9.8}$

$\simeq 10{,}000$ m $= 10$ km

This is more than the height of the Mount Everest. A height greater than this will not be able to withstand the shearing stress due to the weight of the mountain.

21. Initial K.E. of the rocket = Gain in gravitational P.E.

$$\frac{1}{2}mv^2 = -\frac{GMm}{(R+h)} - \left[-\frac{GMm}{R}\right] = \frac{GMmh}{R(R+h)}$$

or $\frac{1}{2}v^2 = -\frac{gR^2h}{R(R+h)} = \frac{gRh}{R+h}$

or $\frac{R+h}{h} = \frac{2gR}{v^2}$ or $\frac{R}{h}+1 = \frac{2gR}{v^2}$

or $h = R\left[\frac{2gR}{v^2}-1\right]^{-1}$

$$= 6.4\times10^6\left[\frac{2\times9.8\times6.4\times10^6}{(10^4)^2}-1\right]^{-1}$$

$= 6.4 \times 10^6 \times (0.2544)^{-1} = 2.5 \times 10^7$ m

$= 2.5 \times 10^4$ km.

22. Given; $r = 1.25$ mm $= 0.125$ cm,

$F = 100 \times 9.8 = 980$ N

N $= 98 \times 10^6$ dyne

$Y = 12.5 \times 10^{11}$ dyne cm^{-2}

As $Y = \frac{F}{A}\cdot\frac{l}{\Delta l}$ or $\frac{\Delta l}{l} = \frac{F}{AY} = \frac{F}{\pi r^2 Y}$

$\therefore$ The percentage increase in length is

$$\frac{\Delta l}{l}\times 100=\frac{F\times 100}{\pi r^2 Y}=\frac{98\times 10^6\times 7\times 100}{22\times(0.125)^2\times 12.5\times 10^{11}}$$

$= 15.965 \times 10^{-2} \approx 0.16\%$.

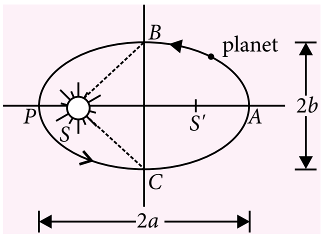
23. (a) Power of observation and curiosity.

(b) According to the law of gravitation, the orange and earth exert equal and opposite forces on each other. But the mass of the earth is extremely large as compared to that of orange, so the acceleration of the earth is very small and is not noticeable.

24. To explain the motion of the planets, Kepler formulated the following three laws.

Law of orbits (first law) : Each planet revolves around the sun in an elliptical orbit with the sun situated at one of the two foci.

The planets move around the sun in an elliptical orbit. An ellipse has two foci S and S', the sun remains at one focus S.

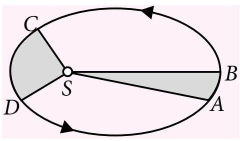


$PA = 2a$ = major axis, $BC = 2b$ = minor axis

Law of areas (second law) : The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time *i.e.*, the areal velocity (area covered per unit time) of a planet around the sun is constant.

Suppose a planet takes same time to go from position A to B as in going from C to D. From Kepler's second law, the areas ASB and CSD must be equal. Clearly, the planet covers a larger distance CD when it is near the sun than AB when it is farther away in the same interval of time.

Hence the linear velocity of a planet is more when it is closer to the sun than its linear velocity when away from the sun.



Laws of periods (third law) : The square of the period of revolution of a planet around the sun is proportional to the cube of the semimajor axis of its elliptical orbit.

If T is the period of revolution of a planet and R is the length of semimajor axis of its elliptical orbit, then

$T^2 \propto R^3$

or $T^2 = kR^3$

Suppose a planet of mass m moves around the sun in a circular orbit of radius r. Let M be the mass of the sun. If v is the orbital velocity of the planet, then the required centripetal force is

$$F=\frac{mv^2}{r}$$

But orbital velocity,

$$v=\frac{\text{Circumference}}{\text{Period of revolution}}=\frac{2\pi r}{T}$$

$$\therefore\quad F=\frac{m}{r}\times\left(\frac{2\pi r}{T}\right)^2=\frac{4\pi^2 mr}{T^2}$$

According to Kepler's law of periods,

$T^2 \propto r^3$

or $T^2 = k\,r^3$

where k is a constant. Therefore,

$$F=\frac{4\pi^2 mr}{k\,r^3}=\frac{4\pi^2}{k}\cdot\frac{m}{r^2}$$

As the force between the sun and the planet is mutual, it should also be proportional to the mass M of the sun.

Hence the factor,

$$\frac{4\pi^2}{k}\propto M \quad\text{or}\quad \frac{4\pi^2}{k}=GM$$

where G is another constant. So we have

$$F=G\frac{Mm}{r^2}$$

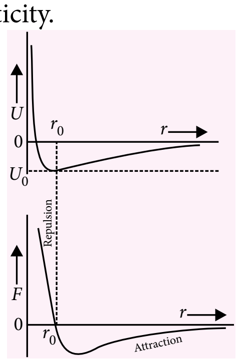
This is Newton's law of gravitation and it is applicable to any two bodies in the universe.

OR

The property of a body, by virtue of which it tends to regain its original size and shape after the applied force is removed is called elasticity.

The atoms in a solid are held together by interatomic forces.

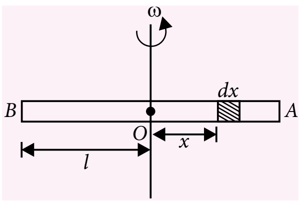
The variations of potential energy U and interatomic force F with interatomic separation r are shown in figure respectively. When the interatomic separation r is large, the potential energy



of the atoms is negative and the interatomic force is attractive. At some particular separation r_0, the potential energy becomes minimum and the interatomic force becomes zero. This separation r_0 is called normal or equilibrium separation.
When separation reduces below r_0, the potential energy increases steeply and the interatomic force becomes repulsive.
Normally, the atoms occupy the positions ($r = r_0$) of minimum potential energy called the positions of stable equilibrium. When a tensile or compressive force is applied on a body, its atoms are pulled apart or pushed closer together to a distance r, greater than or smaller than r_0. When the deforming force is removed, the interatomic forces of attraction / repulsion restore the atoms to their equilibrium positions. The body regains its original size and shape. The stronger the interatomic forces, the smaller will be the displacements of atoms from the equilibrium positions and hence greater is the elasticity (or modulus of elasticity) of the material.

25. Since rod is uniform, so mass per unit length, $\mu = \frac{M}{2l}$. Consider a small element of length dx at distance x of the rod from the axis of rotation.

Mass of small element $= \left(\frac{M}{2l}\right) dx$



Centripetal force acting on this element,

$$dF = \left(\frac{M}{2l}\right) dx \cdot x\omega^2$$

Here, dF is provided by tension in element dx of the rod due to elasticity.
Let tension in the rod be F at a distance x from the axis of rotation.
F is due to centripetal force acting on all the elements from x to l *i.e.*,

$$F = \frac{M\omega^2}{2l}\int_x^l x\,dx = \frac{M\omega^2}{4l}\left(l^2 - x^2\right)$$

If $d(\delta)$ be the extension in the element of length dx at position x, then

$$d(\delta) = \frac{F dx}{YA} \qquad \left[\because\ Y = \frac{F/A}{d(\delta)/dx}\right]$$

Hence, extension in the half of the rod (OA) is given by

$$\delta = \int_0^l d(\delta) = \int_0^l \frac{F dx}{YA}$$

$$= \frac{M\omega^2}{4YAl}\left[l^2(x) - \frac{x^3}{3}\right]_0^l = \frac{M\omega^2}{4YAl}\left[l^3 - \frac{l^3}{3}\right]$$

$$= \frac{M\omega^2 l^2}{6YA}$$

Hence, total extension in entire rod of length $2l$,

$$2\delta = \frac{M\omega^2 l^2}{3YA}$$

OR

When a wire is stretched, interatomic forces come into play which oppose the change. Work has to be done against these restoring forces. The work done in stretching the wire is stored in it as its elastic potential energy.
Suppose a force F is applied on a wire of length l increases its length by Δl. Initially, the internal restoring force in the wire is zero. When the length is increased by Δl. The internal force increases from zero to
F (= applied force).
$\therefore$ Average internal force for an increase in length Δl of wire $= \frac{0+F}{2} = \frac{F}{2}$
Work done on the wire is,

W = Average force × increase in length $= \frac{F}{2} \times \Delta l$

This work done is stored as elastic potential energy U in the wire.

$$\therefore \quad U = \frac{1}{2} F \times \Delta l$$

$$= \frac{1}{2}\text{ Stretching force} \times \text{increase in length}$$

Let A be the area of cross-section of the wire. Then

$$U = \frac{1}{2}\frac{F}{A} \times \frac{\Delta l}{l} \times Al$$

$= \frac{1}{2}$ stress × strain × volume of wire

Elastic potential energy per unit volume of the wire or elastic energy density is

$$u = \frac{U}{\text{Volume}}$$

or $u = \frac{1}{2}$stress×strain

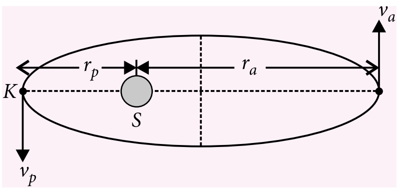
But stress = Young's modulus × strain

$\therefore \quad u = \frac{1}{2}$ Young's modulus × strain^2

Elastic after effect : The delay in regaining the original state by a body on the removal of the deforming force is called elastic after effect.

Elastic fatigue : It is defined as loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

26. As earth orbits the sun, the angular momentum is conserved and areal velocity is constant.



Let M = mass of earth

v_p = velocity of earth at perihelion,

v_a = velocity of earth at aphelion

ω_p = angular velocity of earth at perihelion

ω_a = angular velocity of earth at aphelion

According to law of conservation of angular momentum

$Mv_p r_p = Mv_a r_a \Rightarrow v_p r_p = v_a r_a$

$\Rightarrow (\omega_p r_p) r_p = (\omega_a r_a) r_a \Rightarrow \omega_p r_p^2 = \omega_a r_a^2$

$$\Rightarrow \frac{\omega_p}{\omega_a} = \frac{r_a^2}{r_p^2} \quad \text{...(i)}$$

Now, $r_p = a(1 - e)$, and $r_a = a(1 + e)$

From equation (i),

$$\frac{\omega_p}{\omega_a} = \frac{(1+e)^2}{(1-e)^2} = \frac{(1+0.0167)^2}{(1-0.0167)^2} = 1.0691$$

Let ω = mean angular velocity

$$\therefore \quad \left(\frac{\omega_p}{\omega}\right)\left(\frac{\omega}{\omega_a}\right) = 1.0691 \quad \text{...(ii)}$$

Also, $\omega^2 = \omega_p \omega_a$

$$\Rightarrow \frac{\omega_p}{\omega} = \frac{\omega}{\omega_a} \quad \text{...(iii)}$$

From eqns (ii) and (iii),

$$\Rightarrow \frac{\omega_p}{\omega} = \frac{\omega}{\omega_a} = \sqrt{1.0691} = 1.034$$

If mean angular velocity ω = 1°/day.

then, ω_p = 1.034 °/day and ω_a = 0.967 °/day

∵ 361° corresponds to 24 h.

∴ (360 + 1.034)° corresponds to 24.0023 h = 24 h 8.14".

and (360–0.967)° corresponds to 23.87 h = 23 h 59' 52".

This does not explain the actual variation of the length of the day during the year.

OR

A satellite which revolves around the earth in its equatorial plane with the same angular speed and in the same direction as the earth rotates about its own axis is called a geostationary or synchronous satellite.

The height of a satellite above the earth's surface is given by

$$h = \left[\frac{T^2 R^2 g}{4\pi^2}\right]^{1/3} - R$$

But T = 24 h = 86400 s,

R = radius of the earth = 6400 km,

$g = 9.8 \text{ m s}^{-2} = 0.0098 \text{ km s}^{-2}$

$$h = \left[\frac{(86400)^2 \times (6400)^2 \times 0.0098}{4 \times 9.87}\right]^{1/3} - 6400$$

= 42339 – 6400 = 35939 km.

Necessary conditions for a geostationary satellite.

These are as follows:

(i) It should revolve in an orbit concentric and coplanar with the equatorial plane of the earth.

(ii) Its sense of rotation should be same as that of the earth *i.e.*, from west to east.

(iii) Its period of revolution around the earth should be exactly same as that of the earth about its own axis *i.e.*, 24 hours.

(iv) It should revolve at a height of nearly 36,000 km above the earth's surface.

MPP-6 MONTHLY Practice Problems

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

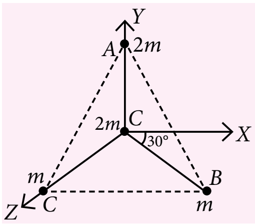
Gravitation

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

1. Masses $2m$, m, m are kept at the vertices of an equilateral triangle of side 1 m, as shown in the figure. Another mass $2m$ is kept at its centroid. The force on the $2m$ at the centroid due to other is



(a) $Gm^2\hat{j}$ (b) $Gm^2\hat{i}$

(c) $2Gm^2\hat{i}$ (d) $6Gm^2\hat{j}$

2. The gravitational field strength at the surface of a certain planet is g. Which of the following is the gravitational field strength at the surface of a planet with twice the radius and twice the mass?

(a) $g/2$ (b) g (c) $2g$ (d) $4g$

3. A planet revolves around the sun in an elliptical orbit. If v_p and v_a are the velocities of the planet at the perigee and apogee respectively, then the eccentricity of elliptical orbit is given by

(a) $\frac{v_p}{v_a}$ (b) $\frac{v_a - v_p}{v_a + v_p}$ (c) $\frac{v_p + v_a}{v_p - v_a}$ (d) $\frac{v_p - v_a}{v_p + v_a}$

4. A satellite is to be placed in equatorial geostationary orbit around earth for communication. The height of such a satellite is

$[M_E = 6 \times 10^{24}$ kg, $R_E = 6400$ km, $T = 24$ h, $G = 6.67 \times 10^{-11}$ N m^2 kg$^{-2}]$

(a) 3.57×10^5 m (b) 3.57×10^6 m

(c) 3.57×10^7 m (d) 3.57×10^8 m

5. The orbital velocity of a satellite orbiting close to earth's surface is (R = radius of earth, g = acceleration due to gravity)

(a) $\sqrt{gR}$ (b) $\sqrt{0.5gR}$

(c) $\sqrt{2gR}$ (d) $2\sqrt{gR}$

6. What happens to the acceleration due to gravity with the increase in altitude from the surface of the earth?

(a) Increases

(b) Decreases

(c) First decreases and then increases

(d) Remains same

7. A projectile is fired vertically upward from the surface of earth with a velocity of kv_e, where v_e is the escape velocity and $k < 1$. Neglecting air resistance, the maximum height to which it will rise, measured from the centre of the earth, is (R = radius of earth)

(a) $\frac{R}{1-k^2}$ (b) $\frac{R}{k^2}$ (c) $\frac{1-k^2}{R}$ (d) $\frac{k^2}{R}$

8. The change in the gravitational potential energy when a body of mass m is raised to a height nR above the surface of the earth is (here R is the radius of the earth)

(a) $\left(\frac{n}{n+1}\right)mgR$ (b) $\left(\frac{n}{n-1}\right)mgR$

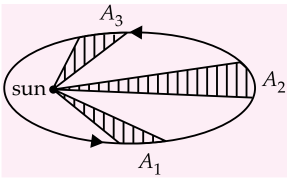
(c) $nmgR$ (d) $\frac{mgR}{n}$

9. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(a) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (b) $\sqrt{\frac{GM}{R}}$

(c) $\sqrt{2\sqrt{2}\frac{GM}{R}}$ (d) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

10. A planet moving around the sun sweeps area A_1 in 2 days, A_2 in 3 days and A_3 in 6 days. Then the relation between A_1, A_2 and A_3 is (Shaded areas are not to scale.)



(a) $3A_1 = 2A_2 = A_3$ (b) $2A_1 = 3A_2 = 6A_3$
(c) $3A_1 = 2A_2 = 6A_3$ (d) $6A_1 = 3A_2 = 2A_3$

11. The acceleration due to gravity at the poles and the equator is g_p and g_e respectively. If the earth is a sphere of radius R_E and rotating about its axis with angular speed ω, then $g_p - g_e$ is given by

(a) $\frac{\omega^2}{R_E}$ (b) $\frac{\omega^2}{R_E^2}$ (c) $\omega^2 R_E{}^2$ (d) $\omega^2 R_E$

12. Imagine a light planet revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$. Then
(a) T^2 is proportional to R^3
(b) T^2 is proportional to $R^{7/2}$
(c) T^2 is proportional to $R^{3/2}$
(d) T^2 is proportional to $R^{7/3}$.

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. Assertion : Orbiting satellite has K.E always less than the magnitude of potential energy.
Reason : For any bound state, the magnitude of potential energy is always twice that of kinetic energy (K.E.)

14. Assertion : A planet move faster, when it is closer to the sun in its orbit and vice-versa.
Reason : Orbital velocity of a planet in an orbit is constant.

15. Assertion : The square of the period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse.
Reason : Sun's gravitational field is inversely proportional to the square of its distance from the planet.

JEE MAIN / JEE ADVANCED

Only One Option Correct Type

16. A simple pendulum has a time period exactly 2 s when used in a laboratory at north pole. What will be the time period if the same pendulum is used in a laboratory at equator? Account for the earth's rotation only. Take $g = \frac{GM}{R^2} = 9.8\ \text{m s}^{-2}$ and radius of earth = 6400 km.
(a) 2 s (b) 1 s (c) 3 s (d) 5 s

17. Find the distance of a point from the earth's centre where the resultant gravitational field due to the earth and the moon is zero. The mass of the earth is 6.0×10^{24} kg and that of the moon is 7.4×10^{22} kg. The distance between the earth and the moon is 4.0×10^5 km.
(a) 4.2×10^5 km (b) 3.6×10^5 km
(c) 2.8×10^5 km (d) 5.4×10^5 km

18. A particle is projected with a velocity $\sqrt{\frac{4gR}{3}}$ vertically upward from the surface of the earth, R being the radius of the earth and g being the acceleration due to gravity on the surface of the earth. The velocity of the particle when it is at one-fourth the maximum height reached by it is

(a) $\sqrt{\frac{gR}{2}}$ (b) $\sqrt{\frac{gR}{3}}$ (c) $\sqrt{gR}$ (d) $\sqrt{\frac{2gR}{3}}$

19. If a particle of mass m is projected with minimum velocity from the surface of a star with kinetic energy $\frac{K_1 GMm}{a}$ and potential energy at surface of the star $-\frac{K_2 GMm}{a}$ towards the star of same mass M and radius a (K_1 and K_2 are constants) to reach the other star. The distance between the centre of the two stars is

(a) $\frac{2a}{(K_2 - K_1)}$ (b) $\frac{4a}{(K_2 - K_1)}$

(c) $\frac{2a}{(K_1 - K_2)}$ (d) $\frac{a}{(K_1 - K_2)}$

More than One Options Correct Type

20. In a solid sphere two small symmetrical cavities are created whose centres lie on a diameter AB of sphere on opposite sides of the centre. If gravitational potential and field are V and E, respectively then

(a) E(centre) = 0

(b) V(centre) is same with and without cavity

(c) V = constant inside the cavity

(d) E = constant inside the cavity

21. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. If acceleration, angular momentum, total energy and linear momentum are a, L, E and p respectively. Then

(a) a is always directed towards the centre of the earth

(b) L will be conserved

(c) E varies periodically with time

(d) p remains constant in magnitude

22. Suppose Newton's law of gravitation for gravitational forces F_1 and F_2 between two masses m_1 and m_2 at positions r_1 and r_2 read

$$F_1 = -F_2 = -\frac{r_{12}}{r_{12}^3} GM_0^2 \left(\frac{m_1 m_2}{M_0^2}\right)^n$$

where M_0 is a constant of dimension of mass, $r_{12} = r_1 - r_2$ and n is a number. In such a case,

(a) The acceleration due to gravity on the earth will be different for different objects

(b) None of the three laws of Kepler will be valid

(c) Only the Kepler's third law will become invalid

(d) All of these

23. The spherical planets A and B have the same mass but densities in the ratio 1 : 8 having escape velocities V_{eA} and V_{eB} respectively. For these planets, the

(g_A = acceleration due to gravity of planet A)

(g_B = acceleration due to gravity of planet B)

(a) $g_A : g_B = 4 : 1$ (b) $g_A : g_B = 1 : 4$

(c) $V_{eA} : V_{eB} = \sqrt{2} : 1$ (d) $V_{eA} : V_{eB} = 1 : \sqrt{2}$

Integer Answer Type

24. Two particles A and B of masses 1 kg and 2 kg respectively are kept at a very large separation. They are released and move under their gravitational attraction. The speed of A is $x \times 10^{-5}$ m s^{-1} and B is 3.6 cm hr^{-1}, find the value of x.

25. If the radius of the earth were to shrink by 2% its mass remaining the same, by how much percentage would the acceleration due to gravity on the earth's surface would increase?

26. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half of the magnitude of escape velocity from the earth. If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, then speed (in m s^{-1}) with which it hits the surface of the earth is $\sqrt{kgR}$. Find the value of k.

(Given, $g = 9.8$ m s^{-2} and $R = 6.4 \times 10^6$ m)

Comprehension Type

A pair of stars rotates about a common centre of mass. One of the stars has a mass M and the other has mass m such that $M = 2m$. The distance between the centres of the stars is d (d being large as compared to the size of either star).

27. The period of rotation of the stars about their common centre of mass (in terms of d, m, G) is

(a) $\sqrt{\frac{4\pi^2}{Gm}d^3}$ (b) $\sqrt{\frac{8\pi^2}{Gm}d^3}$

(c) $\sqrt{\frac{2\pi^2}{3Gm}d^3}$ (d) $\sqrt{\frac{4\pi^2}{3Gm}d^3}$

28. The ratio of kinetic energies of the two stars (K_m/K_M) is

(a) 1 (b) 2 (c) 4 (d) 9

Matrix Match Type

29. Match the following columns (g = acceleration due to gravity).

Column I	Column II
(A) Linear variation of g	(P) Weight increases all places excluding the poles.
(B) g is proportional to the radius of planet	(Q) Free falling body
(C) Weightless	(R) If density of planet is given.
(D) Earth stops rotating about its axis	(S) As we go down from the surface or upto the surface for small height.

	A	B	C	D
(a)	R	S	Q	P
(b)	S	R	Q	P
(c)	S	R	P	Q
(d)	P	Q	R	S

30. Match the following columns.

Column I	Column II
(A) Gravitational field intensity	(P) Negative of the gravitational potential energy gradient
(B) Gravitational potential	(Q) Negative of the potential gradient
(C) Angular momentum is conserved	(R) Gravitational force per unit mass
(D) Gravitational force is proportional	(S) for all central force
	(T) Gravitational potential energy per unit mass

	A	B	C	D
(a)	Q	P, R	T	S
(b)	P, Q	S	Q	T
(c)	Q, R	T	P, S	P
(d)	T	P, Q	S, T	R

◈◈

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……

No. of questions correct ……

Marks scored in percentage ……

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

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Class XII

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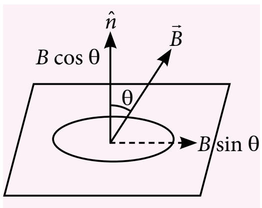
Unit 4 ELECTROMAGNETIC INDUCTION ALTERNATING CURRENT AND EM WAVES

ELECTROMAGNETIC INDUCTION

Electromagnetic induction is the phenomenon of production of emf in a coil, when the magnetic flux linked with the coil is changed. The emf so produced is called induced emf and the current so produced is called induced current.

Magnetic Flux (ϕ)

- The number of magnetic field lines crossing a surface normally is called magnetic flux (ϕ) linked with the surface.



$$\phi = \vec{B}.\vec{A} = BA\cos\theta,$$

where B is the magnetic field, A is the area of the surface and θ is the angle between the direction of the magnetic field and normal to the surface.
- The SI unit of magnetic flux is weber (Wb)
- Magnetic flux can be changed by
  - changing the intensity of the magnetic field.
  - changing the orientation of coil with respect to the magnetic field.
  - changing the area of the closed circuit.

Faraday's Laws of Electromagnetic Induction

- **First law:** Whenever magnetic flux linked with a circuit (a loop of wire or a coil or an electric circuit in general) changes, induced emf is produced. The induced emf lasts as long as the change in the magnetic flux continues.
- **Second law:** The magnitude of the induced emf is directly proportional to the rate of change of the magnetic flux.

Induced emf, $\varepsilon = -\dfrac{d\phi}{dt} = -\dfrac{\phi_2 - \phi_1}{t}$

Lenz's Law

- It states that the induced current produced in a circuit always flows in such a direction that it opposes the change or the cause that produces it.
- Lenz's law can be used to find the direction of induced current. Lenz's law is in accordance with the law of conservation of energy.

Motional Electromotive Force

Motional emf in loop by generated area

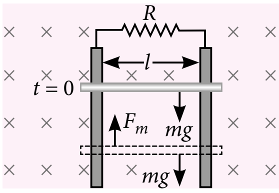
- If conducting rod moves on two parallel conducting rails as then phenomenon of induced emf can also be understood by the concept of generated area (lvt).

Hence induced emf $|\varepsilon| = \dfrac{d\phi}{dt} = Bvl$

- **Induced Current :** $I = \dfrac{\varepsilon}{R} = \dfrac{Bvl}{R}$

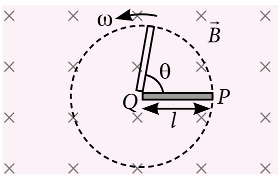
- **Magnetic Force :** $F_m = BIl = B\left(\frac{Bvl}{R}\right)l = \frac{B^2vl^2}{R}$
- **Power dissipated in moving the conductor :** $P_{mech} = P_{ext} = \frac{dW}{dt} = F_{ext}\cdot v = \frac{B^2vl^2}{R}\times v = \frac{B^2v^2l^2}{R}$
- **Electrical Power :** $P_{thermal} = \frac{H}{t} = I^2R = \left(\frac{Bvl}{R}\right)^2\cdot R$

  $\therefore \quad P_{thermal} = \frac{B^2v^2l^2}{R}$
- Motion of conductor rod in a vertical plane
  - Rod will achieve a constant maximum (terminal) velocity v_T if $F_m = mg$



  So,

  $\frac{B^2v_Tl^2}{R} = mg; v_T = \frac{mgR}{B^2l^2}$
- Motional emf due to rotational motion
  - emf induces across the ends of the rod is,



  $\varepsilon = \frac{1}{2}Bl^2\omega = Bl^2\pi\upsilon$

  $= \frac{Bl^2\pi}{T}$

Illustration 1 : A closed coil consists of 500 turns on a rectangular frame of area 4.0 cm^2 and has a resistance of 50 Ω. A uniform magnetic field of 0.2 Wb m^{-2} is perpendicular to its plane. Calculate the amount of charge flowing through the coil if it is rotated through 180°. Will the answer depend on the speed with which coil is rotated?

Soln.: Number of turns in coil, N = 500, Area of coil, A = 4.0 cm^2, Resistance of coil, R = 50 Ω, Intensity of magnetic field, B = 0.2 Wb m^{-2}, Angle of rotation, θ = 180°. Because magnetic field B is perpendicular to the plane of coil, hence magnetic flux linked with each turn of the coil area A, is given by, $\phi_1 = BA$.

When the coil is rotated through 180°, magnetic flux linked with it is given by, $\phi_2 = BA\cos 180° = -BA$

Hence, change in magnetic flux, $\phi = \phi_2 - \phi_1$

$= -BA - (+BA) = -2BA$

According to Faraday's second law, emf induced in a coil consisting of N turns is, $\varepsilon = -N\frac{d\phi}{dt} = \frac{2NBA}{dt}$

If R is the resistance of the coil, then current induced in the coil is $I = \frac{\varepsilon}{R} = \frac{2NBA}{Rdt}$

Hence charge passed through the coil in time dt is

$q = Idt = \frac{2NBA}{R} = \frac{2\times(500)\times 0.2\times(4\times 10^{-4})}{50}$

$= 16\times 10^{-4}$ C

Induced charge will remain same whether the coil is rotated slowly or rapidly because it depends upon the total change in magnetic flux but not the rate of change of magnetic flux.

Eddy Currents

- The currents induced in the body of a conductor, when the magnetic flux linked with the conductor changes are called eddy currents.
- The direction of the eddy currents in the conductor can be found by applying Lenz's law or Fleming's right hand rule.
- **Applications of eddy currents**
  - Induction furance is based on the heating effect of eddy currents.
  - Speedometer is a device used to measure the instantaneous speed of a vehicle.
  - Concept of eddy current is used in energy meter to record the consumption of electricity.

Inductance

- Inductance is analogous to inertia in mechanics, because inductance of an electrical circuit opposes any change of current in the circuit.
- Inductance is a scalar quantity. It has dimensions of [$ML^2T^{-2}A^{-2}$]. SI unit of inductance is henry.
- **Self Inductance**
  - When a current in a coil changes, it induces a back emf in the same coil. The self-induced emf is given by, $\varepsilon = -L\frac{dI}{dt}$, where L is the self-inductance of the coil.
  - $L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{\phi_{total}}{I}$

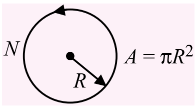
    It is a measure of the inertia of the coil against the change of current through it.
  - The self inductance of a long solenoid, the core of which consists of a magnetic material of permeability μ_r is

    $L = \frac{\mu_0\mu_r N^2A}{l} = \mu_r\mu_0 n^2 Al = \mu_0\mu_r n^2 V$

Here V = volume of solenoid = Al

- Self inductance of a planar coil of radius R

$$L=\frac{\mu_0 N^2 \pi R}{2}$$



- **Mutual inductance**
  - When an emf is produced in a coil because of change in current in a coupled coil, the effect is called mutual inductance.

$$M_{12}=\frac{N_1\phi_1}{I_2} \text{ and } M_{21}=\frac{N_2\phi_2}{I_1}$$

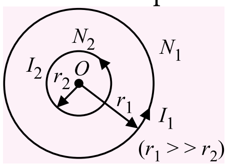
  - For same length and different number of turns per unit length of two solenoids mutual inductance is given by

$M_{12} = M_{21} = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l = M$

r_1 = radius of inner coil

- Mutual inductance of two concentric and coplanar coils $M_{C_1C_2}=\frac{N_2B_1A_2}{I_1}$

$$=\frac{\mu_0 N_1 N_2 \pi r_2^2}{2r_1}$$



- Magnetic energy stored in the inductor, $U_B=\frac{1}{2}LI^2$
- Magnetic energy per unit volume, $u_B=\frac{U_B}{V}=\frac{B^2}{2\mu_0}$

- **Combination of inductance**
  - **Series combination**

$L_S = L_1 + L_2$ (take M = 0). If $M \neq 0$ then $L_S = L_1 + L_2 \pm 2M$.

The plus sign occurs if windings in the two coils are in the same sense, while minus sign occurs if windings are in opposite sense.

  - **Parallel combination,** $\frac{1}{L_p}=\frac{1}{L_1}+\frac{1}{L_2}$

$\therefore \quad L_p=\frac{L_1L_2}{L_1+L_2}$ (take M = 0)

When $M \neq 0$ *i.e.*, they situated close to each other,

$$L_p=\frac{L_1L_2-M^2}{L_1+L_2\pm M}$$

Illustration 2 : Calculate the total energy U_B and the energy density u_B of the magnetic field stored in a solenoid 0.5 m long, having 5000 turns and a currrent 10 A. The radius of the solenoid is 4 cm.

Soln.: Here; l = 0.5 m, N = 5000, r = 4 cm = 4×10^{-2} m

I = 10 A

As $u_B=\frac{B^2}{2\mu_0}$ and $B=\mu_0 nI$, $u_B=\frac{(\mu_0 nI)^2}{2\mu_0}=\frac{1}{2}\mu_0 n^2 I^2$

$$=\frac{1}{2}(4\pi\times10^{-7}\text{ Wb A}^{-1}\text{m}^{-1})\left(\frac{5000}{0.5\text{ m}}\right)^2(10\text{A})^2$$

$$= 2\pi \times 10^3 \text{ J m}^{-3} = 6.28 \times 10^3 \text{ J m}^{-3}$$

Volume of the solenoid,

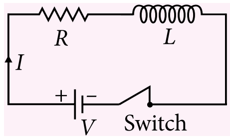
$V = lA = (0.5\text{ m})\pi(4\times10^{-2}\text{ m})^2 = 25\times10^{-4}\text{ m}^3$

Thus, $U_B = u_BV$

$$= (6.28\times10^3\text{ J m}^{-3})(25\times10^{-4}\text{ m}^3) = 15.7\text{ J}$$

Current Growth in *LR* Circuit

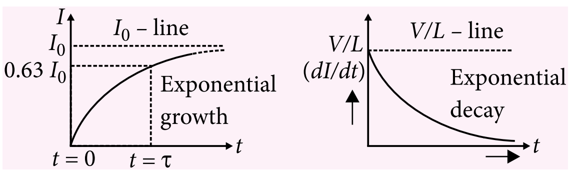
- **Emf equation :** $V=IR+L\frac{dI}{dt}$



- **Current at any instant :** When key is closed the current in circuit increases exponentially with respect to time. The current in circuit at any instant t is given by

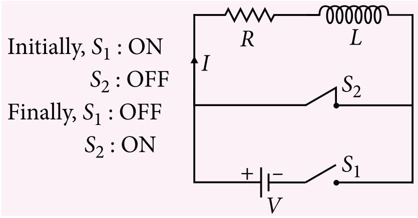
$$I = I_0(1 - e^{-t/\tau})$$

- Just after the closing of key, inductor behaves like open circuit and current in circuit is zero. Thus inductor provide infinite resistance.
- After some time closing of the key inductor behaves like short circuit and current in circuit is constant. Thus inductor provides zero resistance.
  - $I_0=\frac{V}{R}$ (maximum or peak value of current)
  - Peak value of current in circuit does not depend on self inductance of coil.
- **Time constant:** It is a time in which current increases up to 63% or 0.63 times of peak current value $\tau=\frac{L}{R}$



Current Decay in *LR* Circuit

- **Emf equation :** $IR + L\dfrac{dI}{dt} = 0$



- **Current at any instant :** Once current acquires its final maximum steady value, if suddenly required switching positions (S_1 and S_2) are interchanged then current starts decreasing exponentially with respect to time. The current in the circuit at any instant t is given by, $I = I_0\, e^{-t/\tau}$
  - Just after opening of key $t = 0 \Rightarrow I = I_0 = \dfrac{V}{R}$
  - Some time after opening of key $t \to \infty \Rightarrow I_0 \to 0$
- **Time constant (τ) :** It is a time in which current decreases up to 37% or 0.37 times of peak current value $\tau = \dfrac{L}{R}$

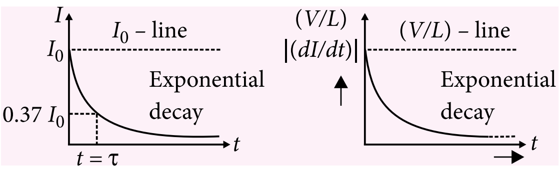
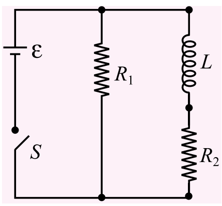


Illustration 3 : An inductor of inductance L = 400 mH and resistors of resistances R_1 = 2 Ω and R_2 = 2 Ω are connected to a battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at t = 0. Find the potential drop across L as a function of time.



Soln.: For the given R, L circuit the potential difference across $AD = V_{BC}$ as they are parallel.

$I_1 = \varepsilon/R_1$

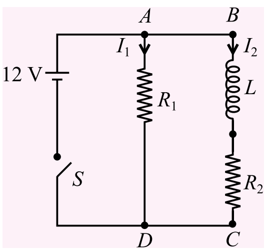
$I_2 = I_0(1 - e^{-t/\tau})$ where τ = mean life or L/R

$\tau = t_0$ (given)

$\varepsilon \text{ (across } BC) = L\dfrac{dI_2}{dt} + R_2 I_2$

$I_2 = I_0(1 - e^{-t/t_0})$

But $I_0 = \dfrac{\varepsilon}{R_2} = \dfrac{12}{2} = 6$ A



$$\tau = t_0 = \frac{L}{R} = \frac{400 \times 10^{-3}\ \text{H}}{2\ \Omega} = 0.2\ \text{s}$$

$\therefore\ I_2 = 6(1 - e^{-t/0.2})$

Potential drop across $L = \varepsilon - R_2 I_2$

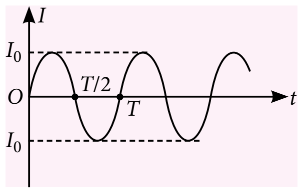
$= 12 - 2 \times 6(1 - e^{-t/0.2}) = 12e^{-t/0.2} = 12e^{-5t}$ V

AC Generator

- In an ac generator, mechanical energy is converted into electrical energy by virtue of electromagnetic induction.
- If the coil of N turns and area A is rotated at υ revolutions per second in a uniform magnetic field B, then the motional emf produced is
  $\varepsilon = NBA\,(2\pi\upsilon)\sin(2\pi\,\upsilon t) = NBA\omega \sin \omega t$
  where we have assumed that at time t = 0, the plane of coil is perpendicular to the field.

ALTERNATING CURRENT

An alternating current is the current whose value changes with time in both magnitude and direction. It is represented by $I = I_0 \sin(\omega t + \phi)$



where $I \to$ instantaneous current

- $I_0 \to$ peak value or maximum value of ac
- $\omega t + \phi \to$ phase at any time t
- $\phi \to$ initial phase or phase constant
- $\omega \to$ angular frequency (where $T = 2\pi/\omega$)

Mean or Average Value of ac Current

- The mean value of ac over half cycle is that value of dc which would send same amount of charge through a circuit as is sent by the the ac through same circuit in the same time.

$$I_{av}(t_2 - t_1) = \int_{t_1}^{t_2} I(t)dt \Rightarrow I_{av} = \frac{\int_{t_1}^{t_2} I(t)dt}{t_2 - t_1}$$

- Average value of $I = I_0 \sin \omega t$
  - **Over first half cycle :** for this $t_1 = 0$, $t_2 = T/2 = \pi/\omega$
    $\Rightarrow \quad I_{av} = \dfrac{2I_0}{\pi} = 0.637 I_0$
  - **Over full cycle :** for this $t_1 = 0$, $t_2 = T = 2\pi/\omega$
    $\Rightarrow \quad I_{av} = 0$

Root Mean Square (rms) Value of ac Current

- The root mean square of any current is defined as that value of steady current (constant), which would

BRAIN MAP CLASS XI — NEWTON'S LAWS OF MOTION

OHM'S LAW AND KIRCHHOFF'S RULE — BRAIN MAP CLASS XII

LAWS OF MOTION AND THEIR CONSEQUENCES

Problem Solving Strategies

- *Identify the unknown forces and accelerations.*
- *Draw FBD of bodies in the system.*
- *Resolve forces into their components.*
- *Apply $\Sigma \vec{F} = M\vec{a}$ in the direction of motion.*
- *Apply $\Sigma \vec{F} = 0$ in the direction of equilibrium.*
- *Write constraint relation if exists.*
- *Solve equations $\Sigma \vec{F} = M\vec{a}$ and $\Sigma \vec{F} = 0$.*

Newton's 1st Law

A body continues its state of rest or motion until unless an external force is acted on it.

- Inertia of rest
- Inertia of motion
- Inertia of direction

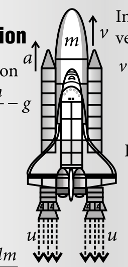
Pseudo Force

$\vec{F}_{ext} + \vec{F}_{Pseudo} = M\vec{a}$

$\vec{F}_{pseudo} = -M\vec{a}_{frame}$

For non-inertial frame of reference

Rocket Propulsion



Acceleration $a = \frac{u}{m}\frac{dm}{dt} - g$

Instantaneous velocity $v = u \log_e \left(\frac{m_0}{m_r}\right) - gt$

Burn out speed $v_{max} = u \log_e \left(\frac{m_0}{m_r}\right)$

Thrust $F = -u\frac{dm}{dt}$

Newton's 2nd Law

The rate of change of linear momentum of a body is directly proportional to the external force applied on the body in the direction of force.

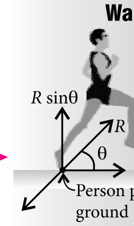
$F = \frac{dp}{dt} = ma$

Newton's 3rd Law

To every action there is always an equal and opposite reaction.

$\vec{F}_{AB} = -\vec{F}_{BA}$

Walking



Horse Cart Type System

For horse cart type system

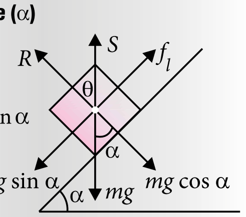
$a = \frac{F_x - f}{M_H + M_{cart}}$ $\begin{cases} F_x = \text{horizontal component of reaction force} \\ f = \text{frictional force} \end{cases}$

Angle of Friction (θ) and Angle of Repose (α)

$S = \sqrt{R^2 + f_l^2}$

$\tan\theta = \frac{f_l}{R} = \mu_s = \tan\alpha$

∴ Numerically, $\theta = \alpha$



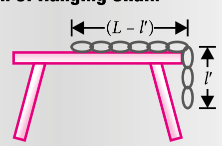
The motion resisted by a bonding between the body and the surface in contact represented by single force called

FRICTION

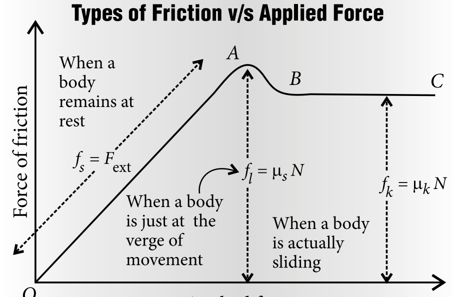
Maximum Length of Hanging Chain

Length of a chain hanging in air

$l' = \frac{\mu L}{1 + \mu}$

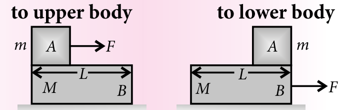


Types of Friction v/s Applied Force



Motion of Two Bodies One Resting on the Other

Force F applied to upper body | Force F applied to lower body



Force applied to upper body:

- **When there is no friction**
  - $a_A = F/m$; $a_B = 0$
  - A will fall from B after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{F}}$
- **Friction present between A and B ($F < f_l$)**
  - Combined system will move together with $a = F/(M + m)$
- **Friction present between A and B ($F > f_l$)**
  - Relative acceleration $a = a_A - a_B = \frac{MF - \mu_k mg(m+M)}{mM}$
  - A will fall from B after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mML}{MF - \mu_k mg(m+M)}}$

Force applied to lower body:

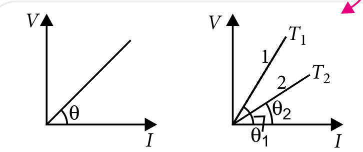
- **When there is no friction**
  - $a_B = F/M$ and $a_A = 0$
  - A will fall from B (backward) after time t
  - $\therefore t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F}}$
- **Friction present between A and B ($F < f_l$)**
  - Both the bodies will move together
  - $a = \frac{F}{M+m}$ and $f_l = \mu_s mg$
  - Pseudo force on the body A, $F' = ma = \frac{mF}{m+M}$
- **Friction present between A and B ($F > f_l$)**
  - Relative acceleration $a = a_A - a_B = -\left[\frac{F - \mu_k g(m+M)}{M}\right]$
  - A will fall from B (backward) after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F - \mu_k g(m+M)}}$

OHM'S LAW

If the physical conditions of the conductor (length, temperature, mechanical strain etc.) remain same, then the current flowing through the conductor is directly proportional to the potential difference across it's two ends *i.e.*, $I \propto V \Rightarrow V = IR$

Basic Features of Ohm's Law

- Vector form of Ohm's law, $\vec{J} = \sigma\vec{E}$ where conductivity $\sigma = \frac{1}{\rho}$ and $\vec{J}$ is the current density.
- Graph between V and I for a metallic conductor

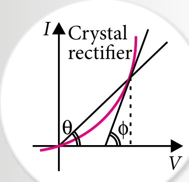


Slope of the line $= \tan\theta = \frac{V}{I} = R$

Here $\tan\theta_1 > \tan\theta_2$ so $R_1 > R_2$ *i.e.* $T_1 > T_2$

V-I curve for non-ohmic substance is not linear

- Static resistance $R_{st} = \frac{V}{I} = \frac{1}{\tan\theta}$
- Dynamic resistance $R_{dyn} = \frac{\Delta V}{\Delta I} = \frac{1}{\tan\phi}$

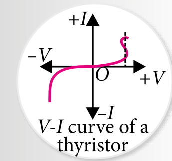


Limitations of Ohm's Law

It is not a universal law that applies everywhere under all conditions. Ohm's law is obeyed by metallic conductors, that too at about normal working temperatures.

Ohm's law is not followed in the following cases

- Materials : Crystal rectifiers, thermistors, thyristors, semi-conductors.
- Conditions :
  - (i) At very high temperatures
  - (ii) At very low temperatures
  - (iii) At very high potential differences.



R is a proportionality constant, known as

Resistance

The property of a substance by virtue of which it opposes the flow of current through it.

$R = \rho\frac{l}{A} = \frac{m}{ne^2\tau}\cdot\frac{l}{A}$

ρ is specific resistance of the material of conductor

Resistivity

It is numerically equal to the resistance of a substance having unit area of cross-section and unit length.

Temperature Dependence

For a conductor then $R_t = R_0(1 + \alpha t + \beta t^2)$

$R_t = R_0(1 + \alpha t)$ $(\beta \approx 0)$

also for resistivity, $\rho_t = \rho_0(1 + \alpha t)$

R_0 = resistance at 0°C, R_t = resistance at t°C, α, β = temperature co-efficients

KIRCHHOFF'S RULE

Junction Rule

At any junction of circuit, the sum of currents entering and leaving must be zero.

$\Sigma I = 0$.

It is based on conservation of charge.

Loop Rule

The algebraic sum of changes in potential around any closed loop must be zero.

$\Sigma\varepsilon - \Sigma IR = 0$

It is based on conservation of energy.

For circuit containing multiple batteries

Grouping of Batteries

Series grouping

For n identical batteries

$I = \frac{n\varepsilon}{nr + R}$ $\begin{cases} \varepsilon = \text{emf} \\ r = \text{Internal resistance} \end{cases}$

If polarity of m batteries is reversed

$I = (n - 2m)\varepsilon/(nr + R)$

Parallel grouping

- With identical batteries : $I = \frac{\varepsilon_{net}}{R_{net}}$, $\varepsilon_{net} = \varepsilon$, $R_{net} = \frac{r}{n} + R$
- With unidentical batteries : $\varepsilon_{net} = \frac{\Sigma(\varepsilon/r)}{\Sigma(1/r)}$, $I = \frac{\varepsilon_{net}}{R_{net}}$

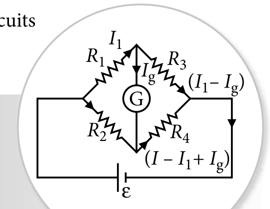
Mixed grouping

- For n rows of identical batteries with m cells in each row. Then, $\varepsilon_{net} = m\varepsilon$, $R_{net} = \frac{mr}{n} + R$, $I = \frac{\varepsilon_{net}}{R_{net}}$

An important application for few circuits

Wheatstone Bridge

- In balanced condition, If $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ then $I_g = 0$.



Guidelines to applying Kirchhoff's rule

Problem Solving Strategies

- *Distribute current at various junctions in the circuit starting from positive terminal.*
- *Pick a point and begin to walk around a closed loop.*
- *Write down the voltage change for that element according to the sign convention.*
- *By applying KVL, select the required number of loops as many as unknowns are available and apply KVL across each loop.*
- *Solve the set of simultaneous equation to find the unknowns.*

generate the same amount of heat in a given resistance in a given time as generated by actual current passing through the same resistance for the same given time.

$$I_{rms}^2 R(t_2 - t_1) = \int_{t_1}^{t_2} [I(t)]^2 R dt \Rightarrow I_{rms} = \sqrt{\frac{\int_{t_1}^{t_2} [I(t)]^2 dt}{t_2 - t_1}}$$

- Root mean square value of $I = I_0 \sin \omega t$
  - **Over first half cycle :** for this $t_1 = 0$, $t_2 = T/2 = \pi/\omega$

$$\Rightarrow \quad I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

  - **Over full cycle :** for this $t_1 = 0$, $t_2 = T = 2\pi/\omega$

$$\Rightarrow \quad I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

- **AC voltage applied to various circuits :**

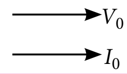
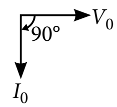
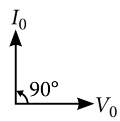
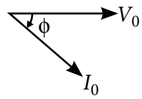
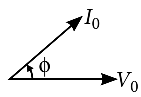
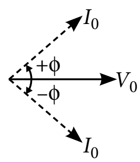
Different types of circuit	Variation of alternating voltage and current	Phase difference (ϕ)	Impendance (Z)	Peak current (I_0)	Phasor diagram
Purely resistive circuit	$V = V_0 \sin \omega t$ $I = I_0 \sin \omega t$	zero	R	$I_0 = V_0/R$	
Purely inductive circuit	$V = V_0 \sin \omega t$ $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$	Lags by $\pi/2$	X_L	$I_0 = V_0/X_L$	
Purely capacitative circuit	$V = V_0 \sin \omega t$ $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$	Leads by $\pi/2$	X_C	$I_0 = V_0/X_C$	
***R-L* circuit**	$V = V_0 \sin \omega t$ $I = I_0 \sin(\omega t - \phi)$	$\tan \phi = \frac{X_L}{R}$	$\sqrt{R^2 + X_L^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + X_L^2}}$	
***R-C* circuit**	$V = V_0 \sin \omega t$ $I = I_0 \sin(\omega t + \phi)$	$\tan \phi = \frac{X_C}{R}$	$\sqrt{R^2 + X_C^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + X_C^2}}$	
***LCR* circuit**	$V = V_0 \sin \omega t$ $I = I_0 \sin(\omega t \pm \phi)$	$\tan \phi = \frac{X_L - X_C}{R}$	$\sqrt{R^2 + (X_L - X_C)^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}$	

Illustration 4 : A 50 Hz ac signal is applied in a circuit of inductance of $(1/\pi)$ H and resistance 2100 Ω. Calculate the impedance offered by the circuit.

Soln.: Here, $L = \left(\frac{1}{\pi}\right)$ H, $\upsilon = 50$ Hz, $R = 2100\ \Omega$

$\therefore \quad \omega = 2\pi\upsilon = 2 \times \pi \times 50 = 100\pi$ rad s^{-1}

Since the given circuit is LR circuit

$\therefore \quad$ Impedance, $Z = \sqrt{R^2 + (\omega L)^2}$

$$Z = \sqrt{(2100)^2 + \left(100\pi \times \frac{1}{\pi}\right)^2}$$

$$= \sqrt{(2100)^2 + (100)^2} = \sqrt{4420000} = 2102\ \Omega$$

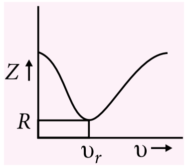
Resonance in Series *LCR* Circuit

- A circuit is said to be resonant when the natural frequency of the circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.
- At resonance,
  - $X_L = X_C$, $V_L = V_C$
  - $\phi = 0$ (Voltage and current are in same phase)
  - $Z_{min} = R$, $I_{max} = \frac{V}{R}$
- Resonance frequency

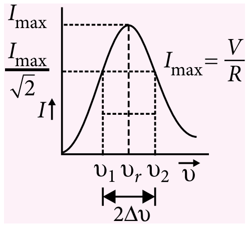
$$\omega_r = \frac{1}{\sqrt{LC}} \text{ or, } \upsilon_r = \frac{1}{2\pi\sqrt{LC}}$$

- Variation of Z with υ
  - If $\upsilon < \upsilon_r$ then $X_L < X_C$, circuit is capacitive, ϕ (negative).
  - At $\upsilon = \upsilon_r$, $X_L = X_C$, circuit is resistive, ϕ = zero.
  - If $\upsilon > \upsilon_r$ then $X_L > X_C$ circuit is inductive, ϕ (positive).

- Variation of I and Z with υ



As υ increases, Z first decreases then increases



As υ increases, I first increases then decreases.

Quality Factor (Q-factor) of Series Resonant Circuit

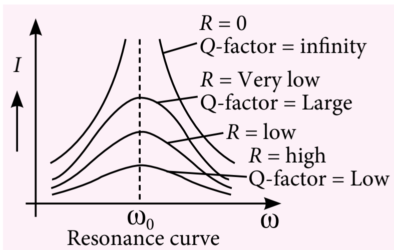
- The characteristic of a series resonant circuit is determined by the quality factor (Q-factor) of the circuit.
- It defines sharpness of I-V curve at resonance. When Q-factor is large, the sharpness of resonance curve is more and vice-versa.
- Q-factor is also defines as

$$Q\text{-factor} = 2\pi \times \frac{\text{Maximum energy stored per cycle}}{\text{Energy dissipation per cycle}}$$

$$= \frac{\text{Resonant frequency}}{\text{Band width}} = \frac{\upsilon_r}{\Delta\upsilon}$$

- $Q\text{-factor} = \dfrac{\omega_r L}{R}$ or $\dfrac{1}{\omega_r CR}$.

$$\Rightarrow \quad Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$



Resonance curve

Illustration 5 : An electrical device draws 2 kW power from ac mains has voltage 223 V (rms). The current differs (lags) in phase by ϕ as $\tan\phi = (-3/4)$ as compared to voltage. Find (a) R (b) $X_C - X_L$ (c) I

Soln.: Here, $P = 2\text{ kW} = 2000\text{ W}$, $V_{rms} = 223\text{ V}$, $\tan\phi = (-3/4)$

(a) As $P = \dfrac{V_{rms}^2}{Z}$, $Z = \dfrac{V_{rms}^2}{P} = 25\,\Omega$

$$\text{As } \tan\phi = \frac{X_C - X_L}{R} = (-3/4),\, X_C - X_L = (-3/4)R$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + [(-3/4)R]^2} = \frac{5}{4}R,$$

$$= 25\,\Omega = \frac{5}{4}R \quad \text{or} \quad R = 20\,\Omega$$

(b) As, $(X_C - X_L) = (-3/4)R = -15\,\Omega$

(c) As $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{223\text{ V}}{25\,\Omega} = 9\text{ A}$

$$\therefore\ I_0 = I_{rms}\sqrt{2} = 12.7\text{ A}$$

Power in AC Circuits

- In dc circuits power is given by $P = VI$. But in ac circuits, since there is some phase angle between voltage and current, power is defined as the product of voltage and that component of the current which is in phase with the voltage.
  Thus $P = VI\cos\phi$; where V and I are *rms* value of voltage and current.
  - **Instantaneous power :** Suppose in a circuit $V = V_0 \sin\omega t$ and $I = I_0 \sin(\omega t + \phi)$ then
    $P_{\text{instantaneous}} = VI = V_0 I_0 \sin\omega t \sin(\omega t + \phi)$
  - **Average power :** The average of instantaneous power in an ac circuit over a full cycle is called average power. It's unit is watt or W

$$P_{av} = V_{rms} I_{rms}\cos\phi = \frac{V_0}{\sqrt{2}}\cdot\frac{I_0}{\sqrt{2}}\cos\phi = \frac{1}{2}V_0 I_0\cos\phi.$$

  - **Apparent or virtual power :** The product of apparent voltage and apparent current in an electric circuit is called apparent power. This is always positive. $P_{app} = V_{rms} I_{rms} = \dfrac{V_0 I_0}{2}$.
- **Power Factor :** It may be defined as cosine of the angle of lag or lead (*i.e.* $\cos\phi$).

AC Circuits	Average power	Power factor
Purely resistive circuit	$V_{rms}I_{rms}$	1
Purely inductive circuit	zero	zero
Purely capacitative circuit	zero	zero
R-L circuit	$\dfrac{V_{rms}I_{rms}R}{\sqrt{R^2 + X_L^2}}$	$\dfrac{R}{\sqrt{R^2 + X_L^2}}$
R-C circuit	$\dfrac{V_{rms}I_{rms}R}{\sqrt{R^2 + X_C^2}}$	$\dfrac{R}{\sqrt{R^2 + X_C^2}}$
RLC circuit	$\dfrac{V_{rms}I_{rms}R}{\sqrt{R^2 + (X_L - X_C)^2}}$	$\dfrac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

Illustration 6 : Power in ac circuit contains an inductor 20 mH, a capacitor 100 μF, a resistor 50 Ω and an *A.C.* source of 12 V, 50 Hz. Find the energy dissipated in the circuit in 1000 s.

Soln.: Here, $V_{rms} = \dfrac{12\text{ V}}{\sqrt{2}}$

$$R = 50\,\Omega, X_L = \omega L = 2\pi\upsilon \cdot L = 2\pi(50\text{ Hz})(20\times10^{-3}\text{ H}) = 2\pi\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon\cdot C}$$

$$= \frac{1}{2\pi(50\text{ Hz})(100\times10^{-6}\text{ F})} = \left(\frac{100}{\pi}\right)\Omega$$

$$\Rightarrow \quad Z = \sqrt{R^2 + (X_L - X_C)^2} = 56\,\Omega$$

Energy used in 1000 s is $(P_{av})(t) = \left(\dfrac{V_{rms}^2 \cdot R}{Z^2}\right)\cdot t$

$$= \left(\frac{12^2\times50}{2\times56^2}\right) \times 1000 = 1.148\times10^3\text{ J}$$

LC Oscillation

- The oscillation of energy between capacitor (electric field energy) and inductor (magnetic field energy) is called *LC* oscillation.
- Frequency of oscillation $\upsilon = \dfrac{1}{2\pi\sqrt{LC}}$
- If charge varies sinusoidally with time *t* as $q = q_0\cos\omega t$ then current varies periodically with *t* as $I = \dfrac{dq}{dt} = q_0\omega\cos\left(\omega t + \dfrac{\pi}{2}\right)$
- If initial charge on the capacitor is q_0 then electrical energy stored in capacitor is $U_E = \dfrac{1}{2}\dfrac{q_0^2}{C}$
- If the capacitor is fully discharged, then total electrical energy is stored in the inductor in the form of magnetic energy.

  $U_B = \dfrac{1}{2}LI_0^2$ where I_0 = maximum current

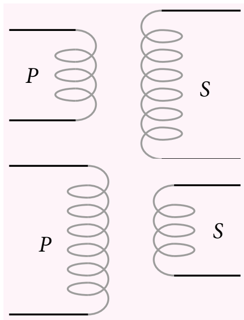
Transformer

- It is a device which raises or lowers the voltage in ac circuits through mutual induction.
- It works on ac only and never on dc.
- It can increase or decrease either voltage or current but not both simultaneously.
- Transformer does not change the frequency of input ac.
- The flux per turn of each coil must be same, *i.e.*, $\phi_S = \phi_P\,;\ -\dfrac{d\phi_S}{dt} = -\dfrac{d\phi_P}{dt}$.
- As in an ideal transformer there is no loss of power, *i.e.*, $P_{out} = P_{in}$ so $V_S I_S = V_P I_P$.

  Hence $\dfrac{N_S}{N_P} = \dfrac{V_S}{V_P} = \dfrac{I_P}{I_S} = k\,;\ k$ = Turn ratio
- For this the law of conservation of energy is held valid.
- **Step up transformer :** It increases voltage and decreases current.

  $V_S > V_P$, $N_S > N_P$, $I_S < I_P$, $R_S > R_P$, $k > 1$
- **Step down transformer :** It decreases voltage and increases current.

  $V_S < V_P$, $N_S < N_P$, $I_S > I_P$, $R_S < R_P$, $k < 1$



- **Efficiency of transformer (η) :** Efficiency is defined as the ratio of output power to input power

  i.e., $\eta\% = \dfrac{P_{out}}{P_{in}}\times100 = \dfrac{V_S I_S}{V_P I_P}\times100$

ELECTROMAGNETIC WAVES

Displacement Current

- Maxwell assumed that a current also flows in the gap between the two plates of a capacitor, during the process of charging, known as displacement current I_D. This displacement current originates due to time varying electric field between the plates of capacitor and is given by

  $$I_D = \varepsilon_0\frac{d\phi_\varepsilon}{dt}$$

  where ϕ_ε is the electric flux linked with the space between the two plates of the capacitor.
- Using the concept of displacement current I_D, Ampere's circuital law can be modified as

  $$\oint \vec{B}\cdot d\vec{l} = \mu_0(I_C + I_D)$$
- Maxwell's equations:
  - $\oint \vec{E}\cdot d\vec{A} = \dfrac{q}{\varepsilon_0}$ (Gauss's law for electricity)
  - $\oint \vec{B}\cdot d\vec{A} = 0$ (Gauss's law for magnetism)
  - $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\phi_B}{dt}$ (Faraday's law)
  - $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_C + \mu_0\varepsilon_0\dfrac{d\phi_E}{dt}$ (Ampere-Maxwell law)

Nature of Electromagnetic Waves

- Electric and magnetic fields oscillate sinusoidally in space and time in an electromagnetic wave. The oscillating electric and magnetic fields, $\vec{E}$ and $\vec{B}$ are perpendicular to each other, and to the direction of propagation of the electromagnetic wave.
- For a wave of frequency υ, wavelength λ and propagating along z-direction, we have

$$E = E_x(t) = E_0 \sin(kz - \omega t)$$

$$= E_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \upsilon t\right)\right] = E_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \frac{t}{T}\right)\right]$$

$$B = B_y(t) = B_0 \sin(kz - \omega t)$$

$$= B_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \upsilon t\right)\right] = B_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \frac{t}{T}\right)\right]$$

They are related by $E_0/B_0 = c$.

- The speed of light, in free space, is related to μ_0 and ε_0 (the free space permeability and permittivity constants) as : $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.
- The speed of light or an electromagnetic wave in a material medium is given by $v = \frac{1}{\sqrt{\mu\varepsilon}}$ where μ is the permeability of the medium and ε its permittivity.

Electromagnetic Spectrum

Different Types of Electromagnetic Waves			
Type	**Wavelength Range**	**Production**	**Detection**
Radio	> 0.1 m	Rapid acceleration and deceleration of electrons in aerials	Receiver's aerials
Microwave	0.1 m – 1 mm	Klystron valve or magnetron valve	Point contact diodes
Infrared	1 mm – 700 nm	Vibration of atoms and molecules	Thermopiles, bolometer, infrared photographic film
Light	700 nm – 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eye, photocells, photographic film
Ultraviolet	400 nm – 1 nm	The inner shell electrons in atoms moving from one energy level to a lower level	Photocells, photographic film
X-rays	1 nm – 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic film, Geiger tubes, ionization chamber
Gamma rays	< 10^{-3} nm	Radioactive decay of the nucleus	Photographic film, Geiger tubes, ionization chamber

Illustration 7 : In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 V m^{-1}.

(a) What is the wavelength of the wave?

(b) What is the amplitude of the oscillating magnetic field?

(c) Find the total average energy density of the electromagnetic field of the wave.

Soln.: Here, $E_0 = 48\text{ V m}^{-1}, \upsilon = 2.0 \times 10^{10}\text{ Hz}, c = 3 \times 10^8\text{ m s}^{-1}$

(a) Wavelength of the wave,

$$\lambda = \frac{c}{\upsilon} = \frac{3\times10^8\text{ m s}^{-1}}{2.0\times10^{10}\text{ s}^{-1}} = 1.5\times10^{-2}\text{ m}$$

(b) Amplitude of the oscillating magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{48\text{ V m}^{-1}}{3\times10^8\text{ m s}^{-1}} = 1.6\times10^{-7}\text{ T}$$

(c) Total average energy density,

$$u_{av} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2}(8.85\times10^{-12})(48)^2\text{ J m}^{-3} = 1.0\times10^{-8}\text{ J m}^{-3}$$

SPEED PRACTICE

1. A transformer (step up) with an 1 : 8 turn ratio has 60 Hz, 120 V across the primary. The load in the secondary is $10^4\ \Omega$. The current in the secondary is
   (a) 1.2 A (b) 0.96 A (c) 12 mA (d) 96 mA

2. A direct current of 5 A is superimposed on an alternating current $I = 10 \sin \omega t$ A flowing through a wire. The effective value of the resulting current will be
   (a) (15/2) A (b) $5\sqrt{3}$ A
   (c) $5\sqrt{5}$ A (d) 15 A

3. For L-R circuit, the time constant is equal to
   (a) twice the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.
   (b) the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.
   (c) half of the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.
   (d) square of the ratio of the energy stored in the magnetic field to the rate of dissipation of energy in the resistance.

4. An electromagnetic wave, going through vacuum is described by $E = E_0 \sin(kx - \omega t)$. Which of the following is independent of wavelength?
   (a) k/ω (b) $k\omega$
   (c) both (a) and (b) are correct
   (d) both (a) and (b) are wrong

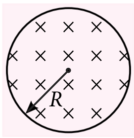
5. A long solenoid that has 800 loops per meter carries a current $I = 3 \sin(400t)$ A. Find the electric field inside the solenoid at a distance of 2 mm from the solenoid axis. Consider only the field tangential to a circle having its center on the axis of the solenoid.
   (a) Zero
   (b) $-960\ \mu_0 \cos(400\ t)$ V m^{-1}
   (c) $-480\ \mu_0 \cos(400\ t)$ V m^{-1}
   (d) $960\ \mu_0 \sin(400\ t)$ V m^{-1}

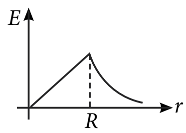
6. An AC source producing emf $V = V_0 [\sin \omega t + \sin 2\omega t]$ is connected in series with a capacitor and a resistor. The current found in the circuit is $I = I_1 \sin(\omega t + \phi_1) + I_2 \sin(2\omega t + \phi_2)$. Then
   (a) $I_1 = I_2$ (b) $I_1 < I_2$ (c) $I_1 > I_2$
   (d) I_1 may be less than, equal to or greater than I_2.

7. A 5 Ω coil, of 100 turns and diameter 6 cm, is placed between the poles of a magnet so that the flux is maximum through its area. When the coil is suddenly removed from the field of the magnet, a charge of 10^{-4} C flows through a 595 Ω galvanometer connected to the coil. The magnetic field between the poles of the magnet is
   (a) 106×10^{-2} T (b) 0.106 T
   (c) 212×10^{-2} T (d) 0.212 T

8. The current across the AC source is given as, $I = I_1 \cos \omega t + I_2 \sin \omega t$. The rms value of current is
   (a) $\dfrac{I_1 + I_2}{2}$ (b) $\dfrac{(I_1 + I_2)^2}{\sqrt{2}}$
   (c) $\sqrt{\dfrac{I_1^2 + I_2^2}{2}}$ (d) $\dfrac{I_1^2 + I_2^2}{2}$

9. A cylindrical space of radius R is filled with a uniform magnetic induction B parallel to the axis of the cylinder. If B changes at a constant rate, the graph showing the variation of induced electric field with distance r from the axis of cylinder is



(a)


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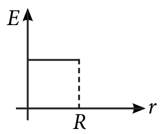
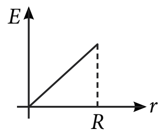
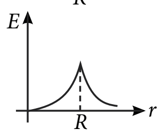
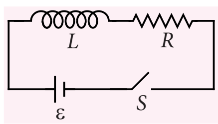


(c)


(d)


10. In the circuit shown in figure, switch S is closed at time $t = 0$. The charge that passes through the battery in one time constant is



   (a) $\dfrac{eR^2\varepsilon}{L}$ (b) $\varepsilon\left(\dfrac{L}{R}\right)$
   (c) $\dfrac{\varepsilon L}{eR^2}$ (d) $\dfrac{eL}{\varepsilon R}$

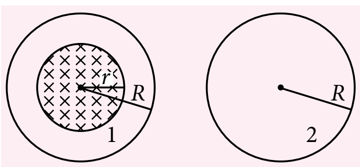
11. A plane electromagnetic wave of wave intensity 6 W m^{-2} strikes a small mirror of area 40 cm^2, held perpendicular to the approaching wave. The momentum transferred by the wave to the mirror will be
(a) 6.4×10^{-7} kg m s^{-1}
(b) 4.8×10^{-8} kg m s^{-1}
(c) 3.2×10^{-9} kg m s^{-1}
(d) 1.6×10^{-10} kg m s^{-1}

12. An electric field of 300 V m^{-1} is confined to a circular area 10 cm in diameter. If the electric field is increasing at the rate of 20 V m^{-1}s^{-1}, the magnitude of magnetic field at a point 15 cm from the centre of the circle will be
(a) 1.85×10^{-15} T (b) 3.70×10^{-16} T
(c) 0.85×10^{-17} T (d) 1.85×10^{-18} T

13. A long solenoid of diameter 0.1 m has 2×10^4 turns per meter. At the centre of the solenoid, a coil of 100 turns and radius 0.01 m is placed with its axis coinciding with the solenoid axis. The current in the solenoid reduces at a constant rate to 0 A from 4 A in 0.05 s. If the resistance of the coil is $10\,\pi^2\,\Omega$, the total charge flowing through the coil during this time is
(a) 16 μC (b) 32 μC (c) 16 π μC (d) 32 π μC
[NEET 2017]

14. In an electromagnetic wave in free space the root mean square value of the electric field is $E_{rms} = 6$ V m^{-1}. The peak value of the magnetic field is
(a) 2.83×10^{-8} T (b) 0.70×10^{-8} T
(c) 4.23×10^{-8} T (d) 1.41×10^{-8} T
[NEET 2017]

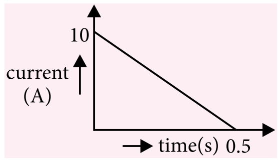
15. A uniform magnetic field is restricted within a region of radius r. The magnetic field changes with time at a rate $\frac{d\vec{B}}{dt}$. Loop 1 of radius $R > r$ encloses the region r and loop 2 of radius R is outside the region of magnetic field as shown in the figure. Then the emf generated is



(a) zero in loop 1 and zero in loop 2
(b) $-\frac{dB}{dt}\pi r^2$ in loop 1 and $-\frac{dB}{dt}\pi r^2$ in loop 2
(c) $-\frac{dB}{dt}\pi R^2$ in loop 1 and zero in loop 2
(d) $-\frac{dB}{dt}\pi r^2$ in loop 1 and zero in loop 2
[NEET Phase II 2016]

16. The potential differences across the resistance, capacitance and inductance are 80 V, 40 V and 100 V respectively in an L-C-R circuit. The power factor of this circuit is
(a) 0.4 (b) 0.5 (c) 0.8 (d) 1.0
[NEET Phase II 2016]

17. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is



(a) 200 Wb
(b) 225 Wb
(c) 250 Wb
(d) 275 Wb
[JEE Main Offline 2017]

18. A small circular loop of wire of radius a is located at the centre of a much larger circular wire loop of radius b. The two loops are in the same plane. The outer loop of radius b carries an alternating current $I = I_0\cos(\omega t)$. The emf induced in the smaller inner loop is nearly
(a) $\frac{\pi\mu_0 I_0}{2}\cdot\frac{a^2}{b}\,\omega\cos(\omega t)$
(b) $\frac{\pi\mu_0 I_0 b^2}{a}\,\omega\cos(\omega t)$
(c) $\frac{\pi\mu_0 I_0}{2}\cdot\frac{a^2}{b}\,\omega\sin(\omega t)$
(d) $\pi\mu_0 I_0\frac{a^2}{b}\,\omega\sin(\omega t)$ **[JEE Main Online 2017]**

19. A sinusoidal voltage of peak value 283 V and angular frequency 320 s^{-1} is applied to a series LCR circuit. Given that $R = 5\ \Omega$, $L = 25$ mH and $C = 1000$ μF. The total impedance, and phase difference between the voltage across the source and the current will respectively be
(a) $10\ \Omega$ and $\tan^{-1}\left(\frac{5}{3}\right)$ (b) $10\ \Omega$ and $\tan^{-1}\left(\frac{8}{3}\right)$
(c) $7\ \Omega$ and $\tan^{-1}\left(\frac{5}{3}\right)$ (d) $7\ \Omega$ and 45°
[JEE Main Online 2017]

20. Magnetic field in a plane electromagnetic wave is given by $\vec{B} = B_0 \sin(kx + \omega t)\hat{j}\,\text{T}$. Expression for corresponding electric field will be
(Where c is speed of light.)

(a) $\vec{E} = -B_0 c \sin(kx + \omega t)\hat{k}$ V m^{-1}

(b) $\vec{E} = B_0 c \sin(kx - \omega t)\hat{k}$ V m^{-1}

(c) $\vec{E} = \frac{B_0}{c} \sin(kx + \omega t)\hat{k}$ V m^{-1}

(d) $\vec{E} = B_0 c \sin(kx + \omega t)\hat{k}$ V m^{-1}

[JEE Main Online 2017]

SOLUTIONS

1. (d): In step up transformer,

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} = 8 \quad \therefore \quad V_S = 8\,V_P = 960\text{ V} = I_S R_S$$

or $\quad I_S = \frac{960}{R_S} = \frac{960}{10^4} = 96 \times 10^{-3}\text{ A} = 96\text{ mA}$

2. (b): Resulting current, $I = (5 + 10 \sin \omega t)$ A

$$I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (5 + 10\sin\omega t)^2 dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T (25 + 100\sin\omega t + 100\sin^2\omega t)\,dt\right]^{1/2}$$

But as $\frac{1}{T}\int_0^T \sin\omega t\, dt = 0$ and $\frac{1}{T}\int_0^T \sin^2\omega t\, dt = \frac{1}{2}$

so, $I_{eff} = \left[25 + \frac{1}{2}\times 100\right]^{1/2} = 5\sqrt{3}$ A

3. (a): In an L–R circuit,

Energy stored in magnetic field $= \frac{1}{2}LI^2$

Rate of dissipation of energy in the resistance $= I^2R$

$$\therefore \quad \frac{2\left(\frac{1}{2}LI^2\right)}{I^2R} = \frac{L}{R} = \tau_L$$

4. (a): The angular wave number, $k = \frac{2\pi}{\lambda}$, where λ is the wavelength. The angular frequency is $\omega = 2\pi\upsilon$

Ratio $\frac{k}{\omega} = \frac{2\pi/\lambda}{2\pi\upsilon} = \frac{1}{\upsilon\lambda} = \frac{1}{c}$ = constant

5. (b): As we know, $\oint E.dl = -\frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\frac{dB}{dt}$

The magnetic field of solenoid $= \mu_0 nI$
$= \mu_0 \times 800 \times 3 \sin(400t)$

Area, $A = \pi r^2 = \pi \times (2 \times 10^{-3})^2$ m^2

$$\therefore E = \frac{-\mu_0 \times 800 \times 3 \times 400\cos(400t) \times \pi \times (2\times 10^{-3})^2}{2\times\pi\times(2\times 10^{-3})}$$

$= -960\,\mu_0 \cos(400\,t)$ V m^{-1}

6. (b): $I_1 = \frac{V_0}{Z_1}$ and $I_2 = \frac{V_0}{Z_2}$

Here, $Z_1 = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$ and $Z_2 = \sqrt{R^2 + \left(\frac{1}{2\omega C}\right)^2}$

As $Z_2 < Z_1 \quad \therefore \quad I_1 < I_2$

7. (d): As the coil is removed, the flux changes from BA, where A is the coil area, to zero. Therefore,

$$|\varepsilon| = N\left|\frac{\Delta\phi}{\Delta t}\right| = N\frac{BA}{\Delta t} \quad \text{.....(i)}$$

Given, $\Delta q = 10^{-4}$ C. By Ohm's law,

$$|\varepsilon| = IR = \frac{\Delta q}{\Delta t}R \quad \text{.....(ii)}$$

where $R = 600\ \Omega$, the total resistance.
From eqns. (i) and (ii), we get

$$B = \frac{R\Delta q}{NA} = \frac{(600\ \Omega)(10^{-4}\text{ C})}{(100)(\pi)(9\times 10^{-4}\text{ m}^2)} = 0.212\text{ T}$$

8. (c): $I = I_1 \cos\omega t + I_2 \sin\omega t$

$$(I^2)_{mean} = \langle I_1^2\cos^2\omega t\rangle + \langle I_2^2\sin^2\omega t\rangle + 2I_1I_2\langle\cos\omega t\sin\omega t\rangle$$

$$= I_1^2 \times \frac{1}{2} + I_2^2 \times \frac{1}{2} + 2I_1I_2 \times 0$$

$$I_{rms} = \sqrt{(I^2)_{mean}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

9. (a): For $r \le R$

$$\oint \vec{E}\cdot\vec{dl} = \left|\frac{d\phi}{dt}\right| \quad \text{or} \quad E(2\pi r) = (\pi r^2)\frac{dB}{dt} \quad \text{or} \quad E \propto r$$

i.e., E-r graph is a straight line passing through origin.

For $r \ge R$

$$\oint \vec{E}\cdot\vec{dl} = \left|\frac{d\phi}{dt}\right| \quad \text{or} \quad E(2\pi r) = (\pi R^2)\left(\frac{dB}{dt}\right)$$

$$E \propto \frac{1}{r}$$

i.e., E-r graph is a rectangular hyperbola.

10. (c): The current at time t is given by
$I = I_0(1 - e^{-t/\tau})$

Here $I_0 = \frac{\varepsilon}{R}$ and $\tau = \frac{L}{R}$

$$\therefore \quad q = \int_0^\tau I dt = \int_0^\tau I_0(1 - e^{-t/\tau})dt$$

$$= \frac{I_0\tau}{e} = \frac{\left(\frac{\varepsilon}{R}\right)\left(\frac{L}{R}\right)}{e} = \frac{\varepsilon L}{eR^2}$$

11. (d): Momentum transferred to the mirror,

$$p = \frac{2U}{c} = \frac{2S_{av}A}{c} = \frac{2\times6\times40\times10^{-4}}{3\times10^8}$$
$= 1.6 \times 10^{-10}$ kg m s^{-1}

12. (d): As we know,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0(I_D + I_C) = \mu_0\left(\varepsilon_0\frac{d\phi}{dt} + 0\right)$$

$$\Rightarrow B\cdot 2\pi R = \mu_0\varepsilon_0\left(\frac{dE}{dt}\right)\left(\pi\frac{d^2}{4}\right)$$

$$\therefore\ B = \frac{\mu_0\varepsilon_0}{2\pi R}\left(\frac{\pi d^2}{4}\right)\frac{dE}{dt}$$

$$= \frac{2\times10^{-7}\times8.85\times10^{-12}\times3.14\times0.01\times20}{4\times0.15}$$

$= 1.85 \times 10^{-18}$ T

13. (b): Given $n = 2 \times 10^4$; $I = 4$ A ; Initially $I = 0$ A

$\therefore\ B_i = 0$ or $\phi_i = 0$

Finally, the magnetic field at the centre of the solenoid is given as

$B_f = \mu_0 nI = 4\pi \times 10^{-7} \times 2 \times 10^4 \times 4 = 32\pi \times 10^{-3}$ T

Final magnetic flux through the coil is given as

$\phi_f = NB_fA = 100 \times 32\,\pi \times 10^{-3} \times \pi \times (0.01)^2$

$\phi_f = 32\pi^2 \times 10^{-5}$ T m^2

Induced charge, $q = \frac{|\Delta\phi|}{R} = \frac{|\phi_f - \phi_i|}{R} = \frac{32\pi^2\times10^{-5}}{10\pi^2}$

$= 32 \times 10^{-6}$ C $= 32$ μC

14. (a): Given: $E_{rms} = 6$ V m^{-1}

$$\frac{E_{rms}}{B_{rms}} = c \text{ or } B_{rms} = \frac{E_{rms}}{c} = \frac{6}{3\times10^8} = 2\times10^{-8}\text{ T}$$

Since, $B_{rms} = \frac{B_0}{\sqrt{2}}$

where B_0 is the peak value of magnetic field.

$\therefore\ B_0 = B_{rms}\sqrt{2} = 2\times10^{-8}\times\sqrt{2}$ T

$B_0 \approx 2.83 \times 10^{-8}$ T

15. (d): Emf generated in loop 1,

$$\varepsilon_1 = -\frac{d\phi}{dt} = -\frac{d}{dt}(\vec{B}\cdot\vec{A}) = -\frac{d}{dt}(BA) = -A\times\frac{dB}{dt}$$

$$\varepsilon_1 = -\left(\pi r^2\frac{dB}{dt}\right)$$

($\because\ A = \pi r^2$ because $\frac{dB}{dt}$ is restricted upto radius r.)

Emf generated in loop 2,

$$\varepsilon_2 = -\frac{d}{dt}(BA) = -\frac{d}{dt}(0\times A) = 0$$

16. (c) : Here, $V_R = 80$ V, $V_C = 40$ V, $V_L = 100$ V

Power factor, $\cos\phi = \frac{R}{Z} = \frac{V_R}{V} = \frac{V_R}{\sqrt{V_R^2 + (V_L - V_C)^2}}$

$$= \frac{80}{\sqrt{(80)^2 + (100-40)^2}} = \frac{80}{100} = 0.8$$

17. (c) : We know, induced emf (ε) is $|\varepsilon| = \frac{d\phi}{dt}$; $IR = \frac{d\phi}{dt}$

Now, $d\phi = R\,Idt$ or $\int d\phi = R\int Idt$

$\therefore$ Change in magnetic flux $= R \times$ area under the current-time graph

$$\Delta\phi = R\times\frac{1}{2}\times10\times0.5 = 100\times\frac{1}{2}\times10\times0.5 = 250\text{ Wb}$$

18. (c) : The induced emf, $\varepsilon = -M\frac{dI}{dt}$ ...(i)

where mutual inductance M is given by, $M = \frac{\mu_0\pi a^2}{2b}$

The current is given by, $I = I_0\cos(\omega t)$

Putting these values in eqn. (i)

$$\varepsilon = \frac{-\mu_0\pi a^2}{2b}\frac{d}{dt}(I_0\cos(\omega t)) = \frac{\mu_0\pi a^2}{2b}I_0\omega\sin(\omega t)$$

$$= \frac{\pi\mu_0 I_0}{2}\frac{a^2}{b}\,\omega\sin(\omega t)$$

19. (d): Here, $\varepsilon_0 = 283$ V, $\omega = 320$ s^{-1}, $R = 5\ \Omega$,

$L = 25$ mH $= 25\times10^{-3}$ H, $C = 1000$ μF $= 10^{-3}$ F

$X_L = \omega L = 320 \times 25 \times 10^{-3} = 8\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{320\times10^{-3}} = \frac{1000}{320} = 3.125\ \Omega$$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{5^2 + (8-3.125)^2} \approx \sqrt{49} = 7\,\Omega$$

Required phase difference, $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

$$\phi = \tan^{-1}\left(\frac{4.875}{5}\right) \approx 45°$$

20. (d): Given : $\vec{B} = B_0\sin(kx + \omega t)\hat{j}$ T

The relation between electric and magnetic field is,

$c = \frac{E}{B}$ or $E = cB$

The electric field component is perpendicular to the direction of propagation and the direction of magnetic field. Therefore, the electric field component along z-axis is obtained as

$\vec{E} = cB_0\sin(kx + \omega t)\,\hat{k}$

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EXAM PREP 2018

CLASS XII

Useful for Medical/Engg. Entrance Exams

CHAPTERWISE MCQs FOR PRACTICE

ELECTROMAGNETIC INDUCTION

1. The magnitude of earth's magnetic field at a place is B_0 and the angle of dip is δ. A horizontal conductor of length l, lying north-south, moves eastwards with a velocity v. The emf induced across the rod is
   (a) zero (b) B_0lv
   (c) $B_0lv\sin\delta$ (d) $B_0lv\cos\delta$

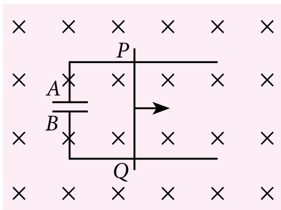
2. A magnetic flux through a stationary loop with a resistance R varies during the time interval τ as $\phi = at(\tau - t)$. What is the amount of heat generated in the loop during that time ?
   (a) $\frac{a^2\tau^3}{4R}$ (b) $\frac{a^2\tau^3}{3R}$ (c) $\frac{a^2\tau^3}{6R}$ (d) $\frac{a^2\tau^3}{2R}$

3. Faraday's laws are consequence of conservation of
   (a) energy
   (b) energy and magnetic field
   (c) charge
   (d) magnetic field

4. A small square loop of wire of side l is placed inside a large square loop of wire of side L ($>> l$). The loops are coplanar and their centres coincide. What is the mutual inductance of the system?
   (a) $2\sqrt{2}\,\frac{\mu_0}{\pi}\frac{l^2}{L}$ (b) $8\sqrt{2}\,\frac{\mu_0}{\pi}\frac{l^2}{L}$
   (c) $2\sqrt{2}\,\frac{\mu_0}{2\pi}\frac{l^2}{L}$ (d) $2\sqrt{2}\,\frac{\mu_0 L^2}{\pi l}$

5. A 50 Hz ac current of peak value 2 A flows through one of the pair of coils. If the mutual inductance between the pair of coils is 150 mH, then the peak value of voltage induced in the second coil is
   (a) 30π V (b) 60π V
   (c) 15π V (d) 300π V

6. A conducting rod PQ of length $L = 1.0$ m is moving with a uniform speed $v = 2.0$ m s^{-1} in a uniform magnetic field $\vec{B} = 4.0$ T directed into the paper. A capacitor of capacity $C = 10$ μF is connected as shown in figure. Then



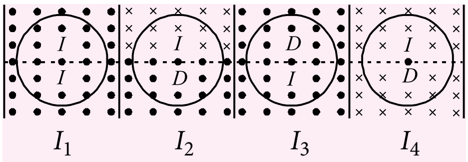
   (a) $q_A = +80$ μC and $q_B = -80$ μC.
   (b) $q_A = -80$ μC and $q_B = +80$ μC.
   (c) $q_A = 0 = q_B$.
   (d) charge stored in the capacitor increases exponentially with time.

7. The mutual inductance between two planar concentric rings of radii r_1 and r_2 (with $r_1 >> r_2$) placed in air is given by
   (a) $\frac{\mu_0\pi r_2^2}{2r_1}$ (b) $\frac{\mu_0\pi r_1^2}{2r_2}$
   (c) $\frac{\mu_0\pi(r_1+r_2)^2}{2r_1}$ (d) $\frac{\mu_0\pi(r_1+r_2)^2}{2r_2}$

8. A uniform magnetic field B points vertically up and is slowly changed in magnitude, but not in direction. The rate of change of the magnetic field is α. A conducting ring of radius r and resistance R is held perpendicular to the magnetic field, and is totally inside it. The induced current in the ring is

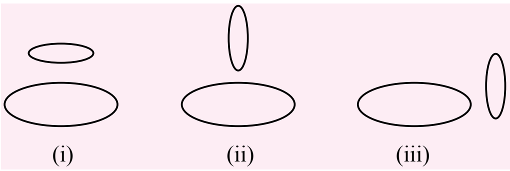
(a) zero (b) $\frac{2\pi rB}{R}$ (c) $\frac{r\alpha}{R}$ (d) $\frac{\pi r^2\alpha}{R}$

9. Four identical circular conducting loops are placed in uniform magnetic fields that are either increasing (I) or decreasing (D) in magnitude at identical rates as shown in figure. Arrange the magnitude of the currents induced in the loops.



(a) $I_1 > I_2 > I_3 > I_4$ (b) $I_1 = I_2 > I_3 > I_4$
(c) $I_1 > I_2 > I_3 = I_4$ (d) $I_1 = I_2 > I_3 = I_4$

10. Two circular coils can be arranged in any of the three situation as shown in the figure. Their mutual inductance will be

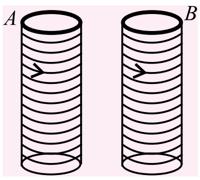


(a) maximum in situation (i)
(b) maximum in situation (ii)
(c) maximum in situation (iii)
(d) the same in all situations.

11. A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the emf induced across the ends of the rod is

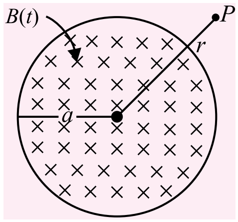
(a) $\frac{5B\omega l^2}{2}$ (b) $\frac{2B\omega l^2}{2}$ (c) $\frac{3B\omega l^2}{2}$ (d) $\frac{4B\omega l^2}{2}$

12. Two metallic rings A and B are identical in shape and size but have different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation between their resistivities and their masses m_A and m_B is



(a) $\rho_A > \rho_B$ and $m_A = m_B$
(b) $\rho_A < \rho_B$ and $m_A = m_B$
(c) $\rho_A > \rho_B$ and $m_A > m_B$
(d) $\rho_A = \rho_B$ and $m_A = m_B$

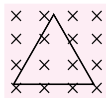
13. A uniform but time varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper, as shown. The magnitude of the induced electric field at point P at a distance r from the centre of the circular region



(a) is zero (b) decreases as $1/r$
(c) increases as r (d) decreases as $1/r^2$

14. Eddy currents are produced when
(a) a metal is kept in varying magnetic field
(b) a metal is kept in steady magnetic field
(c) a circular coil is placed in a magnetic field
(d) through a circular coil, current is passed

15. An equilateral triangular loop having resistance R and length of each side l is placed in a magnetic field which is varying at $\frac{dB}{dt} = 1\ \text{T s}^{-1}$. The induced current in the loop will be



(a) $\frac{\sqrt{3}}{4}\frac{l^2}{R}$ (b) $\frac{4}{\sqrt{3}}\frac{l^2}{R}$

(c) $\frac{\sqrt{3}}{4}\frac{R}{l^2}$ (d) $\frac{4}{\sqrt{3}}\frac{R}{l^2}$

ALTERNATING CURRENT

16. An ac source is of $\frac{200}{\sqrt{2}}$ V, 50 Hz. The value of voltage after $\frac{1}{600}$ s from the start is

(At time $t = 0$, amplitude of ac source is zero.)

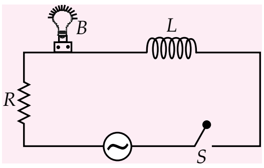
(a) 200 V (b) $\frac{200}{\sqrt{2}}$ V
(c) 100 V (d) 50 V

17. 120 V ac voltage is applied to 10 Ω resistance. The peak voltage across the resistor is
(a) 120 V (b) $120\sqrt{2}$ V
(c) $\frac{120}{\sqrt{2}}$ V (d) none of these

18. When a dc voltage of 200 V is applied to a coil of self inductance $(2\sqrt{3}/\pi)$ H, a current of 1 A flows through it. But by replacing dc source with ac source of 200 V, the current in the coil is reduced to 0.5 A. Then the frequency of ac supply is

(a) 100 Hz (b) 75 Hz (c) 60 Hz (d) 50 Hz

19. Switch S is closed at t = 0. After sufficiently long time, an iron rod is inserted into the inductor L. Then, the light bulb



(a) glows more brightly
(b) gets dimmer
(c) glows with the same brightness
(d) gets momentarily dimmer and then glows more brightly

20. A capacitor and a coil in series are connected to a 6 V ac source. By varying the frequency of the source, maximum current of 600 mA is observed. If the same coil is now connected to a cell of emf 6 V and internal resistance of 2 Ω, the current through it will be

(a) 0.5 A (b) 0.6 A (c) 1.0 A (d) 2.0 A

21. A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. When an alternating emf of 220 V at 50 cycles per second, is applied to it, then wattless component of current in the circuit is

(a) 7 A (b) 5 A (c) 0.7 A (d) 0.5 A

22. A transformer with efficiency 80% works at 4 kW and 100 V. If the secondary voltage is 200 V, then the primary and secondary currents are respectively

(a) 40 A, 16 A (b) 16 A, 40 A
(c) 20 A, 40 A (d) 40 A, 20 A

23. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5 V. 2 s after the connection is made, the current flowing in ampere in the circuit is

(a) $(1 - e^{-1})$ (b) $(1 - e)$
(c) e (d) e^{-1}

24. In an a.c. circuit the voltage applied is $E = E_0 \sin\omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by

(a) $P = \sqrt{2}E_0 I_0$ (b) $P = \frac{E_0 I_0}{\sqrt{2}}$
(c) P = zero (d) $P = \frac{E_0 I_0}{2}$

25. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is

(a) $\pi\sqrt{LC}$ (b) $\frac{\pi}{4}\sqrt{LC}$ (c) $2\pi\sqrt{LC}$ (d) $\sqrt{LC}$

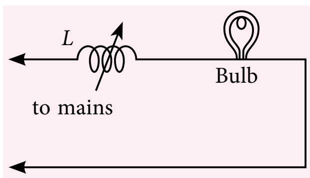
26. Current in an ac circuit is given by
$I = 2\sqrt{2}\sin(\pi t + \pi/4)$ A, then the average value of current during time t = 0 to t = 1 s is

(a) 0 (b) $\frac{4}{\pi}$ A (c) $\frac{4\sqrt{2}}{\pi}$ A (d) $2\sqrt{2}$ A

27. A capacitor and an inductance coil are connected in separate ac circuits with a bulb glowing in both the circuits. The bulb glows more brightly when

(a) an iron rod is introduced into the inductance coil
(b) the number of turns in the inductance coil is increased
(c) separation between the plates of the capacitor is increased.
(d) a dielectric is introduced into the gap between the plates of the capacitor.

28. A typical light dimmer used to dim the stage lights in a theatre consists of a variable induction for L (where inductance is adjustable between zero and L_{max}) connected in series with a light bulb B as shown in figure. The mains electrical supply is 220 V at 50 Hz, the light bulb is rated at 220 V, 1100 W. What L_{max} is required if the rate of energy dissipated in the light bulb is to be varied by a factor of 5 from its upper limit of 1100 W?



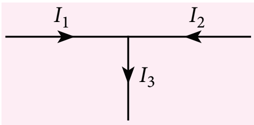
(a) 0.69 H (b) 0.28 H (c) 0.38 H (d) 0.56 H

29. If a direct current of value a ampere is superimposed on an alternative current $I = b\sin\omega t$ A flowing through a wire, what is the effective value of the resulting current in the circuit?

(a) $\left[a^2 - \frac{1}{2}b^2\right]^{1/2}$ (b) $[a^2 + b^2]^{1/2}$

(c) $\left[\frac{a^2}{2} + b^2\right]^{1/2}$ (d) $\left[a^2 + \frac{1}{2}b^2\right]^{1/2}$

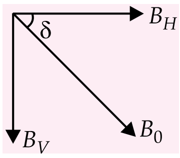
30. If $I_1 = 3 \sin \omega t$ and $I_2 = 4 \cos \omega t$, then I_3 is



(a) $5 \sin(\omega t + 53°)$ (b) $5 \sin(\omega t + 37°)$
(c) $5 \sin(\omega t + 45°)$ (d) $5 \sin(\omega t + 63°)$

SOLUTIONS

1. **(c) :** The vertical component of the earth's magnetic field is $B_V = B_0 \sin\delta$. A conductor lying north-south moving horizontally is normal to B_V. The emf induced is $\varepsilon = B_0 l v \sin\delta$.



2. **(b):** The flux through the stationary loop in the problem is $\phi = at(\tau - t)$

Induced emf,

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}[at(\tau - t)] = -[a\tau - 2at] = (2at - a\tau)$$

The amount of heat generated in the loop during a small time interval dt is

$$dQ = \frac{\varepsilon^2}{R}dt = \frac{(2at - a\tau)^2}{R}dt$$

Hence, the total heat generated is

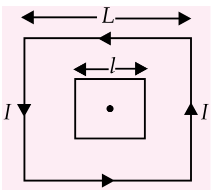
$$Q = \int_0^\tau \frac{(2at - a\tau)^2}{R}dt$$

$$= \frac{1}{R}\int_0^\tau (4a^2t^2 + a^2\tau^2 - 4a^2\tau t)dt$$

$$= \frac{1}{R}\left[\frac{4}{3}a^2t^3 + a^2\tau^2 t - \frac{4}{2}a^2\tau t^2\right]_0^\tau = \frac{1}{3}\frac{a^2\tau^3}{R}$$

3. **(a)**

4. **(a) :** Let the current I be flowing in the larger loop.



The larger loop is made up of four wires each of length L, the field at the centre *i.e.*, at a distance $\frac{L}{2}$ from each wire, will be

$$B = 4 \times \frac{\mu_0 I}{4\pi(L/2)}(\sin 45° + \sin 45°)$$

$$= 4 \times \frac{\mu_0}{4\pi}\frac{2I}{L}\frac{2}{\sqrt{2}} = 2\sqrt{2}\frac{\mu_0}{\pi}\frac{I}{L}$$

Flux linked with smaller loop,

$$\phi_2 = BA_2 = 2\sqrt{2}\frac{\mu_0}{\pi}\frac{I}{L} \times l^2$$

Hence, $M = \frac{\phi_2}{I} \Rightarrow M = 2\sqrt{2}\frac{\mu_0}{\pi}\frac{l^2}{L}$

5. **(a) :** The current flows through the coil 1 is

$I_1 = I_0 \sin\omega t$

where I_0 is the peak value of current.

Magnetic flux linked with the coil 2 is

$\phi_2 = MI_1 = MI_0 \sin\omega t$

where M is the mutual inductance between the two coils.

The magnitude of induced emf in coil 2 is

$$|\varepsilon_2| = \frac{d\phi_2}{dt} = \frac{d}{dt}(MI_0 \sin\omega t) = MI_0\omega\cos\omega t$$

$\therefore$ Peak value of voltage induced in the coil 2 is $= MI_0\omega$

$= 150 \times 10^{-3} \times 2 \times 2\pi \times 50 = 30\pi$ V

6. **(a) :** $q = CV = C(Bvl) = 80\ \mu C$ = constant

Magnetic force on electrons of the conducing rod PQ is towards Q.

Therefore, A is positively charged and B is negatively charged.

7. **(a) :** Magnetic field due to the larger coil at its centre is $B = \frac{\mu_0 I}{2r_1}$

Flux through the inner coil is $\phi = B \times \pi r_2^2$

$$= \frac{\mu_0 I}{2r_1} \times \pi r_2^2$$

But $\phi = MI$.

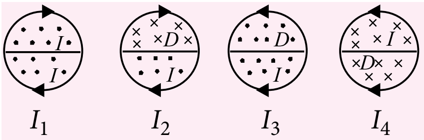
Therefore, $M = \frac{\mu_0 \pi r_2^2}{2r_1}$

8. **(d):** The magnitude of the induced emf in the ring is

$$|\varepsilon| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\frac{dB}{dt} = \pi r^2\alpha$$

The induced current in the ring, $I = \frac{|\varepsilon|}{R} = \frac{\pi r^2\alpha}{R}$

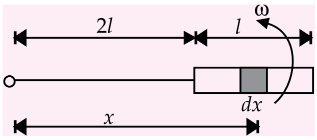
9. (d): For B coming out of the plane of paper, if B is increasing, current is clockwise. $\Delta\phi > 0$, ε opposes (negative). As B is decreasing, current is anticlockwise.
For crosses, *i.e.* B downwards, the currents are opposite.



$\therefore \quad I_1 = I_2 > I_3 = I_4$.

10. (a): Flux linkage is maximum in the arrangement shown in diagram (i). Therefore mutual inductance will be maximum in case (i).

11. (a): Consider a element of length dx at a distance x from the fixed end of the string.



emf induced in the element is $d\varepsilon = B(\omega x)dx$
Hence, the emf induced across the ends of the rod is

$$\varepsilon = \int_{2l}^{3l} B\omega x dx = B\omega\left[\frac{x^2}{2}\right]_{2l}^{3l} = \frac{B\omega}{2}[(3l)^2 - (2l)^2]$$
$$= \frac{5B\omega l^2}{2}$$

12. (b): The magnetic flux is changing at the same rate for both the solenoids.

The induced emf, $\varepsilon = -\frac{d\phi}{dt}$ is the same for both the rings. However, the resistance of A has to be lower, if the masses of the rings A and B are the same, because the repulsion is more for ring A.

13. (b): Magnitude of induced electric field at point P

$$E = \frac{a^2}{2r} \times \frac{dB}{dt}$$

The magnitude of the electric field increases linearly from zero at the centre to $(a/2)(dB/dt)$ at the edge of circular region of radius a and then, decreases inversely with distance.

14. (a): Eddy currents are produced when a metal is kept in a varying magnetic field.

15. (a): $\phi = \frac{\sqrt{3}}{4} l^2 B; \varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{\sqrt{3}}{4} l^2 \frac{dB}{dt}$

Induced current in the loop, $I = \frac{\varepsilon}{R} = \frac{\sqrt{3}l^2}{4R}$

16. (c): $V_{rms} = \frac{200}{\sqrt{2}}$ V

$$V_0 = \sqrt{2}\, V_{rms} = \sqrt{2} \times \frac{200}{\sqrt{2}} = 200 \text{ V}$$
$$V = V_0 \sin 2\pi\upsilon t = 200 \sin\left(2\pi \times 50 \times \frac{1}{600}\right)$$
$$= 200 \sin\frac{\pi}{6} = 100 \text{ V}$$

17. (b): Here, V_{rms} = 120 V ac

Peak voltage, $V_0 = V_{rms}\sqrt{2} = 120\sqrt{2}$ V

18. (d): Resistance of coil, $R = \frac{200 \text{ V}}{1 \text{ A}} = 200\ \Omega$

With ac source, $I = \frac{200}{\sqrt{R^2 + X_L^2}}$

or $\quad 0.5 = \frac{200}{\sqrt{R^2 + X_L^2}}$

or $\quad R^2 + (2\pi\upsilon L)^2 = (400)^2$

or $\quad \left(2\pi\upsilon \times \frac{2\sqrt{3}}{\pi}\right)^2 = (400)^2 - (200)^2 = 200 \times 600$

or $\quad 4\sqrt{3}\upsilon = 2\sqrt{3} \times 100 \quad$ or $\quad \upsilon = 50$ Hz

19. (b): As the rod is inserted, inductance increases and hence the voltage across inductor increases. This causes a drop in the voltage across the bulb and hence it gets dimmer.

20. (a): The maximum current is obtained at resonance where the net impedance is only resistive which is the resistance of the coil only. This gives the resistance of the coil as 10 Ω. Now, this coil along with the internal resistance of the cell gives a current of 0.5 A.

21. (d): Here, $X_L = \omega L = 2\pi\upsilon L = 2\pi \times 50 \times 0.7 = 220\ \Omega$
$R = 220\ \Omega$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{220^2 + 220^2} = 220\sqrt{2}\ \Omega$$
$$I_{rms} = \frac{V_{rms}}{Z} = \frac{220 \text{ V}}{220\sqrt{2}\ \Omega} = \frac{1}{\sqrt{2}} \text{ A}$$
$$\sin\phi = \frac{X_L}{Z} = \frac{220}{220\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Wattless current = $I_{rms} \sin\phi$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2} \text{ A} = 0.5 \text{ A}$$

22. (a): Efficiency of a transformer

$$= \frac{\text{Output power } (P_O)}{\text{Input power}(P_I)} = \frac{V_S I_S}{V_P I_P}$$

$$\therefore \quad \frac{80}{100} = \frac{P_O}{4\times10^3} \Rightarrow P_O = \frac{16}{5}\times10^3 \text{ W} = 3200 \text{ W}$$

Now, $I_S = \frac{P_O}{V_S} = \frac{3200}{200} = 16 \text{ A}$

Also, $P_I = I_P V_P$

or $I_P = \frac{P_I}{V_P} = \frac{4\times10^3 \text{ W}}{100 \text{ V}} = 40 \text{ A}$

23. (a): During the growth of current in LR circuit is given by

$$I = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t}) = \frac{5}{5}(1 - e^{-\frac{5}{10}\times2}) \quad I = (1 - e^{-1})$$

24. (c): Given : $E = E_0 \sin\omega t,\ I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

Since the phase difference between voltage and current is $\frac{\pi}{2}$.

Power consumption $= E_{rms} I_{rms} \cos\phi = 0$

25. (b): Charge on the capacitor at any instant t is

$q = q_0 \cos\omega t$ ...(i)

Energy of a capacitor $= \frac{1}{2} \times$ total energy

$$\frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{q_0^2}{C}\right) \Rightarrow q = \frac{q_0}{\sqrt{2}}$$

$$\therefore \quad \frac{q_0}{\sqrt{2}} = q_0 \cos\omega t\ ;\ \cos\omega t = \frac{1}{\sqrt{2}} \text{ or } \omega t = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} \qquad \left(\because \ \omega = \frac{1}{\sqrt{LC}}\right)$$

26. (b): Average value of current,

$$I_{av} = \frac{\int_0^1 I\,dt}{1} = 2\sqrt{2}\int_0^1 \sin\left(\pi t + \frac{\pi}{4}\right)dt = \frac{4}{\pi}\text{A}$$

27. (d)

28. (b): Resistance of bulb : $R = \frac{V_0^2}{P_0} = \frac{(220)^2}{1100} = 44\ \Omega$

When L is maximum, power consumed will be minimum which is $P = \frac{1100}{5} = 220$ W

Now $P = I_v^2 R\ ;\ 220 = I_v^2 \times 44$

$\Rightarrow \quad I_v = \sqrt{5}$ A

$$I_v = \frac{E_v}{Z}\ ;\ \sqrt{5} = \frac{220}{\sqrt{44^2 + X_L^2}}$$

$X_L = 88\ \Omega$

$2\pi\upsilon L_{max} = 88\ ;\ 2 \times \frac{22}{7} \times 50\ L_{max} = 88$

$\therefore \quad L_{max} = 0.28$ H

29. (d): As current at any instant in the circuit will be $I = I_{dc} + I_{ac} = a + b \sin\omega t$

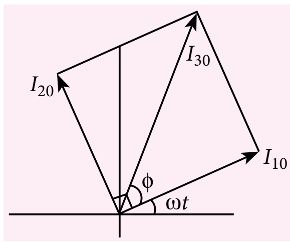
$$\text{so, } I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{1/2} = \left[\frac{1}{T}\int_0^T (a + b\sin\omega t)^2 dt\right]^{1/2}$$

$$I_{eff} = \left[\frac{1}{T}\int_0^T (a^2 + 2ab\ \sin\omega t + b^2 \sin^2 \omega t)dt\right]^{1/2}$$

but as $\frac{1}{T}\int_0^T \sin\omega t\ dt = 0$ and $\frac{1}{T}\int_0^T \sin^2\ \omega t\ dt = \frac{1}{2}$

$$\text{So, } I_{eff} = \left[a^2 + \frac{1}{2}b^2\right]^{1/2}$$

30. (a): $I_3 = I_1 + I_2 = 3 \sin\omega t + 4 \cos\omega t$
$= 3 \sin\omega t + 4 \sin(\omega t + 90°)$



$$I_{30} = \sqrt{I_{10}^2 + I_{20}^2} = \sqrt{3^2 + 4^2} = 5$$

$\tan\phi = \frac{4}{3} \quad \Rightarrow \quad \phi = 53°$

So, $I_3 = I_{30} \sin(\omega t + \phi) = 5 \sin(\omega t + 53°)$

MPP-6 CLASS XI ANSWER KEY

1.	(d)	2.	(a)	3.	(d)	4.	(c)	5.	(a)
6.	(b)	7.	(a)	8.	(a)	9.	(a)	10.	(a)
11.	(d)	12.	(b)	13.	(a)	14.	(c)	15.	(c)
16.	(a)	17.	(b)	18.	(d)	19.	(b)	20.	(a, d)
21.	(a, b)	22.	(a, c)	23.	(b, d)	24.	(2)	25.	(4)
26.	(1)	27.	(d)	28.	(b)	29.	(b)	30.	(c)

ACE YOUR WAY CBSE XII

Series 5

CHAPTERWISE PRACTICE PAPER
Electromagnetic Waves and Optics

Time Allowed : 3 hours | Maximum Marks : 70

GENERAL INSTRUCTIONS

(i) All questions are compulsory.
(ii) Q. no. 1 to 5 are very short answer questions and carry 1 mark each.
(iii) Q. no. 6 to 10 are short answer questions and carry 2 marks each.
(iv) Q. no. 11 to 22 are also short answer questions and carry 3 marks each.
(v) Q. no. 23 is a value based question and carries 4 marks.
(vi) Q. no. 24 to 26 are long answer questions and carry 5 marks each.
(vii) Use log tables if necessary, use of calculators is not allowed.

SECTION - A

1. What does an electromagnetic wave consist of? On what factors does its velocity in vacuum depend?

2. What feature of electromagnetic waves led Maxwell to conclude that light itself is an electromagnetic wave?

3. Do the frequency and wavelength change when light passes from a rarer to a denser medium?

4. What is the shape of the wavefront on earth for sunlight?

5. State Rayleigh's law of scattering.

SECTION - B

6. Explain why does a convex lens behaves as a converging lens when immersed in water ($\mu = 1.33$) and as a diverging lens, when immersed in carbon disulphide ($\mu = 1.6$).

7. Give four basic properties of electromagnetic waves.

8. A plane electromagnetic wave travels in vacuum along *z*-axis. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?

9. A magician during a show makes a glass lens with $\mu = 1.47$ disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

OR

A polaroid (I) is placed in front of a monochromatic source. Another polaroid (II) is placed in front of this polaroid (I) and rotated till no light passes. A third polaroid (III) is now placed in between (I) and (II). In this case, will light emerge from (II). Explain.

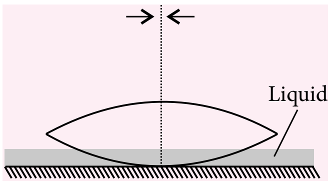
10. Yellow light ($\lambda = 6000$ Å) illuminates a single slit of width 1×10^{-4} m. Calculate (i) the distance between the two dark lines on either side of the central maximum, when the diffraction pattern is viewed on a screen kept 1.5 m away from the slit; (ii) the angular spread of the first diffraction minimum.

SECTION - C

11. How is the working of a telescope different from a microscope?

12. Light of wavelength 550 nm is incident as parallel beam on a slit of width 0.1 mm. Find the angular width and the linear width of the principal maxima in the resulting diffraction pattern on a screen kept at a distance of 1.1 m from the slit. Which of these widths would not change if the screen were moved to a distance of 2.2 m from the slit?

13. An equiconvex lens with radii of curvature of magnitude R each, is put over a liquid layer poured on top of a plane mirror. A small needle, with its tip on the principal axis of the lens, is moved along the axis until its inverted real image coincides with the needle itself. The distance of the needle from the lens is measured to be a. On removing the liquid layer and repeating the experiment the distance is found to be b.



Given that two values of distances measured represent the focal length values in the two cases, obtain a formula for the refractive index of the liquid.

14. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 V m^{-1}.
(a) What is the wavelength of a wave?
(b) What is the amplitude of the oscillating magnetic field?
(c) Show that the average energy density of the $\vec{E}$ field equals the average energy density of the $\vec{B}$ field.

15. (a) Name the electromagnetic waves which are produced during radioactive decay of a nucleus. Write their frequency range.
(b) Welders wear special glass goggles while working. Why? Explain.
(c) Why are infrared waves often called as heat waves? Give their applications.

16. When the oscillating electric and magnetic fields are along the x-and y-direction respectively,
(i) point out the direction of propagation of electromagnetic wave.
(ii) express the velocity of propagation in terms of the amplitudes of the oscillating electric and magnetic fields.
(iii) Write the expression of energy density and momentum carried by the electromagnetic waves.

17. A small bulb is placed at the bottom of a tank containing water to a depth of 80 cm. Find the area of the surface of water through which light from the bulb can emerge out. Refractive index of water is 1.33. Consider the bulb to be a point source. [$\sin^{-1}(0.75) = 48.6°$ and $\tan 48.6° = 1.134$]

18. (a) Why are coherent sources necessary to produce a sustained interference pattern?
(b) In Young's double slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. Find out the intensity of light at a point where path difference is $\lambda/3$.

OR

(a) A mobile phone lies along the principal axis of a concave mirror. Show, with the help of a suitable diagram, the formation of its image. Explain why magnification is not uniform.
(b) Suppose the lower half of the concave mirror's reflecting surface is covered with an opaque material. What effect this will have on the image of the object? Explain.

19. (a) Which of the following, if any, can act as a source of electromagnetic waves?
(i) A charge moving with constant velocity.
(ii) A charge moving in a circular orbit.
(iii) A charge at rest. Give reason.
(b) Identify the part of the electromagnetic spectrum to which waves of frequency :
(i) 10^{20} Hz (ii) 10^{9} Hz

20. A convex lens of focal length 20 cm is placed coaxially with a convex mirror of radius of curvature 20 cm. The two are kept at 15 cm from each other. A point object lies 60 cm in front of the convex lens. Draw a ray diagram to show the formation of the image by the combination. Determine the nature and position of the image formed.

21. What is the angle of minimum deviation for a prism? Deduce the expression for the refractive index of glass prism in terms of the angle of minimum deviation and angle of the prism.

22. Two polaroids are placed 90° to each other. What happens when $(N - 1)$ more polaroids are inserted between two crossed polaroids (at 90° to each other)? Their axes are equally spaced. How does the transmitted intensity behave for large N?

SECTION - D

23. Om and Amber were in a restaurant on a hot summer night. They observed that the restaurant was free from flies due to U.V. lamp with a wire gauge arrangement where the flies were getting attracted and were electrocuted. Amber who is a student of class XII explained about the working of U.V. lamp to Om. Om also had some more queries as follows.

(a) What is the main source of U.V. rays and why U.V. rays are harmful?
(b) Why it was called as U.V. lamp, where it was producing violet light?
(c) What is the importance of violet light in cleaning the insects?

On basis of this paragraph, answer the following questions.

(i) What are the values shown by Amber?
(ii) What are the values shown by Om?
(iii) What are the answers to the questions by Om?

SECTION - E

24. Draw a ray diagram showing the formation of the image by a point object on the principal axis of a spherical convex surface separating two media of refractive indices μ_1 and μ_2, when a point source is kept in rarer medium of refractive index μ_1. Derive the relation between object and image distance in terms of refractive index of the medium and radius of curvature of the surface.
Hence obtain the expression for lens-maker's formula in the case of thin convex lens.

OR

(a) Draw a ray diagram to show the working of a compound microscope. Deduce an expression for the total magnification when the final image is formed at the near point.
(b) In a compound microscope, an object is placed at a distance of 1.5 cm from the objective of focal length 1.25 cm. If the eye piece has a focal length of 5 cm and the final image is formed at the near point, estimate the magnifying power of the microscope.

25. Explain the phenomenon of total internal reflection. State two conditions that must be satisfied for total internal reflection to take place. Derive the relation between the critical angle and the refractive index of the medium. Draw ray diagrams to show how a right angled isosceles prism can be used to (i) deviate ray through 180°, and (ii) to invert it.

OR

State Huygen's principle. Using the geometrical construction of secondary wavelets, explain the refraction of a plane wavefront incident at a plane surface. Hence verify Snell's law of refraction.
Illustrate with the help of diagram the action of (i) convex lens and (ii) concave mirror on a plane wavefront incident on it.

26. (a) How does an unpolarized light incident on a polaroid get polarized?
Describe briefly, with the help of a necessary diagram, the polarization of light by reflection from a transparent medium.
(b) Two polaroids A and B are kept in crossed position. How should a third polaroid C be placed between them so that the intensity of polarized light transmitted by polaroid B reduces to $1/8^{th}$ of the intensity of unpolarized light incident on A?

OR

What are coherent source of light? State two conditions for two light source to be coherent. Derive a mathematical expression for the width of interference fringes obtained in Young's double slit experiment with the help of a suitable diagram.

SOLUTIONS

1. An electromagnetic wave consists of oscillating electric and magnetic fields. The velocity of an electromagnetic wave in vacuum depends on ε_0 (absolute permittivity of free space) and μ_0 (absolute permeability of free space), *i.e.*, $c = 1/\sqrt{\varepsilon_0\mu_0}$.

2. The speed of electromagnetic waves in vacuum from Maxwell's theory was found to be equal to the speed of light in vacuum.

3. When the light travels from a rarer to a denser medium, its frequency remains unchanged but wavelength decreases. It is because, frequency is an inherent property of light. Frequency of light depends on its source of generation.

4. There would be spherical wavefront on earth for sunlight which is treated as a point source, but radius of wavefront is very large as compared to the radius of earth, so it is almost a plane wavefront.

5. According to Rayleigh's law of scattering, the intensity of light of wavelength λ present in the scattered light is inversely proportional to fourth power of wavelength λ. Mathematically, $I \propto \frac{1}{\lambda^4}$

6. Focal length of lens in a liquid is

$$\frac{1}{f} = \left(\frac{\mu - \mu_l}{\mu_l}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

As refractive index of glass of lens is $\mu = 1.5$

(i) In water $\mu > \mu_l$ (= 1.33), so f is positive and hence convex lens behaves as converging lens in water.

(ii) In carbon disulphide $\mu < \mu_l$ (= 1.6), so f is negative and hence convex lens behaves as diverging lens in carbon disulphide.

7. The basic properties of electromagnetic waves are

(i) The electromagnetic waves are produced by accelerated charges and do not require any medium for their propagation.

(ii) The oscillations of $\vec{E}$ and $\vec{B}$ fields are perpendicular to each other as well as to the direction of propagation of the wave. So the electromagnetic waves are transverse in nature.

(iii) All electromagnetic waves travel in free space with the same speed,

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m s}^{-1}$$

(iv) The oscillations of $\vec{E}$ and $\vec{B}$ are in the same phase.

8. In electromagnetic wave, the electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ show their variations perpendicular to the direction of propagation of wave as well as perpendicular to each other. As the electromagnetic wave is travelling along z-axis, hence $\vec{E}$ and $\vec{B}$ show their variation in x-y plane.

The direction of $\vec{E}$ and $\vec{B}$ are along x-axis and y-axis respectively.

Wavelength, $\lambda = \frac{c}{\upsilon} = \frac{3\times10^8 \text{ m s}^{-1}}{30\times10^6 \text{ s}^{-1}} = 10$ m.

9. The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $\mu_1 = \mu_2$.

$$\frac{1}{f} = ({}^1\mu_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

As, $\mu_1 = \mu_2$

$\therefore \quad \frac{1}{f} = 0\,; f = \infty$

The lens in the liquid will act like a plane sheet of glass. No, the liquid is not water. It could be glycerine.

OR

According to question, plane polarized light is emerging from polaroid (I) by monochromatic light. When polaroid (II) is placed in front of polariod (I) and rotated till no light passes through polariod (II). Then, pass axes of (I) and (II) are at 90°.

No light will emerge from (II) when third polaroid is placed between first and second such that the pass axis of (III) is parallel to (I) or (II). In all other cases, some light will emerge from polaroid (II).

10. (i) Here $a = 1 \times 10^{-4}$ m, $D = 1.5$ m

$\lambda = 6000$ Å $= 6000 \times 10^{-10}$ m

The distance between the two dark bands on each side of central band is equal to width of the central bright band, *i.e.*,

$$\frac{2D\lambda}{a} = \frac{2\times1.5\times6000\times10^{-10}}{1\times10^{-4}} = 18 \text{ mm}$$

(ii) Angular spread $= \frac{\lambda}{a} = \frac{6000\times10^{-10}}{1\times10^{-4}}$

$= 6 \times 10^{-3}$

11. Working differences between a telescope and a microscope

(i) A telescope produces high resolution while a microscope produces high magnification.

(ii) The objective of a telescope forms the image of a very far off object at, or within, the focus of its eyepiece. The microscope does the same for a small object kept just beyond the focus of its objective.

(iii) The final image formed by a telescope is magnified relative to its size as seen by the unaided eye while the final image formed by a microscope is magnified relative to its absolute size.

(iv) The objective of a telescope has large focal length and large aperture while the corresponding quantities for a microscope have very small values.

(v) In a telescope, the distance between the objective and eyepiece is adjusted to focus the object at infinity. In a microscope the distance between the objective and eyepiece is fixed, the distance of the objective is changed to focus the object.

12. Here, $\lambda = 550$ nm $= 5.5 \times 10^{-7}$ m,

$a = 0.1$ mm $= 1 \times 10^{-4}$ m, $D = 1.1$ m

Linear width of central maximum is

$$\beta_0 = \frac{2\lambda D}{a} = \frac{2\times5.5\times10^{-7}\times1.1}{1\times10^{-4}} = 12.1\times10^{-3} \text{ m}$$

or $\beta_0 = 12.1$ mm

Angular width of central maximum is

$$\theta_0 = \frac{2\lambda}{a} = \frac{2\times5.5\times10^{-7}}{1\times10^{-4}} = 1.1\times10^{-2} = 0.011 \text{ rad}$$

As angular width θ_0 is independent of distance D of screen from plane of slit, so it will not change when screen is moved to a distance of 2.2 m from the slit.

13. Clearly, equivalent focal length of equiconvex lens and water lens $f = a$

Focal length of equiconvex lens, $f_1 = b$

Focal length f_2 of water lens is given by

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1} = \frac{1}{a} - \frac{1}{b} = \frac{b-a}{ab}$$

or $\quad f_2 = \frac{ab}{b-a}$

The water lens formed between the plane mirror and the equiconvex lens is a planoconcave lens. For this lens,

$R_1 = -R$ and $R_2 = \infty$

Using lens maker's formula,

$$\frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

or $\quad \frac{b-a}{ab} = (\mu-1)\left[\frac{1}{-R} - \frac{1}{\infty}\right]$

or $\quad \mu = 1 + \frac{(a-b)R}{ab}$

14. (a) Wavelength, $\lambda = \frac{c}{\upsilon} = \frac{3\times10^8}{2.0\times10^{10}} = 1.5\times10^{-2}$ m

(b) $B_0 = \frac{E_0}{c} = \frac{48}{3\times10^8} = 1.6\times10^{-7}$ T

(c) Average energy density of $\vec{E}$ field,

$$u_E = \frac{1}{4}\varepsilon_0 E_0^2$$

But $E_0 = cB_0$ and $c^2 = \frac{1}{\mu_0\varepsilon_0}$

$$u_E = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{4}\varepsilon_0 (cB_0)^2$$

$$= \frac{1}{4}\varepsilon_0 \cdot \frac{1}{\mu_0\varepsilon_0} \cdot B_0^2 = \frac{1}{4\mu_0} B_0^2 = u_B.$$

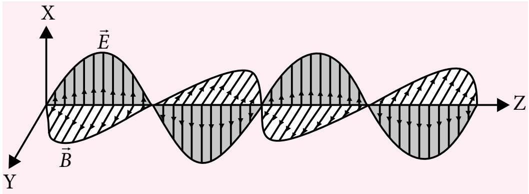
15. (a) The waves produced during radioactive decay are γ-rays and has range 10^{19} Hz to 10^{23} Hz.

(b) Welders wear special glass goggles to protect their eyes from large amount of harmful UV radiation produced by welding arc.

(c) Infrared waves incident on a substance increase the internal energy and hence the temperature of the substance. That is why they are also called heat waves.

Infrared waves are used in remote control of TV, in green houses, in the treatment of muscular complaints, etc.

16. (i)



Propagation of electromagnetic wave is along z-axis.

(ii) Speed of electromagnetic wave can be given as the ratio of magnitude of electric field (E_0) to the magnitude of magnetic field (B_0), *i.e.*,

$$c = \frac{E_0}{B_0}$$

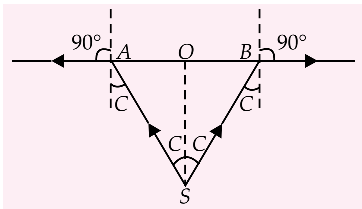
(iii) Energy density carried by the electromagnetic waves is

$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2$; where E and B are electric and magnetic fields of electromagnetic waves.

Momentum of electromagnetic wave is

$p = \frac{U}{c}$; where U is the energy.

17. In figure, the source of light (S) is 80 cm below the surface of water *i.e.*, SO = 80 cm = 0.8 m.



When $\angle i = C$, for SA and SB, $\angle r = 90°$

$\therefore$ Area of the surface to water through which light from the bulb can emerge is area of the circle of radius, *i.e.*,

$$r = \frac{AB}{2} = OA = OB$$

As $\quad \mu = \frac{1}{\sin C}$

$$\sin C = \frac{1}{\mu} = \frac{1}{1.33} = 0.75,\ C = \sin^{-1}(0.75) = 48.6°$$

In ΔOBS, $\tan C = \frac{OB}{OS}$

$\therefore \quad OB = OS \tan C = 0.8 \tan 48.6°$

$r = 0.8 \times 1.134 = 0.907$ m

Area of the surface of water through which light emerges $= \pi r^2 = 3.14 \times (0.907)^2 = 2.584$ m^2

18. (a) Coherent sources are necessary to produce a sustained interference pattern otherwise the phase difference changes very rapidly with time and hence no interference will be observed.

(b) Intensity at a point, $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference

At path difference λ, phase difference,

$$\phi = \frac{2\pi}{\lambda} \times \lambda = 2\pi$$

$\therefore$ Intensity, $K = 4I_0 \cos^2\left(\frac{2\pi}{2}\right)$

[$\because$ Given $I = K$, at path difference λ]

$K = 4I_0$ ...(i)

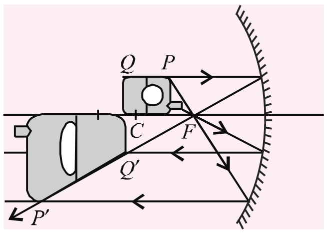
If path difference is $\frac{\lambda}{3}$, then phase difference will be

$$\phi' = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

$\therefore$ Intensity, $I' = 4I_0 \cos^2\left(\frac{2\pi}{6}\right) = \frac{K}{4}$ (Using (i))

OR

(a) The formation of the image of the cell phone is shown in figure. The part which is at R will be imaged at R and will be of the same size, *i.e.*, $Q'C = QC$. The other end P of the mobile phone is highly magnified by the concave mirror.



Thus the different parts of the mobile phone are magnified in different proportions because of their different locations from the concave mirror.

(b) At first sight, it appears that the image will be half of the object, but taking the laws of reflection to be true for all points of the mirror, the image will be of the whole object. However, as the area of the reflecting surface has reduced, the intensity of the image will be low.

19. (a) (i) A charge moving with constant velocity has no acceleration and as such cannot be a source of electromagnetic waves.

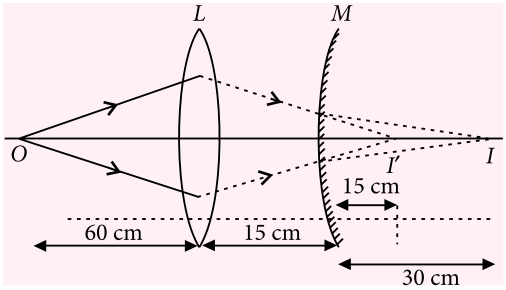
(ii) A charged particle moving in a circular orbit has centripetal acceleration. According to Maxwell's theory of electromagnetism, centripetally accelerated charges revolving with frequency υ should radiate electromagnetic waves of frequency υ.

(iii) A charge at rest has only an electric field. Since it has no acceleration, it cannot be a source of electromagnetic waves.

(b) (i) γ-rays ($\lambda \approx 10^{-12}$ m)

(ii) Radio waves ($\lambda = 0.3$ m)

20.



For the convex lens,

$u = -60$ cm, $f = +20$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}; \frac{1}{v} - \frac{1}{-60} = \frac{1}{20} = +30 \text{ cm}$$

For the convex mirror,

$$u = +(30 - 15) \text{ cm} = 15 \text{ cm}, f = +\frac{20}{2} \text{ cm} = 10 \text{ cm}$$

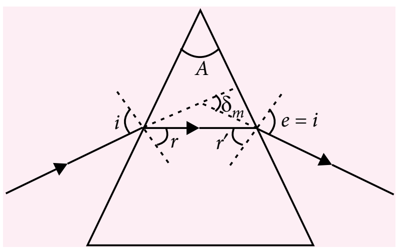
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \frac{1}{v} + \frac{1}{15} = \frac{1}{10} = +30 \text{ cm}$$

Final image is formed at the distance of 30 cm from the convex mirror (or 45 cm from the convex lens) to the right of the convex mirror.

The final image formed is a virtual image.

21. The minimum value of the angle of deviation suffered by a ray of light on passing through a prism is called the angle of minimum deviation.

At minimum deviation, the inside beam travels parallel to base of the prism.



$i = e$ and $r = r'$

$\delta_m = (i + e) - (r + r')$

$\delta_m = 2i - 2r$ ...(i)

Also $r + r' = A = 2r$ ...(ii)

So, angle of incidence using eqns. (i) and (ii),

$$i = \frac{A + \delta_m}{2}, \text{ angle of refraction } r = \frac{A}{2}$$

Now refractive index of the material of prism,

$$^a\mu_g = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin \frac{A}{2}}$$

where A is the refracting angle of the prism and $A = 60°$ for an equiangular prism.

22. Transmitted intensity through first polaroid is

$$I_1 = I_0 \cos^2\theta$$

where I_0 is the original intensity. Similarly, the transmitted intensity through second polaroid will be

$I_2 = I_1 \cos^2\theta = I_0 \cos^4\theta$

If N polaroids are used, then

$I_N = I_0 (\cos\theta)^{2N}$

As the optic axes of the polaroids are equally inclined, so angle of rotation θ is same for each polaroid.

Thus $\frac{I_N}{I_0} = (\cos\theta)^{2N}$

But angle between successive polaroids is

$$\theta = \frac{90°}{N} = \frac{\pi}{2N} \text{ rad}$$

$$\therefore \left(\cos\frac{\pi}{2N}\right)^{2N} = \left(1 - \frac{\pi^2}{8N^2} + ...\right)^{2N} \simeq 1 - \frac{2N\pi^2}{8N^2}$$

which approaches 1 for large N. Hence fractional intensity,

$$\frac{I_N}{I_0} = 1 \quad \text{or} \quad I_N = I_0$$

23. (i) Amber had shown the values of knowledge, observation and ability to explain.

(ii) Om is also observant, quick learner and has inquisitiveness.

(iii) (a) Main source of U.V. rays is Sun and they can cause skin cancer when exposed for a longer duration.

(b) The inside of the fluorescent lamp is coated with fluorescent powder which absorb U.V. rays and emit violet light in the visible region.

(c) Violet light attracts the insects which are electrocuted by the high voltage wire gauge adjusted before the lamp.

24. Refer to point 6.5(5) and 6.6(1) page no. 372 (MTG Excel in Physics).

OR

(a) Refer to point 6.9 (1(iv)) page no. 381 (MTG Excel in Physics).

(b) $u_0 = -1.5$ cm, $f_0 = 1.25$ cm, $f_e = 5$ cm, $D = 25$ cm

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \Rightarrow \frac{1}{v_0} - \frac{1}{-1.5} = \frac{1}{1.25}$$

$$\frac{1}{v_0} = \frac{1}{1.25} - \frac{1}{1.5} \text{ or } v_0 = \frac{1.25 \times 1.5}{0.25}$$

$= 5 \times 1.5 = 7.5$ cm

Magnifying power, $M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$

$$= -\frac{7.5}{1.5}\left(1 + \frac{25}{5}\right) = -5(1 + 5) = -30$$

25. Refer to point 6.4 (1, 2, 3, 5), page no. 371 (MTG Excel in Physics).

OR

Refer to point 6.10(6), 6.11(5) and 6.12(1, 3) page no. 443 (MTG Excel in Physics).

26. (a) Refer to point 6.15 (6, 9) page no. 453 (MTG Excel in Physics).

(b) Let θ be the angle between the pass axis of A and C. Intensity of light passing through $A = \frac{I_0}{2}$

Intensity of light passing through $C = \left(\frac{I_0}{2}\right)\cos^2\theta$

Intensity of light passing through B

$$= \left(\frac{I_0}{2}\right)\cos^2\theta[\cos^2(90 - \theta)]$$

$$= \left(\frac{I_0}{2}\right)\cdot(\cos\theta \cdot \sin\theta)^2 = \frac{I_0}{8} \text{ (Given)}$$

$\therefore$ $\sin 2\theta = 1$, $2\theta = 90°$ or, $\theta = 45°$

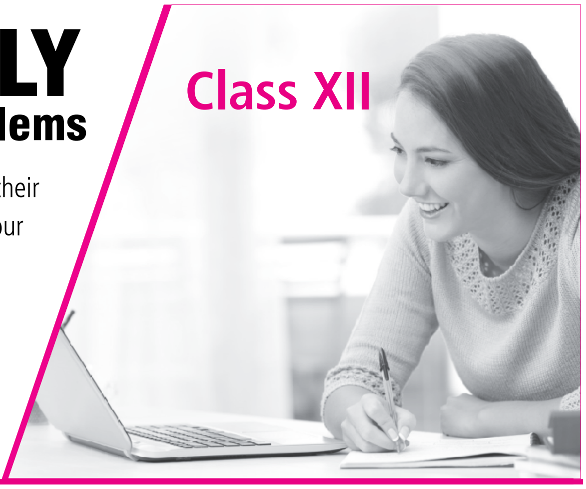
The third polaroid is placed at $\theta = 45°$

OR

Refer to point 6.13 (3, 6, 7) page no. 446 (MTG Excel in Physics.)

◈◈

MPP-6 MONTHLY Practice Problems



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Optics

Total Marks : 120 **Time Taken : 60 min**

NEET / AIIMS

Only One Option Correct Type

1. A real image of a distant object is formed by a plano-convex lens on its principal axis. Spherical aberration
   (a) is absent
   (b) is smaller, if the curved surface of the lens faces the object
   (c) is smaller, if the plane surface of the lens faces the object
   (d) is the same, whichever side of the lens faces the object.

2. When an unpolarised light of intensity I_0 is incident on a polarising sheet, the intensity of the light which does not get transmitted is
   (a) $I_0/2$ (b) $I_0/4$
   (c) zero (d) I_0

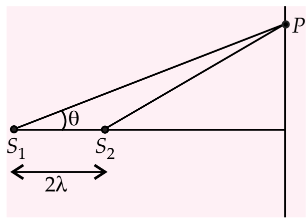
3. The two slits are 1 mm apart from each other and illuminated with a light of wavelength 5×10^{-7} m. If the distance of the screen is 1 m from the slits, then the distance between third dark fringe and fifth bright fringe is
   (a) 1.2 mm (b) 0.75 mm
   (c) 1.25 mm (d) 0.625 mm

4. A square card of side length 1 mm is being seen through a magnifying lens of focal length 10 cm. The card is placed at a distance of 9 cm from the lens. The apparent area of the card through the lens is
   (a) 1 cm^2 (b) 0.81 cm^2
   (c) 0.27 cm^2 (d) 0.60 cm^2

5. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by
   (a) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$ (b) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$
   (c) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$ (d) $\frac{I_m}{9}(4+5\cos\phi)$

6. The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will
   (a) act as a convex lens only for the objects that lie on its curved side.
   (b) act as a concave lens for the objects that lie on its curved side.
   (c) act as a convex lens irrespective of the side on which the object lies.
   (d) act as a concave lens irrespective of side on which the object lies.

7. In Young's double slit experiment, the slits are horizontal. The intensity at a point P as shown in figure is $\frac{3}{4}I_0$, where I_0 is the maximum intensity.



   Then the value of θ is, (Given the distance between the two slits S_1 and S_2 is 2λ)
   (a) $\cos^{-1}\left(\frac{1}{12}\right)$ (b) $\sin^{-1}\left(\frac{1}{12}\right)$
   (c) $\tan^{-1}\left(\frac{1}{12}\right)$ (d) $\sin^{-1}\left(\frac{3}{5}\right)$

8. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and refractive index $\sqrt{2}$. The angle of deviation of the ray is
(a) 0° (b) 12.5° (c) 15° (d) 22.5°

9. A lens having focal length f and aperture of diameter d forms an image of intensity I. Aperture of diameter $\frac{d}{2}$ in central region of lens is covered by a black paper. Focal length of lens and intensity of image now will be respectively
(a) f and $\frac{I}{4}$ (b) $\frac{3f}{4}$ and $\frac{I}{2}$
(c) f and $\frac{3I}{4}$ (d) $\frac{f}{2}$ and $\frac{I}{2}$

10. A planoconvex lens has a maximum thickness of 6 cm. When placed on a horizontal table with the curved surface in contact with the table surface, the apparent depth of the bottom most point of the lens is found to be 4 cm. If the lens is inverted such that the plane face of the lens is in contact with the surface of the table, the apparent depth of the center of the plane face is found to be $\left(\frac{17}{4}\right)$cm. The radius of curvature of the lens is
(a) 68 cm (b) 75 cm (c) 128 cm (d) 34 cm

11. A diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light?
(a) No change.
(b) Diffraction bands become narrower and crowded together.
(c) Band become broader and farther apart.
(d) Bands disappear altogether.

12. The human eye has an approximate angular resolution of $\phi = 5.8 \times 10^{-4}$ rad and typical photoprinter prints a minimum of 300 dpi (dots per inch, 1 inch = 2.54 cm). At what minimal distance z should a printed page be held so that one does not see the individual dots?
(a) 14.5 cm (b) 20.5 cm
(c) 29.5 cm (d) 28 cm

Assertion & Reason Type

Directions : In the following questions, a statement of assertion is followed by a statement of reason. Mark the correct choice as :
(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false.
(d) If both assertion and reason are false.

13. Assertion : A total reflecting prism is used to erect the inverted image without deviation.
Reason: Rays of light incident parallel to base of prism emerge out as parallel rays.

14. Assertion : A single lens produces a coloured image of an object illuminated by white light.
Reason : The refractive index of the material of lens is different for different wavelengths of light.

15. Assertion : When tiny circular obstacle is placed in the path of light from some distance, a bright spot is seen at the centre of the shadow of the obstacle.
Reason : Destructive interference occurs at the centre of the shadow.

JEE MAIN / JEE ADVANCED

Only One Option Correct Type

16. Rays of light from sun fall on a biconvex lens of focal length f and the circular image of sun of image of sun of radius r is formed on the focal plane of the lens. Then
(a) area of image is πr^2 and area is directly proportional to f
(b) area of image is πr^2 and area is directly proportional to f^2.
(c) intensity of image increases if f is increased
(d) if lower half of the lens is covered with black paper area will become half.

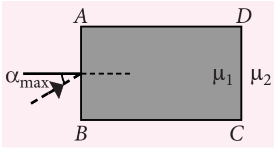
17. A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refractive indices μ_1 and μ_2 respectively ($\mu_2 > \mu_1 > 1$). The lens will diverge a parallel beam of light if it is filled with
(a) air and placed in air
(b) air and immersed in L_1
(c) L_1 and immersed in L_2
(d) L_2 and immersed in L_1.

Solution Senders of Physics Musing

SET-50

1. Prashant Pandey, Nagpur (Maharashtra)
2. Shikha Aggarwal, (Bangalore)
3. Kanika Mukherjee, Bardhaman (West Bengal)

18. A rectangular glass slab $ABCD$ of refractive index μ_1 is immersed in water of refractive index μ_2 ($\mu_1 > \mu_2$). A ray of light is incident at the surface AB of the slab as shown. The maximum value of the angle of incidence α_{max}, such that the ray comes out only from the other surface CD is given by



(a) $\sin^{-1}\left[\frac{\mu_1}{\mu_2}\cos\left(\sin^{-1}\frac{\mu_2}{\mu_1}\right)\right]$

(b) $\sin^{-1}\left[\mu_1\cos\left(\sin^{-1}\frac{1}{\mu_2}\right)\right]$

(c) $\sin^{-1}\frac{\mu_1}{\mu_2}$

(d) $\sin^{-1}\frac{\mu_2}{\mu_1}$

19. In a double slit experiment instead of taking slits of equal widths, one slit is made twice as wide as the other. Then , in the interference pattern

(a) the intensities of both the maxima and the minima increase

(b) the intensity of the maxima increases and the minima has zero intensity

(c) the intensity of the maxima decreases and that of the minima increases

(d) the intensity of the maxima decreases and the minima has zero intensity.

More than One Options Correct Type

20. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45°. The ray undergoes total internal reflection. If μ is the refractive index of the medium with respect to air, select the possible value(s) of μ from the following

(a) 1.3 (b) 1.4 (c) 1.5 (d) 1.6

21. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 m and an eyepiece of focal length 2 cm.

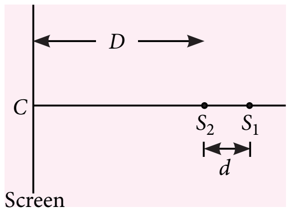
(a) The distance between the objective and the eyepiece is 16.02 m

(b) The angular magnification of the planet is – 800

(c) The image of the planet is inverted

(d) The objective is larger than the eyepiece.

22. Two fixed coherent point sources S_1 and S_2 emitting monochromatic light are arranged in front of a screen as shown in figure. Given $D >> d$.



Both the sources are started at the same time so that there is no initial phase difference. If one moves in the upward direction on the screen, then

(a) if $d = 3\lambda$, there will be minima at C

(b) if $d = 5\lambda/2$, there will be minima at C

(c) $d = 3.6\lambda$, then there will be 9 maxima on screen

(d) $d = 3.6\lambda$, then there will be 8 maxima on screen

23. An object of height 2 cm is kept 2 m in front of a convex lens of focal length 1 m. A plane mirror is placed at 3 m from the lens on its other side. Regarding the nature and magnification of the final image that will be seen by an observer looking towards the mirror through the lens, mark the correct options related to this situation.

(a) Final image will be 4/3 m right of lens

(b) Final image will be 4/3 m left of lens

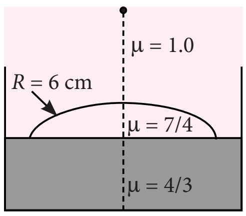
(c) Final image is real and erect

(d) Magnification of final image is 1/3

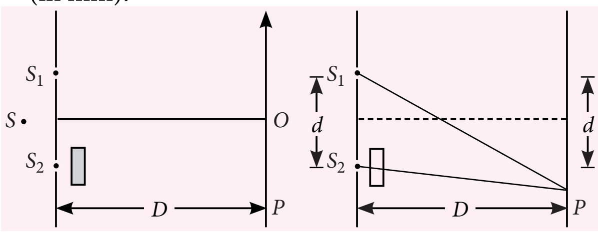
Integer Answer Type

24. A large glass slab (μ = 5/3) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R ?

25. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature R = 6 cm as shown. Consider oil to act as a thin lens. An object S is placed 24 cm above water surface. The location of its image is at x cm above the bottom of the tank. Then x is



26. A Young's double slit experiment is performed in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 nm separation. The lower slit S_2 is covered by a thin glass plate of thickness 10.4 μm and refractive index 1.5. The interface pattern is observed on a screen placed 1.5 m from the slits as shown in figure. Find the location of the central maximum (bright fringe with zero path difference) on the y-axis in the nearest integer form (in mm).



Comprehension Type

White light may be considered to have wavelength from 400 mm to 750 mm. If an oil film of thickness $t = 10^{-4}$ cm. The light is incidence normally and the refractive index of oil is $\mu = 1.4$.

27. If interference is consider in reflected light, the path difference is

(a) $\Delta x = 2\mu t$
(b) $\Delta x = 2\mu t - \lambda/2$
(c) $\Delta x = \lambda/2$
(d) None of the above

28. The wavelengths of visible light for weak reflection are

(a) 400 nm, 466.7 nm, 560 nm, 700 nm
(b) 430 nm, 509.1 nm, 622.2 nm
(c) 400 nm, 509.1 nm, 622.2 nm
(d) None of the above

Matrix Match Type

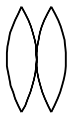
29. A simple telescope used to view distant objects has eyepiece and objective lenses of focal lengths f_e and f_o respectively. Then match the Column I with Column II and select the correct option.

	Column I		Column II
(A)	Intensity of light received by lens	(P)	Radius of aperture
(B)	Angular magnification	(Q)	Dispersion of lens
(C)	Length of telescope	(R)	Focal length of objective lens and eyepiece lens
(D)	Sharpness of image	(S)	Spherical aberration

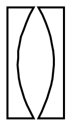
	A	B	C	D
(a)	P, Q, R	P	R	S
(b)	Q, S	P	R	S
(c)	P	R	R	P, Q, S
(d)	S, P	Q	R	S

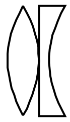
30. Four combinations of two thin lenses are given in Column I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in Column I with their focal length in Column II and select the correct option.

Column I — **Column II**

(A)  — (P) $2r$

(B)  — (Q) $r/2$

(C)  — (R) $-r$

(D)  — (S) r

	A	B	C	D
(a)	P	Q	R	S
(b)	Q	S	R	P
(c)	S	P	Q	R
(d)	Q	P	R	S

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / NEET / AIIMS / JIPMER with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / NEET. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their detailed solutions with their names and complete address. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

PROBLEM Set 51

SINGLE OPTION CORRECT TYPE

1. An electron and a proton are separated by a large distance. The electron starts approaching the proton with energy 2 eV. The proton captures the electron and forms a hydrogen atom in first excited state. The resulting photon is incident on a photosensitive metal of threshold wavelength 4600Å. The maximum kinetic energy of the emitted photoelectron is

(a) 2.4 eV (b) 2.7 eV
(c) 2.9 eV (d) 5.4 eV

2. When a thin transparent sheet of refractive index $\mu = \frac{3}{2}$ is placed near one of the slit in Young's double slit experiment, the intensity at centre of screen reduces to half of maximum intensity. Then, minimum thickness of sheet in terms of wavelength λ of the monochromatic light used,is [Assume there is no absorption by the sheet.]

(a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{8}$ (c) $\frac{\lambda}{2}$ (d) $\frac{\lambda}{3}$

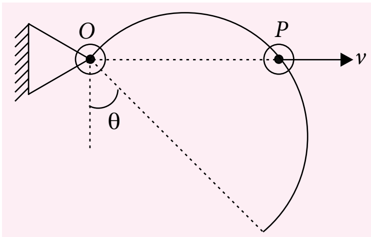
3. A cone of radius r and height h rests on rough horizontal surface, the coefficient of friction between the cone and the surface is μ. A gradually increasing horizontal force F is applied to the vertex of the cone. The largest value of μ for which cone will slide before it topples is

(a) $\mu = \frac{r}{2h}$ (b) $\mu = \frac{2r}{5h}$
(c) $\mu = \frac{r}{h}$ (d) $\mu = \sqrt{\frac{r}{h}}$

4. In a cube, the edge is measured using vernier calliper scale. In this calliper, 9 divisions of the main scale is equal to 10 divisions of vernier scale and 1 main scale division is 1 mm. The main scale division reading is 10 and 1 division of vernier scale was found to be coinciding with the main scale. The mass of the cube is 2.736 g. Calculate the density upto correct 3 significant figures.

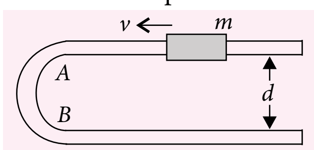
(a) 3.12 g cm^{-3} (b) 2.78 g cm^{-3}
(c) 2.66 g cm^{-3} (d) 4.22 g cm^{-3}

5. A semicircular wire of radius r is supported in its own vertical plane by a hinge at O and smooth peg P as shown in the figure. If peg starts from O and moves with constant speed v along horizontal axis through O. The angular speed of wire at any time, t is



(a) $\frac{v}{\sqrt{4r^2 - v^2t^2}}$ (b) $\frac{v}{2r}$
(c) $\frac{v}{\sqrt{2r^2 - v^2t}}$ (d) None of these

6. A bead of mass m moving with uniform speed v through a frictionless horizontal U-shaped wire as shown in figure. When it enters from A and leaves at B then the magnitude of average force exerted by the bead on semi circular part AB of the wire is

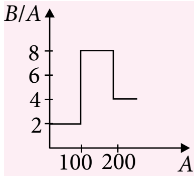


(a) $\frac{8mv^2}{\pi d}$ (b) $\frac{4mv^2}{\pi d}$
(c) zero (d) None of these

By Akhil Tewari, Author Foundation of Physics for JEE Main & Advanced, Professor, IITians PACE, Mumbai.

MULTIPLE OPTIONS CORRECT TYPE

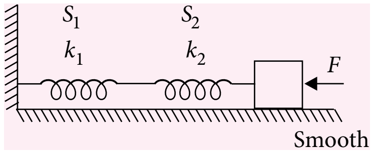
7. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice (s) given below



(a) fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.
(b) fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy.
(c) fission of a nucleus with mass numbers lying in the range of $100 < A < 200$ will release energy when broken into two equal fragments.
(d) fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments.

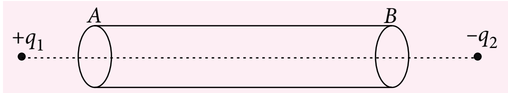
8. If an object is released from rest, we use $g = \frac{2h}{t^2}$ to calculate acceleration due to gravity in two experiments.
First : $t_1 = (0.781 \pm 0.002)\,\text{s}, h_1 = (3.000 \pm 0.003)\,\text{m}$
Second: $t_2 = (0.551 \pm 0.002)\,\text{s}, h_2 = (1.500 \pm 0.003)\,\text{m}$
Then, values of g are [round off to two decimal places]
(a) for first experiment $g = (9.84 \pm 0.06)\,\text{m s}^{-2}$
(b) for second experiment $(9.88 \pm 0.09)\,\text{m s}^{-2}$
(c) for first experiment $g = (9.84 \pm 0.01)\,\text{m s}^{-2}$
(d) for second experiment $g = (9.78 \pm 0.09)\,\text{m s}^{-2}$

9. Initially springs are in natural length. Springs S_1 and S_2 have spring constants k_1 and k_2 respectively. On application of external varying force F causes the block to move slowly a distance x towards wall on smooth floor.



(a) Work done by S_2 on block is $-\frac{1}{2}\left(\frac{k_1k_2}{k_1+k_2}\right)x^2$.
(b) Work done by S_2 on S_1 is $\frac{1}{2}\frac{k_1k_2^2x^2}{(k_1+k_2)^2}$.
(c) Work done by F on block is $\frac{1}{2}\left(\frac{k_1k_2}{k_1+k_2}\right)x^2$.
(d) Work done by S_1 on wall is zero.

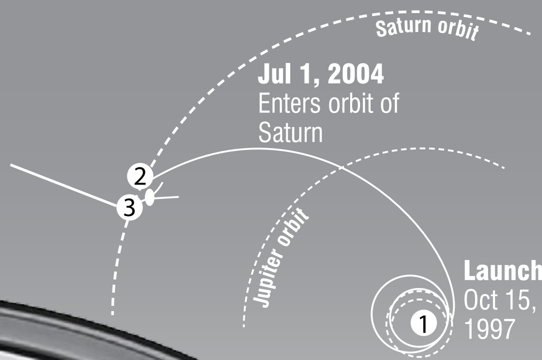
10. A thin conducting rod AB is introduced in between the two point charges $+q_1$ and $-q_2$ as shown in figure. For this situation mark the correct statement(s).



(a) The total force experienced by $+q_1$ is vector of electric force experienced by $+q_1$ due to $-q_2$ and by due to induced charges on rod.
(b) The end A will become negative charged.
(c) The end B will become positive charged.
(d) The total force on $-q_2$ will be greater than as compared to the case without rod.

CASSINI'S DEATH DIVE

MISSION IN NUMBERS



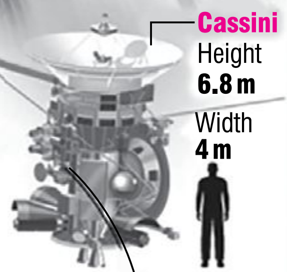
- **7.9 billion km** travelled
- **294 orbits** of Saturn completed
- **635 GB** science data collected
- **4,53,048** images taken
- **2.5 million** commands executed

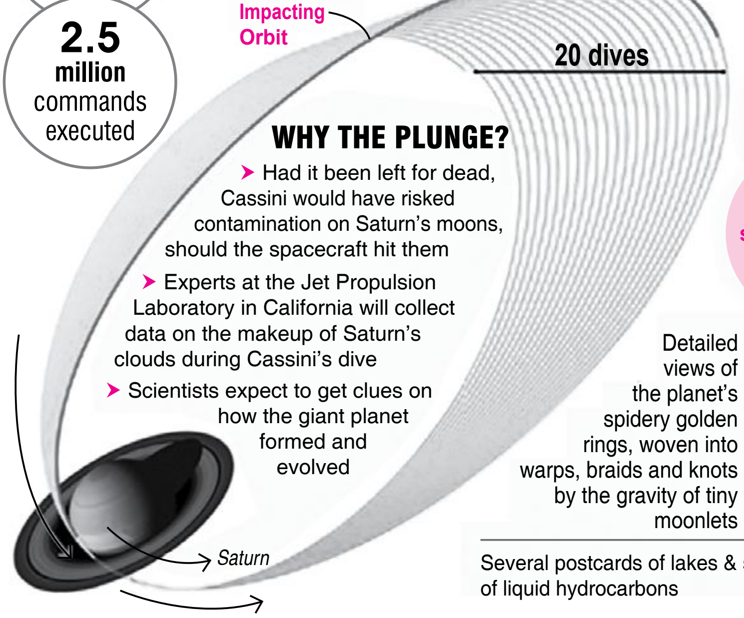
A Nasa illustration of Cassini above Saturn's northern hemisphere before making one of its grand finale dives -Reuters

FINAL MOMENTS

The cameras will turn off on the 14th, after one final look around the environs Cassini has called home for the last 13 years

Most of the spacecraft's instruments will keep working, gathering and analysing samples of the planet's atmosphere as the spacecraft blazes into the clouds





WHY THE PLUNGE?

- Had it been left for dead, Cassini would have risked contamination on Saturn's moons, should the spacecraft hit them
- Experts at the Jet Propulsion Laboratory in California will collect data on the makeup of Saturn's clouds during Cassini's dive
- Scientists expect to get clues on how the giant planet formed and evolved

GREATEST HITS

Movies of the six-sided storm that hugs Saturn's north pole

Detailed views of the planet's spidery golden rings, woven into warps, braids and knots by the gravity of tiny moonlets

The discovery of plumes that look like snow-making machines shooting from the surface of the moon Enceladus

Several postcards of lakes & seas on Titan made of liquid hydrocarbons

2,125 kg on Earth

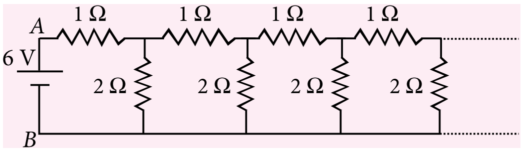
Packet with **cameras,** antennas and other sensors, it's one of the most sophisticated spy robots ever set loose in interplanetary space

- Spacecraft consisted of two parts : An orbiter and a lander – the Huygens probe – to explore Saturn's largest moon, Titan
- It **costs $2.5 billion** to built and launch Cassini and Huygens. Another $1.4 billion was spent to run them for 20 years

Courtsey : The Times of India

JEE WORK OUTS

1. An infinite ladder network of resistances is constructed with 1 Ω and 2 Ω resistances, as shown in figure.

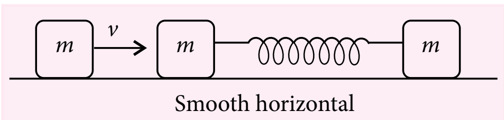


The 6 V battery between A and B has negligible internal resistance.

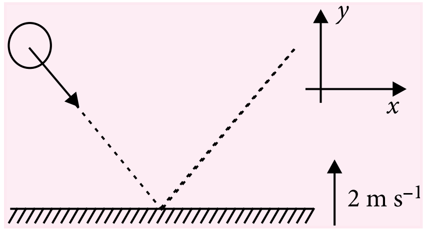
(i) Find the effective resistance between A and B.

(ii) What is the current that passes through the 2 Ω resistance nearest to the battery?

2. A block is moving with speed v towards a system of two blocks system as shown in figure. The first block hits the second block elastically. If the maximum compression is $x = 1$ m. What will be the common velocity by which two blocks system will move together after some time.?
($m = 1$ kg, $k = 2$ N m^{-1}, $v = 2$ m s^{-1})

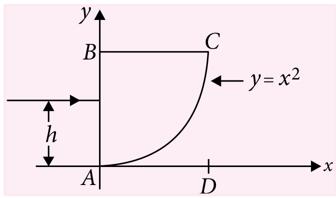


3. Consider a horizontal surface moving vertically upward with velocity 2 m s^{-1}. A small ball of mass 2 kg is moving with velocity $2\hat{i} - 2\hat{j}$ m s^{-1}. If coefficient of restitution is 1/2 and friction coefficient is 1/3. Find horizontal component of velocity of ball after collision.

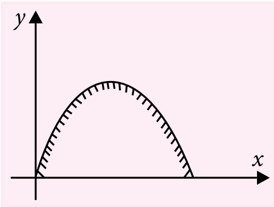


4. A uniform rope of length L and mass m is held at one end and whirled in a horizontal circle with angular velocity ω. You can ignore the force of gravity on rope. Determine the time required for a transverse wave to travel from one end of rope to other.

5. The cross-section of a prism is given in figure. One of the refracting surface is given by $y = x^2$. A ray of light travelling parallel to x-axis is incident normally on face AB and refracted. Find the minimum distance of incidence ray from surface AD to occur total internal reflection. Refractive index for prism is 3.



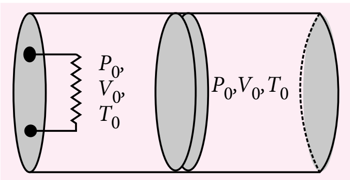
6. A reflecting surface is represented by the equation $y = \frac{2L}{\pi}\sin\left(\frac{\pi}{L}x\right), 0 \le x \le L$ as shown in figure A ray of light moving horizontally becomes vertical after reflecting. Find coordinates of points where ray is incident.



7. A beam of protons with a velocity 4×10^5 m s^{-1} enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix.

8. A light cylindrical vessel is kept on a smooth horizontal surface. Its base area is A. A hole of cross-sectional area a is made just at its bottom of vertical side. Find minimum coefficient of friction needed to prevent sliding of the vessel due to the impact force of the emerging liquid ($a << A$).

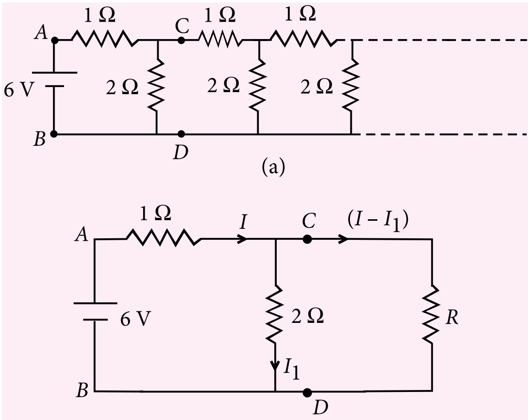
9. A cylindrical container having non-conducting walls is partitioned in two equal parts such that the volume of the each part is equal to V_0. A movable non-conducting piston is kept between the two parts. Gas on left is slowly heated so that the gas on right is compressed upto volume $V_0/8$. Find pressure and temperature on both sides if initial pressure and temperature, were P_0 and T_0 respectively. Also find heat given by the heater to the gas. (Number of moles in each part is n.)



10. Two identical sources of sound S_1 and S_2 produce intensity I_0 at a point P equidistant from each source.
    (a) Determine the intensity of each source at the point P.
    (b) If the power of S_1 is reduced to 64% and phase difference between the two sources is varied continuously, then determine the maximum and minimum intensities at the point P.
    (c) If the power of S_1 is reduced by 64%, then determine the maximum and minimum intensities at the point P.

SOLUTIONS

1. In case of the infinite ladder of resistances, the effective resistance remains the same when one identical item is either added to it or removed from it.
Let the effective resistance between A and B be R. The effective resistance between C and D will also be R.



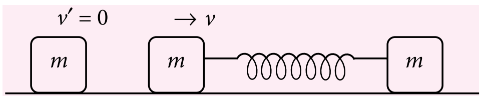
(a)

(b)

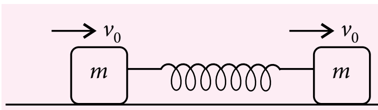
(i) From figure (b), $R = 1 + \frac{2R}{2+R}$

or $2R + R^2 = (2 + R) + 2R$ or $R^2 - R - 2 = 0$
or $(R - 2)(R + 1) = 0$ or $R - 2 = 0$
or $R = 2\ \Omega$.
(ii) Current through nearest 2 Ω resistance.
Apply Kirchhoff's law to the two loops shown in figure (b).
$6 - I - 2I_1 = 0$ or $I = 6 - 2I_1$ ...(i)
$-2(I - I_1) + 2I_1 = 0$ or $-2I + 4I_1 = 0$ ...(ii)
From (i) and (ii), $I = 3$ A, $I_1 = 1.5$ A
∴ Current through nearest 2 Ω resistance = 1.5 A.

2. As the collision is elastic the first block will transfer its speed completely to second block. So, initial scenario will be like this



At maximum compression



(The common speed at maximum compression)

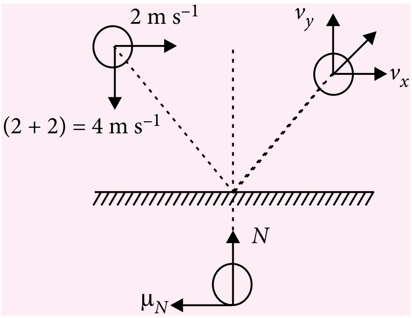
Let the maximum compression is x.
So, by mechanical energy conservation

$$\Rightarrow \frac{1}{2}mv^2 = 2\left(\frac{1}{2}mv_0^2\right) + \frac{1}{2}kx^2$$

$$\Rightarrow mv^2 = 2mv_0^2 + kx^2 \Rightarrow mv^2 - kx^2 = 2mv_0^2$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{mv^2 - kx^2}{2m}} = \sqrt{\frac{1\times4-2\times1}{2\times1}} = 1\,\text{m s}^{-1}$$

3. Take ball with respect to the ground, upward impulse on the ball



$$\int Ndt = p_f - p_i = 2v_y + (2\times4)$$

$$\int Ndt = 2v_y + 8$$

Equation of e along normal direction

$$e = \frac{v_y}{4} \; ; \; v_y = 4e = 2\text{ m s}^{-1}$$

$$\therefore \quad \int Ndt = 4 + 8 = 12$$

Horizontal impulse

$$-\int \mu Ndt = p_f - p_i \quad ; \quad -\mu\int Ndt = 2v_x - 2\times2$$

$$-\frac{1}{3}\times12 = 2v_x - 4 \; ; \quad \therefore \; v_x = 0$$

4. Velocity of transverse wave on rope is $\sqrt{\frac{T}{\mu}}$ where T = tension in string

Variation in tension in a horizontal rotating string.

For small element of rope

$T - (T + dT) = \omega^2 x\, dm$

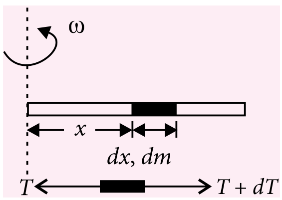
$-dT = \omega^2 x\, dm$

$$\int_{T_0}^{T} dT = \mu\omega^2\int_0^x xdx \; ; \; T - T_0 = -\frac{\mu x^2\omega^2}{2}$$

$$T = T_0 - \frac{\mu x^2\omega^2}{2}$$

At $x = L$, $T = 0$

$$0 = T_0 - \frac{\mu L^2\omega^2}{2}$$



$$T_0 = \frac{\mu\omega^2 L^2}{2} \; ; \; T = \frac{\mu\omega^2(L^2 - x^2)}{2}$$

So, velocity of wave on string

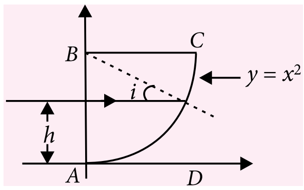
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu\omega^2(L^2-x^2)}{2\mu}} = \omega\sqrt{\frac{L^2-x^2}{2}}$$

$$\frac{dx}{dt} = \frac{\omega}{\sqrt{2}}\sqrt{L^2-x^2} \; ; \; \int_0^L \frac{dx}{\sqrt{L^2-x^2}} = \int_0^t \frac{\omega}{\sqrt{2}}dt$$

$$\left[\sin^{-1}\left(\frac{x}{L}\right)\right]_0^L = \frac{\omega}{\sqrt{2}}t \; ; \; \sin^{-1}(1) = \frac{\omega}{\sqrt{2}}t$$

$$\frac{\pi}{2} = \frac{\omega}{\sqrt{2}}t \; ; \quad \therefore \quad t = \frac{\pi}{\sqrt{2}\omega}$$

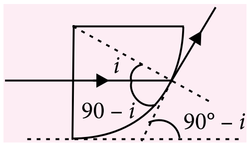
5. We can easily say that as h increases i decreases. For total internal reflection at the point of incidence on curved surface minimum value of i should be θ_c and corresponding to that h will be minimum for total internal refraction.



At just TIR, Snell's law at the curved surface

3 sin$i$ = 1 sin 90° or sin$i$ = 1/3

So, slope of tangent will be 90° – i.

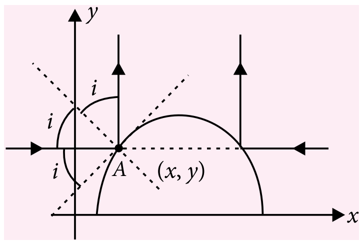


$$\frac{dy}{dx} = \tan(90° - i) = \cot i = \frac{2\sqrt{2}}{1}$$

$$\text{At } y = h, \; \frac{dy}{dx} = 2x \; ; \; 2x = 2\sqrt{2} \; ; \; x = \sqrt{2}\text{ m}$$

$$y = h = (\sqrt{2})^2 = 2\text{ m}$$

6. For reflection at general point A



2 i = 90°

So, i = 45° So, dy / dx at A should be 45°

$$y = \frac{2L}{\pi}\sin\left(\frac{\pi x}{L}\right)$$

$$\frac{dy}{dx} = \frac{2L}{\pi}\cos\left(\frac{\pi x}{L}\right)\times\frac{\pi}{L} = 2\cos\left(\frac{\pi x}{L}\right)$$

$$1 = 2\cos\left(\frac{x\pi}{L}\right)$$

$$\frac{\pi x}{L} = \frac{\pi}{3} \Rightarrow x = L/3 \text{ and } y = \frac{\sqrt{3}}{\pi}L$$

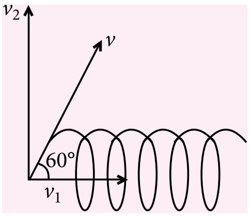
And another point will be the just opposite point so, its coordinates will be $\left(2L/3, \frac{\sqrt{3}L}{\pi}\right)$.

7. The beam of protons enters a uniform magnetic field at an angle of 60° to the magnetic field.

Its velocity $v = 4 \times 10^5$ m s^{-1}.

Resolve v into rectangular components, $v\cos 60°$ and $v\sin 60°$.

v_1 provides horizontal motion while v_2 provides circular motion to the beam of protons.



The protons therefore follow a helical path.

For circular motion :

$$\frac{mv_2^2}{r} = qv_2B \text{ or } r = \frac{mv_2}{qB} = \frac{mv\sin 60°}{qB}$$

$$\text{or} \quad r = \frac{(1.67\times10^{-27})\times(4\times10^5)\times\sqrt{3}}{(1.6\times10^{-19})\times(0.3)\times 2}$$

Pitch of helix, $p = v_1T$, where $T = \frac{2\pi}{\omega} = \frac{2\pi m}{Bq}$

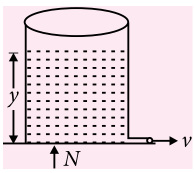
$$= (v\cos\theta)\times\left(\frac{2\pi m}{Bq}\right)$$

$$= \frac{(2\pi)(1.67\times10^{-27})(4\times10^5)\cos 60°}{(0.3)(1.6\times10^{-19})}$$

$p = 4.37 \times 10^{-2}$ m $=$ 0.044 m

8. The velocity of efflux of the liquid is given as

$$v = \sqrt{2gy}$$



The impact force of the emerging liquid on the vessel and liquid content is given by

$$F = \frac{vdm}{dt} = va\rho v = a\rho v^2 = a\rho(\sqrt{2gy})^2 = 2a\rho gy$$

Now consider the condition, to slide the vessel $f \geq F$

$\Rightarrow \ f_{min} = F = f$(say)

Force of friction $= f = F = 2a\rho gy$

$$\Rightarrow \ \mu N = 2a\rho gy \Rightarrow \mu(A\rho gy) = 2a\rho gy \ \therefore \ \mu = \frac{2a}{A}$$

9. Since the process on right is adiabatic therefore PV^{γ} = constant

$$\Rightarrow \ P_0V_0^{\gamma} = P_{final}\left(\frac{V_0}{8}\right)^{\gamma} \Rightarrow P_{final} = 32P_0$$

$$T_0V_0^{\gamma-1} = T_{final}\left(\frac{V_0}{8}\right)^{\gamma-1} \Rightarrow T_{final} = 4T_0$$

$$\left(\because \ \gamma = \frac{5}{3}\right)$$

Let volume of the left part is V_1

$$\Rightarrow 2V_0 = V_1 + \frac{V_0}{8} \Rightarrow V_1 = \frac{15V_0}{8}$$

Since number of moles on the left part remains constant therefore for the left part $\frac{PV}{T}$ = constant.

Final pressure on both sides will be same.

$$\frac{P_0V_0}{T_0} = \frac{P_{final}V_1}{T_{final}} \Rightarrow T_{final} = 60T_0$$

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta Q = \frac{5nR}{2}(60T_0 - T_0) + n\frac{3R}{2}(4T_0 - T_0)$$

$$\Delta Q = \frac{5nR}{2}\times 59T_0 + \frac{3nR}{2}\times 3T_0 = 152\, nRT_0$$

10. (a) Both the sources produce maximum at the point P.

Thus, $I_{max} = I_0 = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

Since the sources are identical, therefore, $I_1 = I_2 = I$.

$I_0 = 4I$ or $I = I_0/4$

(b) Now $I_1 = 0.64I = 0.16\ I_0$

and $I_2 = I = 0.25\ I_0$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 0.81I_0$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = 0.01I_0$$

(c) Now $I_1 = (1 - 0.64)\ I_0 = 0.36\ I = 0.09\ I_0$

and $I_2 = I = 0.25\ I_0$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 0.64\,I_0$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = 0.04\,I_0$$

◈◈

MPP-6 CLASS XII ANSWER KEY

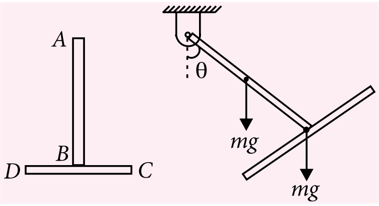
1. (b) 2. (a) 3. (c) 4. (a) 5. (c)
6. (c) 7. (a) 8. (c) 9. (c) 10. (d)
11. (b) 12. (a) 13. (a) 14. (a) 15. (c)
16. (b) 17. (d) 18. (a) 19. (a) 20. (c, d)
21. (a, b, c, d) 22. (b, d) 23. (b, c, d)
24. (6) 25. (2) 26. (4) 27. (b) 28. (a)
29. (c) 30. (b)

PHYSICS MUSING

SOLUTION SET-50

1. **(a,c) :** The basic idea is same as of simple pendulum. It's just there are two forces, one weight of vertical rod and other weight of horizontal rod due to which torque will be produced.

Moment of inertia of rod AB about A is $I_{AB} = \frac{ml^2}{3}$



and moment of inertia of rod CD about A is

$$I_{CD} = \frac{ml^2}{12} + ml^2 = \frac{13ml^2}{12}$$

Total moment of inertia of the system about A,

$$I = \frac{ml^2}{3} + \frac{13ml^2}{12} = \frac{17ml^2}{12}$$

Restoring torque about A

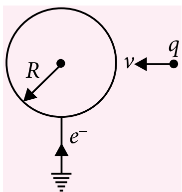
$$\tau_R = mg\sin\theta\frac{l}{2} + mgl\sin\theta = \frac{3mgl}{2}\sin\theta$$

For small value of θ, $\tau_R = \left(\frac{3mgl}{2}\right)\theta = k\theta$

Therefore, time period of pendulum for small angular oscillations,

$$T = 2\pi\sqrt{\frac{I}{k}} = 2\pi\sqrt{\frac{17ml^2 \times 2}{12 \times 3mgl}} = 2\pi\sqrt{\frac{17l}{18g}}$$

2. **(a,c) :** It is based on electric potential energy of a uniformly charge sphere. When the potential on the surface of earth has to be zero, so sphere must get some electrons from the earth to keep the potential on it to be zero.

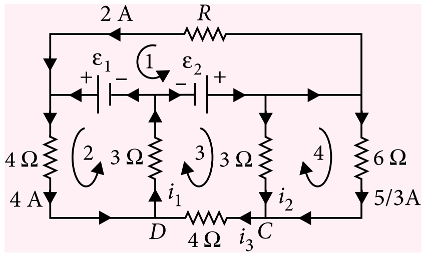


The potential of the grounded sphere has to be zero, due to particle charge the potential of the sphere is positive so negative charge must be possessed by sphere so that total potential of sphere becomes zero. This negative charge is acquired by sphere from the ground, hence current flows into the ground. As the charged particle comes nearer to sphere the potential of sphere due to positive charge increases and hence more electron per unit time will flow from ground to sphere and thus magnitude of current increases.

3. **(a,b,c,d) :** Given that $V_A - V_B = 10$ V

$\Rightarrow$ Current flowing through 6 Ω resistance

$$= \frac{V_A - V_B}{6} = \frac{10}{6}\text{A} = \frac{5}{3}\text{A}$$



At junction C, $i_3 = i_2 + \frac{5}{3}$A ...(i)

At junction D, $i_1 = i_3 + 4$ A ...(ii)

In loop 4,

$6 \times \frac{5}{3} = 3i_2 \Rightarrow i_2 = \frac{10}{3}$A ...(iii)

From equations (i) and (iii)

$$i_3 = \left(\frac{10}{3} + \frac{5}{3}\right)\text{A} = 5\,\text{A}$$

Hence option (a) is correct.

Putting value of i_3 in equation (ii), $i_1 = 9$A

In loop 2,

$\varepsilon_1 = (4 \times 4) + 3i_1 = 16 + (3 \times 9) = 43$ V

Hence option (b) is correct.

In loop 3,

$\varepsilon_2 = 3i_2 + 4i_3 + 3i_1 = 10 + 20 + 27 = 57$ V

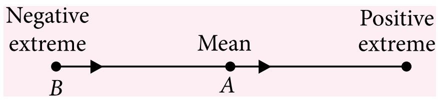
Hence option (c) is correct.

In loop 1, $\varepsilon_2 - \varepsilon_1 = 2R$

$$\therefore \quad R = \frac{(57 - 43)}{2} = 7\ \Omega$$

Hence option (d) is correct.

4. **(a, b, c) :** At $t = 0$, initial position of both particles is indicated below.



For $\omega_B = 2\omega_A \Rightarrow \frac{1}{T_B} = \frac{2}{T_A} \Rightarrow T_A = 2T_B$

So, they can reach at positive extreme at same time

So for $\omega_B = 2\omega_A$, they will meet with zero speed.

For $\omega_B > 2\omega_A$, $T_A > 2T_B$

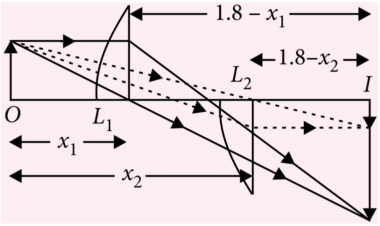
B will catch A before reaching extreme so they will meet and both will start moving in same direction.

For $\omega_B < 2\omega_A \Rightarrow T_B < 2T_A$

Now, it is clear that they will meet when A will be returning from the extreme. So, at the time of meet they will be moving in opposite direction.

5. **(a,b,c) :** Given that magnification at one half is 2. For L_1 object is close so that will be the point with magnification -2 so other half will have $m = -1/2$

For magnification $m = v/u$



For $L_1, m = -2 = \frac{(1.8 - x_1)}{-x_1}$; $x_1 = -0.6$ m

For $L_2, m = -\frac{1}{2} = \frac{(1.8 - x_2)}{-x_2}$; $x_2 = -1.2$ m

$\therefore$ We know that, $m = \frac{f}{f+u}$

$$-2 = \frac{f}{f+0.6} \Rightarrow f = 0.4 \text{ m}$$

6. **(2) :** Applying Bernoulli's equations at pipe (1), (2) and (3)

$$p_3 + \frac{1}{2}\rho v_3^2 = p_2 + \frac{1}{2}\rho v_2^2 = p_1 + \frac{1}{2}\rho v_1^2$$

$$gh_3 + \frac{v_3^2}{2} = gh_2 + \frac{v_2^2}{2} = gh_1 + \frac{v_1^2}{2} [\text{using } p = p_0 + \rho gh] \quad \text{...(i)}$$

Applying equation of continuity at (3), (2) and (1),

$v_3 A_3 = v_2 A_2 = v_1 A_1$

$v_3 \times A = v_2 \times 2 = v_1 \times 1$ ...(ii)

From eqn. (i), we get

$$gh_2 + \frac{v_2^2}{2} = gh_1 + \frac{v_1^2}{2}; 10 \times \frac{35}{100} + \frac{v_2^2}{2} = 10 \times \frac{20}{100} + \frac{v_1^2}{2}$$

$v_2 = 1 \text{ m s}^{-1}, v_1 = 2 \text{ m s}^{-1}$

From eqn. (i), we get

$$gh_3 + \frac{v_3^2}{2} = gh_2 + \frac{v_2^2}{2};\ 10 \times \frac{37.5}{100} + \frac{v_3^2}{2} = 10 \times \frac{35}{100} + \frac{1}{2}$$

$$\Rightarrow v_3^2 = \frac{1}{2} \Rightarrow v_3 = \frac{1}{\sqrt{2}} \text{ m s}^{-1}$$

Comparing with speed of water at cross-section 3, we have $x = 2$

7. **(2) :** According to Moseley's law, for K_α X-rays emission, $\sqrt{\upsilon} = a(Z-1)$

$$\Rightarrow (Z-1)^2 \propto \upsilon \text{ or } (Z-1)^2 \propto \frac{1}{\lambda}$$

$$\Rightarrow \frac{(Z_{Mo}-1)^2}{(Z_{Fe}-1)^2} = \frac{\lambda_{Fe}}{\lambda_{Mo}}$$

$$\Rightarrow \lambda_{Fe} = \lambda_{Mo}\frac{(Z_{Mo}-1)^2}{(Z_{Fe}-1)^2} = 0.75 \times \left(\frac{41}{25}\right)^2 \text{ Å}$$

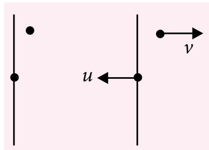
$\therefore \lambda_{Fe} \approx 2$ Å

8. **(9) :** There is no external force so linear momentum will be conserved.

$2m(0) + m(0) = -2mu + mv$

$v = 2u$ ...(i)

Conservation of angular momentum with respect to centre of rod. *i.e.*, $L_i = L_f$



$$\frac{2ml^2}{12} \times \omega + 0 = 0 + mvx \Rightarrow \frac{2\omega l^2}{12} = vx \quad \text{...(ii)}$$

Coefficient of restitution, $e = \frac{u+v}{\omega x}$

Since collision is elastic, $e = 1$.

$\therefore v + u = \omega x$ ...(iii)

From eqns. (i) and (iii) we get

$$3u = \omega x \Rightarrow \omega = \frac{3u}{x}$$

From eqns. (i), (ii) and (iii), we get

$$\frac{2 \times 3u}{12x} \times l^2 = 2ux \Rightarrow x^2 = \frac{l^2}{4}$$

$$\Rightarrow x^2 = \frac{90 \times 90}{4} = (45)^2 \text{ or } x = 45 \text{ cm}$$

Hence, $\frac{x}{5} = \frac{45}{5} = 9$ cm

9. **(c) :** Discharging current through resistance

$$q = Q_0 e^{-t/RC};\ i = -\frac{dq}{dt} = \frac{Q_0}{RC}e^{-t/RC}$$

Charge at capacitor at $t = 2.5 \ln(4)$

$$q = Q_0 e^{-\frac{2.5\ln(4)\times 60}{2000\times 75\times 10^{-3}}} = \frac{Q_0}{4}$$

Loss in energy of capacitor,

$$\Delta U = \frac{Q_0^2}{2C} - \frac{Q_0^2/16}{2C} = \frac{Q_0^2}{2C} \times \frac{15}{16} = \frac{15Q_0^2}{32C},$$

where $Q_0 = CV_0 = 75 \times 10^{-3} \times \frac{640}{3} = 16$ C

Heat supplied to gas is equal to heat loss across resistance.

$$\Delta U = \frac{15Q_0^2}{32C} = \frac{15 \times 16 \times 16}{32 \times 75 \times 10^{-3}} = 1600 \text{ J} = 1.6 \text{ kJ}$$

10. **(a) :** Gas expands at constant pressure so work done will be $W = nR\Delta T$

$\Rightarrow W = 1 \times 8.3 \times 72 = 0.598 \text{ kJ} = 0.6 \text{ kJ}$

$\Delta U = \Delta Q - W = 1.6 \text{ kJ} - 0.6 \text{ kJ} = 1 \text{ kJ}$

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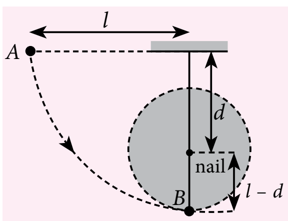
OLYMPIAD PROBLEMS

1. Assume that the de Broglie waves associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2 Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. The energy of the electrons in electron volt and the least value of d for which the standing wave of the type described above can form.
(a) 150 eV and 0.6 nm (b) 160 eV and 0.2 Å
(c) 160 eV and 0.7 nm (d) 150 eV and 0.5 Å

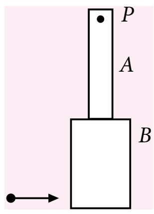
2. In the given system, when the ball of mass m is released, it will swing down the dotted arc. A nail is located at a distance d below the point of suspension.



(a) Velocity when it reaches the lowest point in its swing is $\sqrt{3gl}$.
(b) If the ball is to swing completely around a circle centered along the nail, d must at 0.4 l.
(c) If d = 0.6 l, the change in tension in the string just after it touches the nail is 3 mg.
(d) None of these

3. Three discs A, B and C having radii 2, 4, and 6 cm respectively are coated with carbon black. Wavelengths for maximum intensity for the three discs are 300, 400 and 500 nm respectively. If Q_A, Q_B and Q_C are powers emitted by A, B and C respectively, then
(a) Q_A will be maximum
(b) Q_B will be maximum
(c) Q_C will be maximum
(d) $Q_A = Q_B = Q_C$

4. Two uniform rods A and B of length 0.6 m each and of masses 0.01 kg and 0.02 kg respectively are rigidly joined end to end. The combination is pivoted at the lighter end, P as shown in figure. such that it can freely rotate about point P in a vertical plane. A small object of mass 0.05 kg, moving horizontally, hits the lower end of the combination and sticks to it. The velocity of the object so that the system could just be raised to the horizontal position is

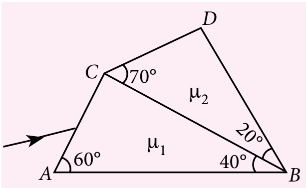


(a) 6.3 m s^{-1} (b) 7.3 m s^{-1}
(c) 85 m s^{-1} (d) None of these

5. A proton accelerated by a potential difference V gets into the uniform electric field of a parallel-plate capacitor whose plates extend over a length l in the motion direction. The field strength varies with time as $E = at$, where a is a constant. Assuming the proton to be non-relativistic, find the angle between the motion directions of the proton before and after its flight through the capacitor; the proton gets in the field at the moment $t = 0$. The edge effects are to be neglected.
(a) $al^2\sqrt{\dfrac{m}{2eV^3}}$ (b) $\dfrac{al^2}{4}\sqrt{\dfrac{m}{2eV^3}}$
(c) $a\sqrt{\dfrac{l^4 m}{8eV^2}}$ (d) $a\sqrt{\dfrac{l^4 m}{8eV^3}}$

6. A prism of refractive index μ_1 and another prism of refractive index μ_2 are stuck together without any gap as shown in figure. The angles of the prisms are as shown. μ_1 and μ_2 depend on λ, the wavelength of light, according to $\mu_1 = 1.20 + \dfrac{10.8\times10^4}{\lambda^2}$ and $\mu_2 = 1.45 + \dfrac{1.80\times10^4}{\lambda^2}$ where λ is in nm.

The wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface and the angle of incidence i on the face AC such that the deviation produced by the combination of prisms is minimum respectively be



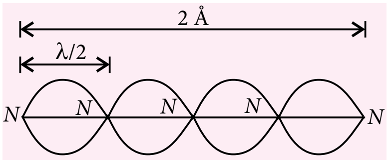
(a) 600 nm and $\tan^{-1}(5/3)$
(b) 500 nm and $\sin^{-1}(4/3)$
(c) 600 nm and $\sin^{-1}(2/3)$
(d) 600 nm and $\sin^{-1}(3/4)$

SOLUTIONS

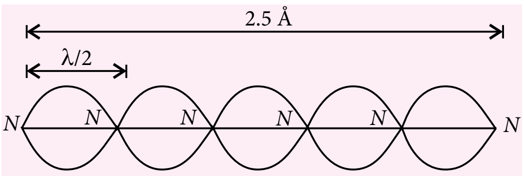
1. (d): Let p loops be formed in distance of 2 Å.

$\therefore$ $(p + 1)$ loops will be formed in distance of 2.5 Å.

Any intermediate value of d is not allowed



p loops



$(p + 1)$ loops

Two neighbouring nodes are separated by $\lambda/2$.

$$\therefore \quad p\left(\frac{\lambda}{2}\right) = 2 \text{ Å} ; (p+1)\frac{\lambda}{2} = 2.5 \text{ Å}$$

$$\therefore \quad \frac{p+1}{p} = \frac{2.5}{2} = \frac{5}{4} \quad \text{or} \quad p = 4$$

$$\therefore \quad \lambda = \frac{2\times 2}{4} = 1 \text{ Å} \quad \therefore \quad \lambda = 1 \text{ Å}$$

(i) de Broglie wavelength $= \lambda = \dfrac{h}{p}$

or $\lambda = \dfrac{h}{\sqrt{2mK}}$ where, K denotes kinetic energy

$\therefore$ $K = \dfrac{h^2}{2m\lambda^2}$ where, $\lambda = 1$ Å $= 10^{-10}$ m.

$$= \frac{(6.63\times10^{-34})^2}{2\times(9.1\times10^{-31})\times(10^{-10})^2} \text{ J} \approx 151 \text{ eV}$$

(ii) Distance d will be minimum when only one loop is formed.

$$\therefore \quad d_{min} = \frac{\lambda}{2} = \frac{1}{2} \text{ Å} = 0.5 \text{ Å}$$

2. (c) : (a) Radius of the circle centered at nail $= l - d$.

To complete the circle centered at nail, the speed at the bottom must be at least $= \sqrt{5g(l-d)}$

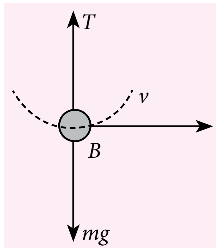
$\because$ Loss in PE = Gain in KE

$$mgl = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{2gl}$$

(b) To complete the circle : $\sqrt{2gl} = \sqrt{5g(l-d)}$

$$\Rightarrow \quad 5l - 5d = 2l \quad \Rightarrow \quad d = \frac{3}{5}l = 0.6\, l$$

(c) Just before touching the nail, the ball is moving in a circle of radius l.



$$\Rightarrow \quad T_{before} - mg = \frac{mv^2}{l}$$

$$= mg + \frac{mv^2}{l}$$

$$= mg + 2mg = 3mg$$

Just after touching the nail, the ball is moving in a circle of radius $(l - d)$.

$$T_{after} - mg = \frac{mv^2}{(l-d)} = mg + \frac{mv^2}{l-d}$$

$$= mg + \frac{m(2gl)}{0.4l} \quad \text{or} \quad T_{after} = 6mg$$

Hence, the tension in the string changes from $3mg$ to $6mg$ as it touches the nail.

3. (b): According to Wien's displacement law, $\lambda T = b$ = Wien's constant.

$$\therefore \quad \lambda_A T_A = b \quad \text{or} \quad T_A = \frac{b}{3\times10^{-7}} = \frac{z}{3}$$

where $z = (b \times 10^7)$

Similarly, $T_B = \dfrac{z}{4}$ and $T_C = \dfrac{z}{5}$

Again, According to Stefan's law,

Q = Power radiated by black body $= A\sigma T^4$

where, A = area of disc $= \pi R^2$.

$$Q_A = (\pi R_A^2) \times \sigma \times (T_A)^4$$

$$\text{or} \quad Q_A = \pi(2 \times 10^{-2})^2 \times \sigma \times \left(\frac{z}{3}\right)^4$$

$$Q_A = \pi\sigma \times 10^{-4} \times z^4 \times \frac{2^2}{3^4}$$

Put $\pi\sigma \times 10^{-4} \times z^4 = k$ = constant

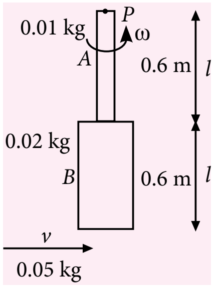
$$\text{or} \quad Q_A = \frac{4k}{81} = 0.049\, k$$

Similarly, $Q_B = \frac{k \times (4)^2}{(4)^4} = \frac{k}{16} = 0.062\ k$

$Q_C = \frac{k \times (6)^2}{(5)^4} = \frac{36\ k}{625} = 0.057\ k.$

Hence Q_B is maximum.

4. **(a) :** The combination of the two rods, A and B, is pivoted at P. It can freely rotate about point P in a vertical plane. A mass 0.05 kg, moving with horizontal velocity v, hits the lower end of the combination and sticks to it.



Angular momentum before collision = $mv \times 2l$

Angular momentum after collision = $I\omega$

I = Moment of inertia of system about P

or $I = I_1 + I_A + I_B$

or $I = m(2l)^2 + m_A\left[\frac{l^2}{12} + \left(\frac{l}{2}\right)^2\right] + m_B\left[\frac{l^2}{12} + \left(\frac{l}{2} + l\right)^2\right]$

or $I = l^2\left[4m + \frac{m_A}{3} + \frac{7m_B}{3}\right]$

or $I = (0.6)^2\left[4 \times 0.05 + \frac{0.01}{3} + \frac{7 \times 0.02}{3}\right] = 0.09\ \text{kg m}^2$

Angular momentum is conserved.

$\therefore\ I\omega = mv \times 2l$

or $\omega = \frac{mv \times 2l}{I} = \frac{0.05 \times v \times 2 \times 0.6}{0.09} = 0.67v$

$\therefore$ Loss of KE = Gain of PE

$\frac{1}{2} I\omega^2 = (mg \times 2l) + \left(m_A \frac{l}{2} g\right) + \left(m_B \frac{3l}{2} g\right)$

$\frac{1}{2} \times 0.09 \times (0.67v)^2$

$= lg\left[(2 \times 0.05) + \left(\frac{0.01}{2}\right) + \left(\frac{0.02 \times 3}{2}\right)\right]$

$\therefore\ 0.02v^2 = 0.6 \times 9.8 \times (0.135) = 0.8$

or $v^2 = 40$ or $v = 6.3\ \text{m s}^{-1}$

5. **(b) :** The electric field inside the capacitor varies with time as, $E = at$.

Hence, electric force on the proton, $F = eat$

$\therefore$ Acceleration of the proton, $w = \frac{eat}{m}$

Now, if t is the time, elapsed by the proton, between the plates, then $t = \frac{l}{v_{||}}$, as no acceleration is effective in this direction. (Here, $v_{||}$ is velocity along the length of the plate.)

$\because$ acceleration, $w = \frac{dv_\perp}{dt}$

or $\int_0^{v_\perp} dv_\perp = \int_0^t w\,dt$, as initially, the component of velocity in the direction, perpendicular to plates, was zero.

or $v_\perp = \int_0^t \frac{eat}{m} dt = \frac{ea}{m}\frac{t^2}{2} = \frac{ea}{2m}\frac{l^2}{v_{||}^2}$

Now, $\tan\alpha = \frac{v_\perp}{v_{||}} = \frac{ea\,l^2}{2mv_{||}^3}$

Velocity acquired by an electron passing through a potential difference V is, $v_{||} = \left(\frac{2eV}{m}\right)^{\frac{1}{2}}$,

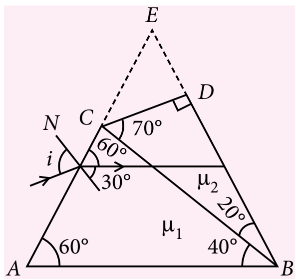
$\tan\alpha = \frac{eal^2}{2m\left(\frac{2eV}{m}\right)^{\frac{3}{2}}} = \frac{al^2}{4}\sqrt{\frac{m}{2eV^3}}$

6. **(d) :** The given prism-system may be taken as a part of an equilateral prism as shown in the figure.

(a) The incident ray will not deviate at BC if $\mu_1 = \mu_2$

$\because\ \mu_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda_0^2}$

$\mu_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda_0^2}$



$\therefore\ \frac{9 \times 10^4}{\lambda_0^2} = 0.25$

or $\lambda_0 = 600$ nm

(b) At the position of minimum deviation, the refracted ray travels parallel to base AB of the deemed prism AEB.

$\therefore\ r = 30°$, (from geometry)

$\therefore\ \mu_1 = \frac{\sin i}{\sin r}$ or $\sin i = \mu_1 \sin r$

or $\sin i = \left[1.20 + \frac{10.8 \times 10^4}{(600)^2}\right](\sin 30°)$

or $\sin i = \frac{3}{4}$ or $i = \sin^{-1}\left(\frac{3}{4}\right)$.

◆◆

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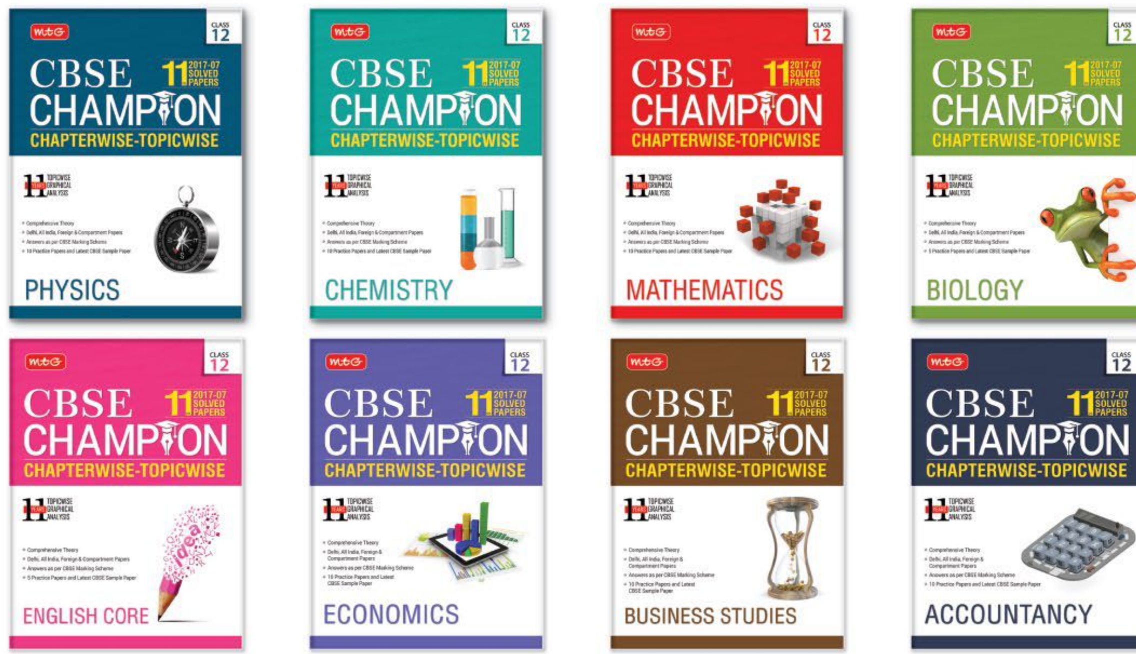

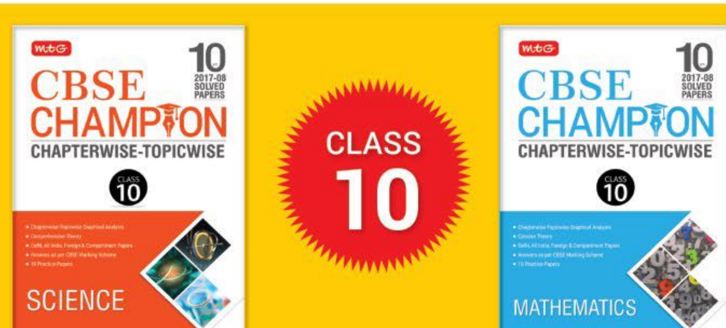

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