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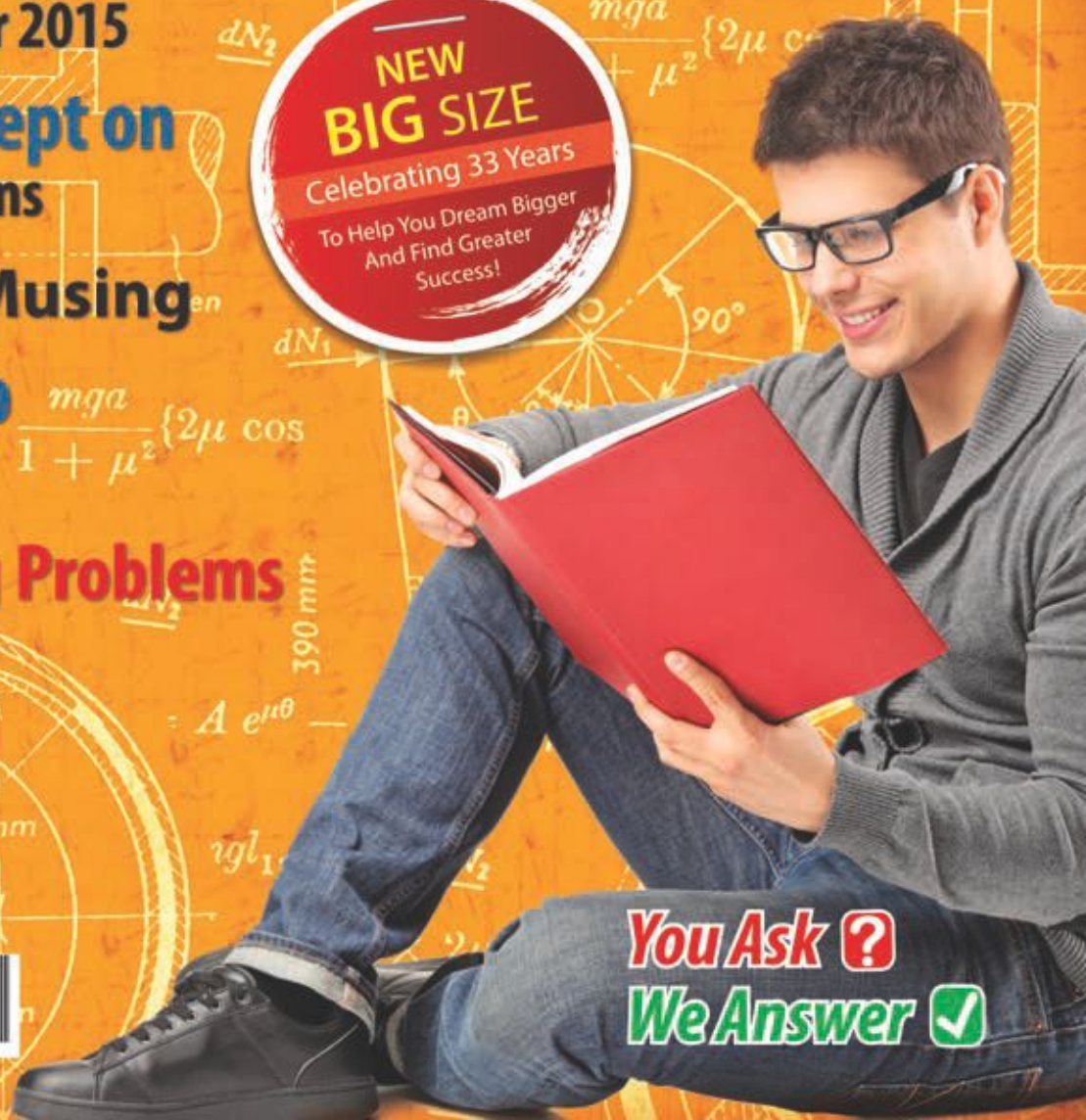
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Editor : Anil Ahlawat

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How to Excel in any Competitive Exams, at any Level

It was a very highly inspiring advice given to the teachers and students by the Hon'ble Prime Minister, Shri Narendra Modi ji himself in a recent radio-talk. Many books have been written on education by very learned authors. These are addressed mainly to higher level students and professors. But the difficulties of the students are not spelt out. The Prime Minister's advice reaches directly the students of all levels.

The first is the fear of exams which all of us have felt. To combat this, the advice is to hold a week-long examination festival, two times a year, with satirical poems on exams, cartoon contents and lectures on the psychological effects of exams with debates to pepper the lectures.

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Anil Ahlawat
Editor

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Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIIMS / Other PMTs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / various PMTs. The detailed solutions of these problems will be published in next issue of Physics For You.

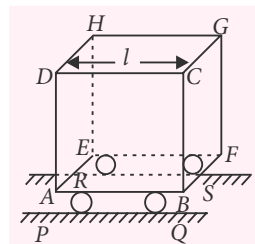
The readers who have solved five or more problems may send their solutions. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

By : Akhil Tewari

PROBLEM Set 21

- A uniform rod of mass m and length l starts rotating with constant angular acceleration α in a horizontal plane about a fixed vertical axis passing through one end. The horizontal component of the net force exerted on the rod by the axis when it has rotated by an angle $\pi/2$, is
 - $m\alpha \frac{l}{2}$
 - $m\alpha \frac{l}{2} \sqrt{1 + \pi^2}$
 - $\frac{m\pi\alpha l}{2}$
 - none of these
- A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R . One of the arcs AB of the ring subtends an angle θ at the centre. The value of the magnetic induction at the centre due to the current in the ring is
 - proportional to $2(180^\circ - \theta)$
 - inversely proportional to r
 - zero, only if $\theta = 180^\circ$
 - zero, for all values of θ .
- A particle is projected with a speed u in air at angle θ with the horizontal. The particle explodes at the highest point of its path into two equal fragments, one of the fragments moving up straight with a speed u . The difference in time in which the two particles fall on the ground is (Assume it is at a height H at the time of explosion.)
 - $\frac{2u}{g}$
 - $\frac{u}{g} \sqrt{u^2 - 2gH}$
 - $\frac{1}{g} \sqrt{u^2 + 2gH}$
 - $\frac{2}{g} \sqrt{u^2 + 2gH}$
- Consider the cube shaped carriage ABCDEFGH of side l and a mass M and it can slide over two frictionless rails PQ and RS. A shot of mass m is thrown from corner A such that it lands at corner F. The angle of projection as seen from the carriage is 45° . While the shot is in the air, the velocity of carriage as seen from the ground is
 - $\frac{m\sqrt{gl\sqrt{2}}}{2(M+m)}$
 - $\frac{m\sqrt{2gl}}{2(M+m)}$
 - $\frac{m\sqrt{gl}}{(M+m)}$
 - $\frac{m\sqrt{2gl}}{(M+m)}$



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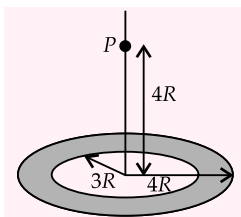
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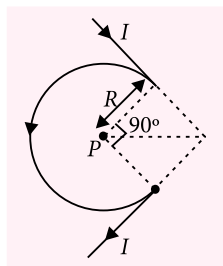
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- (a) the acceleration of the particle will continuously keep on increasing with time
 - (b) particle will execute simple harmonic motion
 - (c) the force on the particle will continuously keep on decreasing with time
 - (d) the acceleration of particle will vary sinusoidally with time.
6. Two rods of equal lengths and equal cross-sectional areas are made of materials whose Young's moduli are in the ratio of 2:3. They are suspended and loaded with the same mass. When stretched and released, they will oscillate with time periods in the ratio of
- (a) $\sqrt{3} : \sqrt{2}$ (b) 3 : 2
(c) $3\sqrt{3} : 2\sqrt{2}$ (d) 9 : 4
7. A thin uniform annular disc (see figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is

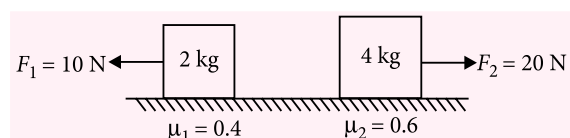


- (a) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$
- (c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2}-1)$
8. An ideal gas is expanded so that amount of heat given is equal to the decrease in internal energy. Find the adiabatic exponent if the gas undergoes the process $TV^{1/5} = \text{constant}$.
- (a) $7/5$ (b) $6/5$
- (c) $8/5$ (d) None of these
9. The magnetic field at the point P is given by



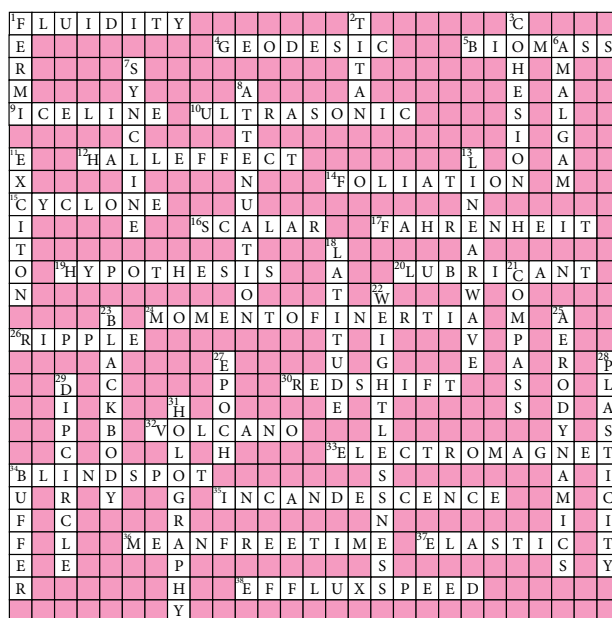
- (a) $\frac{\mu_0 I}{8\pi R}(3\pi+4)\odot$ (b) $\frac{\mu_0 I}{8\pi R}(3\pi+4)\otimes$
(c) $\frac{\mu_0 I}{8\pi R}(3\pi-4)\odot$ (d) $\frac{\mu_0 I}{8\pi R}(3\pi-4)\otimes$

- 10.** Two blocks of masses 2 kg and 4 kg are connected through a massless inextensible string. The co-efficient of friction between 2 kg block and ground is 0.4 and the coefficient of friction between 4 kg block and ground is 0.6. Two forces $F_1 = 10$ N and $F_2 = 20$ N are applied on the blocks as shown in the figure. Calculate the frictional force between 4 kg block and ground. (Assume initially the tension in the string was just zero before forces F_1 and F_2 were applied.)



- (a) 24 N (b) 8 N
(c) 18 N (d) 10 N

SOLUTION OF MARCH 2015 CROSSWORD



WINNER (March 2015)

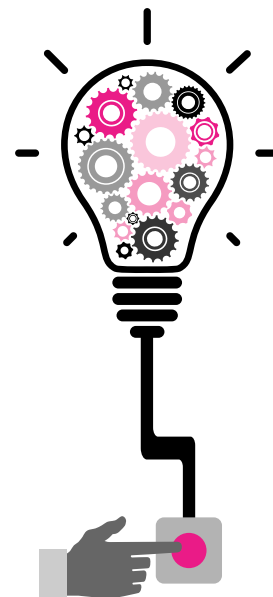
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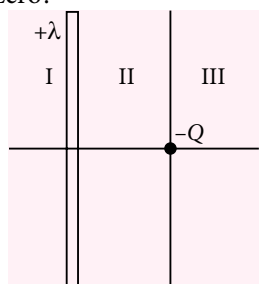
THOUGHT PROVOKING PROBLEMS



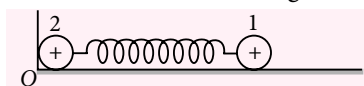
By : Prof. Rajinder Singh Randhawa*

ELECTROSTATICS

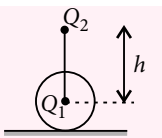
1. An infinitely long conducting wire of charge density $+\lambda$ and a point charge $-Q$ are at a distance from each other. In which of the three regions (I, II or III) are there points that (a) lie on the line passing through point charge perpendicular to the conductor and (b) at which the field is zero?



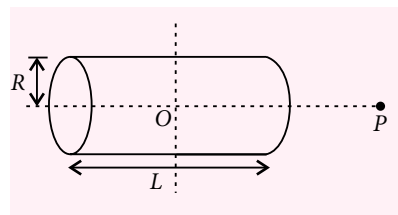
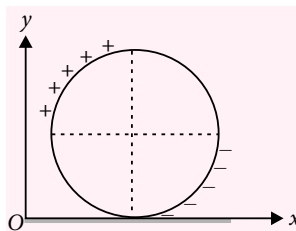
2. Two small identical balls lying on a horizontal plane are connected by a weightless spring. One ball (ball 2) is fixed at O and the other (ball 1) is free. The balls are charged identically as a resultant of which the spring length increases $\eta = 2$ times. Determine the change in frequency?



3. A point charge $Q_1 = -125 \mu\text{C}$ is fixed at the centre of an insulated disc of mass 1 kg. The disc rests on a rough horizontal plane. Another charge $Q_2 = 125 \mu\text{C}$ is fixed vertically above the centre of the disc at a height $h = 1 \text{ m}$. After the disc is displaced slightly in the horizontal direction, find the time period of oscillation of disc.



4. A non-conducting ring of mass m and radius R , the charge per unit length λ is shown in figure. It is then placed on a rough non-conducting horizontal plane. At time $t = 0$, a uniform electric field $\vec{E} = E_0 \hat{i}$ is switched on and the ring starts rolling without sliding. Find the frictional force acting on the ring.
5. A non-conducting solid cylindrical rod of length L and radius R has uniformly distributed charge Q . Find the electric field at point P , a distance L from the centre of the rod.



SOLUTION

1. In the region II, the electric field of wire and point charge point in the same direction, +ve x -axis. So no point can exist where the field is zero. Now, we take a point to the right of the point charge at a distance x from it. Resultant field at this point is

$$\vec{E}_R = \frac{\lambda}{2\pi\epsilon_0(x+a)} \hat{i} + \frac{Q}{4\pi\epsilon_0 x^2} (-\hat{i})$$

Resultant field is zero if

$$\frac{\lambda}{(x+a)} = \frac{Q}{2x^2}$$

$$\text{or } 2\lambda x^2 = Qx + Qa \text{ or } x^2 - \frac{Q}{2\lambda}x - \frac{Qa}{2\lambda} = 0$$

On solving the quadratic equation in x , we get

$$x = \frac{Q}{4\lambda} \pm \sqrt{\frac{Q^2}{16\lambda^2} + \frac{Qa}{2\lambda}}$$

Here, there is only one value of x (with +ve sign) because -ve sign would mean that the point is to the left of point charge.

Now we take a point to the left of wire at a distance x from it.

The resultant field is

$$\vec{E}_R = \frac{\lambda}{2\pi\epsilon_0 x} (-\hat{i}) + \frac{Q}{4\pi\epsilon_0 (a+x)^2} \hat{i}$$

The two fields point in the opposite directions, so resultant field can be zero if,

$$\frac{\lambda}{2\pi\epsilon_0 x} = \frac{Q}{4\pi\epsilon_0 (a+x)^2}$$

$$\text{or } x^2 - \left(\frac{Q}{2\lambda} - 2a\right)x + a^2 = 0$$

On solving the quadratic equation in x , we get

$$x = \frac{1}{2} \left(\frac{Q}{2\lambda} - 2a \right) \pm \sqrt{\frac{1}{4} \left(\frac{Q}{2\lambda} - 2a \right)^2 - a^2}$$

If the discriminant of the quadratic equation is real, we have two points where the field is zero. Discriminant is +ve for $Q \geq 8a\lambda$.

2. When the balls are uncharged, the frequency of oscillation is

$$v_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

where k is the force constant of the spring and m = mass of the oscillating ball (ball 1).

When the balls are identically charged,

$$\frac{1}{4\pi\epsilon_0} \frac{q^2}{(\eta l)^2} = k(\eta l - l) = kl(\eta - 1) \quad \dots(i)$$

where l is the natural length of spring and ηl is the new length of spring after its extension.

$$\text{or } l^3 = \frac{q^2}{4\pi\epsilon_0 \eta^2 (\eta - 1)k} \quad \dots(ii)$$

When the ball 1 is displaced by a small distance x from the equilibrium position to the right, the unbalanced force to the right is given by

$$F_r = \frac{1}{4\pi\epsilon_0} \frac{q^2}{(\eta l + x)^2} - k(\eta l + x - l)$$

Using Newton's law,

$$m \frac{d^2 x}{dt^2} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{\eta^2 l^2} \left(1 + \frac{x}{\eta l}\right)^{-2} - kl(\eta - 1) - kx$$

Expanding binomially and using eqn. (i), we get

$$m \frac{d^2 x}{dt^2} = - \left[\frac{1}{4\pi\epsilon_0} \cdot \frac{2q^2}{\eta^3 l^3} + k \right] x$$

Using eqn. (ii), we get

$$m \frac{d^2 x}{dt^2} = - \left[\frac{1}{4\pi\epsilon_0} \cdot \frac{2q^2}{\eta^3 \frac{q^2}{4\pi\epsilon_0 \eta^2 (\eta - 1)k}} + k \right] x$$

$$= - \left[\frac{2(\eta - 1)}{\eta} k + k \right] x = - \left(\frac{3\eta - 2}{\eta} \right) kx$$

$$\therefore \frac{d^2 x}{dt^2} = - \left(\frac{3\eta - 2}{\eta} \right) \frac{k}{m} x \quad \dots(iii)$$

By definition of simple harmonic motion,

$$\frac{d^2 x}{dt^2} = -\omega^2 x \quad \dots(iv)$$

From eqns. (iii) and (iv), we get

$$\omega^2 = \left(\frac{3\eta - 2}{\eta} \right) \frac{k}{m} \Rightarrow v = \frac{1}{2\pi} \sqrt{\left(\frac{3\eta - 2}{\eta} \right) \frac{k}{m}}$$

$$\therefore \frac{v}{v_0} = \sqrt{\frac{3\eta - 2}{\eta}}$$

Thus frequency is increased $\sqrt{\frac{3\eta - 2}{\eta}}$ times.

Here, $\eta = 2$ so frequency increases $\sqrt{2}$ times.

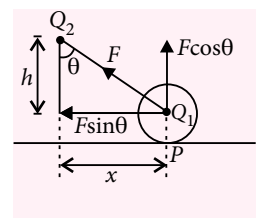
3. Let the radius of the disc be R . If the disc is displaced x , then $\theta = x/R$. The restoring torque τ about the point of contact of the disc with ground,

$$\tau_P = (F \sin \theta)R = I\alpha$$

$$= \left[\frac{MR^2}{2} + MR^2 \right] \alpha$$

$$\text{where, } F = \frac{Q^2}{4\pi\epsilon_0 (h^2 + x^2)}$$

$$\text{and } \sin \theta = \frac{x}{\sqrt{h^2 + x^2}}$$



$$\text{Hence, } \frac{Q^2 x R}{4\pi\epsilon_0 (h^2 + x^2)^{3/2}} = \left[\frac{MR^2}{2} + MR^2 \right] \alpha$$

$$\text{or } \alpha = \frac{Q^2 x}{6\pi\epsilon_0 MR (h^2 + x^2)^{3/2}}$$

$$\text{For } x < h, \alpha \approx -\frac{Q^2 x}{6\pi\epsilon_0 MR h^3} = -\frac{Q^2 \theta R}{6\pi\epsilon_0 MR h^3}$$

Negative sign is being introduced because angular acceleration and angular displacement are opposite to each other.

$$\text{Thus, } \alpha = -\frac{Q^2 \theta}{6\pi\epsilon_0 M h^3}$$

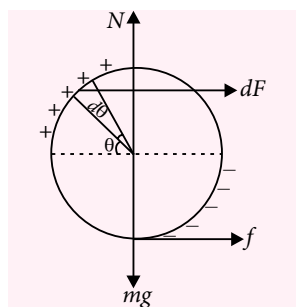
$$\text{Hence, } \omega = \sqrt{\frac{Q^2}{6\pi\epsilon_0 M h^3}}$$

$$\text{or } T = 2\pi \sqrt{\frac{6\pi\epsilon_0 M h^3}{Q^2}} = 2\pi \frac{h}{Q} \sqrt{6\pi\epsilon_0 M h}$$

On substituting the given values, we get

$$T = 0.6 \text{ s}$$

4. Consider a differential element subtending an angle $d\theta$ at the centre and at angle θ as shown in figure.



$$dF = \lambda R d\theta E_0$$

A force of same magnitude but in opposite direction acts on a corresponding element in the region of negative charge.

∴ Equation of motion for pure rolling is

$$\int_0^{\pi/2} 2\lambda R d\theta E_0 R \sin \theta - fR = (mR^2) \alpha$$

$$\text{or } 2\lambda R^2 E_0 - fR = mR^2 \alpha \quad \dots(i)$$

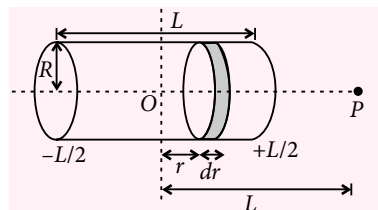
$$\text{and } f = ma \quad \dots(ii)$$

$$\text{and } a = R\alpha \quad \dots(iii)$$

Solving eqns. (i), (ii) and (iii), we get

$$f = \lambda R E_0 \text{ along +ve } x\text{-axis.}$$

5.



Consider a disc of radius R of thickness dr at a distance r from the centre O of the cylinder.

Charge on the disc,

$$dq = \frac{Q}{\pi R^2 L} \times \pi R^2 dr = \frac{Q}{L} dr$$

∴ Electric field due to disc along its axis

$$E_x = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{x}{(x^2 + R^2)^{1/2}} \right]$$

Hence,

$$dE = \frac{(Q/L)dr}{\pi R^2 (2\epsilon_0)} \left[1 - \frac{(L-r)}{[(L-r)^2 + R^2]^{1/2}} \right]$$

$$E = \int dE = \int_{-L/2}^{+L/2} \frac{(Q/L)dr}{\pi R^2 (2\epsilon_0)} \left[1 - \frac{(L-r)}{[(L-r)^2 + R^2]^{1/2}} \right]$$

$$E = \frac{Q}{2\pi R^2 L \epsilon_0} \left[\int_{-L/2}^{+L/2} dr - \int_{-L/2}^{+L/2} \frac{(L-r)dr}{[(L-r)^2 + R^2]^{1/2}} \right]$$

The second integral can be evaluated by substituting $(L-r)^2 + R^2 = t$.

Differentiating both sides, we get,

$$-2(L-r)dr = dt$$

$$\therefore E = \frac{Q}{2\pi R^2 L \epsilon_0} \left[[r]_{-L/2}^{+L/2} + \frac{1}{2} \int_{t}^{dt} \frac{dt}{t^{1/2}} \right]$$

$$= \frac{Q}{2\pi R^2 L \epsilon_0} [L + \sqrt{t}]$$

$$E = \frac{Q}{2\pi R^2 L \epsilon_0} \left[L + \left[(L-r)^2 + R^2 \right]^{1/2} \right]_{-L/2}^{+L/2}$$

$$E = \frac{Q}{2\pi R^2 L \epsilon_0} \left[L + \left(\frac{L^2}{4} + R^2 \right)^{1/2} - \left(\frac{9L^2}{4} + R^2 \right)^{1/2} \right]$$

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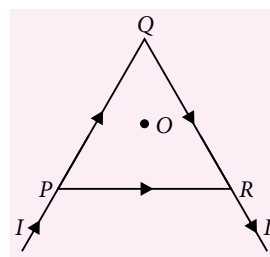
- Suppose speed of light (c), force (F) and kinetic energy (K) are taken as the fundamental units, then the dimensional formula for mass will be
 (a) $[Kc^{-2}]$ (b) $[KF^{-2}]$
 (c) $[cK^{-2}]$ (d) $[Fc^{-2}]$
- A sand bag of mass M is suspended by a rope. A bullet of mass m is fired at it with speed v and gets embedded in it. The loss of kinetic energy of the system is
 (a) $\frac{Mmv^2}{2(M+m)}$ (b) $\frac{Mv^2}{2(M+m)}$
 (c) $\frac{m^2v^2}{2(M+m)}$ (d) $\frac{1}{2}(M+m)v^2$
- A steel wire with cross-section 3 cm^2 has elastic limit $2.4 \times 10^8 \text{ N m}^{-2}$. The maximum upward acceleration that can be given to a 1200 kg elevator supported by this cable wire if the stress is not to exceed one-third of the elastic limit is (Take $g = 10 \text{ m s}^{-2}$)
 (a) 12 m s^{-2} (b) 10 m s^{-2}
 (c) 8 m s^{-2} (d) 7 m s^{-2}
- A body of density ρ at rest is dropped from a height h into a lake of density σ where $\sigma > \rho$. Neglecting all dissipative forces, find the maximum depth to which the body sinks before returning to float on the surface.
 (a) $\frac{h}{\sigma - \rho}$ (b) $\frac{h\rho}{\sigma}$
 (c) $\frac{h\rho}{\sigma - \rho}$ (d) $\frac{h\sigma}{\sigma - \rho}$
- Consider two containers A and B containing identical gases at the same pressure, volume and temperature. The gas in container A is compressed to half of its original volume isothermally while the gas in container B is compressed to half of its original volume adiabatically. The ratio of final pressure of gas in B to that of gas in A is
 (a) $2^{\gamma-1}$ (b) $\left(\frac{1}{2}\right)^{\gamma-1}$
 (c) $\left(\frac{1}{1-\gamma}\right)^2$ (d) $\left(\frac{1}{\gamma-1}\right)^2$
- Soap water drips from a capillary. When the drop breaks away, the diameter of its neck is 1 mm. The mass of the drop is 0.0129 g. Find the surface tension of soapy water. (Take $g = 9.8 \text{ m s}^{-2}$)
 (a) $12.9 \times 10^{-3} \text{ N m}^{-1}$ (b) $31.2 \times 10^{-3} \text{ N m}^{-1}$
 (c) $40.3 \times 10^{-3} \text{ N m}^{-1}$ (d) $58.6 \times 10^{-3} \text{ N m}^{-1}$
- An artificial satellite is moving in a circular orbit around the Earth with a speed equal to half the magnitude of escape velocity from the Earth. The height of the satellite above the Earth's surface is (Take radius of Earth = 6400 km)
 (a) 6000 km (b) 5800 km
 (c) 7500 km (d) 6400 km
- A needle placed 45 cm from a lens forms an image on a screen placed 90 cm on the other side of the lens. Its focal length and the size of image if the size of the needle is 5 cm are respectively
 (a) - 30 cm, 10 cm (b) + 30 cm, - 10 cm
 (c) - 20 cm, 15 cm (d) + 20 cm, - 15 cm
- In Young's double slit experiment distance between two sources is 0.1 mm. The distance of screen from the source is 20 cm. Wavelength of light used is 5460 \AA . Then, angular position of the first dark fringe is
 (a) 0.08° (b) 0.16° (c) 0.20° (d) 0.32°
- When light of wavelength 400 nm is incident on the cathode of a photocell, the stopping potential recorded is 6 V. If the wave of the incident light is increased to 600 nm, then the new stopping potential is
 (a) 1.03 V (b) 2.42 V
 (c) 4.97 V (d) 3.58 V

11. Two particles A and B describe S.H.M. of same amplitude a and frequency ν along the same straight line. The maximum distance between two particles is $\sqrt{3}a$. The initial phase difference between the particles is
(a) $2\pi/3$ (b) $\pi/6$ (c) $\pi/2$ (d) $\pi/4$
12. A racing car moving towards a cliff sounds its horn. The driver observes that the sound reflected from the cliff has a pitch one octave higher than the actual sound of the horn. If ν be the velocity of sound, the velocity of the car is
(a) $\nu/\sqrt{2}$ (b) $\nu/2$ (c) $\nu/3$ (d) $\nu/4$
13. The rate of cooling at 600 K, if surrounding temperature is 300 K is R . Assume that the Stefan's law holds. The rate of cooling at 900 K is
(a) $\frac{16}{3}R$ (b) $2R$ (c) $3R$ (d) $\frac{2}{3}R$
14. The ratio of specific heat of gas at constant pressure to that at constant volume is γ . The change in internal energy of one mole of gas when volume changes from V to $2V$ at constant pressure P is
(a) $\frac{R}{(\gamma-1)}$ (b) PV
(c) $\frac{PV}{(\gamma-1)}$ (d) $\frac{\gamma PV}{\gamma-1}$
15. A boy throws a ball upwards with velocity $u = 15 \text{ m s}^{-1}$. The wind imparts a horizontal acceleration of 3 m s^{-2} to the left. The angle θ with vertical at which the ball must be thrown so that the ball returns to the boy's hand is
(Take $g = 10 \text{ m s}^{-2}$)
(a) $\tan^{-1}(0.4)$ (b) $\tan^{-1}(0.2)$
(c) $\tan^{-1}(0.3)$ (d) $\tan^{-1}(0.15)$
16. A thin uniform rod AB of mass M and length L is hinged at one end A to the level floor. Initially it stands vertically and is allowed to fall freely to the floor in the vertical plane. The angular velocity of the rod, when its end B strikes the floor is (g is acceleration due to gravity)
(a) $\left(\frac{Mg}{L}\right)$ (b) $\left(\frac{Mg}{3L}\right)^{1/2}$
(c) $\left(\frac{g}{L}\right)$ (d) $\left(\frac{3g}{L}\right)^{1/2}$
17. A particle of mass m has half the kinetic energy of another particle of mass $m/2$. If the speed of the heavier particle is increased by 2 m s^{-1} , its new

kinetic energy equals the original kinetic energy of the lighter particle. What is the original speed of the heavier particle?

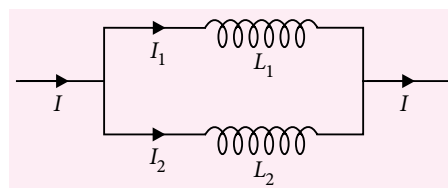
- (a) $(2 - \sqrt{2}) \text{ m s}^{-1}$ (b) $2(1 + \sqrt{2}) \text{ m s}^{-1}$
(c) $(2 + 3\sqrt{2}) \text{ m s}^{-1}$ (d) $4(1 - \sqrt{2}) \text{ m s}^{-1}$

18. An equilateral triangle of side length l is formed from a piece of wire of uniform resistance. The current I is fed as shown in the figure. The magnitude of the magnetic field at its centre O is



- (a) $\frac{\sqrt{3}\mu_0 I}{2\pi l}$ (b) $\frac{3\sqrt{3}\mu_0 I}{2\pi l}$
(c) $\frac{\mu_0 I}{2\pi l}$ (d) zero

19. Two inductors L_1 and L_2 are connected in parallel and a time varying current flows as shown in figure. The ratio of current I_1/I_2 at any time t is



- (a) $\frac{L_2}{L_1}$ (b) $\frac{L_1}{L_2}$
(c) $\frac{L_2^2}{(L_1 + L_2)^2}$ (d) $\frac{L_1^2}{(L_1 + L_2)^2}$

20. An a.c. source is connected across an LCR series circuit with $L = 100 \text{ mH}$, $C = 0.1 \mu\text{F}$ and $R = 50 \Omega$. The frequency of a.c. to make the power factor of the circuit, unity is

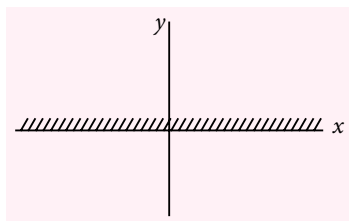
- (a) $\frac{10^4}{2\pi} \text{ Hz}$ (b) $\frac{10^3}{2\pi} \text{ Hz}$
(c) $\frac{10^{-4}}{2\pi} \text{ Hz}$ (d) $\frac{10^{-3}}{2\pi} \text{ Hz}$

21. The electric field (in N C^{-1}) in an electromagnetic wave is given by $E = 50 \sin \omega(t - x/c)$. The energy stored in a cylinder of cross-section 10 cm^2 and

length 100 cm along the x -axis will be

- (a) 5.5×10^{-12} J (b) 1.1×10^{-11} J
(c) 2.2×10^{-11} J (d) 3.3×10^{-11} J

22. In an interference experiment using waves of same amplitude, path difference between the waves at a point on the screen is $\lambda/4$. The ratio of intensity at this point with that at the central bright fringe is
(a) 1 (b) 0.5 (c) 1.5 (d) 2.0
23. A plane mirror is placed along the x -axis facing negative y -axis. The mirror is fixed. A point object is moving with $3\hat{i} + 4\hat{j}$ in front of the plane mirror. The relative velocity of image with respect to its object is



- (a) $-8\hat{j}$ (b) $8\hat{j}$
(c) $3\hat{i} - 4\hat{j}$ (d) $-6\hat{i}$
24. A surface irradiated with light of wavelength 480 nm gives out electrons with maximum velocity v m s $^{-1}$, the cut off wavelength being 600 nm. The same surface would release electrons with maximum velocity $2v$ m s $^{-1}$ if it is irradiated by light of wavelength
(a) 325 nm (b) 360 nm
(c) 384 nm (d) 300 nm
25. The ratio of the de Broglie wavelengths of proton and alpha particle which have been accelerated through same potential difference is
(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $2\sqrt{2}$ (d) $3\sqrt{3}$
26. A plane electromagnetic wave travelling along the x -direction has a wavelength of 3 mm. The variation in the electric field occurs in the y -direction with an amplitude 66 V m $^{-1}$. The equations for the electric and magnetic fields as a function of x and t are

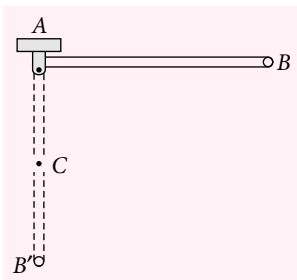
- (a) $E_y = 33 \cos \pi \times 10^{11} (t - \frac{x}{c})$
 $B_z = 1.1 \times 10^{-7} \cos \pi \times 10^{11} (t - \frac{x}{c})$
(b) $E_y = 11 \cos 2\pi \times 10^{11} (t - \frac{x}{c})$
 $B_y = 11 \times 10^{-7} \cos 2\pi \times 10^{11} (t - \frac{x}{c})$

- (c) $E_x = 33 \cos \pi \times 10^{11} (t - \frac{x}{c})$
 $B_x = 11 \times 10^{-7} \cos \pi \times 10^{11} (t - \frac{x}{c})$
(d) $E_y = 66 \cos 2\pi \times 10^{11} (t - \frac{x}{c})$
 $B_z = 2.2 \times 10^{-7} \cos 2\pi \times 10^{11} (t - \frac{x}{c})$

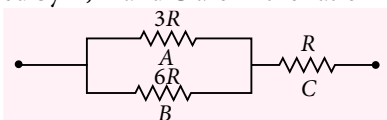
27. The three stable isotopes of neon $^{20}_{10}\text{Ne}$, $^{21}_{10}\text{Ne}$ and $^{22}_{10}\text{Ne}$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u respectively. The average atomic mass of neon is
(a) 11.18 u (b) 15.18 u
(c) 20.18 u (d) 10.18 u
28. Light rays of wavelength 6000 Å and of photon intensity 39.6 W m $^{-2}$ is incident on a metal surface. If only 1% of photons incident on surface emit photoelectrons, then the number of electrons emitted per second per unit area from the surface will be (Take $h = 6.64 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m s $^{-1}$)
(a) 12×10^{18} (b) 10×10^{18}
(c) 12×10^{17} (d) 12×10^{16}
29. A sample contains 10 $^{-2}$ kg each of the two substances A and B with half-lives 4 s and 8 s respectively. Their atomic weights are in the ratio of 1 : 2. Find the ratio of the amounts of A and B after an interval of 16 seconds.
(a) 1 : 4 (b) 4 : 1 (c) 1 : 2 (d) 2 : 1
30. A diode having potential difference 0.5 V across its junction which does not depend on current, is connected in series with resistance of 20 Ω across source. If 0.1 A current passes through resistance, then what is the voltage of the source?
(a) 1.5 V (b) 2.0 V (c) 2.5 V (d) 5 V
31. If ground state ionisation energy of H-atom is 13.6 eV, the energy required to ionize a H-atom from second excited state is
(a) 1.51 eV (b) 3.4 eV
(c) 13.6 eV (d) 12.1 eV
32. Two satellites S_1 and S_2 revolve around a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 h and 8 h respectively. The radius of orbit of S_1 is 10^4 km. When S_2 is closest to S_1 , the speed of S_2 relative to S_1 is
(a) $\pi \times 10^4$ km h $^{-1}$ (b) $2\pi \times 10^4$ km h $^{-1}$
(c) $3\pi \times 10^4$ km h $^{-1}$ (d) $4\pi \times 10^4$ km h $^{-1}$

33. An object initially at rest explodes into three fragments A, B and C. The momentum of A is $P\hat{i}$ and that of B is $\sqrt{3}P\hat{j}$ where P is positive number. The momentum of C is
- $(1 + \sqrt{3})P$ in a direction making 120° with A
 - $2P$ in a direction making 150° with A
 - $2P$ in a direction making 150° with B
 - $(1 + \sqrt{3})P$ in a direction making 150° with B.

34. One end of a uniform rod of length l and mass m is hinged at A. It is released from rest from horizontal position AB as shown in figure. The force exerted by the rod on the hinge when it becomes vertical is

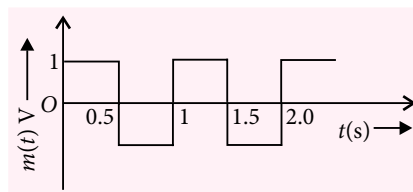


- $\frac{3}{2}mg$
 - $3mg$
 - $5mg$
 - $\frac{5}{2}mg$
35. Two bulbs 40 W and 60 W and rated voltage 240 V are connected in series across a potential difference of 420 V. Which bulb will work at above its rated voltage?
- 40 W
 - 60 W
 - Both 40 W and 60 W
 - None of the bulbs
36. The three resistances A, B and C have values $3R$, $6R$ and R respectively. When some potential difference is applied across the network, the thermal powers dissipated by A, B and C are in the ratio



- 2 : 3 : 4
 - 2 : 4 : 3
 - 4 : 2 : 3
 - 3 : 2 : 4
37. The masses of three wires of copper are in the ratio 1 : 3 : 5 and lengths are in the ratio 5 : 3 : 1. Then the ratio of their electrical resistances are
- 1 : 3 : 5
 - 5 : 3 : 1
 - 1 : 15 : 25
 - 125 : 15 : 1
38. A pendulum bob of mass m carrying a charge q is at rest with its string making an angle θ with the vertical in a uniform horizontal electric field E . The tension in the string is
- $\frac{mg}{\sin \theta}$ and $\frac{qE}{\cos \theta}$
 - $\frac{mg}{\cos \theta}$ and $\frac{qE}{\sin \theta}$
 - $\frac{qE}{mg}$
 - $\frac{mg}{qE}$

39. A modulating signal is a square wave as shown in figure. The carrier wave is given by $c(t) = 2\sin(8\pi t)$ volt. What is the modulation index?

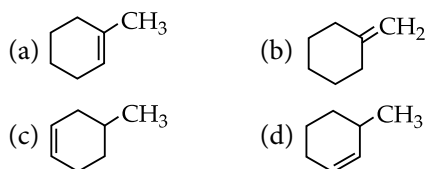


- 0.2
 - 0.3
 - 0.4
 - 0.5
40. A Zener diode when used as a voltage regulator is connected
- in forward bias
 - in reverse bias
 - in parallel with load
 - in series with load
- (i) and (ii) are correct
 - (ii) and (iii) are correct
 - (i) only correct
 - (iv) only correct

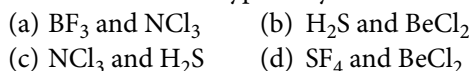
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41. The correct order of first ionisation potential among the following elements Be, B, C, N, O is
- $B < Be < C < O < N$
 - $B < Be < C < N < O$
 - $Be < B < C < N < O$
 - $Be < B < C < O < N$
42. Amongst TiF_6^{2-} , CoF_6^{3-} , Cu_2Cl_2 and $NiCl_4^{2-}$ (At. nos. of Ti = 22, Co = 27, Cu = 29, Ni = 28). The colourless species are
- CoF_6^{3-} and $NiCl_4^{2-}$
 - TiF_6^{2-} and CoF_6^{3-}
 - Cu_2Cl_2 and $NiCl_4^{2-}$
 - TiF_6^{2-} and Cu_2Cl_2
43. Identify 'Z' in the sequence :
- $$C_6H_5NH_2 \xrightarrow[273\text{ K}]{NaNO_2 + HCl} X \xrightarrow{CuCN} Y \xrightarrow[Boil]{H^+/H_2O} Z$$
- C_6H_5CN
 - $C_6H_5CONH_2$
 - C_6H_5COOH
 - $C_6H_5CH_2NH_2$
44. The oxidation number of S in Caro's acid (H_2SO_5) is
- +5
 - +3
 - +2
 - +6
45. The solubility of sulphates in water down the IIA group follows the order $Be > Mg > Ca > Sr > Ba$. This is due to
- increase in melting point
 - increasing molecular mass
 - decreasing lattice energy
 - high heat of solvation of smaller ions.

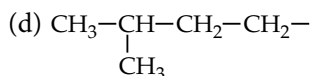
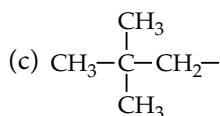
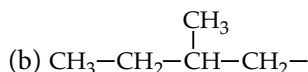
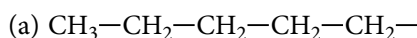
46. A hydrocarbon P of the formula C_7H_{12} on ozonolysis gives a compound Q which undergoes aldol condensation giving 1-acetylcyclopentene. The compound P is



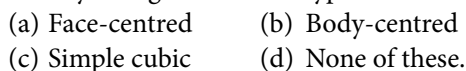
47. Identify the pair of species in which the central atom has the same type of hybridisation.



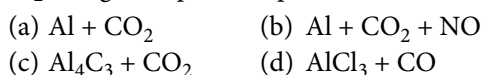
48. The structure of *neo* pentyl group in an organic compound is



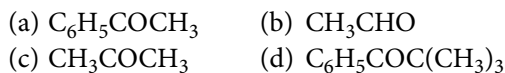
49. The cubic unit cell of aluminium (molar mass 27.0 g mol^{-1}) has an edge length of 405 pm and density 2.70 g cm^{-3} . What type of unit cell is it?



50. Al_2O_3 on heating with carbon in an atmosphere of Cl_2 at high temperature produces



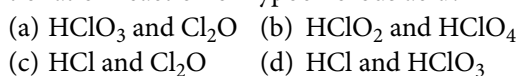
51. Which of the following does not show tautomerism?



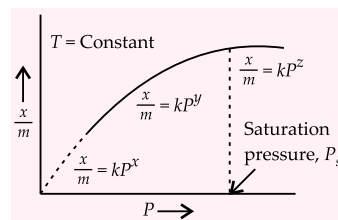
52. Which of the following has least covalent P—H bond?



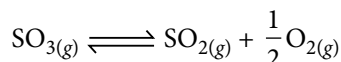
53. What products are expected from the disproportionation reaction of hypochlorous acid?



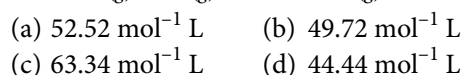
54. In the given Freundlich adsorption isotherm plot, x , y and z are respectively



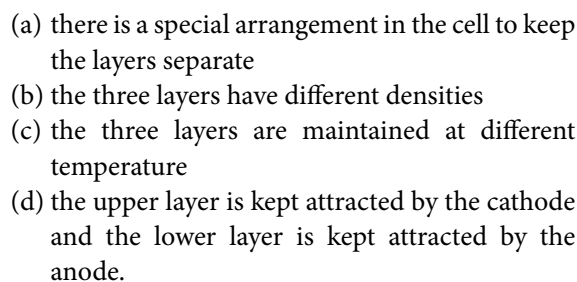
55. The equilibrium constant of the reaction,



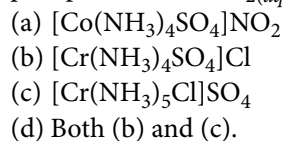
is 0.15 at 900 K. The equilibrium constant for $2SO_{2(g)} + O_{2(g)} \rightleftharpoons 2SO_{3(g)}$ is



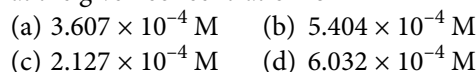
56. In the Hoope's process for refining of aluminium, the fused materials form three different layers and they remain separated during electrolysis also. This is because



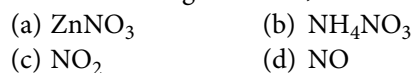
57. Which of the following complexes will give white precipitate with $BaCl_{2(aq)}$?



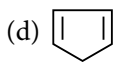
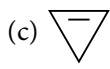
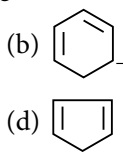
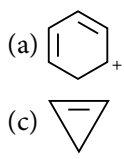
58. At $25^\circ C$, the molar conductivity of 0.001 M hydrofluoric acid is $184.5 \Omega^{-1} \text{ cm}^2 \text{ mol}^{-1}$. If its Λ_m° is $502.4 \Omega^{-1} \text{ cm}^2 \text{ mol}^{-1}$, then equilibrium constant at the given concentration is



59. Zinc on reacting with cold, dil. HNO_3 , gives



60. Which of the following is aromatic?



61. Which of the following statements is not correct?

- (a) Some antiseptics can be added to soaps.
 (b) Dilute solutions of some disinfectants can be used as an antiseptic.
 (c) Disinfectants are antimicrobial drugs.
 (d) Antiseptic medicines can be ingested.

62. Which molecule/ion out of the following does not contain unpaired electrons?

- (a) N_2^+ (b) O_2 (c) O_2^{2-} (d) B_2

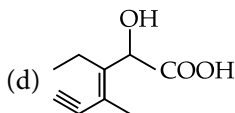
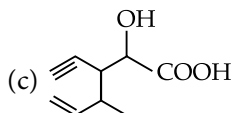
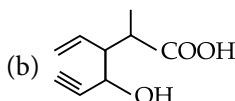
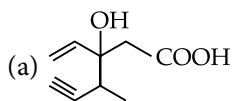
63. Energy of an electron in hydrogen atom is given by

$$E = \frac{13.6}{n^2} \text{ eV. Which one of the following statements is true if } n \text{ is changed from 1 to 3?}$$

Energy will

- (a) decrease three times
 (b) increase three times
 (c) increase nine times
 (d) decrease nine times.

64. Structure of the compound whose IUPAC name is 3-Ethyl-2-hydroxy-4-methylhex-3-en-5-ynoic acid is



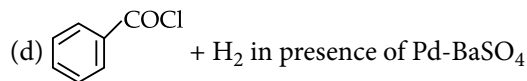
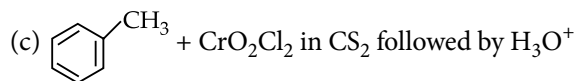
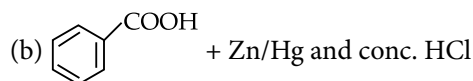
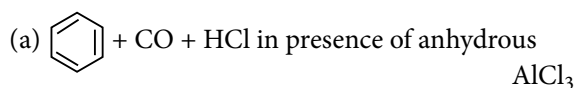
65. The value of Planck's constant is $6.63 \times 10^{-34} \text{ J s}$. The speed of light is $3 \times 10^{17} \text{ nm s}^{-1}$. Which value is closest to the wavelength in nanometer of a quantum of light with frequency of $6 \times 10^{15} \text{ s}^{-1}$?

- (a) 50 (b) 75 (c) 10 (d) 25

66. Identify the correct order of solubility in aqueous medium.

- (a) $Na_2S > CuS > ZnS$ (b) $Na_2S > ZnS > CuS$
 (c) $CuS > ZnS > Na_2S$ (d) $ZnS > Na_2S > CuS$

67. Reaction by which benzaldehyde cannot be prepared



68. A reaction having equal energies of activation for forward and reverse reactions has

- (a) $\Delta H = 0$ (b) $\Delta H = \Delta G = \Delta S = 0$
 (c) $\Delta S = 0$ (d) $\Delta G = 0$

69. Amongst the acids,

- (i) $CH \equiv CCOOH$ (ii) $CH_2 = CHCOOH$ and
 (iii) CH_3CH_2COOH ,

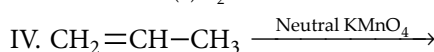
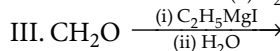
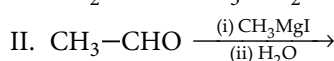
the acid strength follows the sequence

- (a) (i) < (ii) > (iii) (b) (i) > (ii) > (iii)
 (c) (i) = (ii) = (iii) (d) (i) = (ii) > (iii)

70. Ionization potential of hydrogen atom is 13.6 eV. Hydrogen atom in ground state is excited by monochromatic light of energy 12.1 eV. The spectral lines emitted by hydrogen according to Bohr's theory will be

- (a) one (b) two (c) three (d) four.

71. Which of the following reactions will yield 2-propanol?



- (a) I and II (b) II and III
 (c) III and I (d) II and IV

72. The heat liberated when 1.89 g of benzoic acid is burnt in a bomb calorimeter at 25°C and it increases the temperature of 18.94 kg of water by 0.632°C. If the specific heat of water at 25°C is 0.998 cal/g-deg, the value of the heat of combustion of benzoic acid is

- (a) 881.1 kcal (b) 771.12 kcal
 (c) 981.1 kcal (d) 871.2 kcal

73. Which of the following combinations illustrates the law of reciprocal proportions?

- (a) N_2O_3, N_2O_4, N_2O_5 (b) NaCl, NaBr, NaI
 (c) CS_2, CO_2, SO_2 (d) PH_3, P_2O_3, P_2O_5

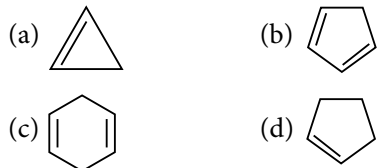
74. Which of the following is formed, when benzaldehyde reacts with alcoholic KCN?

- (a) Benzoin (b) Benzyl alcohol
 (c) Benzoic acid (d) Ethyl benzoate

MATHEMATICS

75. The standard electrode potentials for $\text{Pb}^{2+}|\text{Pb}$ and $\text{Zn}^{2+}|\text{Zn}$ are -0.126 V and -0.763 V respectively. The e.m.f. of the cell $\text{Zn}|\text{Zn}^{2+}(0.1\text{ M})||\text{Pb}^{2+}(0.1\text{ M})|\text{Pb}$ is
 (a) 0.637 V (b) $< 0.637\text{ V}$
 (c) $> 0.637\text{ V}$ (d) 0.889 V

76. Which of the following will be most acidic?

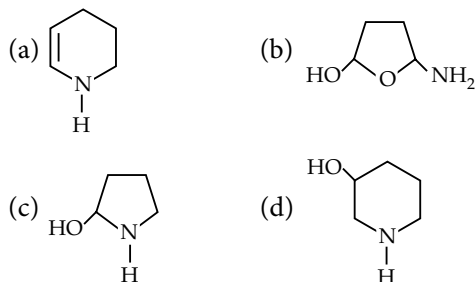
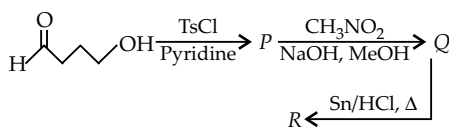


77. Three metal ions (X), (Y), (Z) on treatment with KCN forms yellow, white and reddish-brown precipitate. (X) gives insoluble complex with excess of KCN and no ppt. upon passing H_2S gas. (Y) also gives insoluble complex with excess of KCN but gives yellow ppt. on passing H_2S gas. (Z) gives yellow solution with excess of KCN. Then X, Y and Z respectively are
 (a) Cu^{2+} , Cd^{2+} , Fe^{3+} (b) Cu^{2+} , Fe^{2+} , Cd^{2+}
 (c) Pb^{2+} , Cd^{2+} , Cu^{2+} (d) Fe^{3+} , Pb^{2+} , Fe^{2+}

78. Which of the following represents the correct order of increasing electron gain enthalpy with negative sign for the elements O, S, F and Cl?
 (a) $\text{O} < \text{S} < \text{F} < \text{Cl}$ (b) $\text{F} < \text{S} < \text{O} < \text{Cl}$
 (c) $\text{S} < \text{O} < \text{Cl} < \text{F}$ (d) $\text{Cl} < \text{F} < \text{O} < \text{S}$

79. Potassium permanganate has intense purple colour due to
 (a) weak $d-d$ transitions
 (b) metal to ligand charge transfer
 (c) ligand to metal charge transfer
 (d) both metal and ligand transitions.

80. Identify 'R' in the following series of reactions.



81. If $x + y = \pi + z$, then $\sin^2 x + \sin^2 y - \sin^2 z$ is equal to
 (a) $2 \sin x \sin y \sin z$ (b) $2 \cos x \cos y \cos z$
 (c) $2 \sin x \cos y \cos z$ (d) $2 \sin x \sin y \cos z$

82. Let R be a relation in N defined by $R = \{(x, y) : x + 2y = 8\}$, then range of R is
 (a) $\{2, 4, 6\}$ (b) $\{1, 2, 3, 4, 6\}$
 (c) $\{1, 2, 3\}$ (d) none of these.

83. If R be a relation ' $<$ ' from $A = \{1, 2, 3, 4\}$ to the set $B = \{1, 3, 5\}$ i.e. $(a, b) \in R \Leftrightarrow a < b$, then $R \circ R^{-1}$ equals
 (a) $\{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$
 (b) $\{(3, 3), (5, 3), (3, 5), (5, 5)\}$
 (c) $\{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$
 (d) $\{(4, 5), (3, 4), (3, 3)\}$.

84. The straight lines joining the origin to the points of intersection of the straight line $hx + ky = 2hk$ and the curve $(x - k)^2 + (y - h)^2 = c^2$ are at right angles, then
 (a) $h^2 + k^2 + c^2 = 0$ (b) $h^2 - k^2 - c^2 = 0$
 (c) $h^2 + k^2 - c^2 = 0$ (d) none of these.

85. The exhaustive range of values of a such that the angle between the pair of tangents drawn from (a, a) to the circle $x^2 + y^2 - 2x - 2y - 6 = 0$ lies in the range $(\pi/3, \pi)$ is
 (a) $(1, \infty)$ (b) $(-5, -3) \cup (3, 5)$
 (c) $(-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, \infty)$
 (d) $(-3, -1) \cup (3, 5)$

86. If $\alpha = \tan 27\theta - \tan \theta$ and $\beta = \frac{\sin \theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}$, then

- (a) $\alpha = \beta$ (b) $\alpha = 2\beta$
 (c) $\beta = 2\alpha$ (d) none of these.

87. The graph of $f(x) = \cos x \cos(x + 2) - \cos^2(x + 1)$ is
 (a) a straight line through $(\pi/2, -\sin^2 1)$ and parallel to x -axis
 (b) a parabola with vertex $(1, -\sin^2 1)$
 (c) a straight line passing through origin
 (d) none of these.

88. Seven coupons are selected at random one at a time with replacement from 15 coupons numbered 1 to 15. The probability that the largest number appearing on a selected coupon is 9, is

- (a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$
 (c) $\left(\frac{3}{5}\right)^7$ (d) none of these

89. If the letters of the word MATHEMATICS are arranged arbitrarily, the probability that C comes before E, E before H, H before I and I before S is
 (a) $\frac{1}{75}$ (b) $\frac{1}{24}$ (c) $\frac{1}{120}$ (d) $\frac{1}{720}$
90. Suppose $A_1, A_2, A_3, \dots, A_{30}$ are thirty sets each with five elements and $B_1, B_2, B_3, \dots, B_n$ are n sets each with three elements such that $\bigcup_{i=1}^{30} A_i = \bigcup_{i=1}^n B_i = S$. If each element of S belongs to exactly ten of the A_i 's and exactly 9 of the B_i 's, then the value of n is
 (a) 15 (b) 135 (c) 45 (d) 35
91. The value of $\left(\binom{50}{6} - \binom{5}{1} \binom{40}{6} + \binom{5}{2} \binom{30}{6} - \binom{5}{3} \binom{20}{6} + \binom{5}{4} \binom{10}{6} \right)$ where $\binom{n}{r}$ denotes nC_r , is
 (a) 15625 (b) 0
 (c) 1000000 (d) 2250000
92. The value of $\int_0^{\infty} \left[\frac{3}{x^2+1} \right] dx$ {where $[\cdot]$ is greatest integer function} is
 (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{3}{\sqrt{2}}$
 (c) $\frac{2}{\sqrt{3}}$ (d) none of these
93. The value of $\int_0^3 [x^2 - 2x + 2] dx$, where $[\cdot]$ is greatest integer function is
 (a) $6 - \sqrt{3} - \sqrt{2}$ (b) $6 + \sqrt{3} + \sqrt{2}$
 (c) $8 - \sqrt{3} - \sqrt{2}$ (d) $2 + \sqrt{2}$
94. $f(x)$ is a continuous function such that $f(x+4) = f(x+2) - f(x)$. The value of $\int_{\lambda}^{\lambda+12} f(x) dx$ is
 (a) $\int_0^{12} f(x) dx$ (b) $\int_0^6 f(x) dx$
 (c) $\int_0^8 f(x) dx$ (d) none of these
95. The range of $\int_{-1}^1 \frac{\sin x dt}{1 - 2t \cos x + t^2}$ is $x \in (0, 2\pi)$
 (a) $\{\pi\}$ (b) $\left\{ -\frac{\pi}{2}, \frac{\pi}{2} \right\}$
 (c) $\left\{ \frac{\pi}{2} \right\}$ (d) $\{-\pi, \pi\}$
96. Given a function g continuous everywhere such that $g(1)=5$ and $\int_0^1 g(t) dt = 2$. If $f(x) = \frac{1}{2} \int_0^x (x-t)^2 g(t) dt$, then $f'''(1) - f''(1)$ is
 (a) 2 (b) 4 (c) 3 (d) 5
97. The least value of the function $F(x) = \int_{\pi/6}^x (4 \sin t + 3 \cos t) dt$ in $\left[\frac{\pi}{4}, \frac{3\pi}{4} \right]$
 (a) $\frac{4\sqrt{3} - \sqrt{2}}{2}$ (b) $\frac{4\sqrt{3} - 3 - \sqrt{2}}{2}$
 (c) $\frac{3 + \sqrt{2}}{2}$ (d) $\frac{-4\sqrt{3} + 3 + \sqrt{2}}{2}$
98. For $x^2 - (a+3)|x| + 4 = 0$ to have real solutions, the range of a is
 (a) $(-\infty, -7] \cup [1, \infty)$ (b) $(-3, \infty)$
 (c) $(-\infty, -7]$ (d) $[1, \infty)$
99. If the equation $x^2 + ax + b = 0$ has distinct real roots and $x^2 + a|x| + b = 0$ has only one real root, then which of the following is true?
 (a) $b = 0, a > 0$ (b) $a = 0, b > 0$
 (c) $b > 0, a < 0$ (d) $b < 0, a > 0$
100. If $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then $\det(\text{Adj}(\text{Adj } A)) =$
 (a) 14^4 (b) 14^3
 (c) 14^2 (d) 14
101. The existence of the unique solution of the system of equations $x + y + z = \lambda$, $5x - y + \mu z = 10$, $2x + 3y - z = 6$ depends on
 (a) μ only (b) λ only
 (c) both λ and μ (d) neither λ nor μ
102. If A and B are square matrices such that $B = -A^{-1}BA$, then
 (a) $AB + BA = 0$
 (b) $(A+B)^0 = A^2 + B^2 + AB$
 (c) $(A+B)^2 = A^2 + 2AB + B^2$
 (d) $(A+B)^2 = A + B$
103. If $A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is an orthogonal matrix, then
 (a) $a = 2, b = 1$ (b) $a = -2, b = -1$
 (c) $a = 2, b = -1$ (d) $a = -2, b = 1$

104. If A and B are square matrices of order 3×3 such that A is an orthogonal matrix and B is a skew symmetric matrix, then which of the following statements is true?
 (a) $|AB| = 1$ (b) $|AB| = 0$
 (c) $|AB| = -1$ (d) none of these
105. The least value of the expression $2\log_{10} x - \log_x(0.01)$ for $x > 1$ is
 (a) 10 (b) 2 (c) -0.01 (d) 4
106. The expression $\{x + (x^3 - 1)^{1/2}\}^5 + \{x - (x^3 - 1)^{1/2}\}^5$ is a polynomial of degree
 (a) 5 (b) 6 (c) 7 (d) 8
107. If $f(x) = (a^2 - 3a + 2)\left(\cos^2 \frac{x}{4} - \sin^2 \frac{x}{4}\right) + (a - 1)x + \sin 1$ possesses critical points, then the set of values of ' a ' are
 (a) $(-\infty, 0] \cup [4, \infty)$
 (b) $(-\infty, 0] \cup [4, \infty) \cup \{1\}$
 (c) $(-\infty, 0) \cup (4, \infty) \cup (4, \infty) \cup \{1\}$
 (d) none of these
108. If $x \in (-\infty, -2)$ and $y^3 - 3y + x = 0$, then
 (a) y is not a function of x
 (b) y is not a monotonic function
 (c) y is an increasing function of x
 (d) y is a decreasing function of x
109. If $f(x) = \cot^{-1} x + \frac{1}{2} \ln x$, $x \in \left[\frac{1}{\sqrt{3}}, \sqrt{3}\right]$, then greatest value of $f(x)$ is
 (a) $\frac{\pi}{6} - \frac{1}{4} \ln 3$ (b) $\frac{\pi}{6} + \frac{1}{2} \ln 3$
 (c) $\frac{1}{4} \ln 3 - \frac{\pi}{6}$ (d) $\frac{\pi}{6} + \frac{\ln 3}{4}$
110. The range of $f(x) = \tan^{-1} \sqrt{[x] + [-x]} + \sqrt{3 - |x|} + \frac{1}{x^2}$ is (where $[.]$ denotes G.I.F)
 (a) $\left[\frac{1}{4}, \infty\right)$ (b) $\left\{\frac{1}{4}\right\} \cup [2, \infty)$
 (c) $\left\{\frac{1}{9}, \frac{5}{4}, 1 + \sqrt{2}\right\}$ (d) $\left\{\frac{5}{4}, 1 + \sqrt{2}\right\}$
111. The solution of differential equation $x^2 dy - y^2 dx + xy^2(x - y)dy = 0$ is
 (a) $\ln \left| \frac{xy}{x - y} \right| + \frac{y^2}{2} = C$ (b) $\ln \left| \frac{x + y}{x - y} \right| + \frac{y^2}{2} = C$
 (c) $\ln \left| \frac{x - y}{xy} \right| + \frac{y^2}{2} = C$ (d) $\ln \left| \frac{x - y}{x + y} \right| + \frac{y^2}{2} = C$
112. The area of the region bounded by $y = x \ln x$, $y = 2x - 2x^2$ is
 (a) $7/12$ (b) $5/2$ (c) $1/12$ (d) 4
113. $\int \sin 51x (\sin x)^{49} dx$ equals
 (a) $\frac{\sin 50x (\sin x)^{50}}{50} + C$
 (b) $\frac{\cos 50x (\sin x)^{50}}{50} + C$
 (c) $\frac{\cos 50x (\cos x)^{50}}{50} + C$
 (d) $\frac{\sin 50x (\sin x)^{51}}{51} + C$
114. If f be a polynomial function satisfying $f(x^2 + x + 3) + 2f(x^2 - 3x + 5) = 6x^2 - 10x + 17$ $\forall x \in R$ then
 (a) f is a decreasing function
 (b) $f(x) = 0$ has a root in $(0, 2)$
 (c) $f(x)$ is an odd function
 (d) no such polynomial exist
115. If $|f''(x)| \leq 1 \forall x \in R$ and $f(0) = 0 = f'(0)$ then which of the following cannot be true?
 (a) $f\left(\frac{1}{3}\right) = \frac{1}{5}$ (b) $f\left(-\frac{1}{3}\right) = \frac{1}{12}$
 (c) $f(3) = 4$ (d) $f(-3) = \frac{1}{3}$
116. Let $f(x) = \left[\frac{1}{\sin\{x\}} \right]$, where $[.]$ and $\{.\}$ denote the greatest integer and fractional part function respectively. The range of f is
 (a) the set of integers = I
 (b) the set of natural numbers = N
 (c) the set of whole numbers = W
 (d) $\{2, 3, 4, \dots\}$
117. The chord of contact of tangents from any point of circle $x^2 + y^2 = a^2$ with respect to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$ where $(a, b, c > 0)$ then
 (a) $b < \frac{a + c}{2}$
 (b) $\frac{1}{1 + \log a}, \frac{1}{1 + \log b}, \frac{1}{1 + \log c}$ are in A.P.
 (c) a, b, c are in A.P.
 (d) $b > \sqrt{ac}$
118. The locus of mid-point of the chord of the circle with diameter as minor axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

($a > b$) which subtend right angle at centre of ellipse is

- (a) $x^2 + y^2 = 2b^2$ (b) $x^2 + y^2 = \frac{a^2 + b^2}{2}$
 (c) $2(x^2 + y^2) = b^2$ (d) $x^2 + y^2 = \frac{b^2}{4}$

119. Let α, β, γ are the roots of the equation

$$x^3 + 3x^2 - 6x - 8 = 0. \text{ If } \left(\frac{1}{\alpha}, \alpha\right), \left(\frac{1}{\beta}, \beta\right) \text{ and } \left(\frac{1}{\gamma}, \gamma\right)$$

are the vertices of the triangle, then

- (a) centroid of the triangle is $\left(-\frac{3}{4}, -1\right)$
 (b) orthocentre of the triangle is $\left(-\frac{1}{8}, -8\right)$
 (c) circumcentre of the triangle is $\left(\frac{29}{8}, -\frac{23}{16}\right)$
 (d) centroid of the triangle is $\left(\frac{1}{4}, -1\right)$

120. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals (for some arbitrary constant K)

- (a) $-\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$
 (b) $\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} - \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$
 (c) $-\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$
 (d) $\frac{1}{(\sec x + \tan x)^{11/2}} \left\{ \frac{1}{11} + \frac{1}{7} (\sec x + \tan x)^2 \right\} + K$

121. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a cannot take the value

- (a) -1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

122. Let $f(x) = \begin{cases} x^2 \cos \frac{\pi}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}, x \in \mathbb{R}$, then f is

- (a) differentiable both at $x = 0$ and at $x = 2$
 (b) differentiable at $x = 0$ but not differentiable at $x = 2$
 (c) not differentiable at $x = 0$ but differentiable at $x = 2$
 (d) differentiable at neither at $x = 0$ nor at $x = 2$

123. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

- (a) 75 (b) 150 (c) 210 (d) 243

124. If $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$, then

- (a) $a = 1, b = 4$ (b) $a = 1, b = -4$
 (c) $a = 2, b = -3$ (d) $a = 2, b = 3$

125. The function $f: [0, 3] \rightarrow [1, 29]$, defined by

$$f(x) = 2x^3 - 15x^2 + 36x + 1, \text{ is}$$

- (a) one-one and onto
 (b) onto but not one-one
 (c) one-one but not onto
 (d) neither one-one nor onto

ENGLISH AND LOGICAL REASONING

Directions (Questions 126 to 128) : Read the passage and answer the following questions.

Books are, by far, the most lasting product of human effort. Temples crumble into ruin, pictures and statues decay, but books survive. Time does not destroy the great thoughts which are as fresh today as when they first passed through their author's mind. These thoughts speak to us through the printed page. The only effect of time has been to throw out of currency the bad products. Nothing in literature which is not good can live for long. Good books have always helped man in various spheres of life. No wonder that the world keeps its books with great care.

126. Of the product of human effort, books are the most

- (a) Permanent (b) Important
 (c) Enjoyable (d) Useful.

127. Time does not destroy books because they contain

- (a) Useful material
 (b) Subject matter for education
 (c) High ideals
 (d) Great ideas.

128. "To throw out of currency" means

- (a) Destroy (b) Put out of use
 (c) Extinguish (d) Forget.

Directions (Questions 129 to 130) : Pick out the correct synonyms for each of the following words.

129. Eradicate

- (a) Dedicate (b) Eliminate
 (c) Complicate (d) Indicate

130. Myopic

- (a) Astigmatic (b) Cross-eyed
(c) Blind (d) Short-sighted

Directions (Questions 131 to 132) : In each of the following questions, an idiomatic expression/a proverb has been given, followed by some alternatives. Choose the one which best expresses the meaning of the given idiom or proverb.

131. To take the wind out of another's sails

- (a) To defeat the motives of another
(b) To anticipate another and to gain advantage over him
(c) To manouevre to mislead another on the high seas
(d) To cause harm to another

132. To keep the ball rolling

- (a) To keep the conversation going
(b) To make the best use of
(c) To earn more and more
(d) To work constantly

Directions (Questions 133 to 137) : Rearrange the given five sentences A, B, C, D and E in the proper sequence so as to form a meaningful paragraph and then answer the questions given below them.

- A. The alternative was a blitz by the health workers to popularise preventive measures.
B. This information was considered inadequate.
C. People have been reading about AIDS in the mass media.
D. Nobody is sure as to how effective this would be.
E. People were also not being influenced enough to take preventive measures.

133. Which sentence should come first in the paragraph?

- (a) C (b) D (c) B (d) A

134. Which sentence should come second in the paragraph?

- (a) C (b) D (c) B (d) E

135. Which sentence should come third in the paragraph?

- (a) C (b) D (c) B (d) E

136. Which sentence should come fourth in the paragraph?

- (a) C (b) D (c) B (d) A

137. Which sentence should come last in the paragraph?

- (a) A (b) D (c) B (d) E

Directions (Questions 138 to 140) : In each of the following questions, a word has been written in four different ways out of which only one is correctly spelt. Find the correctly spelt word.

138. (a) Garuntee (b) Guaruntee
(c) Gaurantee (d) Guarantee

139. (a) Benefited (b) Benifitted
(c) Benefeted (d) Benifited

140. (a) Efflorescance (b) Eflorescence
(c) Efflorascence (d) Efflorescence

141. There is a certain relationship between two given words on one side of :: establish a similar relationship on another side of :: by selecting a word from the given options.

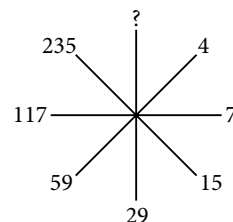
Doctor : Patient :: Politician :

- (a) Masses (b) Voter
(c) Power (d) Chair

142. In the following question, four words have been given, out of which three are alike in some manner, while the fourth one is different. Choose out the odd one.

- (a) Seminar (b) Semicolon
(c) Semifinal (d) Semicircle

143. Find the missing character in the following :



- (a) 327 (b) 386 (c) 438 (d) 469

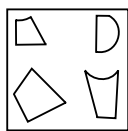
144. Select a figure from amongst the Answer Figures which will continue the same series as established by the five Problem Figures.

Problem Figures

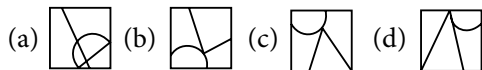


- (a) (b)
(c) (d)

145. In the following question, find out which of the figures (a), (b), (c) and (d) can be formed from the pieces given in fig. (X).



(X)

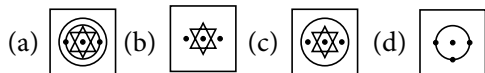
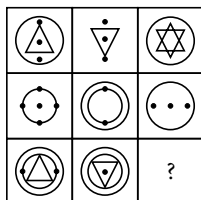


146. Select the missing term.

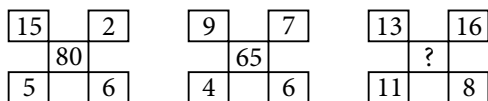
A, D, H, M, ?, Z

- (a) T (b) G (c) N (d) S

147. In the following question, find out which of the answer figures (a), (b), (c) and (d) completes the figure matrix ?



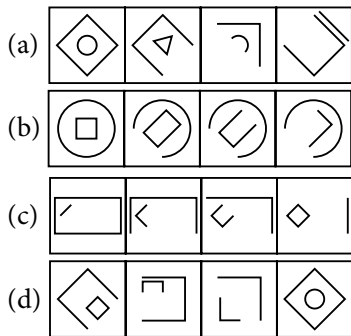
148. In the following question, a set of figures carrying certain characters, is given. Assuming that the characters in each set follow a similar pattern, find the missing character.



- (a) 35 (b) 48 (c) 72 (d) 120

149. In the following question, choose the set of figures which follows the given rule.

Rule : Closed figures gradually become open and open figures gradually become closed.



150. In the following number series, one term is wrong. Find out the wrong term.

1, 3, 10, 21, 64, 129, 356, 777

- (a) 21 (b) 129 (c) 10 (d) 356

SOLUTION

1. (a) : Let $M = kc^x F^y K^z$

where k is a dimensionless constant.

$$\therefore [M^1 L^0 T^0] = [LT^{-1}]^x [MLT^{-2}]^y [ML^2 T^{-2}]^z \\ = M^{y+z} L^{x+y+2z} T^{-x-2y-2z}$$

Applying principle of homogeneity of dimensions,

$$y + z = 1 \quad \dots(i)$$

$$x + y + 2z = 0 \quad \dots(ii)$$

$$-x - 2y - 2z = 0 \quad \dots(iii)$$

Adding (ii) and (iii), we get, $y = 0$

Now, from (i) $z = 1 - y = 1$

from (ii) $x = -y - 2z = 0 - 2$

$$\therefore [M] = [c^{-2} F^0 K^1] = [Kc^{-2}]$$

2. (a) : According to law of conservation of linear momentum, $mv = (M + m) V$
or $V = m v / (M + m)$

$$\text{Initial KE of the system} = \frac{1}{2} m v^2$$

$$\text{Final KE of the system} = \frac{1}{2} (M + m) V^2$$

$$= \frac{1}{2} (M + m) \left(\frac{mv}{M + m} \right)^2 = \frac{1}{2} \frac{(m v)^2}{(M + m)}$$

$$\text{Loss of KE} = \frac{1}{2} m v^2 - \frac{1}{2} \frac{(m v)^2}{(M + m)}$$

$$= \frac{m v^2}{2} \left[\frac{M + m - m}{M + m} \right] = \frac{m M v^2}{2(M + m)}$$

3. (b) : Maximum tension an elevator can tolerate is

$$T = \frac{1}{3} \text{ stress} \times \text{area of cross-section}$$

$$= \frac{1}{3} \times (2.4 \times 10^8) \times (3 \times 10^{-4}) = 2.4 \times 10^4 \text{ N}$$

If a is the maximum upward acceleration of elevator then $T = m(g + a)$

$$\text{or } 2.4 \times 10^4 = 1200(10 + a)$$

On solving, $a = 10 \text{ m s}^{-2}$.

4. (c) : The speed of the body just before entering the liquid is $u = \sqrt{2gh}$. The buoyant force F_B of the lake (i.e., upward thrust of liquid on the body) is greater than the weight of the body W , since $\sigma > \rho$. If V is the volume of the body and a is the acceleration of the body inside the liquid, then $F_B - W = ma$

$$\sigma Vg - \rho Vg = \rho Va$$

$$\text{or } (\sigma - \rho)g = \rho a \text{ or } a = \frac{(\sigma - \rho)g}{\rho}$$

Using the relation, $v^2 = u^2 + 2as$, we have

$$0 = (\sqrt{2gh})^2 - 2g \frac{(\sigma - \rho)}{\rho} s \text{ or } s = \frac{h\rho}{\sigma - \rho}$$

5. (a): For isothermal compression of gas A,

$$P_f = \frac{P_i V_i}{V_f} = 2P_i \quad (\because V_f = \frac{1}{2} V_i)$$

For adiabatic compression of gas B,

$$P'_f = P_i \left(\frac{V_i}{V_f} \right)^\gamma = 2^\gamma P_i$$

$$\text{Thus, } \frac{P'_f}{P_f} = \frac{2^\gamma P_i}{2P_i} = 2^{\gamma-1}$$

6. (c): When the drop breaks away from the capillary, weight of the drop = force of surface tension acting on it due to capillary, i.e.,

$$mg = (\pi D) \times T \text{ or } T = \frac{mg}{\pi D} \quad \dots(i)$$

Here, $m = 0.0129 \text{ g} = 1.29 \times 10^{-5} \text{ kg}$, $g = 9.8 \text{ m s}^{-2}$,
 $D = 1 \text{ mm} = 10^{-3} \text{ m}$

From eqn. (i),

$$T = \frac{(1.29 \times 10^{-5} \text{ kg})(9.8 \text{ m s}^{-2})}{3.14 \times 10^{-3} \text{ m}} = 40.3 \times 10^{-3} \text{ N m}^{-1}$$

$$7. (d): v_0 = \sqrt{\frac{gR^2}{R+h}} \text{ and } v_e = \sqrt{2gR}$$

$$\text{As } v_0 = \frac{v_e}{2},$$

$$\therefore \sqrt{\frac{gR^2}{R+h}} = \frac{\sqrt{2gR}}{2} \text{ or } \frac{gR^2}{R+h} = \frac{gR}{2},$$

$$\Rightarrow h = R = 6400 \text{ km}$$

8. (b): Here, $u = -45 \text{ cm}$, $v = +90 \text{ cm}$

Using thin lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{90} + \frac{1}{45} = \frac{1+2}{90}$$

$$\therefore f = +30 \text{ cm}$$

$$\text{Magnification, } m = \frac{h_2}{h_1} = \frac{v}{u} \text{ or } \frac{h_2}{5} = \frac{90}{-45}$$

$$\therefore \text{Size of image, } h_2 = -10 \text{ cm.} \quad [\because h_1 = 5 \text{ cm}]$$

9. (b): Angular position of first dark fringe,

$$\begin{aligned} \theta_1 &= (2 \times 1 - 1) \frac{\lambda}{2d} = \frac{\lambda}{2d} \\ &= \frac{5460 \times 10^{-10}}{2 \times 0.1 \times 10^{-3}} = 2730 \times 10^{-6} \text{ rad} \\ &= 2730 \times 10^{-6} \times \frac{180^\circ}{\pi} = 0.16^\circ. \end{aligned}$$

10. (c): As $K_{\max} = hv - \phi_0$

$$\text{or } eV_0 = \frac{hc}{\lambda} - \phi_0 \text{ or } V_0 = \frac{hc}{e\lambda} - \frac{\phi_0}{e}$$

$$\begin{aligned} \therefore \Delta V_0 &= (V_0)_1 - (V_0)_2 \\ &= \left[\frac{hc}{e\lambda_1} - \frac{\phi_0}{e} \right] - \left[\frac{hc}{e\lambda_2} - \frac{\phi_0}{e} \right] = \frac{hc}{e} \left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right] \\ &= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}} \left[\frac{1}{4 \times 10^{-7}} - \frac{1}{6 \times 10^{-7}} \right] \\ &= 1.03 \text{ V} \end{aligned}$$

$$\therefore (V_0)_2 = (V_0)_1 - 1.03 = 6 - 1.03 = 4.97 \text{ V}$$

11. (a): $y_1 = a \sin \omega t$ and $y_2 = a \sin (\omega t + \theta)$

$$y_2 - y_1 = a\sqrt{3} = a \sin (\omega t + \theta) - a \sin \omega t$$

$$\begin{aligned} \text{or } \sqrt{3}a &= 2a \cos \frac{(\omega t + \theta) + \omega t}{2} \sin \frac{(\omega t + \theta) - \omega t}{2} \\ &= 2a \cos (\omega t + \theta/2) \sin \theta/2 \end{aligned}$$

For maximum value, $\cos (\omega t + \theta/2) = 1$

$$\therefore \sqrt{3}a = 2a \sin \theta/2$$

$$\text{or } \sin \theta/2 = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \text{ or } \frac{\theta}{2} = \frac{\pi}{3} \text{ or } \theta = \frac{2\pi}{3}.$$

12. (c)

13. (a): Rate of cooling is proportional to $(T^4 - T_0^4)$, as per Stefan's law.

$$\begin{aligned} \therefore \frac{R'}{R} &= \frac{900^4 - 300^4}{600^4 - 300^4} \\ &= \frac{9^4 - 3^4}{6^4 - 3^4} = \frac{3^4(3^4 - 1)}{3^4(2^4 - 1)} = \frac{80}{15} = \frac{16}{3} \end{aligned}$$

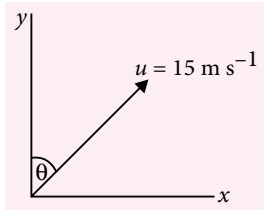
$$\text{or } R' = \frac{16}{3} R.$$

14. (c): As $\frac{C_P}{C_V} = \gamma \therefore \frac{C_P - C_V}{C_V} = \gamma - 1$

$$\text{or } C_V = \frac{C_P - C_V}{\gamma - 1} = \frac{R}{\gamma - 1}$$

$$\begin{aligned} \Delta U &= nC_V dT = n \frac{RdT}{(\gamma - 1)} = \frac{PdV}{\gamma - 1} \\ &= \frac{P(2V - V)}{\gamma - 1} = \frac{PV}{\gamma - 1} \end{aligned}$$

15. (c) : Here, $u_y = u \cos \theta = 15 \cos \theta$
 $u_x = u \sin \theta = 15 \sin \theta$



Time of flight of the ball is

$$T = \frac{2u_y}{g} = \frac{2 \times 15 \cos \theta}{10} = 3 \cos \theta \quad \dots(i)$$

The boy will catch the ball if in time T , displacement of the ball in horizontal direction should also be

zero. So $0 = u_x T - \frac{1}{2} a_x T^2$

$$\text{or } T = \frac{2u_x}{a_x} = \frac{2(15 \sin \theta)}{3} = 10 \sin \theta \quad \dots(ii)$$

From (i) and (ii), $3 \cos \theta = 10 \sin \theta$

$$\text{or } \tan \theta = \frac{3}{10} = 0.3 \text{ or } \theta = \tan^{-1}(0.3).$$

16. (d) : As the rod is hinged at one end, its moment of inertia about this end is $I = \frac{ML^2}{3}$.

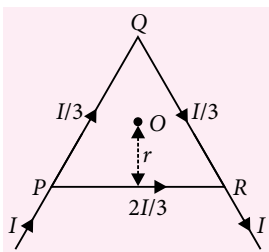
Total energy in upright position = total energy on striking the floor

$$0 + \frac{MgL}{2} = \frac{1}{2} I \omega^2 + 0 = \frac{1}{2} \frac{ML^2}{3} \omega^2$$

$$g = \frac{L\omega^2}{3} \text{ or } \omega = \sqrt{\frac{3g}{L}}$$

17. (b)

18. (d) :



The magnetic field induction at O due to current through PR is

$$B_1 = \frac{\mu_0}{4\pi} \frac{2I/3}{r} [\sin 30^\circ + \sin 30^\circ] = \frac{\mu_0}{4\pi} \frac{2I}{3r}.$$

It is directed outside the paper.

The magnetic field induction at O due to current through PQR is

$$B_2 = 2 \times \frac{\mu_0}{4\pi} \frac{(I/3)}{r} [\sin 30^\circ + \sin 30^\circ] = \frac{\mu_0}{4\pi} \frac{2I}{3r}.$$

It is directed inside the paper.

\therefore Resultant magnetic field induction at O is $B_1 - B_2 = 0$.

19. (a) : As the inductors are in parallel, therefore, induced e.m.f. across the two inductors is the same i.e.

$$\mathcal{E}_1 = \mathcal{E}_2$$

$$L_1 \left(\frac{dI_1}{dt} \right) = L_2 \left(\frac{dI_2}{dt} \right)$$

Integrating both sides, we get

$$L_1 I_1 = L_2 I_2 \quad \therefore \frac{I_1}{I_2} = \frac{L_2}{L_1}$$

20. (a)

21. (b) : Energy contained in a cylinder

$U = \text{average energy density} \times \text{volume}$

$$= \frac{1}{2} \epsilon_0 E_0^2 \times Al$$

$$= \frac{1}{2} \times (8.85 \times 10^{-12}) \times (50)^2 \times (10 \times 10^{-4}) \times 1$$

$$= 1.1 \times 10^{-11} \text{ J}$$

22. (b) : As $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$

$$\text{Here, } \phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}, I_1 = I_2 = I$$

$$\therefore I_R = I + I + 2I \cos \frac{\pi}{2} = 2I$$

At the central bright fringe, $I' = 4I$

$$\therefore \frac{I_R}{I'} = \frac{2I}{4I} = 0.5$$

23. (a) : Velocity of object, $\vec{v}_{ob} = 3\hat{i} + 4\hat{j}$

$$\text{Velocity of image } \vec{v}_{image} = 3\hat{i} - 4\hat{j}$$

Relative velocity of image with respect to its object

$$\vec{v}_{rel} = \vec{v}_{image} - \vec{v}_{ob} = (3\hat{i} - 4\hat{j}) - (3\hat{i} + 4\hat{j}) = -8\hat{j}$$

24. (d) 25. (c)

26. (d) : $E_0 = 66 \text{ V m}^{-1}$

$$B_0 = \frac{E_0}{c} = \frac{66}{3 \times 10^8} = 2.2 \times 10^{-7} \text{ T}$$

Since electromagnetic wave is of transverse nature, hence if electric field is along y -axis the magnetic field must be along z -axis, since the propagation of wave is along x -axis. Thus the equations given in option (d) are correct.

27. (c) : The masses of three isotopes are 19.99 u, 20.99 u, 21.99 u.

Their relative abundances are 90.51%, 0.27% and 9.22%.

∴ Average atomic mass of neon is

$$m = \frac{90.51 \times 19.99 + 0.27 \times 20.99 + 9.22 \times 21.99}{(90.51 + 0.27 + 9.22)}$$

$$= \frac{1809.29 + 5.67 + 202.75}{100} = \frac{2017.7}{100} = 20.18 \text{ u}$$

28. (c) : Useful intensity for the emission of electron is

$$I' = 1\% \text{ of } I = \frac{1}{100} \times 39.6 = 0.396 \text{ W m}^{-2}$$

$$\text{Energy of each photon} = \frac{hc}{\lambda}$$

$$= \frac{(6.64 \times 10^{-34}) \times (3 \times 10^8)}{6000 \times 10^{-10}} = 3.32 \times 10^{-19} \text{ J}$$

Number of photoelectrons emitted per second per unit area

$$= \frac{0.396}{3.32 \times 10^{-19}} \approx 12 \times 10^{17}$$

29. (a) : Let N_0 be the initial amount of a radioactive substance. Then the amount left after n half-lives

$$\text{will be } N = N_0 \left(\frac{1}{2}\right)^n$$

$$\text{For A : } n_A = \frac{t}{T_{1/2}} = \frac{16 \text{ s}}{4 \text{ s}} = 4$$

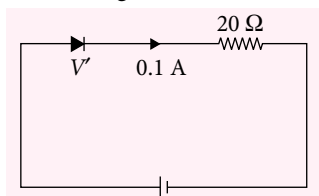
$$N_A = 10^{-2} \text{ kg} \left(\frac{1}{2}\right)^4 = 6.25 \times 10^{-4} \text{ kg}$$

$$\text{For B : } n_B = \frac{16 \text{ s}}{8 \text{ s}} = 2$$

$$\therefore N_B = 10^{-2} \text{ kg} \left(\frac{1}{2}\right)^2 = 2.5 \times 10^{-3} \text{ kg}$$

$$\frac{N_A}{N_B} = \frac{1}{4}$$

30. (c) : The circuit diagram is as shown below :



$$V = V' + IR = 0.5 + 0.1 \times 20 = 0.5 + 2.0 = 2.5 \text{ V}$$

31. (a) : Here $E_\infty - E_1 = 13.6$ or $E_1 = -13.6 \text{ eV}$

For second excited state,

$$E_3 = \frac{E_1}{3^2} = -\frac{13.6}{9} = -1.51 \text{ eV}$$

Energy required to ionise H-atom from second excited state

$$= E_\infty - E_3 = 0 + 1.51 = 1.51 \text{ eV.}$$

$$32. (a) : \frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow v^2 = \frac{GM}{R}$$

$$\text{Also, } v = \frac{2\pi R}{T} \Rightarrow v^2 = \frac{4\pi^2 R^2}{T^2} = \frac{GM}{R}$$

$$\therefore T^2 = \frac{4\pi^2 R^3}{GM}$$

If T_1 and T_2 are the time periods for satellite S_1 and S_2 respectively

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow R_2 = \left(\frac{T_2}{T_1}\right)^{2/3} R_1$$

Here, $T_1 = 1 \text{ h}$, $T_2 = 8 \text{ h}$, $R_1 = 10^4 \text{ km}$

$$\therefore R_2 = \left(\frac{8}{1}\right)^{2/3} \times 10^4 \text{ km} = 4 \times 10^4 \text{ km}$$

$$v_1 = \frac{2\pi R_1}{T_1} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4 \text{ km h}^{-1}$$

$$v_2 = \frac{2\pi R_2}{T_2} = \frac{2\pi \times 4 \times 10^4}{8} = \pi \times 10^4 \text{ km h}^{-1}$$

Relative velocity of S_2 with respect to S_1 is

$$v = v_2 - v_1 = (\pi \times 10^4 - 2\pi \times 10^4) \text{ km h}^{-1}$$

$$|v| = \pi \times 10^4 \text{ km h}^{-1}$$

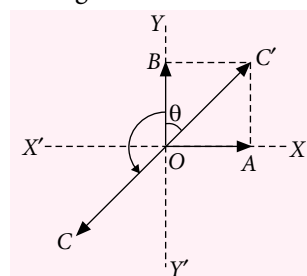
33. (c) : Here $P_x = P$ and $P_y = \sqrt{3} P$

∴ Resultant momentum of A and B

$$P = \sqrt{P_x^2 + P_y^2} = \sqrt{P^2 + (\sqrt{3}P)^2} = 2P$$

It is along OC' .

As is clear from figure



$$\tan \theta = \frac{BC'}{OB} = \frac{OA}{OB} = \frac{P}{\sqrt{3}P} = \frac{1}{\sqrt{3}} \text{ or } \theta = 30^\circ$$

As the object was initially at rest, the vector sum of linear momenta of A, B and C must be zero. Therefore, momentum of $C = 2P$ along OC opposite to OC' . It makes an angle with $B = \angle YOC = \angle YOX' + \angle X'OC = 90^\circ + 60^\circ = 150^\circ$

34. (d) : As the rod rotates about A, therefore, from conservation of mechanical energy, decrease in potential energy = increase in rotational kinetic energy about A

$$mg\left(\frac{l}{2}\right) = \frac{1}{2} I_A \omega^2 = \frac{1}{2} \left(\frac{ml^2}{3} \right) \omega^2 \text{ or } \omega^2 = \frac{3g}{l}$$

Centripetal force of centre of mass of the rod in this position is $= m r \omega^2 = m \frac{l}{2} \frac{3g}{l} = \frac{3mg}{2}$.

If F is the force exerted by the hinge on the rod (upwards), then $F - mg = \frac{3mg}{2}$

$$F = \frac{3mg}{2} + mg = \frac{5}{2} mg.$$

35. (a) : Resistance of 40 W bulb, $R_1 = \frac{(240)^2}{40} \Omega$

Resistance of 60 W bulb, $R_2 = \frac{(240)^2}{60} \Omega$

When bulbs are in series, the effective resistance

$$R = R_1 + R_2 = (240)^2 \left[\frac{1}{40} + \frac{1}{60} \right] = \frac{(240)^2}{24} \Omega$$

$$\text{Current } I = \frac{420 \times 24}{(240)^2} = \frac{21}{120} \text{ A}$$

Potential difference across 40 W bulb

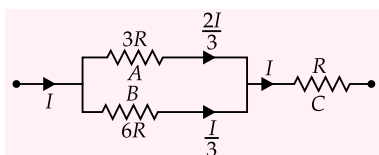
$$= \frac{21}{120} \times \frac{(240)^2}{40} = 252 \text{ V}$$

Potential difference across 60 W bulb

$$= \frac{21}{120} \times \frac{(240)^2}{60} = 168 \text{ V}$$

Since potential difference of 40 W is greater than 240 V, so it will work at above its rated voltage.

36. (c) :



Thermal power in A, $P_A = \left(\frac{2I}{3} \right)^2 3R = \frac{4}{3} I^2 R$

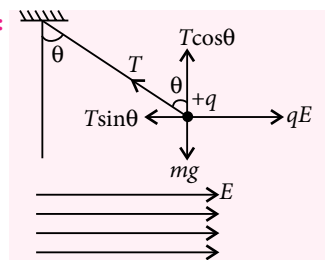
Thermal power in B, $P_B = \left(\frac{I}{3} \right)^2 6R = \frac{2}{3} I^2 R$

Thermal power in C, $P_C = I^2 R$.

$$P_A : P_B : P_C = \frac{4}{3} I^2 R : \frac{2}{3} I^2 R : I^2 R = 4 : 2 : 3$$

37. (d)

38. (b) :



From figure at equilibrium,

$$T \cos \theta = mg \text{ or } T = \frac{mg}{\cos \theta}; T \sin \theta = qE \text{ or } T = \frac{qE}{\sin \theta}$$

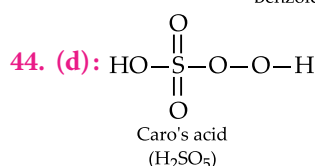
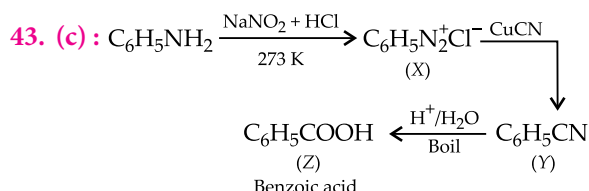
39. (d) : Here, $A_m = 1 \text{ V}$, $A_c = 2 \text{ V}$,

$$\text{Modulation index, } \mu = \frac{A_m}{A_c} = \frac{1}{2} = 0.5$$

40. (b) : The zener diode when used as a voltage regulator is connected in reverse bias and a load is connected in parallel to zener diode for output voltage.

41. (a) : Due to the extra stability of half-filled p -orbitals of N, its first ionisation potential is higher than those of O and C. Further because of higher nuclear charge, first ionisation potential of C is higher than that of Be and B. Amongst Be and B, the first ionisation potential of Be is higher than that of B because in case of Be, an electron is to be removed from $2s^2$ orbital while in case of B, an electron is to be removed from $2p^1$ orbital. Thus, the overall order is $B < Be < C < O < N$.

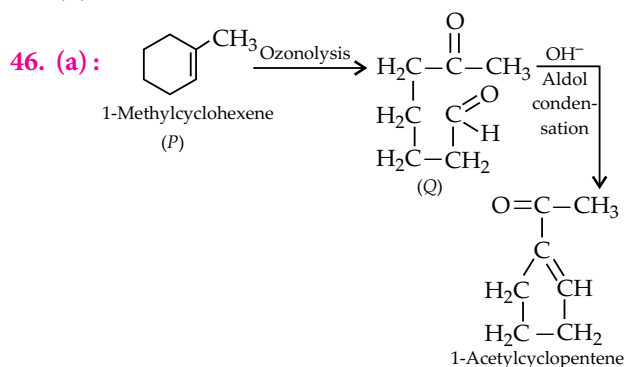
42. (d) : Colour of salts is a property of partially filled d -orbitals. Since TiF_6^{2-} has completely empty and Cu_2Cl_2 has completely filled d -subshells, hence these are colourless salts.



Two oxygen atoms are in peroxo linkage i.e., oxidation number = -1

$$3(-2) + 2(-1) + x + 2(+1) = 0 \Rightarrow -8 + x + 2 = 0 \Rightarrow x = +6$$

45. (d)



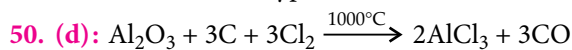
47. (c) : N atom in NCl_3 and S atom in H_2S are sp^3 hybridised.

48. (c)

$$49. (a) : \rho = \frac{Z \times M}{N_0 \times a^3} \text{ or } Z = \frac{\rho \times N_0 \times a^3}{M}$$

$$Z = \frac{2.7 \times 6.023 \times 10^{23} \times (405 \times 10^{-10})^3}{27.0} = 4$$

i.e., number of atoms per unit cell is 4. Hence, unit cell is face-centred type.

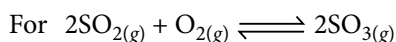
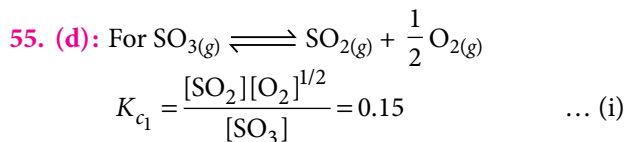


51. (d) : $\text{C}_6\text{H}_5\text{COC}(\text{CH}_3)_3$ does not contain an α -hydrogen and hence does not show tautomerism.

52. (d) : Due to the +ve charge on P, it attracts the electrons of the P—H bond towards itself. As a result, it has some ionic character. In other words, the P—H bond in PH_6^+ is least covalent.

53. (d) : $3\text{HClO}_{(aq)} \longrightarrow \text{HClO}_{3(aq)} + 2\text{HCl}_{(aq)}$
It is a disproportionation reaction of hypochlorous acid where the oxidation number of Cl changes from +1 (in ClO^-) to +5 (in ClO_3^-) and -1 (in Cl^-).

54. (a)



$$K_{c2} = \frac{[\text{SO}_3]^2}{[\text{SO}_2]^2[\text{O}_2]} \quad \dots (ii)$$

By reversing Eq. (i), $\frac{1}{K_{c1}} = \frac{[\text{SO}_3]}{[\text{SO}_2][\text{O}_2]^{1/2}}$

On making square,

$$\left(\frac{1}{K_{c1}} \right)^2 = \frac{[\text{SO}_3]^2}{[\text{SO}_2]^2[\text{O}_2]} = K_{c2} \quad [\text{By Eq. (ii)}]$$

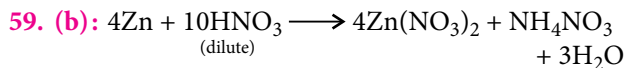
$$\therefore K_{c2} = \left(\frac{1}{0.15} \right)^2 = 44.44 \text{ mol}^{-1} \text{ L}$$

56. (b)

57. (c) : Sulphate ion is present outside the coordination sphere so it can form white ppt. of BaSO_4 with $\text{BaCl}_{2(aq)}$.

58. (c) : $\alpha = \frac{\Lambda_m}{\Lambda_m^\circ} = \frac{184.5}{502.4} = 0.367$

$$K = \frac{C\alpha^2}{1-\alpha} = \frac{0.001\text{M} \times (0.367)^2}{0.633} = 2.127 \times 10^{-4} \text{ M}$$



60. (b) : Aromaticity can be predicted by the use of Huckle's rule which says that $(4n + 2)$ π -electrons are required in delocalisation system to give it aromaticity.

61. (d) 62. (c)

63. (d) : $E \propto \frac{1}{n^2}$ i.e., when $n = 3$; E decreases nine times.

64. (d) 65. (a)

66. (b) : Sodium sulphide is soluble in water. The solubility product (and hence solubility) of ZnS is larger than that of CuS .

67. (b) : Reduction in presence of Zn-Hg and conc. HCl is useful for aldehyde and ketone but carboxylic acid group remains unaffected.

68. (a) : $\Delta H = (E_a)_f - (E_a)_b = 0$

69. (b) : Since sp -hybridized carbon is more electronegative than a sp^2 -hybridized carbon which in turn is more electronegative than sp^3 -hybridized carbon, therefore, $\text{CH} \equiv \text{C} - \text{COOH}$ is a stronger acid than $\text{CH}_2 = \text{CH} - \text{COOH}$ which in turn, is a stronger acid than $\text{CH}_3 - \text{CH}_2 - \text{COOH}$. Thus, the overall order of acid strength is (i) > (ii) > (iii).

70. (c) : Since ionization potential of hydrogen atom is 13.6 eV.

$$\therefore E_1 = -13.6 \text{ eV}$$

Now, $E_n - E_1 = \frac{-13.6}{n^2} - (-13.6) = 12.1$

$$\frac{-13.6}{n^2} + 13.6 = 12.1 \quad \therefore n = 3$$

After absorbing 12.1 eV the electron in H atom is excited to 3^{rd} shell.

Thus, possible transitions are 3

i.e., $3 \rightarrow 2$, $2 \rightarrow 1$ and $3 \rightarrow 1$.

71. (a)

72. (b) : Given : Weight of benzoic acid = 1.89 g;
Temperature of bomb calorimeter = $25^\circ\text{C} = 298 \text{ K}$;
Mass of water (m) = 18.94 kg = 18940 g;
Increase in temperature (t) = 0.632°C and
specific heat of water (s) = 0.998 cal/g-deg.
We know that heat gained by water or heat liberated by benzoic acid (Q) = $ms\Delta t$

$$= 18940 \times 0.998 \times 0.632 = 11946.14 \text{ cal}$$

...Contd. on Page no. 76



GO!!

EXAM ON
24TH
MAY

READY STEADY

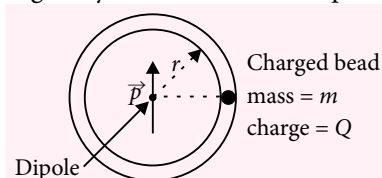
PAPER-1

SECTION-1

One or More Than One Options Correct Type

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONE or MORE THAN ONE are correct.

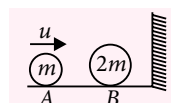
1. A small charged bead can slide on a circular frictionless, insulating wire frame. A point like dipole is fixed at the centre of circle, dipole moment is \vec{p} . Initially the bead is on the plane of symmetry of the dipole. Bead is released from rest. Ignore the effect of gravity. Select the correct options.



- (a) Magnitude of velocity of bead as function of its angular position is $\sqrt{\frac{Qp \cos \theta}{2\pi\epsilon_0 m r^2}}$.
- (b) Normal force exerted by the string on bead is zero.
- (c) If the wire frame were not present bead executes circular motion and returns to initial point after tracing a complete circle.
- (d) Bead would move along a circular path until it reaches the opposite its starting position and then executes periodic motion.
2. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are the same. The two bodies emit total radiant power of the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A, by $1.0 \mu\text{m}$. If the temperature of A is 5802 K,

- (a) the temperature of B is 1934 K
- (b) $\lambda_B = 1.5 \mu\text{m}$
- (c) the temperature of B is 11604 K
- (d) $\lambda_B = 2.0 \mu\text{m}$

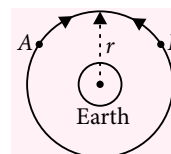
3. The two balls A and B as shown in figure are of masses m and $2m$, respectively.



The ball A moves with velocity u toward right while B is at rest. The wall at extreme right is fixed. Coefficient of restitution for collision between two balls is $\frac{1}{2}$ and between either ball and wall is 1. Then speeds of A and B after all possible collisions are

- (a) $v_A = \frac{u}{2}$ (b) $v_A = \frac{u}{4}$
- (c) $v_B = \frac{u}{8}$ (d) $v_B = \frac{u}{4}$

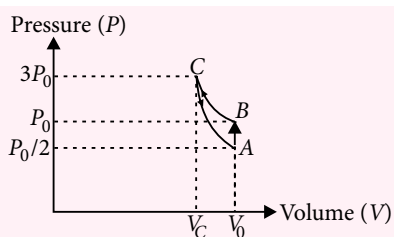
4. Consider two satellites A and B of equal mass m , moving same circular orbit about earth, but in opposite sense as shown in figure. The orbital radius is r .



The satellites undergoes a collision which is perfectly inelastic. For this situation, mark out the correct statement(s). [Take mass of earth as M]

- (a) The total energy of the two satellites plus earth system just before collision is $-\frac{GMm}{r}$.
- (b) The total energy of the two satellites plus earth system just after collision is $-\frac{2GMm}{r}$.
- (c) The total energy of the two satellites plus earth system just after collision is $-\frac{GMm}{2r}$.
- (d) The combined mass (two satellites) will fall towards the earth just after collision.

5. A charged particle with velocity $\vec{v} = x\hat{i} + y\hat{j}$ moves in a magnetic field $\vec{B} = y\hat{i} + x\hat{j}$. The force acting on the particle has magnitude F . Which one of the following statements is/are correct?
- No force will act on charged particle if $x = y$.
 - If $x > y$, $F \propto (x^2 - y^2)$.
 - If $x > y$, the force will act along z -axis.
 - If $y > x$, the force will act along y -axis.
6. One mole of an ideal gas is carried through a thermodynamic cycle as shown in the figure. The cycle consists of an isochoric, an isothermal and an adiabatic processes. The adiabatic exponent of the gas is γ . Choose the correct option(s).



- $\gamma = \frac{\ln 6}{\ln 3}$
 - $\gamma = \frac{\ln 5}{\ln 3}$
 - BC is adiabatic
 - AC is adiabatic
7. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A} \times \vec{L}$ where \vec{A} is a constant vector, and \vec{L} is the angular momentum of the body about that point. From this it follows that
- $\frac{d\vec{L}}{dt}$ is perpendicular to \vec{L} at all instants of time.
 - the component of \vec{L} in the direction of \vec{A} does not change with time.
 - the magnitude of \vec{L} does not change with time.
 - \vec{L} does not change with time.
8. A metallic sphere of radius r remote from all other bodies is irradiated with a radiation wavelength λ which is capable of causing photoelectric effect. Mark out the correct statement(s).
- The maximum potential gained by the sphere will be independent of its radius.
 - The net positive charge appearing on the sphere after a long time will depend on the radius of the sphere.
 - The kinetic energy of the most energetic electrons emanating from the sphere will keep declining with time.
 - The kinetic energy of the most energetic electrons emanating from the sphere initially will be independent of the radius of the sphere.

9. When a charge of amount Q is given to an isolated metal plate X of surface area A , its surface charge density becomes σ_1 . When an isolated identical plate Y is brought close to X the surface charge density on X becomes σ_2 . When Y is earthed the surface charge density on X becomes σ_3 . Then

- $\sigma_1 = \frac{Q}{A}$
- $\sigma_1 = \frac{Q}{2A}$
- $\sigma_1 = \sigma_2$
- $\sigma_3 = \frac{Q}{A}$

10. An inductor of inductance 2 mH is connected across a charged capacitor of capacitance $5 \mu\text{F}$ and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of Q is $200 \mu\text{C}$.

- When $Q = 100 \mu\text{C}$, then the value of $\left| \frac{dI}{dt} \right| = 10^4 \text{ A s}^{-1}$.
- When $Q = 200 \mu\text{C}$, then the value of $I = 2 \text{ A}$.
- Maximum value of $I = 3 \text{ A}$.
- When I is equal to one-half the maximum value then the value of $|Q|$ is $173 \mu\text{C}$.

SECTION-2

One Integer Value Correct Type

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

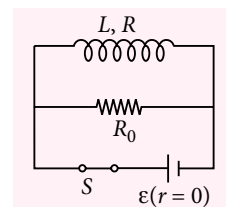
11. Two identical steel cubes each of mass 50 g and side 1 cm collide head-on face to face with a speed of 10 cm s^{-1} each. The maximum compression of each cube is $n \times 10^{-7} \text{ m}$, then find n .

[Young's modulus for steel = $2 \times 10^{11} \text{ N m}^{-2}$]

12. The internal energy of monatomic ideal gas is $1.5 nRT$. One mole of helium is kept in a cylinder of cross-section 8.5 cm^2 . The cylinder is closed by a light frictionless piston. The gas is heated slowly in a process during which a total of 42 J heat is given to the gas. The temperature rises through 2°C . The distance moved by the piston is given as $\alpha \times 10^{\beta} \text{ m}$ in scientific notation. Find the value of $\alpha + \beta$.

[Take $R = \frac{25}{3}$ in SI unit, atmospheric pressure = 100 kPa]

13. A coil of inductance $L = 2 \mu\text{H}$ and resistance $R = 1 \Omega$ is connected to a source of constant emf $\epsilon = 3 \text{ V}$ as shown in the figure.



A resistance $R_0 = 2 \Omega$ is connected in parallel with the coil. Find the amount of heat generated (in μ J) after the switch S is disconnected.

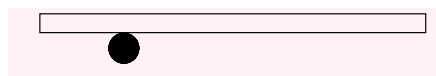
14. A cylindrical wooden float whose base area is 4 m^2 and height 1 m drifts on the water surface in vertical position. Density of wood is 500 kg m^{-3} and that of water is 1000 kg m^{-3} . What minimum work (in kJ) must be performed to take the float out the water?
15. Light of wavelength 627 nm illuminates two slits. The minimum path difference between the waves from the slits for the resultant intensity to fall to 25% of the central maximum is $(200 + n)\text{nm}$. Find the value of n .
16. Consider two point masses m and λm located at points, $x = a$ and $x = \mu a$ respectively. Assuming that the sum of the two masses is constant, what is the value of λ for which the magnitude of the gravitational force is maximum?
17. A X-ray tube is working at a potential difference of 20 kV . The potential difference is decreased to 10 kV . It is found that difference of the wavelength of K_α X-ray and the most energetic continuous X-ray becomes 4 times of the difference prior to the change of voltage. The atomic number of the target element is $11x$. Find the value of x .

18. Find the ratio of the fundamental tone frequencies of two identical strings after one of them was stretched by $n_1 = 2\%$ and the other by $n_2 = 9\%$. Also the tension is assumed to be proportional to the elongation.

19. Electromagnetic radiation whose electric component varies with time as $E = C_1(C_2 + C_3 \cos \omega t) \cos \omega_0 t$, here C_1 , C_2 and C_3 are constants, is incident on lithium and liberates photoelectrons. If the kinetic energy of most energetic electrons be 2.6 eV , the work function of lithium is (in eV). [Take : $\omega_0 = 2.4\pi \times 10^{15} \text{ rad s}^{-1}$ and $\omega = 0.8\pi \times 10^{14} \text{ rad s}^{-1}$, Plank's constant $h = 6.6 \times 10^{-34} \text{ MKS}$]

20. A thin rod of mass m and length $2l$ is placed horizontal and perpendicular to a horizontal rough nail, as shown in figure and set free. The point of contact of the rod with the nail is $l/3$ distance away from the centre of rod. If the rod starts slipping when it forms an angle θ with the horizontal and the coefficient of friction of rod with nail is μ , then

find $\frac{\mu}{\tan \theta}$.



PAPER-2

SECTION-1

Only One Option Correct Type

This section contains 10 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

1. What is the minimum attainable pressure of the gas in the process $T = T_0 + \alpha V^2$, where T_0 and α are the positive constants, and V is the volume of one mole?
- (a) $2R\alpha\sqrt{T_0}$ (b) $2R\sqrt{\alpha T_0}$
- (c) $R\sqrt{2\alpha T_0}$ (d) 0
2. The differential equation of charging of a capacitor is as given below :
- $$\frac{1}{K_1} \frac{dq}{dt} + K_2 q = K_3$$

The time constant and steady state charge are respectively

- (a) $\frac{1}{K_1}$ and K_3 (b) $\frac{1}{K_1 K_2}$ and $\frac{K_3}{K_2}$
- (c) $\frac{K_2}{K_1}$ and $K_2 K_3$ (d) $\frac{1}{K_1 K_2}$ and $\frac{K_3}{K_1}$

3. A chain of mass M and length l is suspended vertically with its lower end touching a weighing scale. The chain is released and falls freely onto the scale. Neglecting the size of the individual links, what is the reading of the scale when a length x of the chain has fallen?

- (a) $\frac{Mgx}{l}$ (b) $\frac{2Mgx}{l}$
- (c) $\frac{3Mgx}{l}$ (d) $\frac{4Mgx}{l}$

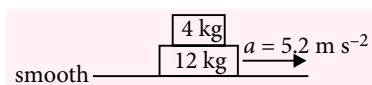
4. A point mass m is suspended at the end of massless wire of length L and cross-sectional area A . If Y is the Young's modulus of elasticity of the material of wire. The frequency of simple harmonic motion along the vertical line is

- (a) $\frac{1}{2\pi} \sqrt{\frac{A}{mLY}}$ (b) $\frac{1}{2\pi} \sqrt{\frac{YA^2}{mL^3}}$
- (c) $\frac{1}{2\pi} \sqrt{\frac{YA}{mL}}$ (d) $\frac{1}{2\pi} \sqrt{\frac{YL}{m}}$

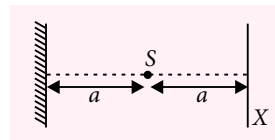
5. A curved rectangular bar forms a resistor. The curved sides are concentric circular arcs. If ρ is the resistivity of the material of bar, l_0 is the length of inner arc of radius r_0 , $(r_0 + b)$ is the radius of the outer arc, and a is the width of the bar. The electric resistance of the bar across its rectangular ends is

- (a) $\frac{\rho l_0}{ar_0}$ (b) $\frac{\rho l_0}{ar_0 \ln \left[1 + \frac{b}{r_0} \right]}$
 (c) $\frac{2\rho l_0}{ar_0 \ln \left[1 + \frac{b}{r_0} \right]}$ (d) None of these

6. A 4 kg block is placed on top of a long 12 kg block, which is accelerating along a smooth horizontal table at $a = 5.2 \text{ m s}^{-2}$ under application of an external constant force. Let minimum coefficient of friction between the two blocks which will prevent the 4 kg block from sliding is μ_0 and coefficient of friction between blocks is only half of this minimum value (i.e., $\mu_0/2$). Find the amount of heat generated due to sliding between the two blocks during the time in which 12 kg block moves 10 m starting from rest.



- (a) 26 J (b) 12 J
 (c) 52 J (d) 48 J
7. What should be the minimum value of refractive index of a prism of refractive angle A , so that there is no emergent ray irrespective of angle of incidence?
- (a) $\sin \frac{A}{2}$ (b) $\cos \frac{A}{2}$
 (c) $\operatorname{cosec} \frac{A}{2}$ (d) $\sec \frac{A}{2}$
8. A particle is projected from the origin in such a way that it passes through a given point $P(a, b)$. The minimum required speed to do so is
- (a) bg (b) ag
 (c) $\sqrt{ag + g\sqrt{a^2 + b^2}}$ (d) $\sqrt{bg + g\sqrt{a^2 + b^2}}$
9. A point source of light S is placed in front of a perfect reflecting mirror as shown in the figure. X is a screen. The intensity at the centre of screen is found to be I . If the mirror is removed, then the intensity at the centre of screen would be



- (a) I (b) $\frac{10I}{9}$
 (c) $\frac{9I}{10}$ (d) $2I$

10. Inside a long straight uniform wire of round cross-section there is a long round cylindrical cavity whose axis is parallel to the axis of the wire and displaced from latter by a distance d . If a direct current of density \vec{j} flows along the wire, then magnetic field inside the cavity will be

- (a) 0 (b) $\frac{1}{2} \mu_0 \vec{j} \times \vec{d}$
 (c) $\mu_0 \vec{j} \times \vec{d}$ (d) $\frac{3}{2} \mu_0 \vec{j} \times \vec{d}$

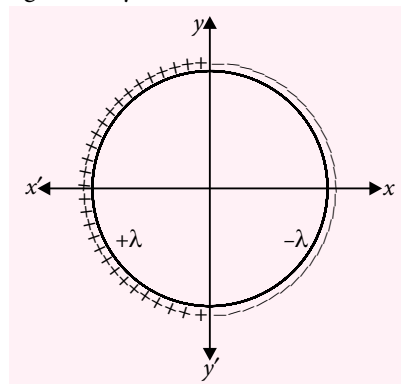
SECTION-2

Comprehension Type (Only One Option Correct)

This section contains 3 paragraphs, each describing theory, experiments, data etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given options (a), (b), (c) and (d).

Paragraph for questions 11 and 12

A thin ring of radius R metres is placed in x - y plane such that its centre lies on origin. The half ring in region $x < 0$ carries uniform linear charge density $+\lambda \text{ C m}^{-1}$ and the remaining half ring in region $x > 0$ carries uniform linear charge density $-\lambda \text{ C m}^{-1}$.



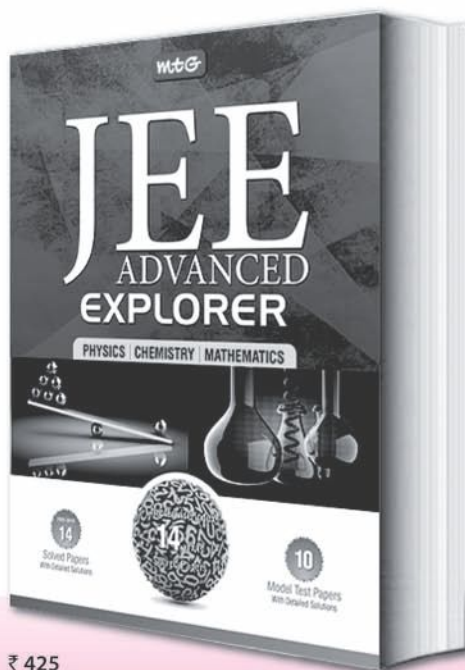
11. The electric potential (in volts) at point P whose coordinates are $(0 \text{ m}, +\frac{R}{2} \text{ m})$ is
- (a) $\frac{1}{4\pi\epsilon_0} \frac{\lambda}{2}$ (b) 0
 (c) $\frac{1}{4\pi\epsilon_0} \frac{\lambda}{4}$ (d) cannot be determined

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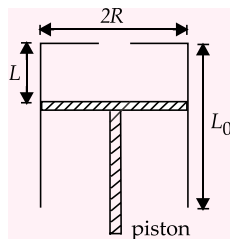
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12. The dipole moment of the ring in C m is

- (a) $-(2\pi R^2 \lambda) \hat{i}$ (b) $(2\pi R^2 \lambda) \hat{i}$
(c) $-(4R^2 \lambda) \hat{i}$ (d) $(4R^2 \lambda) \hat{i}$

Paragraph for questions 13 and 14

A fixed thermally conducting cylinder has a radius R and height L_0 . The cylinder is open at its bottom and has a small hole at its top. A piston of mass M at a distance L from the top surface is as shown in the figure. The atmospheric pressure is P_0 .



13. The piston is now pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and the piston will then be

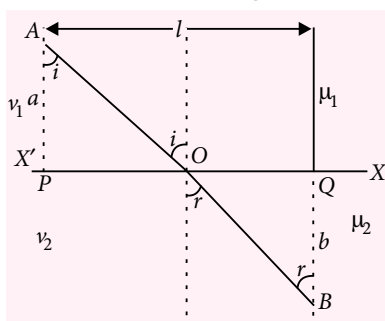
- (a) P_0 (b) $\frac{P_0}{2}$
(c) $\frac{P_0}{2} + \frac{Mg}{\pi R^2}$ (d) $\frac{P_0}{2} - \frac{Mg}{\pi R^2}$

14. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

- (a) $\left(\frac{2P_0 \pi R^2}{\pi R^2 P_0 + Mg} \right) (2L)$ (b) $\left(\frac{P_0 \pi R^2 - Mg}{\pi R^2 P_0} \right) (2L)$
(c) $\left(\frac{P_0 \pi R^2 + Mg}{\pi R^2 P_0} \right) (2L)$ (d) $\left(\frac{P_0 \pi R^2}{\pi R^2 P_0 - Mg} \right) (2L)$

Paragraph for questions 15 and 16

A ray of light goes from point A in a medium where the speed of light is v_1 to a point B in a medium where the speed of light is v_2 as shown in the figure. The path of the rays in the two is shown in figure.



15. The time taken for the light to go from the point A to the point B in the figure is

- (a) $\frac{a \sin i}{v_1}$ (b) $\frac{b \sin r}{v_2}$
(c) $\frac{v_2 a \sin r}{v_1 b \sin r}$ (d) $\frac{a \sec i}{v_1} + \frac{b \sec r}{v_2}$

16. The slope of $i - r$ curve is

- (a) $\frac{-b \cos^2 i}{a \cos^2 r}$ (b) $\frac{a \cos r}{b \cos i}$
(c) $\frac{a \sin^2 r}{b \sin^2 i}$ (d) $\frac{-a \tan i}{b \tan r}$

SECTION-3

Matching List Type (Only One Option Correct)

This section contains four questions, each having two matching lists. Choices for the correct combination of elements from List-I and List-II are given as options (a), (b), (c) and (d), out of which one is correct.

17. In case of motion of a particle of mass m in a vertical circle of radius r with the help of a string, velocity at lowest point is v_0 . Assume the vertical distance of any said point from lowest point to be h . Match the Column I and Column II.

Column I		Column II	
P.	Tension in the string at any point	1.	$2mg$
Q.	Minimum value of v_0 for looping the loop	2.	$6mg$
R.	Difference of tensions at the lowest and the highest points	3.	$\sqrt{5gr}$
S.	Difference of tensions at the lowest and the highest points in case the motion is uniform	4.	$\frac{m}{r}(v_0^2 - 3gh + gr)$

Code :

- (a) P - 4, Q - 3, R - 2, S - 1
(b) P - 1, Q - 3, R - 4, S - 2
(c) P - 2, Q - 3, R - 1, S - 4
(d) P - 3, Q - 2, R - 4, S - 1

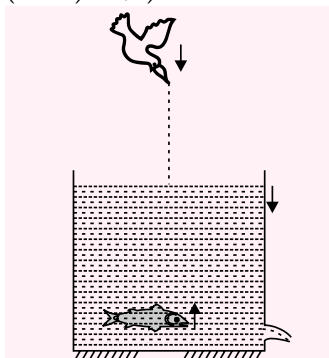
18. Match the quantities given in Column-I with their values given in Column-II.

Column I		Column II	
P.	n^{th} orbital radius in Bohr model	1.	$\frac{e^2}{2\epsilon_0 nh}$
Q.	n^{th} orbital speed in Bohr model	2.	$\frac{-me^4}{8\epsilon_0^2 n^2 h^2}$
R.	n^{th} orbital total energy in Bohr model	3.	$\frac{-me^4}{4\epsilon_0^2 n^2 h^2}$
S.	n^{th} orbital potential energy in Bohr model	4.	$\frac{\epsilon_0 n^2 h^2}{\pi me^2}$

Code :

- (a) P - 4, Q - 1, R - 2, S - 3
 (b) P - 1, Q - 3, R - 4, S - 2
 (c) P - 2, Q - 3, R - 1, S - 4
 (d) P - 3, Q - 2, R - 4, S - 1

19. A bird in air is diving vertically over a tank with speed 6 cm s^{-1} . Base of tank is silvered. A fish in the tank is rising upward along the same line with speed 4 cm s^{-1} . Water level is falling at rate of 2 cm s^{-1} . (Take : μ (water) = $4/3$)



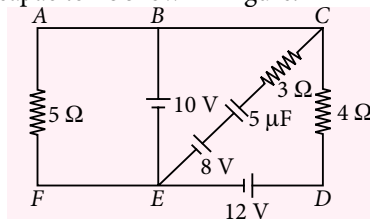
Column I		Column II	
P.	Speed of the image of fish as seen by the bird directly	1.	12 cm s^{-1}
Q.	Speed of the image of fish formed after reflection from the mirror as seen by the bird	2.	4 cm s^{-1}
R.	Speed of image of bird relative to the fish looking upwards	3.	9 cm s^{-1}

S.	Speed of image of bird relative to the fish looking downwards in the mirror	4.	3 cm s^{-1}
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Code :

- (a) P - 4, Q - 1, R - 2, S - 3
 (b) P - 1, Q - 2, R - 4, S - 3
 (c) P - 3, Q - 1, R - 2, S - 4
 (d) P - 3, Q - 4, R - 1, S - 2

20. A network consisting of three resistors, three batteries, and a capacitor is shown in figure.



Column I		Column II	
P.	Current in branch EB is	1.	2 A
Q.	Current in branch CB is	2.	1.5 A
R.	Current in branch AF is	3.	0.5 A
S.	Current in branch EC is	4.	0 A

Code :

- (a) P - 4, Q - 1, R - 2, S - 3
 (b) P - 2, Q - 1, R - 4, S - 3
 (c) P - 2, Q - 3, R - 1, S - 4
 (d) P - 3, Q - 2, R - 1, S - 4

SOLUTIONS

PAPER-1

1. (a,b,d) : Applying energy conservation principle, increase in KE of charged bead + decrease in electrostatic potential energy = 0

$$\frac{1}{2}mv^2 + Q\left(\frac{p \cos \theta}{4\pi\epsilon_0 r^2}\right) = 0$$

$$\text{or } v = \sqrt{\frac{-2Qp \cos \theta}{4\pi\epsilon_0 mr^2}} \quad \left\{ \frac{\pi}{2} \leq \theta \leq \pi \right\}$$

Circular motion of bead requires a centripetal force.

$$E_r = -\frac{\partial V}{\partial r} = \frac{2p \cos \theta}{4\pi\epsilon_0 r^3}$$

$$\therefore QE_r = \frac{2pQ \cos \theta}{4\pi\epsilon_0 r^3} = \frac{mv^2}{r}$$

thus wire frame does not exert any force on the bead

to sustain circular motion. Bead will reach the point opposite its starting position and then repeatedly retrace its path executing a periodic motion.

2. (a,b) : Radiant power of body $A = e_A \sigma T_A^4 A$

Radiant power of body $B = e_B \sigma T_B^4 A$

The two powers are equal.

$$\therefore e_B \sigma T_B^4 A = e_A \sigma T_A^4 A$$

$$\text{or } T_B^4 = \left(\frac{e_A}{e_B}\right) T_A^4 \text{ or } T_B^4 = \left(\frac{0.01}{0.81}\right) \times (5802)^4$$

$$\text{or } T_B^4 = \left(\frac{1}{3}\right)^4 \times (5802)^4 \text{ or } T_B = \frac{5802}{3} = 1934 \text{ K}$$

\therefore Option (a) is correct.

According to Wien's displacement law,

$$\lambda T = \text{constant} \therefore \lambda_A T_A = \lambda_B T_B$$

$$\text{or } \frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A} \text{ or } \frac{\lambda_A}{\lambda_B} = \frac{1934}{5802} = \frac{1}{3} \text{ or } \lambda_B = 3\lambda_A$$

$$\text{Given : } \lambda_B - \lambda_A = 1.0 \times 10^{-6} \text{ m}$$

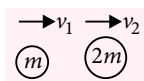
$$\text{or } 3\lambda_A - \lambda_A = 10^{-6} \text{ or } 2\lambda_A = 10^{-6}$$

$$\text{or } \lambda_A = 0.5 \times 10^{-6} \text{ m}$$

$$\therefore \lambda_B = 3 \times 0.5 \times 10^{-6} = 1.5 \times 10^{-6} \text{ m}$$

Hence option (b) is correct.

3. (a,d) : After 1st collision between the 2 balls



$$mv_1 + 2mv_2 = mu \quad \dots(i)$$

$$\text{and } \frac{v_2 - v_1}{0 - u} = -\frac{1}{2} \Rightarrow v_2 - v_1 = \frac{u}{2} \quad \dots(ii)$$

$$\text{From (i) and (ii), } v_2 = \frac{u}{2}, v_1 = 0$$

After collision of ball B with wall, direction of velocity is interchanged only.

Finally after collision of balls

$$mv'_1 + 2mv'_2 = 2m \times \frac{u}{2}$$

$$\Rightarrow v'_1 + 2v'_2 = u \quad \dots(iii)$$

$$\text{Also } \frac{v'_1 - v'_2}{0 - \frac{u}{2}} = -\frac{1}{2}$$

$$\Rightarrow v'_1 - v'_2 = \frac{u}{4} \quad \dots(iv)$$

From (iii) and (iv),

$$v'_2 = \frac{u}{4}, v'_1 = \frac{u}{2}$$

\therefore Speeds of A and B after all possible collisions,

$$v_A = 0, \frac{u}{2}; v_B = \frac{u}{2}, \frac{u}{4}$$

4. (a,b,d) : Just before collision, the total energy of two satellites is,

$$E = -\frac{GMm}{2r} - \frac{GMm}{2r} = -\frac{GMm}{r}$$

Let orbital velocity be v , then from momentum conservation,

$$mv - mv = 2m \times v_1$$

$$\Rightarrow v_1 = 0$$

As velocity of combined mass just after collision is zero, the combined mass will fall towards earth. At this instant, the total energy of the system only consists of the gravitational potential energy given

$$\text{by } U = -\frac{GM \times 2m}{r}$$

5. (a, b, c) : If $x = y$, then $\vec{v} \parallel \vec{B}$, i.e., $\vec{F} = 0$.

$$\vec{F} = q(x^2 - y^2)\hat{k}$$

If $x > y$, the force is along z -axis.

6. (a, d) : For isothermal process $PV = \text{constant}$ and for adiabatic process $PV^\gamma = \text{constant}$ where $\gamma > 1$. First we assume that BC is isothermal and CA is adiabatic.

$$\Rightarrow P_0 V_0 = 3P_0 V_C \Rightarrow V_C = \frac{V_0}{3} \text{ [for process BC]}$$

$$\text{and } 3P_0 \left(\frac{V_0}{3}\right)^\gamma = \frac{P_0}{2} V_0^\gamma \Rightarrow \gamma = \frac{\ln 6}{\ln 3} \text{ [for process CA]}$$

Now, we assume that process BC is adiabatic and CA is isothermal.

$$3P_0 V_C = \frac{P_0 V_0}{2} \Rightarrow V_C = \frac{V_0}{6} \text{ [for process CA]}$$

$$\text{and } 3P_0 \left(\frac{V_0}{6}\right)^\gamma = P_0 V_0^\gamma \Rightarrow 6^\gamma = 3 \text{ [for process BC]}$$

$$\Rightarrow \gamma = \frac{\ln 3}{\ln 6} < 1 \text{ [not possible]}$$

Hence, process AC is adiabatic and $\gamma = \frac{\ln 6}{\ln 3}$

7. (a,b,c) : $\vec{\tau} = \vec{A} \times \vec{L}$ i.e., $\frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$

This relation implies that $\frac{d\vec{L}}{dt}$ is perpendicular to

\vec{A} and \vec{L} . Therefore, option (a) is correct.

$$\vec{L} \cdot \vec{L} = L^2$$

Differentiating with respect to time, we get

$$\vec{L} \cdot \frac{d\vec{L}}{dt} + \frac{d\vec{L}}{dt} \cdot \vec{L} = 2L \frac{dL}{dt} \Rightarrow 2\vec{L} \cdot \frac{d\vec{L}}{dt} = 2L \frac{dL}{dt}$$

Since $\vec{L} \perp \frac{d\vec{L}}{dt}$ so, $\vec{L} \cdot \frac{d\vec{L}}{dt} = 0$

Therefore $\frac{dL}{dt} = 0$ and hence L does not change with time. So option (c) is correct.

Since L is not changing with time, therefore it is the case when direction of \vec{L} is changing but its magnitude is constant and $\vec{\tau}$ is perpendicular to \vec{L} at all points.

8. (a, b, c, d) : Maximum potential will be equal to the stopping potential which depends on λ and nature of material.

$$\text{As } V = \frac{kQ}{r} \Rightarrow Q = \frac{rV}{k}$$

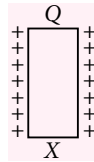
Since V and k are constant, maximum positive charge appearing depends on r .

As the sphere gets charged (which goes on increasing), it applies a force on the emanating electrons thus reduces the velocity of emanating electrons.

Initially the sphere is uncharged, thus KE_{\max} of emanating electron is independent of radius of sphere.

9. (b, c, d) : Initially charge Q will be distributed uniformly on plate X of surface area A , a plate has 2 surfaces.

$$\therefore \sigma_1 = \frac{Q}{2A}$$

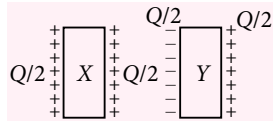


As plate Y is brought closer, the charge is induced on it but there is no effect on the plate X .

$$\text{Charge on outer surface of } X = \frac{Q}{2}$$

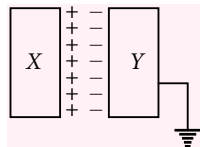
\Rightarrow distribution of charges on X is

$$\sigma_2 = \frac{Q}{2A} = \sigma_1$$



When Y is earthed, the new charge distribution will be as shown in the figure

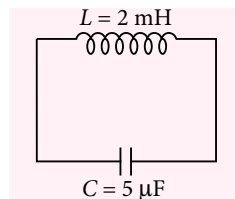
$$\therefore \sigma_3 = \frac{Q}{A}$$



10. (a, d) : The charge stored in the capacitor oscillates simple harmonically.

Here Q_0 = maximum value of charge
 $= 200 \mu\text{C} = 2 \times 10^{-4} \text{ C}$

$$\text{and } \omega = \frac{1}{\sqrt{LC}}$$



$$= \frac{1}{\sqrt{(2 \times 10^{-3} \text{ H})(5 \times 10^{-6} \text{ F})}} = 10^4 \text{ s}^{-1}$$

Let at $t = 0$, $Q = Q_0$ then,

$$Q(t) = Q_0 \cos(\omega t) \quad \dots(i)$$

$$\Rightarrow I(t) = \frac{dQ}{dt} = -Q_0 \omega \sin(\omega t) \text{ and} \quad \dots(ii)$$

$$\frac{dI(t)}{dt} = -Q_0 \omega^2 \cos(\omega t) \quad \dots(iii)$$

$$\text{When } Q = 100 \mu\text{C} = \frac{Q_0}{2}$$

$$\cos(\omega t) = \frac{1}{2}, \quad (\text{from eqn. (i)})$$

$$\therefore \left| \frac{dI}{dt} \right| = (2 \times 10^{-4} \text{ C})(10^4 \text{ s}^{-1})^2 \left(\frac{1}{2} \right)$$

$$\Rightarrow \left| \frac{dI}{dt} \right| = 10^4 \text{ A s}^{-1}$$

When $Q = 200 \mu\text{C} = Q_0$,

$\cos(\omega t) = 1$, i.e., $\omega t = 0, 2\pi, \dots$

At this time, $I(t) = -Q_0 \omega \sin(\omega t)$

$$\Rightarrow I(t) = 0 \quad (\because \sin 0^\circ = \sin 2\pi = 0)$$

$$I(t) = -Q_0 \omega \sin(\omega t)$$

The maximum value of I is $Q_0 \omega$

$$\Rightarrow I_{\max} = Q_0 \omega = (2 \times 10^{-4} \text{ C})(10^4 \text{ s}^{-1}) = 2 \text{ A}$$

(d) From energy conservation,

$$\frac{1}{2} L I_{\max}^2 = \frac{1}{2} L I^2 + \frac{1}{2} \frac{Q^2}{C}$$

$$\Rightarrow Q = \sqrt{LC(I_{\max}^2 - I^2)}$$

For $I = \frac{I_{\max}}{2} = 1 \text{ A}$, we get

$$Q = \sqrt{(2 \times 10^{-3})(5 \times 10^{-6})(2^2 - 1^2)}$$

$$\Rightarrow Q = \sqrt{3} \times 10^{-4} \text{ C} = 1.732 \times 10^{-4} \text{ C} \approx 173 \mu\text{C}$$

11. (5) : From Hooke's law : $\frac{F}{A} = Y \frac{\Delta L}{L}$ where A is the

surface area and L is length of the side of the cube. If k is spring or compression constant, then $F = k \Delta L$

$$\therefore k = Y \frac{A}{L} = YL$$

$$\text{Initial KE} = 2 \times \frac{1}{2} m v^2 = 5 \times 10^{-4} \text{ J}$$

$$\text{Final PE} = 2 \times \frac{1}{2} k (\Delta L)^2$$

$$\therefore \Delta L = \sqrt{\frac{\text{KE}}{k}} = \sqrt{\frac{\text{KE}}{YL}} = \sqrt{\frac{5 \times 10^{-4}}{2 \times 10^{11} \times 0.01}} = 5 \times 10^{-7} \text{ m}$$

12. (1) : The change in internal energy of the gas is

$$\Delta U = 1.5nR(\Delta T) = 1.5 \times 1 \times \frac{25}{3} \times 2 = 25 \text{ J}$$

The heat given to the gas = 42 J

The work done by the gas is

$$\Delta W = \Delta Q - \Delta U = 42 \text{ J} - 25 \text{ J} = 17 \text{ J}$$

If the distance moved by the piston is x , the work done is $\Delta W = (100 \text{ kPa})(8.5 \text{ cm}^2)x$

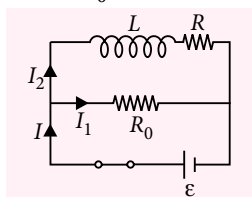
Thus, $(10^5 \text{ N m}^{-2})(8.5 \times 10^{-4} \text{ m}^2)x = 17 \text{ J}$

or $x = 0.2 \text{ m} = 2 \times 10^{-1} \text{ m}$

13. (3) : Initially, after a steady current is set up, the current, flowing is as shown.

In steady condition $I_{20} = \frac{\epsilon}{R}$, $I_{10} = \frac{\epsilon}{R_0}$

When the switch is disconnected, the current through R_0 changes from I_{10} to the right to I_{20} to the left.



(The current in the inductance cannot change suddenly). We then have the equation,

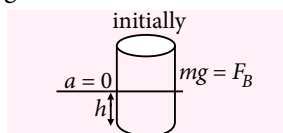
$$L \frac{dI_2}{dt} + (R + R_0)I_2 = 0$$

This equation has the solution $I_2 = I_{20}e^{-t(R+R_0)/L}$

The heat dissipated in the coil is,

$$\begin{aligned} Q &= \int_0^\infty I_2^2 R dt = I_{20}^2 R \int_0^\infty e^{-2t(R+R_0)/L} dt \\ &= RI_{20}^2 \times \frac{L}{2(R+R_0)} = \frac{L\epsilon^2}{2R(R+R_0)} = 3 \mu\text{J} \end{aligned}$$

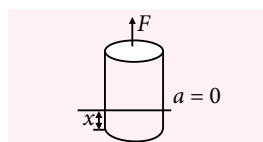
14. (5) : Applying Newton's law in vertical direction



$$\Rightarrow dSH \times g = \rho Sh \times g \Rightarrow h = \frac{dH}{\rho}$$

Now when force F is applied, for minimum work $a = 0$

(\because for $a = 0$, F is minimum)



$$F - mg + \rho Sxg = 0; F = mg - \rho Sxg$$

$$W = \int F dx = \int (mg - \rho Sxg) dx = mg \int dx - \rho Sg \int x dx$$

$$\begin{aligned} W &= mgh - \frac{\rho Sgh^2}{2} = (\rho Sh)gh - \frac{\rho Sgh^2}{2} \\ &= \frac{\rho Sgh^2}{2} = \frac{\rho Sg}{2} \left(\frac{dH}{\rho} \right)^2 = \frac{Sgd^2H^2}{2\rho} \end{aligned}$$

Here $S = 4 \text{ m}^2$, $H = 1 \text{ m}$, $d = 500 \text{ kg m}^{-3}$,
and $\rho = 1000 \text{ kg m}^{-3}$

Putting these values, we get

$$W = 5 \text{ kJ}$$

15. (9) : As $I = I_0 \cos^2 \left(\frac{\phi}{2} \right)$,

Here $I = \frac{I_0}{4}$

$$\therefore \cos \left(\frac{\phi}{2} \right) = \frac{1}{2} \Rightarrow \frac{\phi}{2} = \frac{\pi}{3} \Rightarrow \phi = \frac{2\pi}{3}$$

$$\text{or } \frac{\Delta x}{\lambda} \times 2\pi = \frac{2\pi}{3} \Rightarrow \Delta x = \frac{627}{3} \text{ nm}$$

$$\Delta x = 209 \text{ nm} = (200 + 9) \text{ nm}$$

16. (1) : Gravitational force, $F = \frac{Gm_1m_2}{r^2}$

Here, $m_1 = m$, $m_2 = \lambda m$, $r = \mu a - a = a(\mu - 1)$

Also, $m_1 + m_2 = \text{constant (C)}$

$$m + \lambda m = C; m = \frac{C}{(1 + \lambda)}$$

$$\text{So, } F = \frac{Gm^2\lambda}{a^2(\mu - 1)^2} = \frac{GC^2}{a^2(\mu - 1)^2} \cdot \frac{\lambda}{(1 + \lambda)^2}$$

$$\frac{dF}{d\lambda} = \frac{GC^2}{a^2(\mu - 1)^2} \cdot \frac{(1 + \lambda)^2 \times 1 - \lambda \times 2(1 + \lambda)}{(1 + \lambda)^4}$$

For F to be maximum, $\frac{dF}{d\lambda} = 0$

$$\therefore (1 + \lambda)^2 - 2\lambda(1 + \lambda) = 0$$

$$\text{or } (1 + \lambda)(1 + \lambda - 2\lambda) = 0$$

$$\text{or } (1 + \lambda)(1 - \lambda) = 0 \quad \therefore \lambda = 1 \quad (\because \lambda \neq -1)$$

17. (5) : $\lambda_{K_\alpha} = \frac{4}{3R(Z-1)^2}$ and $\lambda_{\min} = \frac{hc}{eV}$

As per question,

$$\lambda_{\min(10 \text{ kV})} - \lambda_{K_\alpha} = 4 (\lambda_{\min(20 \text{ kV})} - \lambda_{K_\alpha})$$

$$\Rightarrow \frac{hc}{e \times 10^3} \left[\frac{4}{20} - \frac{1}{10} \right] = 3 \times \frac{4}{3R(Z-1)^2}$$

On solving, we get $Z = 55$

$$\therefore x = 5$$

18. (2) : Frequency of stretched wire is given by

$$v = \frac{1}{2l} \sqrt{\frac{T}{\mu}} = \frac{1}{2l} \sqrt{\frac{Tl}{m}} \quad \left(\because \mu = \frac{m}{l} \right)$$

Here,

For the first wire, new length $l_1 = l + n_1 l$

new tension, $T_1 = \alpha n_1 l$

New frequency of first wire

$$v_1 = \frac{1}{2(l+n_1 l)} \sqrt{\frac{(\alpha n_1 l)(l+n_1 l)}{m}}$$

Similarly,

$$v_2 = \frac{1}{2(l+n_2 l)} \sqrt{\frac{(\alpha n_2 l)(l+n_2 l)}{m}}$$

$$\begin{aligned} \therefore \frac{v_2}{v_1} &= \frac{(1+n_1)}{(1+n_2)} \sqrt{\frac{n_2(1+n_2)}{n_1(1+n_1)}} \\ &= \sqrt{\frac{n_2(1+n_1)}{n_1(1+n_2)}} = \sqrt{\frac{0.09(1+0.02)}{0.02(1+0.09)}} = 2.05 \\ &\approx 2 \end{aligned}$$

19. (4) : The given wave is superposition of 3 waves with frequency, ω_0 , $\omega + \omega_0$, $\omega_0 - \omega$

$$\omega_{\max} = (\omega_0 + \omega)$$

$$\therefore E_{\max} = h\nu_{\max} = h \frac{(\omega_0 + \omega)}{2\pi}$$

Now, $h\nu_{\max} = KE_{\max} + \phi$

$$\begin{aligned} \Rightarrow \phi &= \frac{h}{2\pi} (\omega_0 + \omega) - KE_{\max} \\ &= \frac{6.6 \times 10^{-34}}{2\pi} \frac{(2.4\pi + 0.8\pi) \times 10^{15}}{1.6 \times 10^{-19}} \text{ eV} - 2.6 \text{ eV} \\ &= 4 \text{ eV} \end{aligned}$$

20. (2) : Till the rod does not slip, the centre of mass of the rod is in circular motion around the nail.

Thus from conservation of energy

$$mg \frac{l}{3} \sin \theta = \frac{1}{2} I \omega^2$$

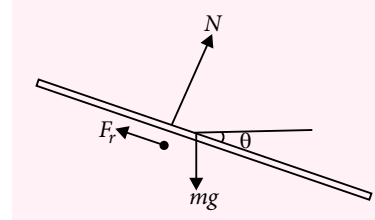
$$\text{where } I = \frac{4ml^2}{12} + \frac{ml^2}{9} = \frac{4ml^2}{9}$$

$$\text{thus } \omega = \sqrt{\frac{3g}{2l} \sin \theta}$$

$$\text{Now, } m \frac{l}{3} \omega^2 = F_r - mg \sin \theta \quad \dots(i)$$

$$\text{or } F_r = \frac{3}{2} mg \sin \theta$$

Angular acceleration of rod at an angle θ



$$\alpha = \frac{mg \left(\frac{l}{3} \right) \cos \theta}{(4/9)ml^2}$$

$$\Rightarrow \alpha = \frac{3g}{4l} \cos \theta$$

and linear acceleration of centre of mass

$$\text{From pure rolling, } a = \left(\frac{l}{3} \right) \alpha$$

$$\text{thus } a = \frac{g}{4} \cos \theta$$

$$\text{Also } mg \cos \theta - N = ma$$

$$\Rightarrow N = \frac{3mg}{4} \cos \theta \quad \dots(ii)$$

$$\text{Now, } \mu = \frac{F_r}{N} = 2 \tan \theta; \frac{\mu}{\tan \theta} = 2.$$

PAPER-2

1. (b) : $T = T_0 + \alpha V^2$

$$T = T_0 + \alpha \frac{R^2 T^2}{P^2} \quad (\text{as, } V = RT/P \text{ for one mole of gas})$$

$$\text{or, } \alpha \frac{R^2 T^2}{P^2} = T - T_0 \quad \text{or, } P^2 = \frac{\alpha R^2 T^2}{T - T_0}$$

$$\text{or, } P = \sqrt{\alpha R T (T - T_0)}^{-1/2} \quad \dots(i)$$

After differentiating, we get,

$$\frac{dP}{dT} = \sqrt{\alpha R} \left[(T - T_0)^{-1/2} - \frac{1}{2} T (T - T_0)^{-3/2} \right]$$

$$\text{Condition for minimum pressure, } \frac{dP}{dT} = 0$$

$$\therefore \sqrt{\alpha R} \left[(T - T_0)^{-1/2} - \frac{1}{2} T (T - T_0)^{-3/2} \right] = 0$$

$$\text{or, } (T - T_0) - \frac{1}{2} T = 0.$$

$$\text{or, } T = 2T_0 \quad \dots(ii)$$

From (i) and (ii), we get,

$$P_{\min} = \sqrt{\alpha R \cdot 2T_0 (2T_0 - T_0)^{-1/2}} = 2R \sqrt{\alpha T_0}$$

2. (b) : Consider the circuit

$$\varepsilon - \frac{q}{C} - IR = 0$$

$$\text{and } I = \frac{dq}{dt}$$

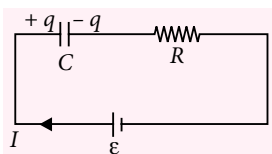
$$\Rightarrow \varepsilon C = \frac{dq}{dt} \cdot RC + q$$

$$\Rightarrow RC \frac{dq}{dt} + q = \varepsilon C$$

$$\Rightarrow q = \varepsilon C(1 - e^{-t/RC}) = q_0(1 - e^{-t/RC}) \quad \dots(i)$$

$$\text{Comparing eqn. (i) with } \frac{1}{K_1} \cdot \frac{dq}{dt} + K_2 q = K_3,$$

$$\text{we get } \tau = \frac{1}{K_1 K_2}, q_0 = \frac{K_3}{K_2}$$



3. (c) : Mass per unit length, $\lambda = \frac{M}{l}$.

The descending part of the chain is in free fall, also its every point has descended by a distance x .

So, speed of each point, $v = \sqrt{2gx}$

Assume a very small distance dx falls in a short interval of time dt .

Normal exerted on the falling part,

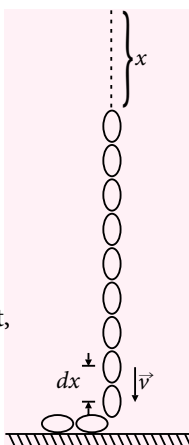
$$N = -\frac{dp_x}{dt} = \frac{-(0 - (\lambda dx)v)}{dt}$$

$$= \lambda v^2 = \lambda(2gx) = 2\lambda gx$$

Normal due to x part of the chain on the weighing machine, $N' = \lambda gx$

Reading of the scale $W = N + N' = 3\lambda gx$

$$= \frac{3Mgx}{l}$$

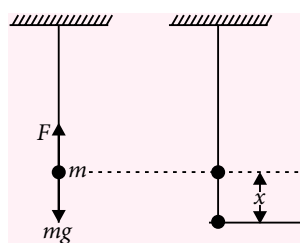


4. (c) : Let l be the increase in the length of the wire due to the force $F = mg$.

Then

$$\text{Stress} = \frac{F}{A}, \quad \text{Strain} = \frac{l}{L}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{mgL}{lA} \quad \therefore F = mg = \frac{YlA}{L}$$



This force is acting upward in the equilibrium state. If the mass is pulled down a little through a distance x , such that the total extension in the string is $(l + x)$, then force in wire acting upwards is

$$= \frac{YA(l+x)}{L}$$

and downward force is $F = mg$. The restoring force is the net downward force.

Restoring force

$$= mg - \frac{YA}{L}(l+x) = \frac{YAl}{L} - \frac{YA}{L}(l+x) = -\frac{YAx}{L}$$

$$\therefore \text{Acceleration of the mass} = \frac{\text{Force}}{\text{mass}} = -\frac{YAx}{mL}$$

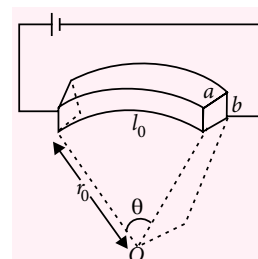
$$\therefore \omega = \sqrt{\frac{YA}{mL}} = 2\pi\nu \quad \text{or} \quad \nu = \frac{1}{2\pi} \sqrt{\frac{YA}{mL}}$$

5. (b) : Let us consider an elemental portion of the resistor. The element considered is a circular arc of radius r and thickness dr . The resistance of this element would be

$$dR = \frac{\rho \times r\theta}{adr}$$

$$\text{here } l_0 = r_0\theta$$

$$\text{So, } dR = \frac{\rho r \times l_0}{ar_0 \times dr}$$

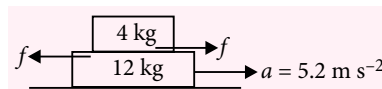


If we divide the entire resistor in these elemental portions, then these elemental resistors are joined in parallel, equivalent resistance of which is given by

$$\frac{1}{R} = \int \frac{1}{dR} = \int_{r_0}^{r_0+b} \frac{ar_0 dr}{\rho l_0 r} = \frac{ar_0}{\rho l_0} \times \ln \frac{r_0+b}{r_0}$$

$$\Rightarrow R = \frac{\rho l_0}{ar_0 \ln \left[1 + \frac{b}{r_0} \right]}$$

6. (c) : First assume that blocks have common acceleration, for both blocks to move together acceleration of 4 kg block must be 5.2 m s^{-2} .



$$a = 5.2 \text{ m s}^{-2}$$

$$\therefore f = 4 \times 5.2 \text{ kg m s}^{-2}$$

$$\text{or } \mu_0 mg = m(5.2 \text{ m s}^{-2})$$

$$\mu_0 = 0.52$$

If $\mu = \frac{1}{2}(0.52) = 0.26$, the acceleration of 4 kg block due to friction is $a_1 = \mu g = 2.6 \text{ m s}^{-2}$

As there is relative motion between blocks

$$S_{\text{rel}} = u_{\text{rel}}t + \frac{1}{2}a_{\text{rel}}t^2$$

$$a_{\text{rel}} = a_1 - a = 2.6 - 5.2 = -2.6 \text{ m s}^{-2},$$

$$\therefore S_{\text{rel}} = \frac{1}{2}(-2.6)t^2$$

Time of motion can be determined from motion of lower block.

$$\text{For 12 kg block, } S = \frac{1}{2}(5.2)t^2 = 10 \text{ m (given)}$$

$$\Rightarrow t = \sqrt{\frac{20}{5.2}} \text{ s}$$

$$\therefore S_{\text{rel}} = -5 \text{ m}$$

Work done by friction is given by

$$W_f = \mu mg S_{\text{rel}} = 0.26 \times 4 \times 10 \times (-5) = -52 \text{ J}$$

$$\therefore \text{Heat generated} = 52 \text{ J}$$

7. (c) : If the ray just emerges from face AC,

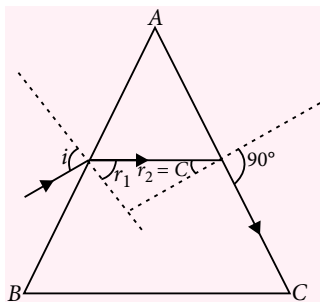
$$e = 90^\circ \text{ and } r_2 = C \quad \dots(i)$$

From Snell's law at face AB, we have

$$1 \sin i = n \sin r_1 \quad \dots(ii)$$

$$\text{and } A = r_1 + r_2 = r_1 + C \quad \dots(iii)$$

From eqn. (ii) n is minimum when r_1 is maximum, i.e., $r_1 = C$. In this case $i = 90^\circ$.



From eqn. (iii), $A = 2C$ or $C = A/2$

$$\text{From eqn. (ii), } \sin C = \frac{1}{n}; \sin \frac{A}{2} = \frac{1}{n}$$

$$\therefore n = \operatorname{cosec} \frac{A}{2}$$

8. (d) : Equation of trajectory of a particle is

$$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$$

If projectile passes through the point (a, b) ,

$$b = a \tan \alpha - \frac{ga^2}{2u^2 \cos^2 \alpha} = a \tan \alpha - \frac{ga^2}{2u^2} (1 + \tan^2 \alpha)$$

$$\text{or } ga^2 \tan^2 \alpha - 2au^2 \tan \alpha + (ga^2 + 2bu^2) = 0$$

This quadratic equation in $\tan \alpha$ must give real roots for a particle to pass through (a, b) .

Thus Discriminant ≥ 0

$$\text{i.e., } 4a^2u^4 - 4ga^2(ga^2 + 2bu^2) \geq 0$$

$$\text{or } u^4 - 2gbu^2 - g^2a^2 \geq 0$$

$$\text{or } u^4 - 2gbu^2 + b^2g^2 \geq b^2g^2 + a^2g^2$$

$$\text{or } (u^2 - bg)^2 \geq (b^2 + a^2)g^2$$

$$\text{or } u \geq \sqrt{bg + g\sqrt{a^2 + b^2}}$$

$$\therefore u_{\text{min}} = \sqrt{bg + g\sqrt{a^2 + b^2}}$$

9. (c) : Let the power of light source be P , then intensity at any point on the screen is due to light rays directly received from source and that due to after reflection from the mirror

$$\begin{aligned} \Rightarrow I &= \frac{P}{4\pi a^2} + \frac{P}{4\pi \times (3a)^2} \\ &= \frac{P}{4\pi a^2} \left(1 + \frac{1}{9}\right) = \frac{P}{4\pi a^2} \left(\frac{10}{9}\right) \end{aligned}$$

When mirror is taken away,

$$I_1 = \frac{P}{4\pi a^2} = \frac{9I}{10}$$

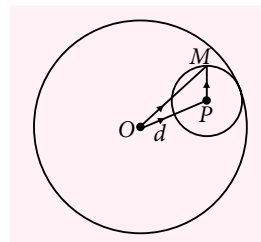
10. (b) : From Ampere's law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{s}$

Inside the conductor at a distance r from its axis

$$B(2\pi r) = \mu_0 J(\pi r^2)$$

$$B = \frac{1}{2}\mu_0 J r \text{ or } \vec{B} = \frac{1}{2}\mu_0 \vec{J} \times \vec{r}$$

Now, conductor has cavity so we can assume that current in it is the superposition of positive current and negative current (in place of cavity).

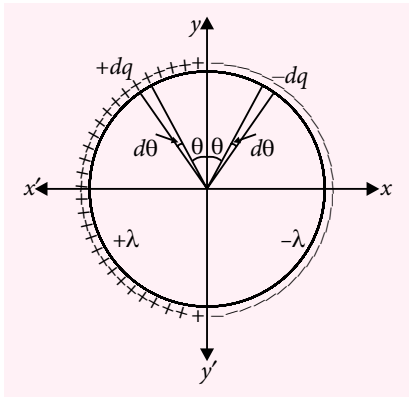


Required magnetic field at point M

\vec{B}_N = Magnetic field at M due to whole conductor
- Magnetic field at M due to cavity shaped conductor

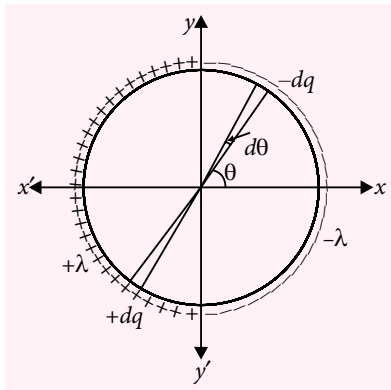
$$\begin{aligned} \vec{B}_N &= \frac{1}{2}\mu_0 (\vec{J} \times \vec{OM}) - \frac{1}{2}\mu_0 (\vec{J} \times \vec{PM}) \\ &= \frac{1}{2}\mu_0 \vec{J} \times (\vec{OM} - \vec{PM}) = \frac{1}{2}\mu_0 \vec{J} \times \vec{d}. \end{aligned}$$

11. (b) : Consider two small elements of ring having charges $+dq$ and $-dq$ symmetrically located about y -axis.



The potential due to this pair at any point on y -axis is zero. The sum of potential due to all such possible pairs is zero at all points on y -axis. Hence potential at $P\left(0, \frac{R}{2}\right)$ is zero.

12. (c) : Consider two small elements of ring having charge $+dq$ and $-dq$ as shown in figure.



The pair constitutes a dipole moment.

$$dp = dq \cdot 2R$$

The net dipole moment of system is vector sum of dipole moments of all such pairs of elementary charges.

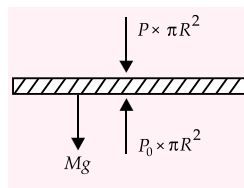
By symmetry the resultant dipole moment is along negative x -direction.

\therefore Net dipole moment

$$\begin{aligned} \vec{p} &= - \int_{-\pi/2}^{+\pi/2} (dp \cos \theta) \hat{i} \\ &= - \int_{-\pi/2}^{+\pi/2} (2\lambda R^2 \cos \theta d\theta) \hat{i} = -4R^2 \lambda \hat{i} \end{aligned}$$

13. (a)

14. (d) Let the pressure of the trapped air at the equilibrium be P . Draw the forces on the piston. For the equilibrium of the piston,

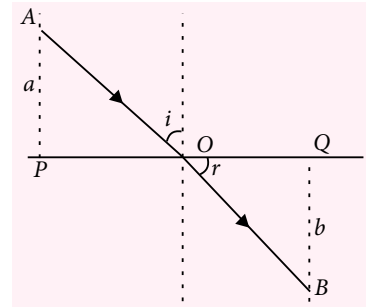


$$Mg + P(\pi R^2) = P_0 \pi R^2$$

$$\text{Also, } P_0(2L\pi R^2) = P(x\pi R^2)$$

$$\therefore x = \left(\frac{P_0 \pi R^2}{P_0 \pi R^2 - Mg} \right) (2L)$$

15. (d) : Total time taken = time taken (t_1) from A to O in medium 1 + time taken (t_2) from O to B in medium 2.



$$t_1 = \frac{AO}{v_1} = \frac{a \sec i}{v_1}$$

$$t_2 = \frac{BO}{v_2} = \frac{b \sec r}{v_2}$$

$$t = t_1 + t_2 = \frac{a \sec i}{v_1} + \frac{b \sec r}{v_2}$$

16. (a) : To optimize time, $\frac{dt}{dr} = 0$

$$\Rightarrow \frac{d}{dr} \left(\frac{a \sec i}{v_1} + \frac{b \sec r}{v_2} \right) = 0$$

$$\Rightarrow \frac{a \sec i \tan i}{v_1} \frac{di}{dr} + \frac{b \sec r \tan r}{v_2} = 0$$

$$\Rightarrow \frac{di}{dr} = - \frac{b \sec r \tan r}{v_2} \times \frac{v_1}{a \sec i \tan i}$$

$$= - \frac{b}{a} \times \frac{v_1}{v_2} \times \frac{\sin r}{\sin i} \times \frac{\cos^2 i}{\cos^2 r}$$

$$= - \frac{b}{a} \cdot \frac{\mu_2}{\mu_1} \times \frac{\mu_1}{\mu_2} \times \frac{\cos^2 i}{\cos^2 r} = - \frac{b \cos^2 i}{a \cos^2 r}$$

17. (a) : P \rightarrow 4, Q \rightarrow 3, R \rightarrow 2, S \rightarrow 1

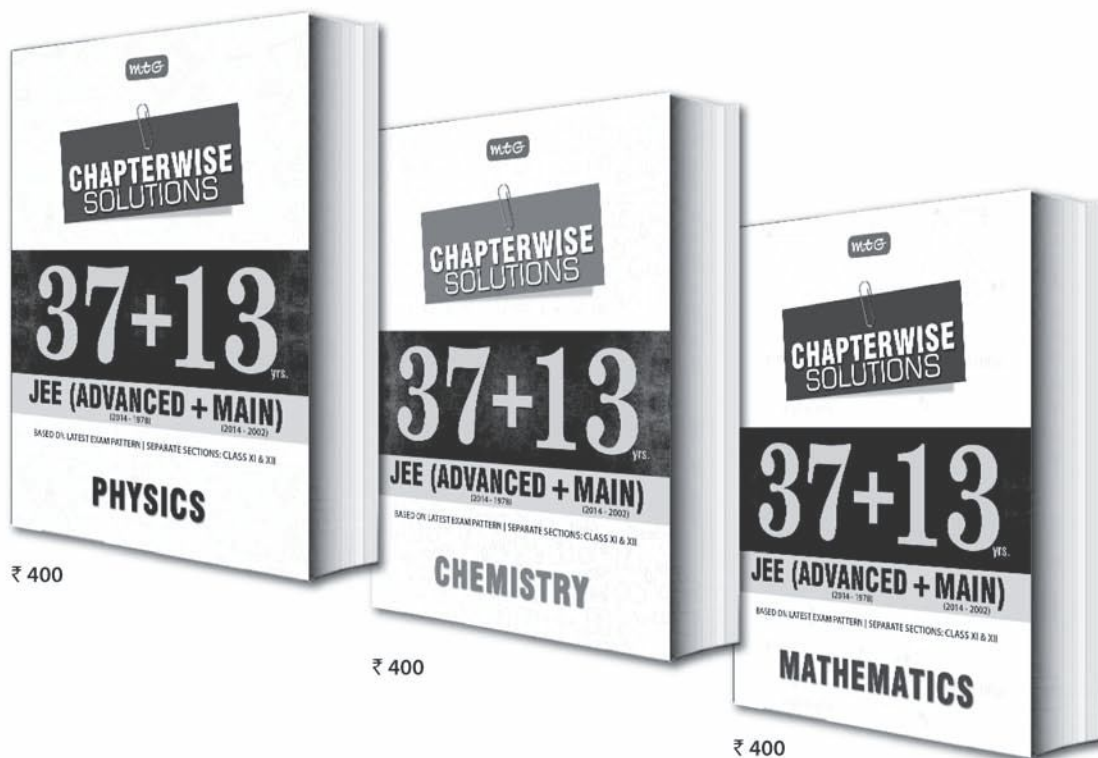
18. (a) : P \rightarrow 4, Q \rightarrow 1, R \rightarrow 2, S \rightarrow 3

19. (d) : P \rightarrow 3, Q \rightarrow 4, R \rightarrow 1, S \rightarrow 2

20. (c) : P \rightarrow 2, Q \rightarrow 3, R \rightarrow 1, S \rightarrow 4



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WAVES

Every piece of music you hear, from Hindustani classical to film songs, depends on performers producing waves and your detection of those waves.

Wave Motion

A means of transferring momentum and energy from one point to another without any actual transportation of matter

Transverse and Longitudinal Waves

- A transverse wave is one in which the disturbance occurs perpendicular to the direction of travel of the wave.
- A longitudinal wave is one in which the disturbance occurs parallel to the line of travel of the wave.

Velocity of Longitudinal Waves

- Velocity of longitudinal waves in a solid of bulk modulus κ , modulus of rigidity η and density ρ is

$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$

- Velocity of longitudinal waves in a long solid rod of Young's modulus Y and density ρ is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

- Velocity of longitudinal waves in a fluid of bulk modulus κ and density ρ is

$$v = \sqrt{\frac{\kappa}{\rho}}$$

- Newton's formula for the velocity of sound in a gas is

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

here, P = pressure of the gas

- Laplace formula for the velocity of sound in a gas is

$$v = \sqrt{\frac{\kappa_{ps}}{\rho}} = \sqrt{\frac{\epsilon P}{\rho}}$$

- Intensity of sound waves

$$I = \frac{1}{2} \frac{v^2 A^2 \kappa}{v} = \frac{2\pi^2 \kappa}{v} A^2 v^2 = \frac{P_0^2 v}{2\kappa} = \frac{P_0^2}{2\rho v}$$

Factors Affecting Velocity of Sound through Gases

- Effect of density, $v \propto \frac{1}{\sqrt{\rho}}$

$$\text{i.e., } \frac{v_2}{v_1} = \sqrt{\frac{\rho_1}{\rho_2}}$$

- Effect of temperature, $v \propto \sqrt{T}$

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273}}$$

- No change in velocity of sound with change in pressure provided temperature is kept constant.

Doppler's Effect

- If v , v_o , v_s and v_m are the velocities of sound, observer, source and medium respectively, then the apparent frequency,

$$v' = \frac{v \pm v_m \pm v_o}{v \pm v_m \pm v_s} \times v$$

- If the medium is at rest, ($v_m = 0$) then

$$v' = \frac{v \pm v_o}{v \pm v_s} \times v$$

- Upper sign on v_s (or v_o) is used when source (observer) moves towards the observer (source) while lower sign is used when it moves away.

Progressive Wave Parameters

- Displacement, $y = A \sin(\omega t + kx)$

$$y = A \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right) = A \sin \frac{2\pi}{\lambda} (vt + x)$$

- Phase, $\phi = 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right) + \phi_0$

where ϕ_0 is the initial phase.

- Phase change with time,

$$\Delta\phi = \frac{2\pi}{T} \Delta t.$$

- Phase change with position,

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x.$$

- Instantaneous particle velocity,

$$u = \frac{dy}{dt} = \frac{2\pi A}{T} \cos 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right)$$

- Velocity amplitude,

$$u_0 = \frac{2\pi A}{T} = \omega A$$

- Instantaneous particle acceleration,

$$a = \frac{du}{dt} = -\frac{4\pi^2 A}{T^2} \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right) = -\omega^2 y$$

- Acceleration amplitude,

$$a_0 = \frac{4\pi^2 A}{T^2} = \omega^2 A$$

Wave Travelling Along a String

- Speed, v ; $\sqrt{\frac{T}{m}}$,

where, T = tension in the string,
 m = mass per unit length.

- Average rate at which kinetic energy or potential energy transported = $\frac{1}{4} \frac{v^2 A^2 T}{v}$

- Average power transmitted along the string by a sine wave

$$P_{av} = \frac{1}{2} \frac{v^2 A^2 T}{v} = 2\pi^2 m v A^2 v^2$$

Principle of Superposition of Waves

- According to the principle of superposition of waves, when any number of waves interact at a point in a medium, the net displacement of the point at a given time is the algebraic sum of the displacements due to each wave at that instant of time.

Stationary Waves

- The stationary wave formed by the superposition of incident wave and reflected wave is given by

$$y = 2A \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi t}{T}$$

Nodes are formed at the positions

$$x = 0, \frac{\phi}{2}, \lambda, \frac{3\lambda}{2}, \dots$$

and anti nodes are formed at

$$x = \frac{\phi}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$$

Organ Pipes

- Open organ pipe
Fundamental mode,

$$\sigma_1 = \frac{v}{2L} = v \quad (\text{First harmonic})$$

Second mode, $v_2 = 2v$

(Second harmonic or first overtone)

$$n^{\text{th}} \text{ mode, } v_n = \frac{nv}{2L}$$

(n^{th} harmonic or $(n-1)^{\text{th}}$ overtone)

- Closed organ pipe

Fundamental mode,

$$\sigma_1 = \frac{v}{4L} = v \quad (\text{First harmonic})$$

Second mode, $v_2 = 3v$

(Third harmonic or first overtone)

Third mode, $v_3 = 5v$

(Fifth harmonic or second overtone)

n^{th} mode, $v_n = (2n-1)v$

[($2n-1$)th harmonic or $(n-1)^{\text{th}}$ overtone]

- Laplace correction $e = 0.6r$ (in closed pipe) and $2e = 1.2r$ (in open pipe)

$$v = n \left[\frac{v}{2(l + 1.2r)} \right] \quad (\text{in open pipe})$$

$$v = n \left[\frac{v}{4(l + 0.6r)} \right] \quad (\text{in closed pipe})$$

Modes of Vibration of Strings

- String fixed at both ends

Frequency of vibration

$$\sigma = \frac{nv}{2L} = \frac{n}{2L} \sqrt{\frac{T}{m}}$$

where L = length of string

n = mode of vibration

Fundamental frequency

$$\sigma_0 = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{T}{m}}$$

Second harmonic or 1st overtone, $v_2 = 2v_0$

Third harmonic or 2nd overtone, $v_3 = 3v_0$ and so on.

- String fixed at one end

Frequency of vibration

$$v = \left(n + \frac{1}{2} \right) \frac{v}{2L} = \frac{\left(n + \frac{1}{2} \right)}{2L} \sqrt{\frac{T}{m}}$$

Fundamental frequency,

$$\sigma_0 = \frac{v}{4L} = \frac{1}{4L} \sqrt{\frac{T}{m}}$$

- Law of length

$vL = \text{constant}$

or $v_1 L_1 = v_2 L_2$

Beats

- Beat frequency = Number of beats/sec = Difference in frequencies of two sources.

$$v_{\text{beat}} = (v_1 - v_2) \text{ or } (v_2 - v_1)$$

$$\therefore v_2 = v_1 \pm v_{\text{beat}}$$

- The \pm sign is decided by loading/filing any of the prongs of either tuning fork.

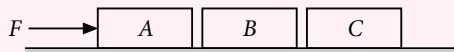
- On loading a fork, its frequency decreases and on filing, its frequency increases.

Target AIPMT



Practice Paper

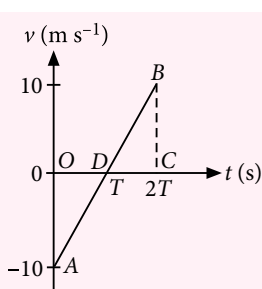
- A car starts from station and moves along the horizontal road by a machine delivering constant power. The distance covered by the car in time t is proportional to
(a) t^2 (b) $t^{3/2}$ (c) $t^{2/3}$ (d) t^3
- The dimensions of a rectangular block measured with callipers having least count of 0.01 cm are 5 mm \times 10 mm \times 5 mm. The maximum percentage error in the measurement of the volume of the block is
(a) 5% (b) 10% (c) 15% (d) 20%
- The figure shows the velocity (v) of a particle plotted against time (t).
- If the position vector of a particle is given by :
 $\vec{r} = (4 \cos 2t) \hat{i} + (4 \sin 2t) \hat{j} + (6t) \hat{k}$ m, its acceleration at $t = \pi/4$ s in m s^{-2} is
(a) $16 \hat{i}$ (b) $-16 \hat{k}$ (c) $-16 \hat{j}$ (d) $16 (\hat{i} + \hat{j})$
- Three identical blocks, each having a mass m are pushed by a force F on a frictionless table as shown in figure. (i) What is the acceleration of the blocks?



- (ii) What is the net force on the block A? (iii) What force does A apply on B? (iv) What force does B apply on C?

- | | | | |
|------------|-------|--------|-------|
| (i) | (ii) | (iii) | (iv) |
| (a) $F/3m$ | $F/3$ | $2F/3$ | $F/3$ |
| (b) $F/4m$ | $F/3$ | $2F/3$ | $F/3$ |
| (c) $F/3m$ | $F/4$ | $2F/3$ | $F/3$ |
| (d) $F/3m$ | $F/3$ | $2F/4$ | $F/3$ |

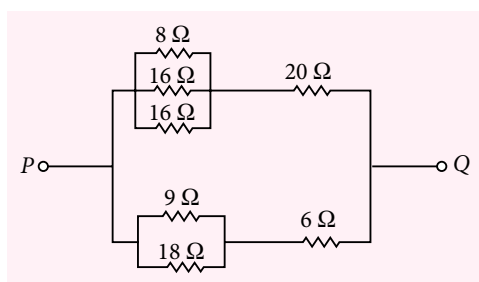
- A large plane sheet of charge having surface charge density $5.0 \times 10^{-16} \text{ C m}^{-2}$ lies in the X-Y plane. The electric flux through a circular area of radius 0.1 m, if the normal to the circular area makes an angle of 60° with the Z-axis is
[Given : $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$.]
(a) $4.44 \times 10^{-7} \text{ N m}^2 \text{ C}^{-1}$
(b) $3.33 \times 10^{-7} \text{ N m}^2 \text{ C}^{-1}$
(c) $2.22 \times 10^{-7} \text{ N m}^2 \text{ C}^{-1}$
(d) $1.11 \times 10^{-7} \text{ N m}^2 \text{ C}^{-1}$
- A $4 \mu\text{F}$ capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged $2 \mu\text{F}$ capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?
(a) $2.67 \times 10^{-2} \text{ J}$ (b) $3.67 \times 10^{-2} \text{ J}$
(c) $2.67 \times 10^{-4} \text{ J}$ (d) $5.67 \times 10^{-4} \text{ J}$



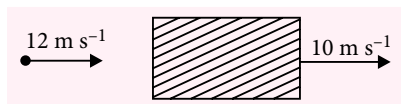
Mark out the incorrect statement.

- (a) The displacement of the particle in time $2T$ is zero.
(b) The initial and final speeds of the particle are the same.
(c) The acceleration of the particle remains constant throughout the motion.
(d) The particle does not change its direction of motion.
- Two stones having different masses m_1 and m_2 are projected at angles θ and $(90^\circ - \theta)$ with same velocity from the same point. The ratio of their maximum heights is
(a) 1 : 1 (b) 1 : $\tan \theta$
(c) $\tan \theta$: 1 (d) $\tan^2 \theta$: 1

9. The equivalent resistance of the arrangement of resistances shown in the given figure between points P and Q is



- (a) $6\ \Omega$ (b) $8\ \Omega$ (c) $24\ \Omega$ (d) $16\ \Omega$
10. A long solenoid carrying a current produces a magnetic field B along its axis. If the current is doubled and the number of turns per cm is halved, then new value of the magnetic field is
(a) B (b) $2B$ (c) $4B$ (d) $B/2$
11. A Carnot engine operates with a source at 500 K and sink at 375 K . Engine consumes 600 kcal of heat per cycle. The heat rejected to sink per cycle is
(a) 250 kcal (b) 350 kcal
(c) 450 kcal (d) 550 kcal
12. A wire of cross section 4 mm^2 is stretched by 0.1 mm by a certain weight. How far will the wire of same material and length but of area 8 mm^2 stretch under the action of same force?
(a) 0.05 mm (b) 0.10 mm
(c) 0.15 mm (d) 0.20 mm
13. A light particle moving horizontally with a speed of 12 m s^{-1} strikes a very heavy block moving in the same direction at 10 m s^{-1} . The collision is one dimensional and elastic. After the collision, the particle will



- (a) move at 2 m s^{-1} in its original direction
(b) move at 8 m s^{-1} in its original direction
(c) move at 8 m s^{-1} opposite to its original direction
(d) move at 12 m s^{-1} opposite to its original direction.
14. A satellite is placed in a circular orbit around earth at such a height that it always remains stationary with respect to earth surface. In such case, its height from the earth surface is
(a) 32000 km (b) 36000 km
(c) 3400 km (d) 4800 km

15. Two pendulums of length 1 m and 16 m start vibrating one behind the other from the same stand. At some instant, the two are in the mean position in the same phase. The time period of shorter pendulum is T . The minimum time after which the two threads of the pendulum will be one behind the other is

(a) $T/4$ (b) $T/3$ (c) $4T/3$ (d) $4T$

16. A denotes the cross-sectional area of a cubical tank, h the depth of an orifice of area of cross-section a , below the liquid surface. The velocity of the liquid flowing through the orifice is

(a) $\sqrt{2gh}$ (b) $\sqrt{2gh} \sqrt{\left(\frac{A^2}{A^2 - a^2}\right)}$
(c) $\sqrt{2gh} \sqrt{\left(\frac{A}{A - a}\right)}$ (d) $\sqrt{2gh} \sqrt{\left(\frac{A^2 - a^2}{A^2}\right)}$

17. The electric potential V at any point (x, y, z) in space is given by $V = 4x^2\text{ V}$. The electric field at the point $(1\text{ m}, 0, 2\text{ m})$ in V m^{-1} is

(a) $-8\hat{i}$ (b) $+8\hat{i}$ (c) $-16\hat{i}$ (d) $16\hat{k}$

18. A convex lens of focal length f is placed somewhere in between an object and a screen. The distance between object and screen is x . If numerical value of magnification produced by lens is m , focal length of lens is

(a) $\frac{mx}{(m+1)^2}$ (b) $\frac{mx}{(m-1)^2}$
(c) $\frac{(m+1)^2}{m}x$ (d) $\frac{(m-1)^2}{m}x$

19. A transformer with efficiency 80% works at 4 kW and 100 V . If the secondary voltage is 200 V , then the primary and secondary currents are respectively

(a) $40\text{ A}, 16\text{ A}$ (b) $16\text{ A}, 40\text{ A}$
(c) $20\text{ A}, 40\text{ A}$ (d) $40\text{ A}, 20\text{ A}$

20. The coefficient of friction between the ground and the wheels of a car moving on a horizontal road is 0.5 . If the car starts from rest, what is the minimum distance in which it can acquire a speed of 72 km h^{-1} without slipping? (Take $g = 10\text{ m s}^{-2}$).

(a) 10 m (b) 20 m (c) 30 m (d) 40 m

21. A block of mass 2 kg initially at rest is dropped from a height of 1 m into a vertical spring having force constant 490 N m^{-1} . The maximum distance through which the spring will be compressed, is

(a) 0.11 m (b) 0.33 m
(c) 0.22 m (d) 0.44 m

22. A body of mass m is raised to a height h from the surface of the earth where the acceleration due to gravity is g . If R is the radius of the earth and $h \ll R$, then the loss in weight due to variation in g is approximately

(a) $\frac{2mgh}{R}$ (b) $\frac{2mgR}{h}$ (c) $\frac{mgR}{h}$ (d) $\frac{mgh}{R}$

23. The earth's magnetic field at a given point is $0.5 \times 10^{-5} \text{ Wb m}^{-2}$. This field is to be annulled by magnetic induction at the centre of a circular loop of radius 5 cm. The current required to be flown in the loop is nearly

(a) 0.2 A (b) 0.4 A (c) 4 A (d) 40 A

24. A railway track running north-south has two parallel rails 1.0 m apart. If the horizontal component of earth's magnetic field at that place is $0.3 \times 10^{-4} \text{ Wb m}^{-2}$ and angle of dip is 60° , the value of induced emf between the rails, when a train passes at a speed of 90 km h^{-1} is

(a) $1.3 \times 10^{-3} \text{ V}$ (b) $5.2 \times 10^{-4} \text{ V}$
(c) $5.2 \times 10^{-3} \text{ V}$ (d) $1.3 \times 10^{-4} \text{ V}$

25. The primary and secondary coils of a transformer have 50 and 1500 turns respectively. If the magnetic flux ϕ linked with the primary coil is given by $\phi = \phi_0 + 4t$, where ϕ is in weber, t is time in second and ϕ_0 is a constant, the output voltage across the secondary coil is

(a) 90 V (b) 120 V (c) 220 V (d) 30 V

26. Electromagnetic waves travel in a medium which has relative permeability 1.3 and relative permittivity 2.14. Then the speed of the electromagnetic wave in the medium will be

(a) $3.6 \times 10^6 \text{ m s}^{-1}$ (b) $1.8 \times 10^2 \text{ m s}^{-1}$
(c) $3.6 \times 10^8 \text{ m s}^{-1}$ (d) $1.8 \times 10^8 \text{ m s}^{-1}$

27. In a common emitter amplifier, the resistance of the output circuit is 500 times the resistance of the input circuit. If $\alpha = 0.98$, then find the voltage gain and the power gain.

(a) 23500, 1300500 (b) 24500, 1200500
(c) 22505, 1300500 (d) 23500, 1200500

28. A parallel beam of monochromatic light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is

(a) zero (b) $\frac{\pi}{2}$ (c) π (d) 2π

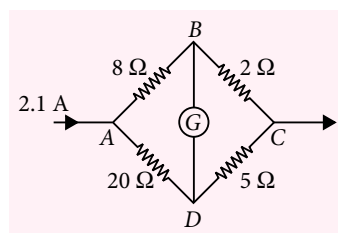
29. Two cells of same emf ε but of different internal resistances r_1 and r_2 are connected in series with an external resistance R . The potential drop across the first cell is found to be zero. The external resistance R is

(a) $r_1 + r_2$ (b) $r_1 - r_2$
(c) $r_2 - r_1$ (d) r_1^2 / r_2

30. A certain gas at atmospheric pressures is compressed adiabatically so that its volume becomes half of its original volume. The resulting pressure in N m^{-2} is (Take $\gamma = 1.4$ for air)

(a) $1.013 \times 10^3 \times (2)^{1.4}$ (b) $1.013 \times 10^5 \times (2)^{1.4}$
(c) $\frac{1.013 \times 10^5}{(2)^{1.4}}$ (d) $\frac{1.013 \times 10^3}{(2)^{1.4}}$

31. In the given figure, when galvanometer shows no deflection the current flowing through 5Ω resistance will be



(a) 0.5 A (b) 0.6 A (c) 1.5 A (d) 2.0 A

32. A current of 5.0 A is passed through the coil of a galvanometer having 500 turns and each turn has an average area of $3 \times 10^{-4} \text{ m}^2$. If a torque of 1.5 N m is required for this coil carrying same current to set it parallel to a magnetic field, the strength of the magnetic field is

(a) 20 T (b) 25 T (c) 23 T (d) 21 T

33. Water is boiled in flat bottom kettle placed on a stove. The area of the bottom is 3000 cm^2 and the thickness is 2 mm. If the amount of steam produced is 1 g min^{-1} , the difference of temperature between the inner and outer surfaces of the bottom is (Given : Thermal conductivity of the material of kettle is $0.5 \text{ cal } ^\circ\text{C}^{-1} \text{ s}^{-1} \text{ cm}^{-1}$)

(a) $2.1 \times 10^{-3} ^\circ\text{C}$ (b) $3.1 \times 10^{-3} ^\circ\text{C}$
(c) $1.2 \times 10^{-3} ^\circ\text{C}$ (d) $2.5 \times 10^{-3} ^\circ\text{C}$

34. Four identical spheres each of mass M and radius 10 cm each are placed on a horizontal surface touching one another so that their centres are located at the corners of a square of side 20 cm. What is the distance of their centre of mass from centre of any sphere?

(a) 5 cm (b) 10 cm (c) 20 cm (d) $10\sqrt{2} \text{ cm}$

35. An aeroplane is flying horizontally with a velocity of 216 km h^{-1} and at a height of 1960 m. When it is vertically above a point A on the ground, a bomb is released from it. The bomb strikes the ground at point B. The distance AB is (ignoring air resistance)

- (a) 1200 m (b) 0.33 km
(c) 3.33 km (d) 33 km

36. An electron of mass m , when accelerated through a potential difference V , has de Broglie wavelength λ . The de Broglie wavelength associated with a proton of mass M accelerated through the same potential difference, will be

- (a) $\lambda \frac{M}{m}$ (b) $\lambda \frac{m}{M}$ (c) $\lambda \sqrt{\frac{M}{m}}$ (d) $\lambda \sqrt{\frac{m}{M}}$

37. When in hydrogen like ion, electron jumps from $n = 3$ to $n = 1$, the emitted photon has frequency $2.7 \times 10^{15} \text{ Hz}$. When electron jumps from $n = 4$ to $n = 1$, the frequency is

- (a) $1.6 \times 10^{15} \text{ Hz}$ (b) $2.8 \times 10^{15} \text{ Hz}$
(c) $6.4 \times 10^{15} \text{ Hz}$ (d) $4.8 \times 10^{15} \text{ Hz}$

38. Half-life of a substance is 20 minutes. Find the time between 33% decay and 67% decay.

- (a) 40 min (b) 20 min
(c) 30 min (d) 25 min

39. In a transistor the collector current is always less than the emitter current because

- (a) collector side is reverse biased and the emitter side is forward biased
(b) a few electrons are lost in the base and only remaining ones reach the collector
(c) collector being reverse biased, attracts less electrons
(d) collector side is forward biased and emitter side is reverse biased.

40. In Young's double slit experiment, first slit has width four times the width of the second slit. The ratio of the maximum intensity to the minimum intensity in the interference fringe system is

- (a) 2 : 1 (b) 4 : 1 (c) 9 : 1 (d) 8 : 1

41. Which of the following statements are true?

- (I) All radioactive elements decay exponentially with time.
(II) Half life time of a radioactive element is the time required for one half of the radioactive atoms to disintegrate.
(III) Age of the earth can be determined by radioactive dating.
(IV) Half life time of a radioactive element is fifty percent of its average life period.

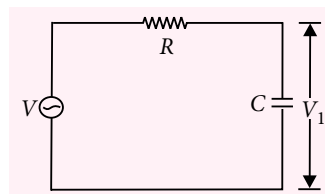
Select the correct answer using the codes given below.

- (a) I and II (b) I, III and IV
(c) I, II and III (d) II and III

42. An interference pattern has maximum and minimum intensities in the ratio of 36 : 1, then the ratio of amplitudes is

- (a) 5 : 7 (b) 7 : 4 (c) 4 : 7 (d) 7 : 5

43. In the circuit shown, the voltage V_1 , across capacitor C



- (a) is in phase with the source voltage V
(b) leads the source voltage V by 90°
(c) leads the source voltage V by an angle between 0° and 90°
(d) lags behind the source voltage V by an angle between 0° and 90° .

44. A bat flies at a steady speed of 4 m s^{-1} emitting 90 kHz sound waves and is flying towards a wall. It detects a reflected signal at a frequency (Take speed of sound = 340 m s^{-1})

- (a) 90.1 kHz (b) 91.1 kHz
(c) 92.1 kHz (d) 93.1 kHz

45. A block of mass 10 kg is moving horizontally with a speed of 1.5 m s^{-1} on a smooth plane. If a constant vertical force 10 N acts on it, the displacement of the block from the point of application of the force at the end of 4 second is

- (a) 5 m (b) 20 m (c) 12 m (d) 10 m

SOLUTIONS

1. (b) : Power $P = \frac{W}{t} = \frac{Fs}{t} = \frac{mas}{t}$ ($\because F = ma$)

$$P = \frac{mvs}{t^2}, \quad \left(\because a = \frac{v}{t} \right)$$

$$P = \frac{ms^2}{t^3} \quad \left(\because v = \frac{s}{t} \right)$$

or $Pt^3 = ms^2$
 $\therefore s \propto t^{3/2}$

2. (a) : Required percentage error

$$= \left(\frac{0.1}{5} + \frac{0.1}{10} + \frac{0.1}{5} \right) \times 100 = 5\%$$

3. (d) : Displacement = velocity \times time. In time 0 to $2T$, the displacement = - Area of ΔOAD + Area of

$\Delta DBC = 0$. Initial and final speeds are the same as per graph.

The slope of velocity-time graph represents acceleration. Here, the velocity-time graph is a straight line inclined to time axis, hence has equal acceleration throughout. The particle changes its direction of motion after time T .

4. (d) : Maximum height for mass m_1 , $H_1 = \frac{u^2 \sin^2 \theta}{2g}$

and for mass m_2 ,

$$H_2 = \frac{u^2 \sin^2(90^\circ - \theta)}{2g},$$

$$\therefore \frac{H_1}{H_2} = \frac{\sin^2 \theta}{\sin^2(90^\circ - \theta)} = \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{\tan^2 \theta}{1}$$

$$\therefore H_1 : H_2 = \tan^2 \theta : 1.$$

5. (c) : As $\vec{r} = (4 \cos 2t) \hat{i} + (4 \sin 2t) \hat{j} + 6t \hat{k}$,

$$\begin{aligned} \text{Velocity, } \vec{v} &= \frac{d\vec{r}}{dt} = [4(-\sin 2t) \cdot (2)] \hat{i} \\ &\quad + [4(\cos 2t) \cdot (2)] \hat{j} + 6 \hat{k} \\ &= (-8 \sin 2t) \hat{i} + (8 \cos 2t) \hat{j} + 6 \hat{k} \end{aligned}$$

$$\begin{aligned} \text{Acceleration, } \vec{a} &= \frac{d\vec{v}}{dt} \\ &= [-8(\cos 2t)(2)] \hat{i} + [8(-\sin 2t)(2)] \hat{j} \\ &= (-16 \cos 2t) \hat{i} + (-16 \sin 2t) \hat{j} \end{aligned}$$

When $t = \pi/4$ s

$$\begin{aligned} \vec{a} &= (-16 \cos \pi/2) \hat{i} + (-16 \sin \pi/2) \hat{j} \\ &= (-16 \times 0) \hat{i} + (-16 \times 1) \hat{j} = -16 \hat{j} \text{ m s}^{-2} \end{aligned}$$

6. (a) : Let a be the common acceleration. Then

$$F = 3m \times a \text{ or } a = F/3m$$

Net force on block A will be

$$F_1 = m \times a = m \times \frac{F}{3m} = \frac{F}{3}$$

Force applied by A on B,

$$F_2 = (m_1 + m_2)a = 2m \times \frac{F}{3m} = \frac{2F}{3}$$

Force applied by B on C,

$$F_3 = m \times a = m \times \frac{F}{3m} = \frac{F}{3}$$

7. (a) : Here, $\sigma = 5.0 \times 10^{-16} \text{ C m}^{-2}$, $r = 0.1 \text{ m}$, $\theta = 60^\circ$
Field due to a plane sheet of charge,

$$E = \frac{\sigma}{2\epsilon_0}$$

Flux through circular area,

$$\begin{aligned} \phi_E &= E \Delta S \cos \theta = \frac{\sigma}{2\epsilon_0} \times \pi r^2 \cos \theta \\ &= \frac{5.0 \times 10^{-16} \times 3.14 \times (0.1)^2 \cos 60^\circ}{2 \times 8.85 \times 10^{-12}} \\ &= 4.44 \times 10^{-7} \text{ N m}^2 \text{ C}^{-1} \end{aligned}$$

8. (a) : Initial electrostatic energy of the $4 \mu\text{F}$ capacitor is

$$U_i = \frac{1}{2} CV^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (200)^2 = 8 \times 10^{-2} \text{ J}$$

Charge on $4 \mu\text{F}$ capacitor

$$= CV = 4 \times 10^{-6} \times 200 = 8 \times 10^{-4} \text{ C}$$

When the $4 \mu\text{F}$ and $2 \mu\text{F}$ capacitors are connected together, both attain a common potential V' . Thus

$$V' = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{8 \times 10^{-4} \text{ C}}{(4+2) \times 10^{-6} \text{ F}} = \frac{400}{3} \text{ V}$$

Final electrostatic energy of the combination,

$$\begin{aligned} U_f &= \frac{1}{2} \times (4+2) \times 10^{-6} \times \left(\frac{400}{3}\right)^2 \text{ J} = \frac{16}{3} \times 10^{-2} \text{ J} \\ &= 5.33 \times 10^{-2} \text{ J} \end{aligned}$$

\therefore Electrostatic energy of the first capacitor lost in the form of heat and electromagnetic radiation is $\Delta U = U_i - U_f = (8 - 5.33) \times 10^{-2} \text{ J} = 2.67 \times 10^{-2} \text{ J}$.

9. (b) : Equivalent resistance of 8Ω , 16Ω and 16Ω connected in parallel is given by

$$\frac{1}{R'} = \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16} = \frac{1}{4} \quad \therefore R' = 4 \Omega$$

Total resistance in upper arm = $4 + 20 = 24 \Omega$

Equivalent resistance of 9Ω and 18Ω connected in

parallel is $R'' = \frac{9 \times 18}{9 + 18} = 6 \Omega$

Total resistance in lower arm = $6 + 6 = 12 \Omega$

Now 24Ω and 12Ω resistances are in parallel.

$$\therefore R_{PQ} = \frac{24 \times 12}{24 + 12} = 8 \Omega$$

10. (a) : As $\frac{B'}{B} = \frac{\mu_o(n/2)2I}{\mu_o nI} = 1 \quad \therefore B' = B$.

11. (c) : The efficiency of the engine is

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{375}{500} = 25\%.$$

Heat consumed per cycle, $Q_1 = 600 \text{ kcal}$

\therefore Heat rejected to the sink is

$$Q_1 - \eta Q_1 = 600 - 0.25 \times 600 = 450 \text{ kcal}$$

12. (a) : $A_1 = 4 \text{ mm}^2 = 4 \times 10^{-6} \text{ m}^2$, $\Delta L_1 = 0.1 \times 10^{-3} \text{ m}$

$$A_2 = 8 \times 10^{-6} \text{ m}^2, Y_2 = Y_1, L_2 = L_1, F_2 = F_1, \Delta L_2 = ?$$

$$\text{As } \Delta L_1 = \frac{F_1 L_1}{A_1 Y_1}, \Delta L_2 = \frac{F_2 L_2}{A_2 Y_2}$$

$$\therefore \frac{\Delta L_2}{\Delta L_1} = \frac{A_1}{A_2} = \frac{4 \times 10^{-6}}{8 \times 10^{-6}} = \frac{1}{2}$$

$$\text{or } \Delta L_2 = \frac{\Delta L_1}{2} = 0.05 \times 10^{-3} \text{ m} = 0.05 \text{ mm}$$

13. (b) : Here, $u_1 = 12 \text{ m s}^{-1}$, $u_2 = 10 \text{ m s}^{-1}$

Let v_1 be velocity of the light particle after collision

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2} \quad \dots(i)$$

Given : $m_1 \ll m_2$

m_1 can be ignored compared to m_2

From equation (i), we get

$$v_1 = -u_1 + 2u_2$$

Substituting the values, we get

$$v_1 = -12 \text{ m s}^{-1} + 2(10 \text{ m s}^{-1}) \\ = 8 \text{ m s}^{-1} \text{ in its original direction}$$

$$\begin{aligned} \text{14. (b) : As } h &= \left(\frac{T^2 R^2}{4 \pi^2} g \right)^{1/3} - R \\ &= \left[\frac{(24 \times 60 \times 60)^2 \times (6.4 \times 10^6)^2 \times 9.8}{4 \times (22/7)^2} \right]^{1/3} - 6.4 \times 10^6 \\ &= 3.6 \times 10^7 \text{ m} = 36000 \text{ km} \end{aligned}$$

15. (c) : Let T_1 be the time period of longer pendulum

$$\therefore \frac{T_1}{T} = \sqrt{\frac{l_1}{l}} = \sqrt{\frac{16}{1}} = 4 \quad \text{or} \quad T_1 = 4T$$

Let after time t , the pendulum be in the same phase. It will be so then

$$\frac{t}{T_1} = \frac{t}{T} - 1 = \frac{t - T}{T}$$

$$\text{or } \frac{t}{4T} = \frac{t - T}{T} \quad \text{or } t = 4t - 4T \quad \text{or } 3t = 4T \therefore t = 4T/3$$

16. (b)

17. (a) : $\therefore V = 4x^2$

$$\therefore \vec{E} = - \left(\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right) = -8x \hat{i}$$

Hence, value of \vec{E} at (1 m, 0, 2 m) will be

$$\vec{E} = -8 \times 1 \hat{i} = -8 \hat{i} \text{ V m}^{-1}$$

18. (a) : Here, $x = -u + v$...(i)

$$\text{As } m = \frac{f}{f + u} = \frac{f - v}{f}$$

and image is real, magnification is negative.

$$\therefore -m = \frac{f}{f + u} \Rightarrow u = \frac{-(m+1)f}{m} \quad \dots(ii)$$

$$\text{and } -m = \frac{f - v}{f} \Rightarrow v = (m+1)f \quad \dots(iii)$$

$$\text{Putting in eqn. (i) } x = \frac{(m+1)}{m} f + (m+1) f$$

$$\text{Solving, we get, } f = \frac{mx}{(m+1)^2}$$

19. (a) : Here, $\eta = 80\%$, $E_p I_p = 4 \text{ kW} = 4000 \text{ W}$

$$E_p = 100 \text{ V}, E_s = 200 \text{ V}$$

$$I_p = \frac{E_p I_p}{E_p} = \frac{4000}{100} = 40 \text{ A}$$

$$\eta = \frac{E_s I_s}{E_p I_p} \Rightarrow \frac{80}{100} = \frac{200 I_s}{4000} = \frac{I_s}{20}$$

$$\text{or } I_s = \frac{20 \times 80}{100} = 16 \text{ A}$$

20. (d) : Here $\mu = 0.5$, $u = 0$

$$v = 72 \text{ km h}^{-1} = 72 \times \frac{5}{18} = 20 \text{ m s}^{-1}$$

$$\text{As } \mu = \frac{f}{R} = \frac{ma}{mg} = \frac{a}{g} \quad (\because f = ma, \text{ without slipping})$$

$$\therefore a = \mu g = 0.5 \times 10 = 5 \text{ m s}^{-2}$$

$$\text{As } v^2 - u^2 = 2as$$

$$\therefore (20)^2 - 0^2 = 2 \times 5 \times s \quad \text{or } s = \frac{400}{10} = 40 \text{ m}$$

21. (b) : Here $m = 2 \text{ kg}$, $h = 1 \text{ m}$, $k = 490 \text{ N m}^{-1}$

Let the spring be compressed through distance x .

Then the block falls through a height $(h + x)$.

Gain in P.E. of the spring = Loss in P.E. of the block

$$\frac{1}{2} kx^2 = mg(h + x)$$

$$\text{or } \frac{1}{2} \times 490 \times x^2 = 2 \times 9.8 \times (1 + x)$$

$$\text{or } 12.5x^2 - x - 1 = 0$$

$$\therefore x = \frac{1 \pm \sqrt{1 + 4 \times 12.5}}{2 \times 12.5} = \frac{1 \pm \sqrt{51}}{25} = 0.33 \text{ m}$$

22. (a) : For $h \ll R$,

$$g_h = g \left(1 - \frac{2h}{R} \right) = g - \frac{2gh}{R}$$

$$\text{or } g - g_h = \frac{2gh}{R}$$

\therefore Loss in weight due to variation in g

$$= mg - mg_h = m(g - g_h)$$

$$= \frac{m \times 2gh}{R} = \frac{2mgh}{R}$$

23. (b) : As $B = \frac{\mu_0 I}{2R} = B_H$

$$\therefore I = \frac{2B_H R}{\mu_0} = \frac{2 \times 0.5 \times 10^{-5} \times 0.05}{4\pi \times 10^{-7}} = 0.3978 \approx 0.4 \text{ A.}$$

24. (a) : Here, $l = 1.0 \text{ m}$, $B_H = 0.3 \times 10^{-4} \text{ Wb m}^{-2}$, $\delta = 60^\circ$
 $B_V = B_H \tan \delta = 0.3 \times 10^{-4} \tan 60^\circ$
 $= 0.3 \times 10^{-4} \times 1.732 = 0.52 \times 10^{-4} \text{ Wb m}^{-2}$
 $v = 90 \text{ km h}^{-1} = \frac{90 \times 1000}{3600} = 25 \text{ m s}^{-1}$

$\therefore \epsilon = B_V lv = 0.52 \times 10^{-4} \times 1.0 \times 25 = 1.3 \times 10^{-3} \text{ V}$

25. (b) : Flux linked with the primary coil,

$$\phi = \phi_0 + 4t$$

Voltage across primary,

$$V_p = \frac{d\phi}{dt} = 0 + 4 \times 1 = 4 \text{ V}$$

Voltage across secondary,

$$V_s = \frac{N_s}{N_p} \cdot V_p = \frac{1500}{50} \times 4 = 120 \text{ V}$$

26. (d) : $v = \frac{c}{\sqrt{\mu_r \epsilon_r}}$
 $= \frac{3 \times 10^8}{\sqrt{1.3 \times 2.14}} = \frac{3 \times 10^8}{1.67} = 1.8 \times 10^8 \text{ m s}^{-1}$

27. (b) : Given $\alpha = 0.98$

and $\frac{R_{\text{out}}}{R_{\text{in}}} = 500$

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.98}{1 - 0.98} = 49$$

$$\text{Voltage gain} = (\beta) \left(\frac{R_{\text{out}}}{R_{\text{in}}} \right) = (49)(500) = 24500$$

$$\text{Power gain} = (\beta^2) \left(\frac{R_{\text{out}}}{R_{\text{in}}} \right) = (49)^2 (500) = 1200500$$

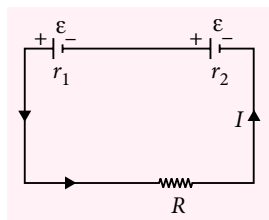
28. (d) : At the first minimum of the diffraction pattern, the rays coming from the two edges of the slit have a path difference $= \lambda$ and phase difference $= 2\pi$.

29. (b) : As both cells are in series, the circuit current

$$I = \frac{\epsilon + \epsilon}{r_1 + r_2 + R} = \frac{2\epsilon}{r_1 + r_2 + R}$$

As terminal potential drop across 1st cell is zero, hence

$$V_1 = \epsilon - Ir_1 = \epsilon - \frac{2\epsilon}{(r_1 + r_2 + R)} r_1 = 0$$



$$\Rightarrow \epsilon = \frac{2\epsilon r_1}{(r_1 + r_2 + R)} \text{ or } r_1 + r_2 + R = 2r_1 \text{ or } R = (r_1 - r_2)$$

30. (b) : Let the original volume, $V_1 = V$

\therefore Final volume $V_2 = V/2$

Initial pressure, $P_1 = 1.013 \times 10^5 \text{ N m}^{-2}$

Let P_2 be the final pressure after compression

$$\therefore P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\text{or } P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = P_1 \left(\frac{V}{V/2} \right)^{1.4}$$

$$P_2 = 1.013 \times 10^5 \times (2)^{1.4} \text{ N m}^{-2}$$

31. (b) : Let I be the current through arm ADC. Then current through arm ABC $= (2.1 - I)$. As there is no deflection in the galvanometer, hence

$$(20 + 5)I = (8 + 2)(2.1 - I)$$

$$\text{or } 25I = 21 - 10I \text{ or } 35I = 21$$

$$\text{or } I = 21/35 = 3/5 = 0.6 \text{ A}$$

32. (a) : The magnetic moment of a current loop

$$M = NI A = 500 \times 0.5 \times 3 \times 10^{-4} = 0.075 \text{ A m}^2$$

$$\text{Also, } \vec{\tau} = \vec{M} \times \vec{B} \text{ or } |\tau| = MB \sin \theta$$

where, θ = angle between \vec{M} and \vec{B} .

Here, $\theta = 90^\circ$

$$\therefore \tau = MB \sin 90^\circ$$

$$\text{or } B = \frac{\tau}{M} = \frac{1.5}{0.075} = 20 \text{ T}$$

33. (c) : Mass of steam produced per second,

$$\frac{dm}{dt} = \frac{1}{60} \text{ g s}^{-1}$$

$$\text{Heat transferred per second, } \frac{dQ}{dt} = L \frac{dm}{dt}$$

$$\Rightarrow \frac{dQ}{dt} = 540 \times \frac{1}{60} \text{ cal s}^{-1}$$

$$\text{Also } \frac{dQ}{dt} = \frac{KA \Delta T}{d}$$

$$\therefore \Delta T = \frac{dQ}{dt} \times \frac{d}{KA}$$

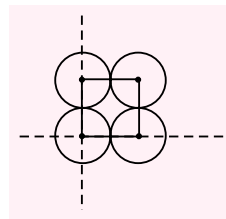
$$= \frac{9 \times 0.2}{0.5 \times 3000}$$

$$= 1.2 \times 10^{-3} ^\circ \text{C}$$

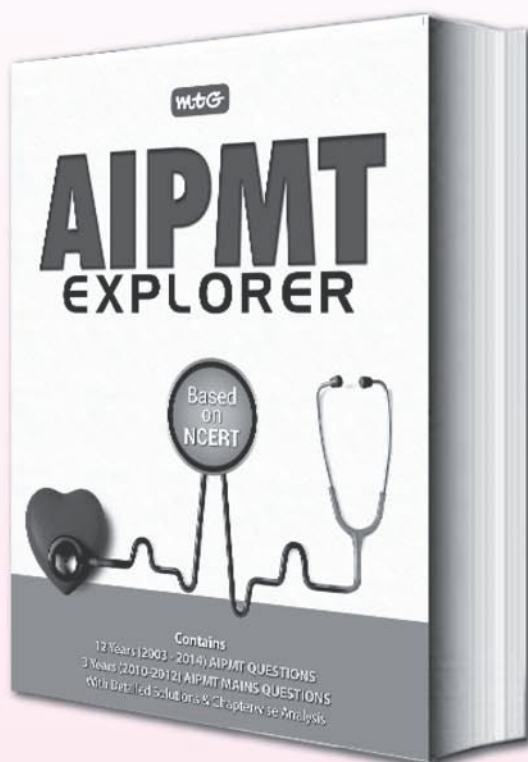
34. (d) : As shown in figure

$$x_{CM} = \frac{M \times 0 + M \times 20 + M \times 20 + M \times 0}{4M} = 10 \text{ cm}$$

Similarly, $y_{CM} = 10 \text{ cm}$



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Hence, distance of centre of mass from centre of any one sphere, $r = \sqrt{(10-0)^2 + (10-0)^2} = 10\sqrt{2}$ cm

35. (a) : Horizontal velocity of aeroplane,

$$u = \frac{216 \times 1000}{60 \times 60} = 60 \text{ m s}^{-1}$$

$$\text{Time of flight, } T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20 \text{ s}$$

Horizontal range, $AB = uT = 60 \times 20 = 1200 \text{ m}$

36. (d) : For an electron accelerated through potential

$$\text{difference } V, \lambda = \frac{h}{\sqrt{2meV}}$$

For a proton accelerated through the same potential

$$\text{difference } V, \lambda' = \frac{h}{\sqrt{2MeV}}$$

$$\therefore \frac{\lambda'}{\lambda} = \sqrt{\frac{m}{M}} \quad \text{or} \quad \lambda' = \lambda \sqrt{\frac{m}{M}}$$

37. (b) : When the electron jumps from $n = 3$ to $n = 1$

orbit,

$$v = Rc \left[\frac{1}{1^2} - \frac{1}{3^2} \right] \quad \text{or} \quad 2.7 \times 10^{15} = Rc \times \frac{8}{9} \quad \dots(i)$$

When the electron jumps from $n = 4$ to $n = 1$

$$v' = Rc \left[\frac{1}{1^2} - \frac{1}{4^2} \right] = Rc \times \frac{15}{16} \quad \dots(ii)$$

From eqn (i) and (ii)

$$\frac{v'}{2.7 \times 10^{15}} = \frac{15}{16} \times \frac{9}{8}$$

$$\text{or } v' = \frac{15}{16} \times \frac{9}{8} \times 2.7 \times 10^{15} = 2.8 \times 10^{15} \text{ Hz.}$$

38. (b) : As $\frac{N}{N_0} = e^{-\lambda t_1}$

$$\text{In first case, } \frac{1}{3} = e^{-\lambda t_1}$$

$$\text{In second case, } \frac{2}{3} = e^{-\lambda t_2}$$

$$\therefore t_1 - t_2 = \frac{\log_e 3}{\lambda} - \frac{\log_e (3/2)}{\lambda}$$

$$= \frac{1}{\lambda} (\log_e 3 - \log_e 1.5) = \frac{1}{\lambda} \log_e \frac{3}{1.5} = \frac{1}{\lambda} \log_e 2$$

$$= 20 \text{ min} \quad (\because T_{1/2} = \frac{1}{\lambda} \log_e 2)$$

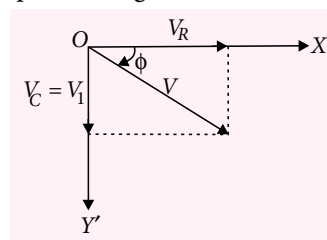
39. (b) : When majority charge carriers cross the emitter-base junction, few electrons combine with holes, and remaining charge carriers reach the collector. So $I_C < I_E$.

$$40. (c) : \text{As } \frac{W_1}{W_2} = 4 \quad \therefore \frac{I_1}{I_2} = \frac{a^2}{b^2} = 4 \quad \text{or} \quad \frac{a}{b} = 2$$

$$\therefore \frac{I_{\max}}{I_{\min}} = \frac{(a+b)^2}{(a-b)^2} = \frac{\left(\frac{a}{b}+1\right)^2}{\left(\frac{a}{b}-1\right)^2} = \frac{(2+1)^2}{(2-1)^2} = \frac{9}{1}$$

41. (c) 42. (d)

43. (d) : The phasor diagram is as shown in the figure.



44. (c) : Here, velocity of bat, $v_s = 4 \text{ m s}^{-1}$
velocity of sound, $v = 340 \text{ m s}^{-1}$

Frequency emitted by the bat, $v = 90 \text{ kHz}$

As source (bat) is moving towards the wall, the apparent frequency of sound striking the wall is

$$v' = v \left[\frac{v}{v - v_s} \right] \quad \dots(i)$$

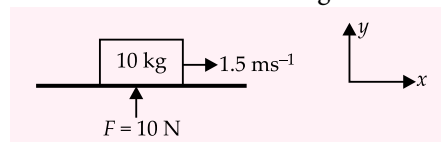
On reflection, wall acts as source and bat is the observer

$$\therefore v'' = v' \left[\frac{v + v_o}{v} \right] = v \left[\frac{v + v_o}{v - v_s} \right] \quad [\text{Using (i)}]$$

$$= 90 \left[\frac{340 + 4}{340 - 4} \right] = \frac{90 \times 344}{336} = 92.1 \text{ kHz}$$

45. (d) : Here, $m = 10 \text{ kg}$, $F = 10 \text{ N}$

The situation is as shown in the figure.



Acceleration along vertical direction

$$a_y = \frac{F}{m} = \frac{10 \text{ N}}{10 \text{ kg}} = 1 \text{ m s}^{-2}$$

Distance travelled by the block in 4 s in vertical direction is

$$s_y = \frac{1}{2} a_y t^2 = \frac{1}{2} \times (1 \text{ m s}^{-2})(4 \text{ s})^2 = 8 \text{ m}$$

Distance travelled by the block in 4 s in horizontal direction is $s_x = (1.5 \text{ m s}^{-1})(4 \text{ s}) = 6 \text{ m}$

The displacement of the block at the end of 4 s is

$$= \sqrt{s_x^2 + s_y^2} = \sqrt{(8 \text{ m})^2 + (6 \text{ m})^2} = 10 \text{ m}$$

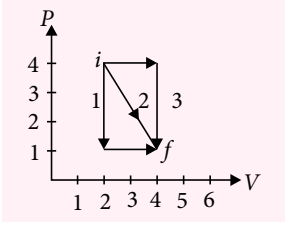


PRACTICE PAPER AIIMS

Exam on : 1st June

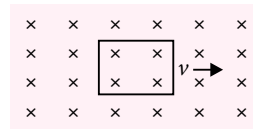
TIME TO THINK SMART!

- A particle covers 4 m, 5 m, 6 m and 7 m in 3rd, 4th, 5th and 6th second respectively. The particle starts
 - with an initial non-zero velocity and moves with uniform acceleration
 - from rest and moves with uniform velocity
 - with an initial velocity and moves with uniform velocity
 - from rest and moves with uniform acceleration.
- A body of mass M is dropped from a height h on a sand floor. If the body penetrates a distance x into the sand, the average resistance offered by the sand to the body is
 - $Mg\left(\frac{h}{x}\right)$
 - $Mg\left(1 + \frac{h}{x}\right)$
 - $Mgh + Mgx$
 - $Mg\left(1 - \frac{h}{x}\right)$
- A 5 μF capacitor is charged by a 220 V supply. It is then disconnected from the supply and is connected to another uncharged 2.5 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?
 - 0.02 J
 - 0.04 J
 - 0.121 J
 - 0.081 J
- A gas expands from i to f along the three paths indicated. The work done along the three paths denoted by W_1 , W_2 and W_3 have the relationship



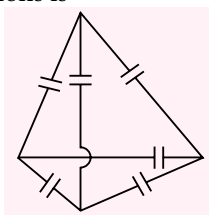
- $W_1 < W_2 < W_3$
 - $W_2 < W_1 = W_3$
 - $W_2 < W_1 < W_3$
 - $W_1 > W_2 > W_3$
- If the radius of the earth were to shrink by two percent, its mass remaining the same, the acceleration due to gravity on the earth's surface would
 - decrease by 2%
 - increase by 2%
 - increase by 4%
 - decrease by 4%
- A transparent cube contains a small air bubble. Its apparent distance is 2 cm when seen through one face and 5 cm when seen through opposite face. If the refractive index of the material of the cube is 1.5, then real length of the edge of cube must be
 - 7 cm
 - 7.5 cm
 - 10.5 cm
 - $\frac{14}{3}$ cm
- The photoelectrons emitted from a given cathode on the incidence of a given monochromatic beam of light have
 - energy spread with a lower limit
 - energy spread with an upper limit
 - energy spread with no sharp limits
 - a definite energy only.
- The ground state energy of hydrogen atom is -13.6 eV. What is the potential energy of the electron in this state?
 - 0 eV
 - -27.2 eV
 - 1 eV
 - 2 eV
- Whenever a stream of electrons collides with a stream of photons, which of the following is not conserved?
 - Linear momentum
 - Total energy
 - Number of photons
 - Number of electrons
- A Ge specimen is doped with Al. The concentration of acceptor atoms is 10^{21} atoms m^{-3} . Given that the intrinsic concentration of electron-hole pair is equivalent to 10^{19} m^{-3} , the concentration of electrons in the specimen is
 - 10^{17} m^{-3}
 - 10^{15} m^{-3}
 - 10^4 m^{-3}
 - 10^2 m^{-3}

11. A person is standing in an elevator. In which situation he finds his weight less?
- When the elevator moves upward with constant acceleration
 - When the elevator moves downward with constant acceleration
 - When the elevator moves upward with uniform velocity
 - When the elevator moves downward with uniform velocity.
12. The centre of mass of a solid cone along the line from the centre of the base to the vertex is at
- one-fourth of the height
 - one-third of the height
 - one-fifth of the height
 - none of these
13. If the radii of circular paths of two particles of same masses are in the ratio of 1 : 2, then to have a constant centripetal force, their velocities should be in a ratio of
- $1 : \sqrt{2}$
 - $\sqrt{2} : 1$
 - 4 : 1
 - 1 : 4
14. Two projectiles are projected with the same velocity. If one is projected at an angle of 30° and the other at 60° to the horizontal, then the ratio of maximum height reached, is
- 3 : 1
 - 1 : 3
 - 1 : 2
 - 2 : 1
15. Two blocks of masses $m_1 = 4$ kg and $m_2 = 2$ kg are connected to the ends of a string which passes over a massless, frictionless pulley. The total downward thrust on the pulley is nearly
- 27 N
 - 54 N
 - 0.8 N
 - zero
16. A square plate of 10 cm side moves parallel to another plate with a velocity of 10 cm s^{-1} , both plates immersed in water. If the viscous force is 200 dyne and viscosity of water is 0.01 poise, what is their separation distance?
- 0.05 cm
 - 1 cm
 - 0.07 cm
 - 7 cm
17. Two point charges repel each other with a force of 100 N. One of the charges is increased by 10% and other is reduced by 10%. The new force of repulsion at the same distance would be
- 100 N
 - 121 N
 - 99 N
 - 140 N
18. In hydrogen atom, the electron is making $6.6 \times 10^{15} \text{ rev s}^{-1}$ around the nucleus of radius of 53 \AA . The magnetic field produced at the centre of the orbit is nearly
- 0.125 Wb m^{-2}
 - 1.25 Wb m^{-2}
 - 125 Wb m^{-2}
 - 12.5 Wb m^{-2}
19. A coil of 1000 turns is wound on a book and this book is lying on the table. The vertical component of earth's magnetic field is $0.6 \times 10^{-4} \text{ T}$ and the area of the coil is 0.05 m^2 . The book is turned over once about a horizontal axis in 0.1 s. Thus average emf induced in the coil is
- 0.03 V
 - 0.06 V
 - 0.6 V
 - zero
20. A man has a concave shaving mirror of focal length 0.2 m. How far should the mirror be held from his face in order to give an image of two fold magnification?
- 0.1 m
 - 0.2 m
 - 0.3 m
 - 0.4 m
21. In Young's double slit experiment, the two slits are separated by 0.1 mm and they are 0.5 m from the screen. The wavelength of light used is 5000 \AA . What is the distance between 7th maxima and 11th minima on the screen?
- 5.65 mm
 - 6.75 mm
 - 8.75 mm
 - 7.8 mm
22. Three different objects of masses m_1 , m_2 and m_3 are allowed to fall from the same point O along three different frictionless paths. The speeds of the three objects, on reaching the ground, will be in the ratio of
- $m_1 : m_2 : m_3$
 - $m_1 : 2m_2 : 3m_3$
 - $1/m_1 : 1/m_2 : 1/m_3$
 - 1 : 1 : 1
23. The wavefront of a distant source of unknown shape is approximately
- spherical
 - cylindrical
 - elliptical
 - plane
24. A conducting square loop of side l and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnetic field B exists along the perpendicular to the plane of the loop as shown in figure. The current induced in the loop is



- $\frac{Blv}{R}$ clockwise
- $\frac{Blv}{R}$ anticlockwise
- $\frac{2Blv}{R}$ anticlockwise
- zero

25. If the capacitance of each capacitor is C , then effective capacitance of the given network across any two junctions is



- (a) $2C$ (b) C (c) $\frac{C}{2}$ (d) $5C$
26. A 5 litre vessel contains 2 mole of oxygen gas at a pressure of 8 atm. The average translational kinetic energy of an oxygen molecule under this condition is
(a) 8.4×10^{-14} J (b) 4.98×10^{-21} J
(c) 7.4×10^{-16} J (d) 4.2×10^{-23} J
27. If the radius of a potentiometer wire is increased four times, keeping its length constant, then the value of its potential gradient will become
(a) half (b) two times
(c) four times (d) unchanged
28. A particle of mass m , charge Q and kinetic energy T enters a transverse uniform magnetic field of induction \vec{B} . After 3 seconds, the kinetic energy of the particle will be
(a) T (b) $4T$ (c) $3T$ (d) $2T$
29. A series AC circuit has a resistance of 4Ω and a reactance of 3Ω . The impedance of the circuit is
(a) 5Ω (b) 7Ω (c) $\frac{12}{7} \Omega$ (d) $\frac{7}{12} \Omega$
30. The dimensions of $\frac{1}{2} \epsilon_0 E^2$ (where ϵ_0 is the permittivity of free space and E is electric field) is
(a) $[MLT]$ (b) $[ML^2T^{-2}]$
(c) $[ML^{-1}T^{-2}]$ (d) $[ML^2T^{-1}]$
31. If two electric bulbs, each designed to operate with a power of 500 W in 220 V line, are put in series in a 110 V line, what will be the power generated by each bulb?
(a) 31.25 W (b) 21.25 W
(c) 11.25 W (d) 9.25 W
32. An electric field \vec{E} and magnetic field \vec{B} exist in a region. If these fields are not perpendicular to each other, then the electromagnetic wave
(a) will not pass through the region
(b) will pass through region
(c) may pass through the region
(d) nothing is definite.
33. A radioactive isotope X has a half-life of 3 s. At $t = 0$ s, a given sample of this isotope X contains 8000 atoms. Find the time t_1 , when 1000 atoms of isotope X remains in the sample.
(a) 2 s (b) 4 s (c) 7 s (d) 9 s
34. The current gain of a transistor in a common base arrangement is 0.98. Find the change in collector current corresponding to a change of 5.0 mA in emitter current.
(a) 4.9 mA (b) 2.2 mA
(c) 3.3 mA (d) 3.9 mA
35. A steamer moves with velocity 3 km h^{-1} in and against the direction of river water whose velocity is 2 km h^{-1} . Find its total time for total journey, if the steamer travels 2 km in the direction of the stream and then come back to the same place.
(a) 2 h (b) 2.2 h (c) 2.4 h (d) 3 h
36. An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rpm, the acceleration of a point on the tip of the blade is
(a) 23.7 m s^{-2} (b) 47.4 m s^{-2}
(c) 50.55 m s^{-2} (d) 1600 m s^{-2}
37. An athlete completes one round of a circular track of radius R in 40 s. What will be his displacement at the end of 2 minute 20 seconds?
(a) zero (b) $2R$ (c) $2\pi R$ (d) $7\pi R$
38. The function $\sin \omega t - \cos \omega t$ represents
(a) a simple harmonic motion with a period $\frac{\pi}{\omega}$.
(b) a simple harmonic motion with a period $\frac{2\pi}{\omega}$.
(c) a periodic, but not simple harmonic motion with a period $\frac{\pi}{\omega}$.
(d) a periodic, but not simple harmonic motion with a period $\frac{2\pi}{\omega}$.
39. A superconducting material is
(a) ferromagnetic (b) ferroelectric
(c) diamagnetic (d) paramagnetic
40. Two forces of equal magnitude F act at a point. If the angle between them is θ , then the magnitude of the resultant force is
(a) $F\sqrt{2(1 - \sin \theta)}$ (b) $F\sqrt{2(1 + \sin \theta)}$
(c) $2F \sin\left(\frac{\theta}{2}\right)$ (d) $2F \cos\left(\frac{\theta}{2}\right)$

Directions : In the following questions (41-60), a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false.

(d) If both assertion and reason are false.

41. Assertion : A body becomes weightless at the centre of earth.

Reason : As the distance from centre of earth decreases, acceleration due to gravity increases.

42. Assertion : Converging property of convergent lens does not remain same in all mediums.

Reason : Property of lens whether the ray is diverging or converging depends on the surrounding medium.

43. Assertion : Thin films such as soap bubble or a thin layer of oil on water show beautiful colours when illuminated by white light.

Reason : It happens due to the interference of light reflected from the upper surface of the thin film.

44. Assertion : The centre of mass of a two particle system lies on the line joining the two particles, being closer to the heavier particle.

Reason : Product of mass of one particle and its distance from centre of mass is numerically equal to product of mass of other particle and its distance from centre of mass.

45. Assertion : We can change the temperature of a system without giving (or taking) heat to (or from) it.

Reason : According to principle of conservation of energy, total energy of a system should remain conserved.

46. Assertion : As the drift velocity decreases the current flowing through the conductor decreases.

Reason : The current flowing through a conductor is directly proportional to the drift velocity.

47. Assertion : Earth's magnetic field inside a closed iron box is less as compared to its walls.

Reason : The magnetic permeability of iron is low.

48. Assertion : The mass of a nucleus can be either less than or more than the sum of the masses of nucleons present in it.

Reason : The density of nuclear matter is dependent on the size of the nucleus.

49. Assertion : Stopping potential depends upon the frequency of incident light but is independent of the intensity of the light.

Reason : The maximum kinetic energy of the photoelectrons is proportional to stopping potential.

50. Assertion : Ocean waves hitting a beach are always found to be nearly normal to the shore.

Reason : Ocean waves are longitudinal waves.

51. Assertion : Work done by friction on a body sliding down an inclined plane is negative.

Reason : Work done is less than zero, if angle between force and displacement is acute or both are in same direction.

52. Assertion : When a particle moves in a circle with a uniform speed, its velocity and acceleration both changes.

Reason : The centripetal acceleration in circular motion is independent on angular velocity of the body.

53. Assertion : The graph between velocity and displacement for a simple harmonic motion is an ellipse.

Reason : Velocity does not change uniformly with displacement in simple harmonic motion.

54. Assertion : The shape of a liquid drop is spherical.

Reason : The pressure inside the drop is greater than that outside.

55. Assertion : All nuclei are not of same size.

Reason : Size of nucleus depends on atomic mass.

56. Assertion : Electron hole combination in base results in increase of base current.

Reason : When base region has larger width, the collector current increases.

57. Assertion : When charges are shared between two bodies, there occurs no loss of charge. However, there is a loss in electrical energy.

Reason : Electrostatic potential energy does not come under the preview of the conservation law of energy.

58. Assertion : The bridges are declared unsafe after a long use.

Reason : The bridges lose their elastic strength with time.

59. Assertion : The binding energy of a satellite depend upon the mass of the satellite.

Reason : Binding energy is the negative value of total energy of satellite.

- 60. Assertion :** The velocity of a body at the bottom of an inclined plane of given height is more when it slides down the plane compared to when it rolls down the same plane.

Reason : In rolling down, a body acquires both, kinetic energy of translation and kinetic energy of rotation.

SOLUTIONS

1. (a) : As $S_n = u + \frac{a}{2}(2n-1)$

$$4 = u + \frac{a}{2}(2 \times 3 - 1) \quad \text{or} \quad 4 = u + \frac{5a}{2} \quad \dots(i)$$

$$5 = u + \frac{a}{2}(2 \times 4 - 1) \quad \text{or} \quad 5 = u + \frac{7a}{2} \quad \dots(ii)$$

$$\text{Subtracting (ii) from (i) we get, } 1 = \frac{7a}{2} - \frac{5a}{2} = \frac{2a}{2} = a$$

$$\text{Again } 4 = u + \frac{5}{2} \quad \text{or} \quad u = 4 - \frac{5}{2} = 1.5 \text{ m s}^{-1}$$

So, the initial velocity is non-zero and acceleration is uniform.

2. (b) : If the body strikes the sand floor with a velocity v , then

$$Mgh = \frac{1}{2}Mv^2$$

With this velocity v , when body passes through the sand floor it comes to rest after travelling a distance x .

Let F be the resisting force acting on the body. Net force in downwards direction

$$= Mg - F$$

Work done by all the forces is equal to change in kinetic energy

$$\therefore (Mg - F)x = 0 - \frac{1}{2}Mv^2$$

$$(Mg - F)x = -Mgh \quad \text{or} \quad Fx = Mgh + Mgx$$

$$\therefore F = Mg \left(1 + \frac{h}{x} \right)$$

3. (b) : Here, $C_1 = 5 \mu\text{F} = 5 \times 10^{-6} \text{ F}$, $V_1 = 220 \text{ V}$
Energy stored in the capacitor,

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 5 \times 10^{-6} \times (220)^2 = 0.121 \text{ J}$$

When this charged capacitor is connected to another uncharged capacitor of capacitance $C_2 = 2.5 \mu\text{F}$, they will share charges, till the potential difference across their plates become equal.

Total charge on the two capacitors,

$$q = C_1 V_1 + C_2 V_2 = 5 \times 10^{-6} \times 220 = 11 \times 10^{-4} \text{ C}$$

Total capacitance of the two capacitors,

$$C = C_1 + C_2 = 5 \mu\text{F} + 2.5 \mu\text{F} = 7.5 \mu\text{F} = 7.5 \times 10^{-6} \text{ F}$$

Therefore, common potential of the two capacitors,

$$V = \frac{q}{C} = \frac{11 \times 10^{-4}}{7.5 \times 10^{-6}} = \frac{440}{3} \text{ V}$$

Energy stored in the combination,

$$U_2 = \frac{1}{2} C V^2 = \frac{1}{2} \times 7.5 \times 10^{-6} \times \left(\frac{440}{3} \right)^2 = 0.081 \text{ J}$$

Therefore, energy lost by C_1 in the form of heat and electromagnetic radiation,

$$U_1 - U_2 = 0.121 - 0.081 = 0.04 \text{ J}$$

4. (a) : Work done, $W = \text{Area under } P\text{-}V \text{ graph}$

5. (c) : As, $g = \frac{GM}{R^2}$, If R decreases then g increases.

Taking logarithm of both the sides,

$$\log g = \log G + \log M - 2 \log R$$

Differentiating it we get, $\frac{dg}{g} = 0 + 0 - \frac{2dR}{R}$

$$= -2 \left(\frac{-2}{100} \right) = \frac{4}{100}$$

$$\therefore \% \text{ increase in } g = \frac{dg}{g} \times 100 = \frac{4}{100} \times 100 = 4\%$$

6. (c) : Here, $\mu = \frac{\text{real depth}}{\text{apparent depth}}$

$$\text{or } 1.5 = \frac{\text{real depth}}{2 \text{ cm} + 5 \text{ cm}}$$

$$\therefore \text{Real depth} = 1.5 \times 7 = 10.5 \text{ cm}$$

7. (b)

8. (b) : Potential energy = $2 \times \text{total energy} = 2 \times (-13.6) = -27.2 \text{ eV}$

9. (c) : In a photon-electron collision, a photon may be absorbed or a new photon may be released. Here number of photons is not conserved but linear momentum, total energy and number of electrons are conserved.

10. (a) : Here, $n_i = 10^{19} \text{ m}^{-3}$, $n_h = 10^{21} \text{ m}^{-3}$

$$\text{As } n_i^2 = n_e n_h$$

$$\therefore n_e = \frac{n_i^2}{n_h} = \frac{10^{19} \times 10^{19}}{10^{21}} = 10^{17} \text{ m}^{-3}$$

11. (b) : When the elevator moves downward with acceleration a , the apparent weight,

$$W' = m(g - a), \quad W = mg$$

$$\therefore W' < W$$

12. (a) : The centre of mass of a solid cone lies on its axis at a distance of $h/4$ from its base.

13. (a) : Centripetal force, $F = \frac{mv^2}{r}$

$$\therefore v = \sqrt{\frac{Fr}{m}}$$

For same F and m , $v \propto \sqrt{r}$

$$\therefore \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{1}{2}} = 1 : \sqrt{2}$$

14. (b) : Maximum height, $H = \frac{u^2 \sin^2 \theta}{2g}$

$$\therefore \frac{H_1}{H_2} = \frac{\sin^2 30^\circ}{\sin^2 60^\circ} = \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3}$$

15. (b) : Here, $T = \frac{2m_1 m_2}{m_1 + m_2} g = \frac{2 \times 4 \times 2 \times 10}{4 + 2}$

$$= \frac{160}{6} = 26.6 \approx 27 \text{ N}$$

Total downward thrust on the pulley
 $= 2T = 2 \times 27 = 54 \text{ N}$

16. (a) : Here, side of the square plate,
 $L = 10 \text{ cm}$

Area of the plate, $A = L^2 = 100 \text{ cm}^2$,

$dv = 10 \text{ cm s}^{-1}$, $F = 200 \text{ dyne}$,

$\eta = 0.01 \text{ poise}$, $dx = ?$

As $F = \eta A \frac{dv}{dx}$

$$dx = \frac{\eta A dv}{F} = \frac{0.01 \times 100 \times 10}{200} = 0.05 \text{ cm}$$

17. (c)

18. (a) : $B = \frac{\mu_0}{4\pi} \frac{2\pi I}{r} = \frac{\mu_0}{2} \frac{1}{r} \times (e v)$

$$= \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-19} \times 6.6 \times 10^{15}}{2 \times 53 \times 10^{-10}}$$

$$= 0.125 \text{ Wb m}^{-2}$$

19. (b) : Here, $|\epsilon| = \frac{d\phi}{dt} = \frac{NBA (\cos 0^\circ - \cos 180^\circ)}{dt}$

$$= \frac{2 NBA}{dt} = \frac{2 \times 1000 \times 0.6 \times 10^{-4} \times 0.05}{0.1}$$

$$= 0.06 \text{ V}$$

20. (a) : As magnification, $m = \frac{f}{f - u}$

$$2 = \frac{-0.2}{-0.2 - u}$$

$$\text{or } 2 = \frac{0.2}{0.2 + u} \text{ or } 0.4 + 2u = 0.2$$

$$\text{or } 2u = 0.2 - 0.4 = -0.2$$

$$\therefore u = -0.1 \text{ m}$$

21. (c) : Here, $d = 0.1 \text{ mm} = 10^{-4} \text{ m}$

$D = 0.5 \text{ m}$, $\lambda = 5000 \text{ \AA} = 5.0 \times 10^{-7} \text{ m}$

$$\therefore \Delta x = (x_{11})_{\text{dark}} - (x_7)_{\text{bright}}$$

$$= \frac{(2 \times 11 - 1)\lambda D}{2d} - \frac{7\lambda D}{d}$$

$$\Delta x = \frac{7\lambda D}{2d} = \frac{7 \times 5 \times 10^{-7} \times 0.5}{2 \times 10^{-4}} = 8.75 \times 10^{-3} \text{ m}$$

$$= 8.75 \text{ mm}$$

22. (d) : When different objects allowed to fall from the same height, they attain the same speed on reaching the ground, independent of their masses or the frictionless paths.

23. (d)

24. (d) : As the magnetic flux is not changing through the loop, hence no current is induced in the loop.

25. (a) : The given network is a balanced Wheatstone bridge with one capacitor in parallel with this bridge.

26. (b) : Here, $P = 8 \text{ atm} = 8 \times 10^5 \text{ N m}^{-2}$,

$V = 5 \text{ litre} = 5 \times 10^{-3} \text{ m}^3$, $n = 2$

According to ideal gas equation

$$PV = nRT$$

$$\text{or } T = \frac{PV}{nR} = \frac{(8 \times 10^5 \text{ N m}^{-2})(5 \times 10^{-3} \text{ m}^3)}{(2)(8.31 \text{ J mol}^{-1} \text{ K}^{-1})} = 240.7 \text{ K}$$

The average translational kinetic energy of a gas molecule is

$$E = \frac{3}{2} k_B T$$

$$\therefore E = \frac{3}{2} \times 1.38 \times 10^{-23} \text{ J K}^{-1} \times 240.7 \text{ K}$$

$$= 4.98 \times 10^{-21} \text{ J}$$

27. (d) : Potential gradient is independent of area of cross-section of wire.

28. (a) : When a charged particle having kinetic energy T is subjected to a transverse uniform magnetic field, it describes a circular path in the magnetic field without any change in its speed. Thus, the kinetic energy of the charged particle remains T at all times.

29. (a) : Here, resistance, $R = 4 \Omega$

Reactance $X = 3 \Omega$

The impedance of the circuit is

$$Z = \sqrt{(R)^2 + (X)^2} = \sqrt{(4 \Omega)^2 + (3 \Omega)^2} = 5 \Omega$$

30. (c) : Dimensions of $[\epsilon_0] = [M^{-1}L^{-3}T^4A^2]$

Dimension of $[E] = [MLT^{-3}A^{-1}]$

\therefore Dimensions of

$$\left[\frac{1}{2}\epsilon_0 E^2\right] = [M^{-1}L^{-3}T^4A^2] \times [MLT^{-3}A^{-1}]^2 \\ = [ML^{-1}T^{-2}]$$

31. (a) : Power of electric bulb, $P = 500$ W

Operating voltage $V = 220$ V

Let R be the resistance of bulb.

$$\text{Here, } P = \frac{V^2}{R}$$

$$\therefore R = \frac{V^2}{P} = \frac{(220)^2}{500} = 96.8 \, \Omega$$

According to Ohm's law, $V = IR$

For series connection, current remains same

$$\therefore \frac{V_1}{V_2} = \frac{R_1}{R_2}$$

Here $R_1 = R_2 \therefore V_1 = V_2$

Also $V_1 + V_2 = 110$ V (\because series combination)

\therefore Voltage across each bulb = 55 V

Power generated by each bulb

$$= \frac{V_1^2}{R_1} = \frac{(55)^2}{96.8} = 31.25 \text{ W}$$

32. (c) : The electromagnetic wave being packets of energy moving with speed of light may pass through the region.

33. (d) : Decay constant $\lambda = \frac{\log_e 2}{T_{1/2}} = \frac{0.693}{3} = 0.231 \text{ s}^{-1}$

Number of atoms remain undecayed, $N = N_0 e^{-\lambda t_1}$

$$\text{or } t_1 = \frac{1}{\lambda} \log_e \frac{N_0}{N} = \frac{1}{0.231} \log_e 8 = \frac{2.079}{0.231} = 9 \text{ s}$$

34. (a) : Given, $\alpha = 0.98$ and $\Delta I_E = 5.0$ mA

From the definition,

$$\alpha = \frac{\Delta I_C}{\Delta I_E}$$

Change in collector current

$$\Delta I_C = \alpha \times \Delta I_E = 0.98 \times 5.0 \text{ mA} = 4.9 \text{ mA}$$

35. (c) : The velocity of streamer while moving downstream = $3 \text{ km h}^{-1} + 2 \text{ km h}^{-1} = 5 \text{ km h}^{-1}$ and while moving upstream

$$= 3 \text{ km h}^{-1} - 2 \text{ km h}^{-1} = 1 \text{ km h}^{-1}.$$

$$\text{Total time taken} = \frac{2 \text{ km}}{5 \text{ km h}^{-1}} + \frac{2 \text{ km}}{1 \text{ km h}^{-1}} \\ = 0.4 \text{ h} + 2 \text{ h} = 2.4 \text{ h}$$

36. (b) : Here, $r = 30 \text{ cm} = 0.3 \text{ m}$,

$$v = 120 \text{ rpm} = \frac{120}{60} \text{ rps}$$

$$\text{Acceleration} = r\omega^2 = r4\pi^2 v^2 \quad (\because \omega = 2\pi v)$$

$$= 0.3 \times 4 \times \left(\frac{22}{7}\right)^2 \times \left(\frac{120}{60}\right)^2 = 47.4 \text{ m s}^{-2}$$

37. (b) : Total time of motion on circular track = $2 \times 60 \text{ s} + 20 \text{ s} = 140 \text{ s}$.

Time period of revolution = 40 s.

Therefore displacement in time ($= 3 \times 40 = 120 \text{ s}$) = zero as athlete will be reaching at the starting point.

Thus, Displacement in 140 s = Displacement in 20 s = $2R$.

$$\begin{aligned} 38. (b) : \sin \omega t - \cos \omega t &= \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin \omega t - \frac{1}{\sqrt{2}} \cos \omega t \right) \\ &= \sqrt{2} \left(\sin \omega t \cos \frac{\pi}{4} - \cos \omega t \sin \frac{\pi}{4} \right) \\ &= \sqrt{2} \sin \left(\omega t - \frac{\pi}{4} \right) \end{aligned}$$

It represents a simple harmonic motion with a period $\frac{2\pi}{\omega}$.

39. (c) : A superconductor is a diamagnetic material.

40. (d) : Here, $F_1 = F_2 = F$

The magnitude of the resultant force is

$$\begin{aligned} F_R &= \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta} \\ &= \sqrt{F^2 + F^2 + 2(F)(F) \cos \theta} \\ &= \sqrt{2F^2 + 2F^2 \cos \theta} = F\sqrt{2(1 + \cos \theta)} \\ &= F\sqrt{2 \left(2 \cos^2 \frac{\theta}{2} \right)} = 2F \cos \left(\frac{\theta}{2} \right) \end{aligned}$$

41. (c) : Variation of g with depth from surface of earth is given by

$$g' = g \left(1 - \frac{d}{R} \right)$$

At the centre of earth, $d = R$

$$\therefore g' = g \left(1 - \frac{R}{R} \right) = 0$$

\therefore Apparent weight of body = $mg' = 0$

Assertion is true but reason is false.

42. (a) : In air or water, a convex lens made of glass behaves as a convergent lens but when it is placed in carbon disulfide, it behaves as a divergent lens. Therefore, when a convergent lens is placed inside a transparent medium of refractive index greater than that of material of lens, it behaves as divergent lens. It simply concludes that property of a lens whether the rays is diverging or converging depends on the surrounding medium.

43. (c) : The beautiful colours are seen on account of interference of light reflected from the upper and the lower surfaces of the thin films.

44. (a) : The centre of mass of a body or a system of bodies is the point that moves as though all of the mass were concentrated there and all external forces were applied there. The position of centre of mass

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

45. (a) : We can change the temperature of a body without giving (or taking) heat to (or from) it. For example, in an adiabatic compression temperature rises and in an adiabatic expansion temperature falls, although no heat is given or taken from the system in the respective changes.

46. (a) : Consider a conductor of length l and area of cross section A . Time taken by the free electrons to cross the conductor,

$$t = l/v_d.$$

$$\text{Hence, current, } I = \frac{q}{t} = \frac{Al \times ne}{l/v_d}$$

$$\text{or, } I = Anev_d$$

$$\therefore I \propto v_d.$$

Thus current is directly proportional to drift velocity.

47. (c) : As the magnetic permeability ($\mu = B/H$) of iron is high, therefore, lines of force prefer to pass through or concentrate in the iron walls of the box.

48. (d) : The mass of a nucleus is always less than the sum of the masses of the nucleons present in it. When nucleons combine to form a nucleus, some energy is liberated, and this is the binding energy of the nucleus. The mass of the nucleus cannot be more than the total mass of the nucleons because then stable nucleus cannot be formed.

49. (b) : Stopping potential is a measure of maximum kinetic energy of emitted photoelectron ($eV_0 = K_{\max}$) and K_{\max} depends upon the frequency of incident light but is independent of intensity.

50. (c) : Ocean waves are the combination of transverse and longitudinal waves travelling in concentric circles of ever increasing radius. When they hit the shore, their radius of curvature is so large that they can be treated as plane waves. Hence they hit the shore nearly normal to the shore.

51. (c) : When a body slides down on an inclined plane, the angle between direction of motion, i.e. displacement and frictional force is equal to 180° (Friction always acts in a direction opposite to the direction of the motion).

$\therefore W = \vec{F} \cdot \vec{s} = Fs \cos \theta = Fs \cos 180^\circ$ which is negative.

i.e. work done by friction on a body sliding down an inclined plane is negative.

52. (c) : In uniform circular motion, the magnitude of velocity and acceleration remains same, but due to change in direction of motion, the direction of velocity and acceleration changes. Also the centripetal acceleration is given by $a = \omega^2 r$.

53. (a) : In SHM, $v = \omega \sqrt{a^2 - y^2}$ or $v^2 = \omega^2 a^2 - \omega^2 y^2$.

Dividing both sides by $\omega^2 a^2$, $\frac{v^2}{\omega^2 a^2} + \frac{y^2}{a^2} = 1$. This is the equation of an ellipse.

54. (b)

55. (a) : The radius of nucleus is given by $R = R_0 A^{1/3}$ where R_0 is a constant. For different nuclei, mass number A is different, therefore R is different.

56. (c) 57. (c)

58. (a) : A bridge during its use undergoes alternating strains for a large number of times each day, depending upon the movement of vehicles on it. When a bridge is used for long time, it loses its elastic strength. Due to which the amount of strain in the bridge for a given stress will become large and ultimately, the bridge may collapse.

59. (b) : Binding energy is the minimum energy required to free a satellite from the gravitational attraction. It is the negative value of total energy of satellite. Let a satellite of mass m be revolving around earth of mass M_e and radius R_e
Total energy of satellite = $PE + KE$

$$= \frac{-GM_e m}{R_e} + \frac{1}{2}mv^2$$

$$= \frac{-GM_e m}{R_e} + \frac{GM_e m}{2R_e}$$

$$= -\frac{GM_e m}{2R_e}$$

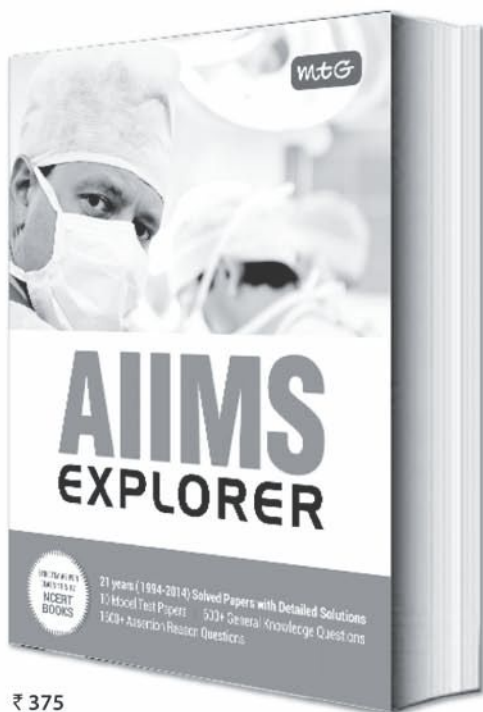
\therefore Binding energy of satellite = - (total energy of satellite) which depend on mass of the satellite.

60. (b) : In sliding down, the entire potential energy is converted only into linear kinetic energy. In rolling down, a part of some potential energy is converted into kinetic energy of rotation. Therefore, velocity acquired is less.

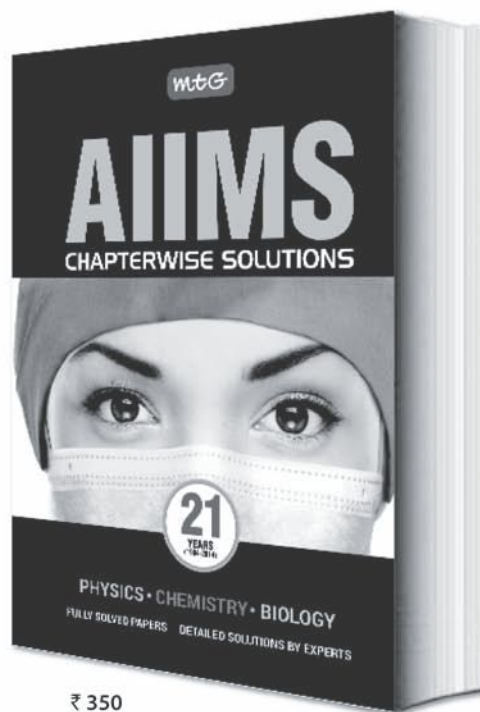




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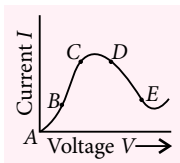
Maximum Marks : 70

GENERAL INSTRUCTIONS

- All questions are compulsory. There are 26 questions in all.
- This question paper has five sections: Section A, Section B, Section C, Section D and Section E.
- Section A contains five questions of one mark each, Section B contains five questions of two marks each, Section C contains twelve questions of three marks each, Section D contains one value based question of four marks and Section E contains three questions of five marks each.
- There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all the three questions of five marks weightage. You have to attempt only one of the choices in such questions.
- You may use the following values of physical constants wherever necessary.
 $c = 3 \times 10^8 \text{ m/s}$, $h = 6.63 \times 10^{-34} \text{ J s}$, $e = 1.6 \times 10^{-19} \text{ C}$, $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$, $\epsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$, $1/4\pi\epsilon_0 = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$,
 $m_e = 9.1 \times 10^{-31} \text{ kg}$, mass of neutron $= 1.675 \times 10^{-27} \text{ kg}$, mass of proton $= 1.673 \times 10^{-27} \text{ kg}$
Avogadro's number $= 6.023 \times 10^{23}$ per gram mole, Boltzmann constant $= 1.38 \times 10^{-23} \text{ J K}^{-1}$

SECTION-A

- What is the electric flux through a cube of side 1 cm which encloses an electric dipole?
- A concave lens of refractive index 1.5 is immersed in a medium of refractive index 1.65. What is the nature of the lens?
- How are side bands produced?
- Graph showing the variation of current versus voltage for a material GaAs is shown in the figure. Identify the region of
 - negative resistance
 - where Ohm's law is obeyed.
- Define capacitor reactance. Write its S.I. units.



SECTION-B

- Show that the radius of the orbit in hydrogen atom varies as n^2 , where n is the principal quantum number of the atom.
- Distinguish between 'intrinsic' and 'extrinsic' semiconductors.
- Use the mirror equation to show that an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.

OR

Find an expression for intensity of transmitted light when a polaroid sheet is rotated between two

crossed polaroids. In which position of the polaroid sheet will the transmitted intensity be maximum?

- Use Kirchhoff's rules to obtain conditions for the balance condition in a Wheatstone bridge.
- A proton and an α -particle have the same de-Broglie wavelength. Determine the ratio of
 - their accelerating potentials
 - their speeds.

SECTION-C

- Draw a block diagram of a detector for AM signal and show, using necessary processes and the waveforms, how the original message signal is detected from the input AM wave.
- A cell of emf 'E' and internal resistance 'r' is connected across a variable load resistor R. Draw the plots of the terminal voltage V versus (i) R and (ii) the current I.
It is found that when $R = 4 \Omega$, the current is 1 A and when R is increased to 9Ω , the current reduces to 0.5 A. Find the values of the emf E and internal resistance r.
- Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the value of C_1 and C_2 . Also calculate the charge on each capacitor in parallel combination.

14. State the principle of working of a galvanometer.

A galvanometer of resistance G is converted into a voltmeter to measure upto V volts by connecting a resistance R_1 in series with the coil. If a resistance R_2 is connected in series with it, then it can measure upto $V/2$ volts. Find the resistance, in terms of R_1 and R_2 , required to be connected to convert it into a voltmeter that can read upto 2 V. Also find the resistance G of the galvanometer in terms of R_1 and R_2 .

15. With what considerations in view, a photodiode is fabricated? State its working with the help of a suitable diagram.

Eventhough the current in the forward bias is known to be more than in the reverse bias, yet the photodiode works in reverse bias. What is the reason?

16. Draw a circuit diagram of a transistor amplifier in CE configuration.

Define the terms: (i) Input resistance and (ii) Current amplification factor. How are these determined using typical input and output characteristics?

17. Answer the following questions :

- (a) In a double slit experiment using light of wavelength 600 nm, the angular width of the fringe formed on a distant screen is 0.1° . Find the spacing between the two slits.
- (b) Light of wavelength 500 Å propagating in air gets partly reflected from the surface of water. How will the wavelengths and frequencies of the reflected and refracted light be affected?

18. An inductor L of inductance X_L is connected in series with a bulb B and an ac source. How would brightness of the bulb change when (i) number of turn in the inductor is reduced, (ii) an iron rod is inserted in the inductor and (iii) a capacitor of reactance $X_C = X_L$ is inserted in series in the circuit. Justify your answer in each case.

19. Name the parts of the electromagnetic spectrum which is

- (a) suitable for radar systems used in aircraft navigation.
- (b) used to treat muscular strain.
- (c) used as a diagnostic tool in medicine.
- Write in brief, how these waves can be produced.

20. (i) A giant refracting telescope has an objective lens of focal length 15 m. If an eye piece of focal length 1.0 cm is used, what is the angular magnification of the telescope?
- (ii) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon

is 3.48×10^6 m and the radius of lunar orbit is 3.8×10^8 m.

21. Write Einstein's photoelectric equation and mention which important features in photoelectric effect can be explained with the help of this equation.

The maximum kinetic energy of the photoelectrons gets doubled when the wavelength of light incident on the surface changes from λ_1 to λ_2 . Derive the expressions for the threshold wavelength λ_0 and work function for the metal surface.

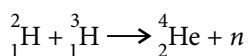
22. In the study of Geiger-Marsdon experiment on scattering of α particles by a thin foil of gold, draw the trajectory of α -particles in the coulomb field of target nucleus. Explain briefly how one gets the information on the size of the nucleus from this study.

From the relation $R = R_0 A^{1/3}$, where R_0 is constant and A is the mass number of the nucleus, show that nuclear matter density is independent of A .

OR

Distinguish between nuclear fission and fusion. Show how in both these processes energy is released.

Calculate the energy release in MeV in the deuterium-tritium fusion reaction :



Using the data :

$$m({}_1^2\text{H}) = 2.014102 \text{ u}$$

$$m({}_1^3\text{H}) = 3.016049 \text{ u}$$

$$m({}_2^4\text{He}) = 4.002603 \text{ u}$$

$$m_n = 1.008665 \text{ u}$$

$$1 \text{ u} = 931.5 \text{ MeV}/c^2$$

SECTION-D

23. A group of students while coming from the school noticed a box marked "Danger H.T. 2200 V" at a substation in the main street. They did not understand the utility of a such a high voltage, while they argued, the supply was only 220 V. They asked their teacher this question the next day. The teacher thought it to be an important question and therefore explained to the whole class.

Answer the following questions :

- (i) What device is used to bring the high voltage down to low voltage of a.c. current and what is the principle of its working?
- (ii) Is it possible to use this device for bringing down the high dc voltage to the low voltage? Explain.
- (iii) Write the values displayed by the students and the teacher.

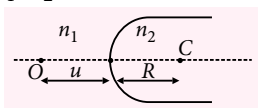
SECTION-E

24. (a) Using Huygens's construction of secondary wavelets explain how a diffraction pattern is obtained on a screen due to a narrow slit on which a monochromatic beam of light is incident normally.
- (b) Show that the angular width of the first diffraction fringe is half that of the central fringe.
- (c) Explain why the maxima at $\theta = \left(n + \frac{1}{2}\right) \frac{\lambda}{a}$ become weaker and weaker with increasing n .

OR

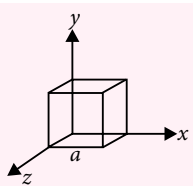
- (a) A point object 'O' is kept in a medium of refractive index n_1 in front of a convex spherical surface of radius of curvature R which separates the second medium of refractive index n_2 from the first one, as shown in the figure.

Draw the ray diagram showing the image formation and deduce the relationship between the object distance and the image distance in terms of n_1 , n_2 and R .



- (b) When the image formed above acts as a virtual object for a concave spherical surface separating the medium n_2 from n_1 ($n_2 > n_1$), draw this ray diagram and write the similar (similar to (a)) relation. Hence obtain the expression for the lens maker's formula.
25. (a) An electric dipole of dipole moment \vec{p} consists of point charges $+q$ and $-q$ separated by a distance $2a$ apart. Deduce the expression for the electric field \vec{E} due to the dipole at a distance x from the centre of the dipole on its axial line in terms of the dipole moment \vec{p} . Hence show that in the limit $x \gg a$, $\vec{E} \rightarrow 3\vec{p}/(4\pi\epsilon_0 x^3)$.

- (b) Given the electric field in the region $\vec{E} = 2xi$, find the electric flux through the cube and the charge enclosed by it.

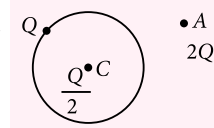


OR

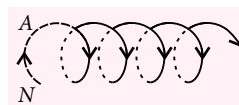
- (a) Explain, using suitable diagrams, the difference in the behaviour of a (i) conductor and (ii) dielectric in the presence of external electric field. Define the terms polarization of a dielectric and write its relation with susceptibility.
- (b) A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $\frac{Q}{2}$ is placed at its centre C and an other

charge $+2Q$ is placed outside the shell at a distance x from the centre as shown in the figure.

Find (i) the force on the charge at the centre of shell and the point A , (ii) the electric flux through the shell.



26. (a) State ampere's circuital law. Use this law to obtain the expression for the magnetic field inside an air cored toroid of average radius ' r ', having ' n ' turns per unit length and carrying steady current I .
- (b) An observer to the left of a solenoid of N turns each of cross section area ' A ' observes that a study current I in it flows in the clockwise direction. Depict the magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic moment $m = NIA$.



OR

- (a) Define mutual inductance and write its S.I. units.
- (b) Derive an expression for the mutual inductance of two long co-axial solenoids of same length wound one over the other.
- (c) In an experiment, two coils C_1 and C_2 are placed close to each other. Find out the expression for emf induced in the coil C_1 due to a change in the current through the coil C_2 .

SOLUTION

1. According to Gauss's law, net flux through a closed surface,

$$\phi_E = \oint \vec{E} \cdot d\vec{s} = \frac{q_{en}}{\epsilon_0}$$

Total charge enclosed, $q_{en} = 0$
as net charge on dipole is zero.

$$\therefore \phi_E = \frac{0}{\epsilon_0} = 0$$

2. Focal length of a concave lens is negative.

Using lens maker's formula,

$$\frac{1}{f} = \left(\frac{\mu_l}{\mu_m} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

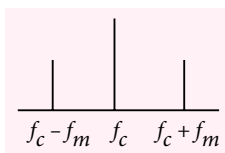
Here, $\mu_l = 1.5$, $\mu_m = 1.65$

Also, $\frac{\mu_l}{\mu_m} < 1$, so $\left(\frac{\mu_l}{\mu_m} - 1 \right)$ is negative and focal

length of the given lens becomes positive. Hence, it behaves as a convex lens.

3. Side bands are produced during the modulation.

When a message signal is super imposed on a carrier wave then there exists the sum and difference of the two frequencies of different waves.



These are called side bands. In amplitude modulation, side bands are :

Lower side band frequency = $f_c - f_m$

Upper side band frequency = $f_c + f_m$

4. (i) Region DE has negative resistance property because current decreases with increase in voltage or slope of DE is negative.
(ii) Region BC obeys Ohm's law because current varies linearly with the voltage.

5. Capacitor reactance is meaningful for an a.c. electrical circuit. It is the resistance offered by a capacitor when it is connected to an electrical circuit.

Mathematically,

$$X_C = \frac{1}{2\pi\nu C}$$

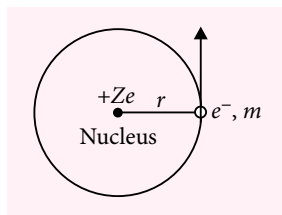
Where ν = frequency of a.c. source

C = capacitance of the capacitor.

Ohm (Ω) is the SI unit of capacitor reactance.

6. According to Bohr's theory, a hydrogen atom consists of a nucleus with a positive charge Ze , and a single electron of charge $-e$, which revolves around it in a circular orbit of radius r .

Here Z is the atomic number and for hydrogen $Z = 1$. The electrostatic force of attraction between the hydrogen nucleus and the electron is



$$F = \frac{k e \cdot e}{r^2} = \frac{k e^2}{r^2}$$

To keep the electron in its orbit, the centripetal force on the electron must be equal to the electrostatic attraction. Therefore,

$$\frac{mv^2}{r} = \frac{ke^2}{r^2}$$

$$\text{or } mv^2 = \frac{ke^2}{r} \quad \dots(i)$$

$$\text{or } r = \frac{ke^2}{mv^2} \quad \dots(ii)$$

where m is the mass of the electron, and v , its speed in an orbit of radius r .

Bohr's quantisation condition for angular momentum is

$$L = mvr = \frac{nh}{2\pi} \quad \text{or} \quad r = \frac{nh}{2\pi mv} \quad \dots(iii)$$

From equation (ii) and (iii), we get

$$\frac{ke^2}{mv^2} = \frac{nh}{2\pi mv} \quad \text{or} \quad v = \frac{2\pi ke^2}{nh} \quad \dots(iv)$$

Substituting this value of v in equation (iii), we get

$$r = \frac{nh}{2\pi m} \cdot \frac{nh}{2\pi ke^2}; \Rightarrow r = \frac{n^2 h^2}{4\pi^2 m k e^2} \quad \therefore r \propto n^2$$

7.

	Intrinsic Semiconductors	Extrinsic Semiconductors
1.	These are pure semiconducting tetravalent crystals.	These are semi-conducting tetravalent crystals doped with impurity atoms of group III or V.
2.	Their electrical conductivity is low.	Their electrical conductivity is high.
3.	There is no permitted energy state between valence and conduction bands.	There is permitted energy state of the impurity atom between valence and conduction bands.

8. From mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

Now for a concave mirror, $f < 0$ and for an object on the left of the mirror, $u < 0$

$$\therefore 2f < u < f \quad \text{or} \quad \frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$$

$$\text{or} \quad -\frac{1}{2f} < -\frac{1}{u} < -\frac{1}{f}$$

$$\text{or} \quad \frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < \frac{1}{f} - \frac{1}{f} \quad \text{or} \quad \frac{1}{2f} < \frac{1}{v} < 0$$

This implies that $v < 0$ so that image is formed on left. Also the above inequality implies.

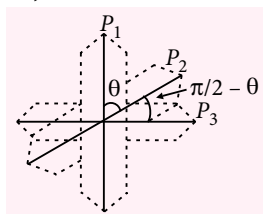
$$2f > v$$

or $|2f| < |v|$ $[\because 2f \text{ and } v \text{ are negative}]$
i.e., the real image is formed beyond $2f$.

OR

Let I_0 be the intensity of polarised light after passing through the first polariser P_1 . Then the intensity of light after passing through second polarised P_2 will be

$$I = I_0 \cos^2 \theta,$$



Let P_2 be the polaroid sheet rotated between P_1 and P_3 . Let I_0 be intensity of unpolarised light incident on polaroid P_1 the outgoing intensity will after P_1 will be $I_1 = I_0/2$

Let at a moment angle between polarises P_1 and P_2 is θ . The outgoing intensity will be

$$I_2 = I_1 \cos^2 \theta = \frac{I_0}{2} \cos^2 \theta$$

As the angle between polariser P_1 and P_3 is $\pi/2$ and angle between P_1 and P_2 is θ . So the angle between P_2 and P_3 is $(\pi/2 - \theta)$.

Outcoming intensity after P_3 is

$$I_3 = I_2 \cos^2 (\pi/2 - \theta)$$

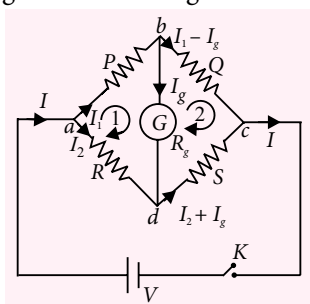
$$I_3 = \frac{I_0}{2} \cos^2 \theta \sin^2 \theta$$

$$\text{or } I_3 = \frac{I_0}{2} \left[\frac{1}{2} \sin 2\theta \right]^2$$

Maximum outcoming intensity is received, when $\theta = \pi/4$

$$I_3 = \frac{I_0}{2} \left[\frac{1}{2} \right]^2 = \frac{I_0}{8}$$

9. Let four resistances P , Q , R and S are arranged to form a bridge as shown in figure.



Applying Kirchhoff's law in loop-1, $abda$

$$I_1 P + I_g R_g - I_2 R = 0 \quad \dots(i)$$

Applying again Kirchhoff's law in loop-2, $bcd b$

$$(I_1 - I_g) Q - (I_2 + I_g) S - I_g R_g = 0 \quad \dots(ii)$$

Wheatstone bridge is said to be in balanced condition if electric current I_g flowing through

the galvanometer becomes zero. So in balanced Wheatstone bridge, $I_g = 0$ and equations (i) and (ii) become

$$I_1 P - I_2 R = 0 \text{ or } I_1 P = I_2 R \quad \dots(iii)$$

$$I_1 Q - I_2 S = 0 \text{ or } I_1 Q = I_2 S \quad \dots(iv)$$

Dividing equation (iii) by (iv), we get

$$\frac{P}{Q} = \frac{R}{S}$$

This gives us equation of balanced Wheatstone bridge.

10. de Broglie wavelength of a particle of mass m and charge q accelerating through a potential V is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}} \quad \dots(i)$$

- (i) Here, $m_p = m$, $q_p = e$, $m_\alpha = 4m_p = 4m$, $q_\alpha = 2q_p = 2e$
From eqn. (i)

$$\therefore \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}}$$

$$1 = \sqrt{\frac{4m \times 2e \times V_\alpha}{m \times e \times V_p}} \quad (\because \lambda_p = \lambda_\alpha)$$

$$\therefore \frac{V_p}{V_\alpha} = \frac{8}{1}; V_p : V_\alpha = 8 : 1$$

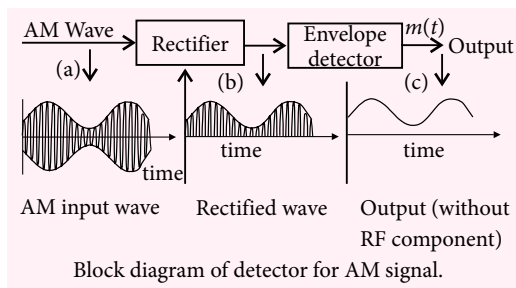
- (ii) Again from eqn. (i)

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\therefore \frac{\lambda_p}{\lambda_\alpha} = \frac{m_\alpha v_\alpha}{m_p v_p}; 1 = \frac{4mv_\alpha}{mv_p}$$

$$\frac{v_p}{v_\alpha} = 4; v_p : v_\alpha = 4 : 1$$

11. Detection is the process of recovering the modulating signal from the modulated carrier wave. The modulated carrier wave contains the frequencies ω_c , $\omega_c - \omega_m$ and $\omega_c + \omega_m$. The original message signal $m(t)$ of angular frequency ω_m can be obtained from AM signal by using a simple method shown in the form of the block diagram.

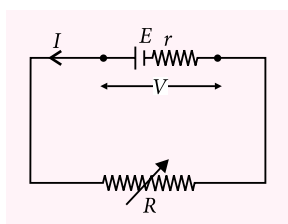


The modulated signal of the form given in (a) of above figure is passed through a rectifier. The rectifier conducts during the positive half cycles only. Hence, the output signal shown in (b), is still amplitude modulated, but consists of positive half cycles only.

In order to retrieve the original message signal $m(t)$, the signal is passed through an envelope detector (which may consist of a simple RC circuit). The output of envelope detector is then the original message signal $m(t)$ as shown in (c) of above figure.

12. Given situation is shown in figure

$$I = \frac{E}{r+R}$$

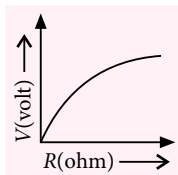


Terminal voltage,

$$V = E - Ir$$

- (i) V versus R ,

$$V = E - Ir = E - \frac{E}{r+R}r = \frac{ER}{r+R}$$



- (ii) V versus I ,

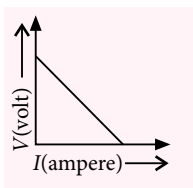
$$V = E - Ir$$

When $R = 4 \Omega$, then $I_1 = 1 \text{ A}$

$$\therefore 1 = \frac{E}{r+4}; r+4 = E \quad \dots(i)$$

When $R = 9 \Omega$, then $I = 0.5 \text{ A} = \frac{1}{2} \text{ A}$

$$\therefore \frac{1}{2} = \frac{E}{r+9} = \frac{r+4}{r+9}$$



[Using eqn. (i)]

$$r+9 = 2r+8, r = 1 \Omega$$

From eqn. (i)

$$\text{emf, } E = 1 + 4 = 5 \text{ V}$$

13. When two capacitors C_1 and C_2 are in parallel,

Equivalent capacitance, $C_p = C_1 + C_2$

$$\text{Energy stored, } U_p = \frac{1}{2} C_p V^2 = \frac{1}{2} (C_1 + C_2) V^2$$

Here, $U_p = 0.25 \text{ J}$, $V = 100 \text{ V}$

$$C_1 + C_2 = \frac{2U_p}{V^2} = \frac{2 \times 0.25}{(100)^2}$$

$$\therefore C_1 + C_2 = 5 \times 10^{-5} \quad \dots(i)$$

When C_1 and C_2 are connected in series

$$\text{Equivalent capacitance, } C_s = \frac{C_1 C_2}{C_1 + C_2}$$

$$\text{Energy stored, } U_s = \frac{1}{2} C_s V^2 = \frac{1}{2} \left(\frac{C_1 C_2}{C_1 + C_2} \right) V^2$$

Here, $U_s = 0.045 \text{ J}$

$$\therefore C_1 C_2 = \frac{2U_s (C_1 + C_2)}{V^2}$$

$$= \frac{2 \times 0.045 \times 5 \times 10^{-5}}{10^4} = 4.5 \times 10^{-10}$$

$$C_1 - C_2 = \sqrt{(C_1 + C_2)^2 - 4C_1 C_2}$$

$$= \sqrt{(5 \times 10^{-5})^2 - 4 \times 4.5 \times 10^{-10}}$$

$$C_1 - C_2 = 2.64 \times 10^{-5} \quad \dots(ii)$$

Solving eqn. (i) and (ii), we get

$$C_1 = 38.2 \mu\text{F}, C_2 = 11.8 \mu\text{F}$$

When capacitors are connected in parallel they have different amount of charge and given by

$$Q_1 = C_1 V = 38.2 \times 10^{-6} \times 100 = 38.2 \times 10^{-4} \text{ C}$$

$$Q_2 = C_2 V = 11.8 \times 10^{-6} \times 100 = 11.8 \times 10^{-4} \text{ C}$$

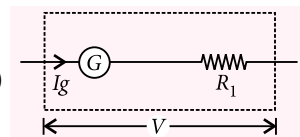
14. Principle of working of a moving coil galvanometer :

A current carrying coil placed in a magnetic field experiences a torque, the magnitude of which depends on the strength of current.

Galvanometer as a voltmeter :

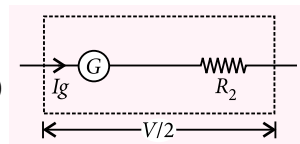
Case (i)

$$R_1 = \frac{V}{I_g} - G \quad \dots(i)$$



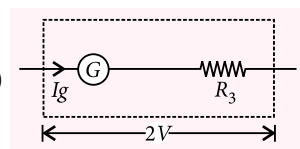
Case (ii)

$$R_2 = \frac{V}{2I_g} - G \quad \dots(ii)$$



Case (iii)

$$R_3 = \frac{2V}{I_g} - G \quad \dots(iii)$$



I_g = current through galvanometer which is fixed.

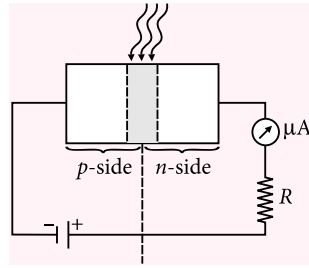
From eqn. (i) and (ii), we get

$$R_1 - R_2 = \frac{V}{2I_g}, G = R_1 - 2R_2$$

Put these values in eqn. (iii), we get

$$R_3 = 4(R_1 - R_2) - (R_1 - 2R_2) = 3R_1 - 2R_2$$

15. The photodiode is special-purpose diode. It is fabricated with a transport window to expose its junction to light radiations. It works in reverse bias condition below the breakdown voltage.



When light of frequency ν is incident on the junction, such that the energy of its photons is greater than the band gap of the semiconductor (i.e., $h\nu > E_g$), additional electron-hole pairs are created due to the conduction band. The photogenerated charge carriers increase the conductivity of the semiconductor. Larger the intensity of incident light, larger would be the increase in the conductivity of semiconductor.

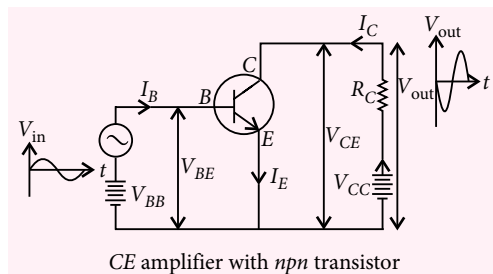
Consider the case of an n -type semiconductor. Obviously, the majority carrier density (n) is considerably larger than the minority hole density p (i.e., $n \gg p$). On illumination, let the excess electrons and holes generated be Δn and Δp , respectively:

$$n' = n + \Delta n$$

$$p' = p + \Delta p.$$

Here n' and p' are the electron and hole concentrations at any particular illumination and n and p are carrier concentrations when there is no illumination. Remember $\Delta n = \Delta p$ and $n \gg p$. Hence, the fractional change in the majority carriers (i.e., $\Delta n/n$) would be much less than that in the minority carriers (i.e., $\Delta p/p$). In general, we can state that the fractional change due to the photo-effects on the minority carrier dominated reverse bias current is more easily measurable than the fractional change in the forward bias current. Hence, photodiodes are preferably used in the reverse bias condition for measuring light intensity.

16.

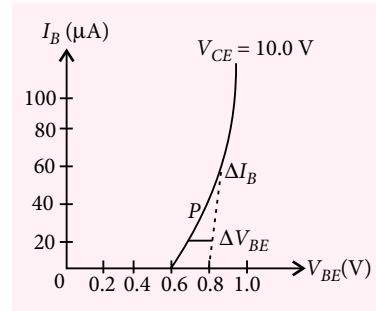


Input resistance (r_i): This is defined as the ratio of change in base-emitter voltage (ΔV_{BE}) to the resulting change in base current (ΔI_B) at constant collector-emitter voltage (V_{CE}).

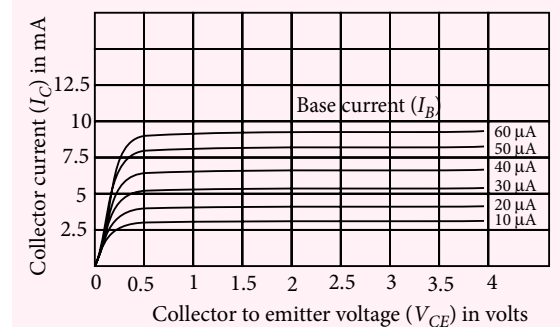
$$r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B} \right)_{V_{CE}}$$

Current amplification factor (β): This is defined as the ratio of the change in collector current to the change in base current at a constant collector-emitter voltage (V_{CE}) when the transistor is in active state.

$$\beta_{ac} = \left(\frac{\Delta I_C}{\Delta I_B} \right)_{V_{CE}}$$



To find the input resistance from input characteristic, mark a point P , draw a tangent on this point. The reciprocal of the slope of the tangent gives input resistance.



From output characteristic, find the ratio of change in collector current for the corresponding change in the base current. It gives the current amplification factor.

17. (a) : Angular width, $\theta = \frac{\lambda}{d}$ or $d = \frac{\lambda}{\theta}$

Here, $\lambda = 600 \text{ nm} = 6 \times 10^{-7} \text{ m}$

$$\theta = 0.1^\circ = \frac{0.1 \times \pi}{180} \text{ rad} = \frac{\pi}{1800} \text{ rad}$$

$$d = ?$$

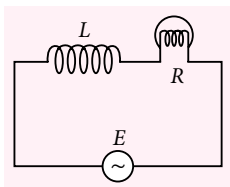
$$\therefore d = \frac{6 \times 10^{-7} \times 1800}{\pi} = 3.44 \times 10^{-4} \text{ m}$$

- (b) : Frequency of a light depends on its source only. So, the frequencies of reflected and refracted light will be same as that of incident light. Reflected light is in the same medium (air) so its wavelength remains same as 500 \AA .

Wavelength of refracted light, $\lambda_r = \frac{\lambda}{\mu_w}$
 μ_w = refractive index of water.

So, wavelength of refracted wave will be decreased.

18. Inductive reactance, $X_L = \omega L$



Impedance of the circuit,

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{\omega^2 L^2 + R^2}$$

- (i) When the number of turns in an inductor coil decreases then its inductance L decreases. So, the net impedance of the circuit decreases and current through the bulb (circuit) increases. Hence brightness ($I^2 R$) of bulb increases.
- (ii) When an iron rod is inserted in the inductor, then its inductance L increases. So, Z will increase and current through the bulb will decrease. Hence, brightness of the bulb will decrease.
- (iii) A capacitor is connected in the series in the circuit, so its impedance,

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$Z = R \quad (\because X_L = X_C)$$

This is the case of resonance so maximum current will flow through the circuit. Hence brightness of the bulb will increase.

19. (a) : Microwave are suitable for radar systems used in aircraft navigation.

These wave are produced by special vacuum tubes, namely klystrons, magnetrons and Gunn diodes.

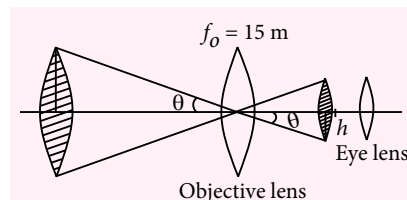
- (b) Infrared waves are used to treat muscular pain. These waves are produced by hot bodies and molecules.

- (c) X-ray are used as a diagnostic tool in medicine. These are produced when high energy electrons are stopped suddenly on a metal of high atomic number.

20. (i) : Here, $f_o = 15 \text{ m} = 1500 \text{ cm}$ and $f_e = 1.0 \text{ cm}$
 angular magnification by the telescope in normal adjustment

$$m = \frac{f_o}{f_e} = \frac{1500 \text{ cm}}{1.0 \text{ cm}} = 1500$$

- (ii) The image of the moon by the objective lens is formed on its focus only as the moon is nearly infinite distance as compared to focal length.



Height of object

$$\text{i.e., Radius of moon } R_m = \frac{3.48}{2} \times 10^6 \text{ m}$$

$$R_m = 1.74 \times 10^6 \text{ m}$$

Distance of object = Radius of lunar orbit

$$R_o = 3.8 \times 10^8 \text{ cm}$$

Distance of image for objective lens is the focal length of objective lens, $f_o = 15 \text{ m}$

Radius of image of moon by objective lens can be calculated.

$$\tan \theta = \frac{R_m}{R_o} = \frac{h}{f_o}$$

$$h = \frac{R_m \times f_o}{R_o} = \frac{1.74 \times 10^6 \times 15}{3.8 \times 10^8}$$

$$h = 6.87 \times 10^{-2} \text{ m}$$

Diameter of the image of moon,

$$D_I = 2h = 13.74 \times 10^{-2} \text{ m} = 13.74 \text{ cm}$$

21. Einstein's photoelectric equation

$$K_{\max} = \frac{1}{2} mv^2 = h\nu - \phi_0 = h\nu - h\nu_0 \quad \dots(i)$$

Here, K_{\max} = Maximum Kinetic energy of photoelectron with speed v

ν = frequency of incident light

$\phi_0 (= h\nu_0)$ = work function of the metal

Important features of photoelectric equation.

- (i) K_{\max} depends linearly on ν and is independent of intensity of incident light. This happen due quantum nature of light.

- (ii) Since K_{\max} must be positive, equation implies that photoelectric emission is possible only if $h\nu > \phi_0 = h\nu_0$ or $\nu > \nu_0$

Thus there exists a threshold frequency $\nu_0 (= \phi_0/h)$ for the metal surface, below which no photoelectric emissions possible.

From eqn. (i)

$$K_{\max} = \frac{hc}{\lambda} - \phi_0$$

According to question,

$$K_{\max} = \frac{hc}{\lambda_1} - \phi_0 \quad \dots(ii)$$

$$2K_{\max} = \frac{hc}{\lambda_2} - \phi_0 \quad \dots(iii)$$

From eqn. (ii) and (iii),

$$2\left(\frac{hc}{\lambda_1} - \phi_0\right) = \frac{hc}{\lambda_2} - \phi_0$$

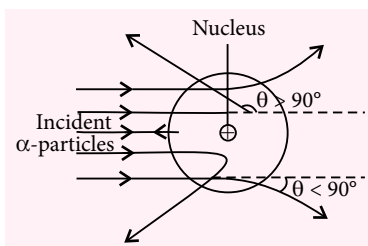
$$\phi_0 = \frac{2hc}{\lambda_1} - \frac{hc}{\lambda_2} = hc\left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

Also, $\phi_0 = \frac{hc}{\lambda_0} \quad \therefore \frac{hc}{\lambda_0} = hc\left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right)$

or $\frac{1}{\lambda_0} = \frac{2\lambda_2 - \lambda_1}{\lambda_1\lambda_2}; \lambda_0 = \frac{\lambda_1\lambda_2}{2\lambda_2 - \lambda_1}$

22. Trajectory of α particles in coulomb field of target nucleus.

Only a small fraction of the number of incident α -particles (1 in 8000) rebound back.



This shows that the number of α -particles undergoing head-on collision is small. This implies that the entire positive charge of the atom is concentrated in a small volume.

So, this experiment is an important way to determine an upper limit on the size of nucleus.

$$\text{Density of nucleus} = \frac{\text{mass of nucleus}}{\text{volume}}$$

$$\rho = \frac{A \times 1 \text{ amu}}{\frac{4}{3} \pi R^3}$$

where $R = R_0 A^{1/3}$

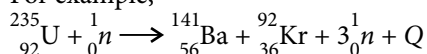
$$\text{Density } \rho = \frac{A \times 1 \text{ amu}}{\frac{4}{3} \pi R_0^3 A} = \frac{1 \text{ amu}}{\frac{4}{3} \pi R_0^3}$$

$$\rho = 2.97 \times 10^{17} \text{ kg m}^{-3}$$

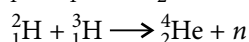
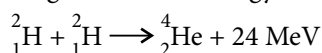
so, nuclear density is constant irrespective of mass number or size.

OR

Nuclear fission : It is the process in which a heavy nucleus ($A > 230$) when excited gets split up into two smaller nuclei of nearly comparable masses. For example,



Nuclear fusion : It is the process of fusion of two smaller nuclei into a heavier nucleus with the liberation of large amount of energy. For example,



Mass defect,

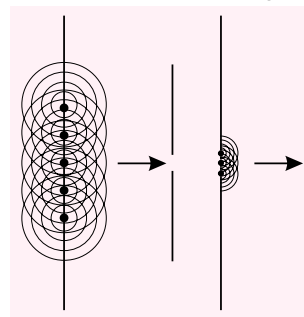
$$\Delta m = (2.014102 + 3.016049) - (4.002603 + 1.008665) = 0.018883$$

Energy released,

$$Q = 0.018883 \times 931.5 \text{ MeV} = 17.589 \text{ MeV}$$

- 23. (i) :** Transformer is a device which is used to bring the high voltage down to low voltage of a.c. current. It works on the principle of mutual induction of two coils in a transformer.
- (ii) Transformer does not work for dc voltage. A dc current gives constant magnetic field and constant magnetic flux through the coil of fixed area of cross-section. As there is no change in magnetic flux so there is no induced emf in the coil.
- (iii) The value displayed by the students are gaining knowledge and curiosity to learn new things. The values displayed by the teacher are providing good education and helpful.

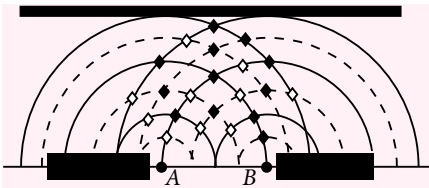
- 24. (a)** Waves diffract when they encounter obstacles. Applying Huygens principle it becomes clear. A wavefront impinging on a barrier with a slit in it, only the points on the wavefront that move into the slit can continue emitting forward moving waves - but because a lot of the wavefront has been blocked by the barrier, the points on the edges of the hole emit waves that bend round the edges.



Before the wavefront strikes the barrier the wavefront generates another forward moving wavefront. Once the barrier blocks most of the wavefront the forward moving wavefront bends around the slit because the secondary waves they would need to interfere with to create a straight wavefront have been blocked by the barrier.

Each point on the wavefront moving through the slit acts like a point source. We can think about some of the effect of this if we analyse what happens when two point sources are close together and emit wavefronts with the same wavelength and frequency. These two point sources represent the point sources on the two edges of the slit and we can call the source A and source B as shown in the figure.

Each point source emits wavefronts from the edge of the slit. In the diagram we show a series of wavefronts emitted from each point source. The continuous lines show peaks in the waves emitted by the point sources and the dotted lines represent troughs. We label the places where constructive interference (peak meets a peak or trough meets a trough) takes place with a solid diamond and places where destructive interference (trough meets a peak) takes place with a hollow diamond. When the wavefronts hit a barrier there will be places on the barrier where constructive interference takes place and places where destructive interference happens.



The measurable effect of the constructive or destructive interference at a barrier depends on what type of waves we are dealing with.

- (b) Refer point 6.14 (4 (ii)), (MTG Excel in Physics)
 (c) On increasing the value of n , the part of slit contributing to the maximum decreases. Hence, the maximum becomes weaker.

OR

- (a) Refer point 6.5 (5 (ii) and (iii)),
 (MTG Excel in Physics)
 (b) Refer point 6.6 (1), (MTG Excel in Physics)

25. (a) Refer point 1.4 (3), (MTG Excel in Physics)

(b) $\vec{E} = 2x\hat{i}$

So flux passes through faces of cube which are perpendicular to x -axis.

The magnitude of electric field at the left face ($x = 0$),
 $E_L = 0$

The magnitude of electric field at the right face, ($x = a$), $E_R = 2a$

So, net flux, $\phi = \vec{E} \cdot \Delta\vec{s}$

$$= E_L \Delta s \cos 180^\circ + E_R \Delta s \cos 0^\circ$$

$$= 0 + 2a \times a^2 = 2a^3$$

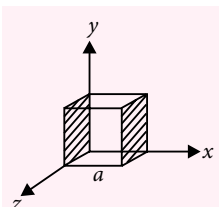
Assume enclosed charge is q .

Use Gauss's law, $\phi = \frac{q}{\epsilon_0}$; $q = \epsilon_0 \phi$

$$\therefore q = 2a^3 \epsilon_0$$

OR

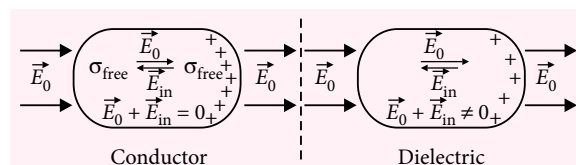
- (a) When a conductor is placed in an external electric field, the free charges present inside the conductor redistribute themselves in such



a manner that the electric field due to induced charges opposes the external field within the conductor. This happens until a static situation is achieved i.e., when the two fields cancel each other and the net electrostatic field in the conductor becomes zero.

Dielectrics are non-conducting substances i.e., they have no charge carriers. Thus, in a dielectric, free movement of charges is not possible. It turns out that the external field induces dipole moment by reorienting molecules of the dielectric. The collective effect of all the molecular dipole moments is the net charge on the surface of the dielectric which produce a field that opposes the external field, unlike a conductor in an external electric field. However, the opposing field so induced does not exactly cancel the external field. It only reduces it. The extent of the effect depends on the nature of the dielectric.

The effect of electric field on a conductor and a dielectric is shown below :

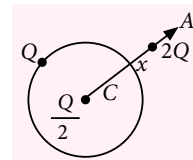


The dipole moment per unit volume is called polarisation and is denoted by P . For linear isotropic dielectrics, $P = \chi E$ where χ is electric susceptibility of the dielectric medium.

- (b) The electric field inside a spherical conducting shell is zero.

So force experienced by the charge $\frac{Q}{2}$ at the point C.

$$F = \frac{Q}{2} E = Q \times 0 = 0$$



Force experienced by charge $2Q$ at point A,

$$F_A = 2Q E_A = 2Q \left(\frac{Q}{4\pi\epsilon_0 x^2} \right)$$

26. (a) Refer point 3.2 ((1), (4)), (MTG Excel in Physics)

OR

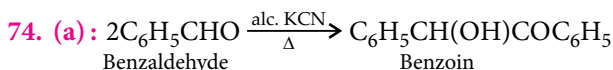
- (a) Refer point 4.2 (2(i)), (MTG Excel in Physics)
 (b) Refer point 4.2 (2(vi)), (MTG Excel in Physics)
 (c) Refer point 4.2 (2(ii)), (MTG Excel in Physics)



Since 1.89 g of acid liberates 11946.14 cal of heat, therefore heat liberated by 122 g (mol. wt. of benzoic acid) of acid

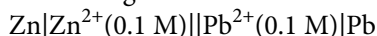
$$= \frac{11946.14 \times 122}{1.89} = 771126.5 \text{ cal} = 771.12 \text{ kcal}$$

73. (c)



The reaction is known as benzoin condensation.

75. (a): The cell is given as



$$\therefore E_{\text{cell}}^{\circ} = E_{\text{right}}^{\circ} - E_{\text{left}}^{\circ} \\ = -0.126 - (-0.763) \text{ V} = 0.637 \text{ V}$$

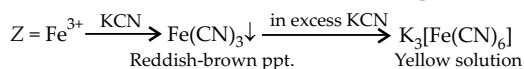
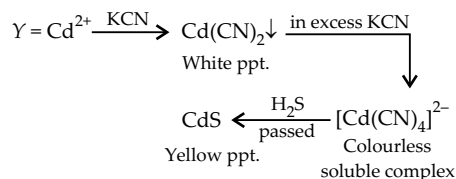
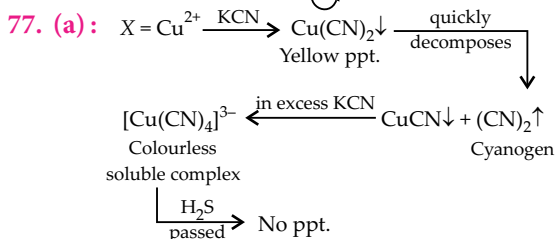
Now, using Nernst equation

$$E_{\text{cell}} = E_{\text{cell}}^{\circ} - \frac{0.059 \text{ V}}{2} \log \frac{[\text{Pb}^{2+}]}{[\text{Zn}^{2+}]}$$

$$E_{\text{cell}} = 0.637 \text{ V} - \frac{0.059 \text{ V}}{2} \log \frac{[0.1]}{[0.1]}$$

$$\therefore E_{\text{cell}} = 0.637 \text{ V}$$

76. (b): Compound which can stabilise the charge after removal of proton, will be more acidic.



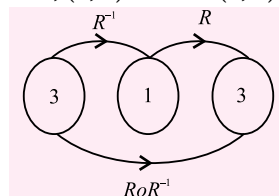
78. (a): 3^{rd} period elements have more ΔH_{eg} than 2^{nd} period elements and in a period it increases.

79. (c) 80. (a)

81. (d): $\sin^2 x + \sin^2 y - \sin^2 z$
 $= \sin^2 x + \sin(y+z) \sin(y-z)$
 $= \sin^2 x + \sin(y+z) \sin(\pi - x)$
 $= \sin x [\sin x + \sin(y+z)]$
 $= \sin x [\sin(\pi + z - y) + \sin(y+z)]$
 $= \sin x [\sin(y+z) - \sin(z-y)] = 2 \sin x \cos z \sin y$

82. (c): Let $R = \{(x, y) : x + 2y = 8, x, y \in \mathbb{N}\}$
 $\therefore x + 2y = 8$ (which must be a natural number)
 $\Rightarrow y = \frac{8-x}{2} \therefore x = \{2, 4, 6\} \therefore y = \{3, 2, 1\}$
 $\therefore R = \{(x, y) : x + 2y = 8\}$
 $\Rightarrow R = \{(2, 3), (4, 2), (6, 1)\} \therefore \text{Range of } R = \{1, 2, 3\}$

83. (b): $A = \{1, 2, 3, 4\}, B = \{1, 3, 5\}$
 $R = \{(a, b) : a < b, a \in A, b \in B\}$
 $= \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$
 $\therefore R^{-1} = \{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$
 Now $(3, 1) \in R^{-1}, (1, 3) \in R \therefore (3, 3) \in \text{RoR}^{-1}$.



$$\begin{aligned} (3, 1) \in R^{-1}, (1, 5) \in R &\Rightarrow (3, 5) \in \text{RoR}^{-1} \\ (5, 1) \in R^{-1}, (1, 3) \in R &\Rightarrow (5, 3) \in \text{RoR}^{-1} \\ (3, 2) \in R^{-1}, (2, 3) \in R &\Rightarrow (3, 3) \in \text{RoR}^{-1} \\ (3, 2) \in R^{-1}, (2, 5) \in R &\Rightarrow (3, 5) \in \text{RoR}^{-1} \\ (5, 2) \in R^{-1}, (2, 3) \in R &\Rightarrow (5, 3) \in \text{RoR}^{-1} \\ (5, 2) \in R^{-1}, (2, 5) \in R &\Rightarrow (5, 5) \in \text{RoR}^{-1} \\ (5, 3) \in R^{-1}, (3, 5) \in R &\Rightarrow (5, 5) \in \text{RoR}^{-1} \\ (5, 4) \in R^{-1}, (4, 5) \in R &\Rightarrow (5, 5) \in \text{RoR}^{-1} \\ \therefore \text{RoR}^{-1} &= \{(3, 3), (3, 5), (5, 3), (5, 5)\}. \end{aligned}$$

84. (c): Making the equation of the curve homogeneous with the help of the line, we get

$$x^2 + y^2 - 2(kx + hy) \left(\frac{hx + ky}{2hk} \right) + (h^2 + k^2 - c^2) \left(\frac{hx + ky}{2hk} \right)^2 = 0$$

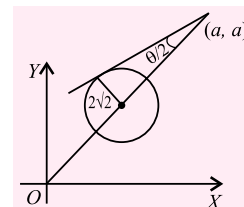
or $4h^2k^2x^2 + 4h^2k^2y^2 - 4hk^2x(hx + ky) - 4h^2ky(hx + ky) + (h^2 + k^2 - c^2)(h^2x^2 + k^2y^2 + 2hkxy) = 0$
 This is the equation of the pair of lines joining the origin to the points of intersection of the given line and the curve. They will be at right angles if coefficient of x^2 + coefficient of y^2 = 0.
 $(h^2 + k^2)(h^2 + k^2 - c^2) = 0$ since $[h^2 + k^2 \neq 0]$
 $\Rightarrow h^2 + k^2 = c^2$.

85. (d): The centre of the circle is (1, 1) and radius = $2\sqrt{2}$.

From (a, a) must lie outside the circle, so

$$2a^2 - 4a - 6 > 0 \\ \Rightarrow a < -1 \text{ or } a > 3$$

Now, $\tan \frac{\theta}{2} = \frac{2\sqrt{2}}{\sqrt{2a^2 - 4a - 6}}$.



$$\text{As } \frac{\pi}{3} < \theta < \pi \Rightarrow \frac{\pi}{6} < \frac{\theta}{2} < \frac{\pi}{2}$$

$$\therefore \frac{2\sqrt{2}}{\sqrt{2a^2 - 4a - 6}} > \frac{1}{\sqrt{3}} \Rightarrow \sqrt{a^2 - 2a - 3} < 2\sqrt{3}$$

$$\therefore a^2 - 2a - 15 < 0 \Rightarrow -3 < a < 5$$

$$\therefore a \in (-3, -1) \cup (3, 5).$$

$$\begin{aligned} 86. (b): \beta &= \frac{\sin \theta \cos \theta}{\cos 3\theta \cos \theta} + \frac{\sin 3\theta \cos 3\theta}{\cos 9\theta \cos 3\theta} + \frac{\sin 9\theta \cos 9\theta}{\cos 27\theta \cos 9\theta} \\ \therefore 2\beta &= \frac{\sin 2\theta}{\cos 3\theta \cos \theta} + \frac{\sin 6\theta}{\cos 9\theta \cos 3\theta} + \frac{\sin 18\theta}{\cos 27\theta \cos 9\theta} \\ &= \frac{\sin(3\theta - \theta)}{\cos 3\theta \cos \theta} + \frac{\sin(9\theta - 3\theta)}{\cos 9\theta \cos 3\theta} + \frac{\sin(27\theta - 9\theta)}{\cos 27\theta \cos 9\theta} \\ &= \tan 3\theta - \tan \theta + (\tan 9\theta - \tan 3\theta) + \tan 27\theta - \tan 9\theta \\ &= \tan 27\theta - \tan \theta = \alpha \quad \therefore \alpha = 2\beta. \end{aligned}$$

$$\begin{aligned} 87. (a): f(x) &= \cos x \cos(x+2) - \cos^2(x+1) \\ &= \cos(x+1-1)\cos(x+1+1) - \cos^2(x+1) \\ &= \cos^2(x+1) - \sin^2 1 - \cos^2(x+1) \text{ by using } \cos^2 A - \sin^2 B = \cos(A+B) \cdot \cos(A-B) \\ f(x) &= -\sin^2 1 \text{ (constant)} \\ \Rightarrow x &= \frac{\pi}{2} \Rightarrow f(x) = -\sin^2 1 \\ \Rightarrow f(x) &\text{ represents a straight line through } \left(\frac{\pi}{2}, -\sin^2 1\right) \text{ which is parallel to } x\text{-axis as } f(x) = -\sin^2 1 \text{ is a constant.} \end{aligned}$$

88. (d): Each coupon can be selected in 15 ways. The total number of ways of choosing 7 coupons is 15^7 . If largest number is 9, then the selected numbers have to be from 1 to 9 excluding those consisting of only 1 to 8.

$$\text{Desired probability is } \frac{9^7 - 8^7}{15^7} = \left(\frac{3}{5}\right)^7 = \left(\frac{8}{15}\right)^7$$

$$89. (c): \text{The total numbers of arrangements is } \frac{11!}{2!2!2!} = \frac{11!}{8}$$

The number of arrangements in which C, E, H, I, S appear in that order = $\binom{11}{5} \frac{6!}{2!2!2!} = \frac{11!}{8 \cdot 5!}$

$$\therefore \text{Required Probability} = \frac{11!}{8 \cdot 5!} \div \frac{11!}{8} = \frac{1}{5!} = \frac{1}{120}$$

$$90. (c): \sum_{i=1}^{30} n(A_i) = 5 \times 30 = 150$$

Suppose S has m elements
 $\Rightarrow 150 = 10m \Rightarrow m = 15$

$$\sum_{i=1}^n n(B_i) = 3n = 9m \Rightarrow n = 3m = 45$$

$$\begin{aligned} 91. (d): {}^{50}C_6 - {}^5C_1 {}^{40}C_6 + {}^5C_2 {}^{30}C_6 - {}^5C_3 {}^{20}C_6 + {}^5C_4 {}^{10}C_6 \\ = \text{coefficient of } x^6 \text{ in } [{}^5C_0 (1+x)^{50} - {}^5C_1 (1+x)^{40} \\ + {}^5C_2 (1+x)^{30} - {}^5C_3 (1+x)^{20} + {}^5C_4 (1+x)^{10} - {}^5C_5 (1+x)^0] \\ = \text{coefficient of } x^6 \text{ in } [(1+x)^{10} - 1]^5 \\ = {}^5C_1 \cdot ({}^{10}C_1)^4 ({}^{10}C_2) = 2250000 \end{aligned}$$

$$92. (b): y = \frac{3}{x^2 + 1} \Rightarrow x^2 = \frac{3-y}{y} \quad \dots(i)$$

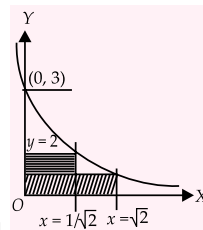
When $x = 0, y = 3; x = \infty, y = 0$

$$x^2 = 2 \Rightarrow x = \sqrt{2}$$

$$2(x^2 + 1) = 3 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \frac{1}{\sqrt{2}}$$

$$\text{Area} = \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right) \cdot 1 + \left(\frac{1}{\sqrt{2}} - 0\right) \cdot 2$$

$$\Rightarrow \text{Definite integral} = \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$



93. (c)

$$\begin{aligned} 94. (a): f(x+4) &= f(x+2) - f(x) \\ \Rightarrow f(x+6) &= f(x+4) - f(x+2) \\ &= f(x+2) - f(x) - f(x+2) = -f(x) \\ f(x+12) &= f(x) \end{aligned}$$

$$\int_{\lambda}^{\lambda+12} f(x) dx = \int_0^{12} f(x) dx$$

$$95. (b): \int_{-1}^1 \frac{\sin x dt}{\sin^2 x + (t - \cos x)^2}$$

$$= \frac{\sin x}{\sin x} \tan^{-1} \left(\frac{t - \cos x}{\sin x} \right) \Big|_{-1}^1$$

$$= \tan^{-1} \left(\tan \frac{x}{2} \right) + \tan^{-1} \left(\cot \frac{x}{2} \right)$$

$$0 < x < \pi \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{2}$$

$$\tan^{-1} \left(\tan \frac{x}{2} \right) + \tan^{-1} \left(\cot \frac{x}{2} \right) = \frac{x}{2} + \frac{\pi}{2} - \frac{x}{2} = \frac{\pi}{2}$$

$$\pi < x < 2\pi \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \pi$$

$$\tan^{-1} \left(\tan \frac{x}{2} \right) + \tan^{-1} \left(\cot \frac{x}{2} \right) = \frac{x}{2} - \pi + \frac{\pi}{2} - \frac{x}{2} = -\frac{\pi}{2}$$

$$\text{The range is } \left\{ -\frac{\pi}{2}, \frac{\pi}{2} \right\}$$

$$96. (c): f(x) = \frac{1}{2} \int_0^x (x-t)^2 g(t) dt$$

$$f'(x) = \frac{1}{2} \int_0^x 2(x-t) g(t) dt; \quad f''(x) = \int_0^x g(t) dt$$

$$f'''(x) = g(x) \Rightarrow f'''(1) = 5$$

$$f''(1) = 2; \quad f'''(1) - f''(1) = 3$$

97. (b): $F'(x) = 4 \sin x + 3 \cos x > 0 \quad \forall x \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$\therefore F(x)$ is strictly increasing

$$\therefore F(x)_{\text{least}} = \int_{\pi/6}^{\pi/4} (4 \sin t + 3 \cos t) dt$$

$$= 3 \left[\frac{1}{\sqrt{2}} - \frac{1}{2} \right] - 4 \left[\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \right]$$

$$= 3 \left(\frac{\sqrt{2}-1}{2} \right) - 4 \left(\frac{\sqrt{2}-\sqrt{3}}{2} \right)$$

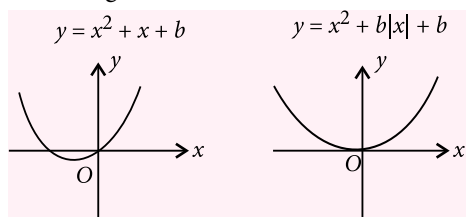
$$= \frac{3\sqrt{2}-3-4\sqrt{2}+4\sqrt{3}}{2} = \frac{4\sqrt{3}-\sqrt{2}-3}{2}$$

98. (d): $a = \frac{x^2+4}{|x|} - 3 = |x| + \frac{4}{|x|} - 3$

$$\geq 2\sqrt{|x| \cdot \frac{4}{|x|}} - 3 = 4 - 3 = 1$$

$$\Rightarrow a \geq 1$$

99. (a): The equation $x^2 + ax + b = 0$ has distinct real roots and $x^2 + a|x| + b = 0$ has only one real root. So one root of the equation $x^2 + ax + b = 0$ is zero and other root negative.



100. (a): $\det(\text{Adj}(\text{Adj} A)) = |A|^{n^2 - 2n + 1}$

$$n = 3, |A| = 14, \det(\text{Adj}(\text{Adj} A)) = 14^4$$

101. (a): $\begin{bmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \lambda \\ 10 \\ 6 \end{bmatrix}$

$$\begin{aligned} R_2 &\rightarrow R_2 - 5R_1 \\ R_3 &\rightarrow R_3 - 2R_1 \end{aligned} \left\{ \begin{bmatrix} 1 & 1 & 1 \\ 0 & -6 & \mu-5 \\ 0 & 0 & \mu-23 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \lambda \\ 10-5\lambda \\ 46-17\lambda \end{bmatrix} \right.$$

$$R_3 \rightarrow 6R_3$$

$$R_3 \rightarrow R_3 + R_2$$

$$\mu - 23 \neq 0, \quad \mu \neq 23$$

102. (a): $B = -A^{-1}BA$

$$AB = -A(A^{-1}BA) \Rightarrow AB = -(AA^{-1})(BA)$$

$$\Rightarrow AB = -IBA \quad \text{or} \quad AB + BA = 0$$

103. (b): $AA^T = I$

$$\Rightarrow a = -2, b = -1$$

104. (b): A is orthogonal $\Rightarrow |A| = \pm 1$

B is skew symmetric matrix of odd order $\Rightarrow |B| = 0$

$$\therefore |AB| = |A||B| \Rightarrow |AB| = 0$$

105. (d): $2 \log_{10} x - \log_x(10)^{-2} = 2 \log_{10} x + 2 \log_x 10$

$$= 2 \log_{10} x + \frac{2}{\log_{10} x} \geq 4$$

106. (c): $(a+b)^5 + (a-b)^5 = 2[a^5 + 10a^3b^2 + 5ab^4]$

$$[x + (x^3 - 1)^{1/2}]^5 + [x - (x^3 - 1)^{1/2}]^5$$

$$= 2[x^5 + 10x^3(x^3 - 1) + 5x(x^3 - 1)^2]$$

\therefore given expression is a polynomial of degree 7.

107. (b): Given :

$$f(x) = (a^2 - 3a + 2) \left(\cos^2 \frac{x}{4} - \sin^2 \frac{x}{4} \right) + (a-1)x + \sin 1$$

$$\Rightarrow f'(x) = -\frac{1}{2}(a-1)(a-2) \sin \frac{x}{2} + (a-1)$$

Now, $f'(x) = 0 \Rightarrow a = 1, \sin \frac{x}{2} = \frac{2}{a-2}$

But $\left| \sin \frac{x}{2} \right| \leq 1 \Rightarrow |a-2| \geq 2$

$$\Rightarrow a \in (-\infty, 0] \cup [4, \infty) \cup \{1\}$$

108. (d): $y^3 - 3y + x = 0$

$$x = -y^3 + 3y < -2 \Rightarrow y^3 - 3y - 2 > 0$$

$$\Rightarrow (y+1)^2(y-2) > 0 \Rightarrow y > 2$$

$$\frac{dy}{dx} = \frac{1}{3(1-y^2)} < 0$$

109. (d): $f(x) = \cot^{-1} x + \frac{1}{2} \ln x$

$$f'(x) = -\frac{1}{1+x^2} + \frac{1}{2x} = \frac{-2x+1+x^2}{2x(1+x^2)} = \frac{(1-x)^2}{2x(1+x^2)}$$

f is increasing $\Rightarrow f_{\max} = f(\sqrt{3}) = \frac{\pi}{6} + \frac{1}{4} \ln 3$

110. (c): $D_f = \{-3, -2, -1, 1, 2, 3\}$

$$\text{Range} = \left\{ \frac{1}{9}, \frac{5}{4}, 1 + \sqrt{2} \right\}$$

111. (a): D.E. is $\frac{dy}{y^2} - \frac{dx}{x^2} + y \left(\frac{1}{y} - \frac{1}{x} \right) dy = 0$

$$\Rightarrow \frac{dy}{y^2} - \frac{dx}{x^2} + y dy = 0 \Rightarrow -\ln \left| \frac{1}{y} - \frac{1}{x} \right| + \frac{y^2}{2} = C$$

$$\Rightarrow \ln \left| \frac{xy}{x-y} \right| + \frac{y^2}{2} = C$$

$$112. (a) : \int_0^1 (-x \ln x + 2x - 2x^2) dx$$

$$= \frac{1}{4} + 1 - \frac{2}{3} = \frac{3+12-8}{12} = \frac{7}{12}$$

$$113. (a) : \sin 51x (\sin x)^{49} = (\sin x \cdot \cos 50x) (\sin x)^{49}$$

$$+ \cos x \sin 50x (\sin x)^{49}$$

$$= \frac{d}{dx} ((\sin x)^{50} \sin 50x)$$

$$= \frac{50}{50}$$

114. (b) : It is obvious from the given functional relation that degree of polynomial is 1. Putting $f(x) = ax + b$
we get $a = 2, b = -3$ i.e. $f(x) = 2x - 3$

115. (a) : According to L.M.V.T,

$$\left| \frac{f'(x) - f'(0)}{x} \right| = |f''(C_1)| \leq 1 \Rightarrow |f'(x)| \leq |x|$$

$$\text{Again, } \left| \frac{f(x) - f(0)}{x} \right| = |f'(C_2)| \leq |x| \Rightarrow |f(x)| \leq |x^2|$$

116. (b) : $0 \leq \{x\} < 1$; $0 \leq \sin\{x\} < \sin 1$

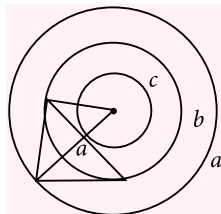
$$\frac{1}{\sin 1} < \frac{1}{\sin\{x\}} < \infty \text{ or } \left[\frac{1}{\sin\{x\}} \right] \in \{1, 2, 3, \dots\}$$

$$117. (a) : \frac{1}{2} a \sqrt{b^2 - c^2}$$

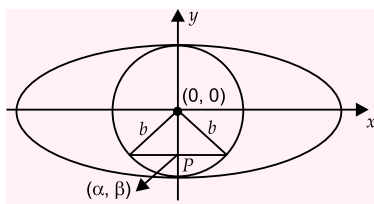
$$= \frac{1}{2} b \sqrt{a^2 - b^2}$$

$$\Rightarrow b = -\sqrt{ac}$$

\therefore A.M. \geq G.M.



$$118. (c) : \alpha^2 + \beta^2 = \frac{b^2}{2}$$



119. (c) : $\Sigma \alpha = -3, \Sigma \alpha\beta = -6, \alpha\beta\gamma = 8$

Since $\left(\frac{1}{\alpha}, \alpha\right), \left(\frac{1}{\beta}, \beta\right)$ and $\left(\frac{1}{\gamma}, \gamma\right)$ lie on the hyperbola $xy = 1$. Therefore, orthocentre will be $\left(-\alpha\beta\gamma, -\frac{1}{\alpha\beta\gamma}\right)$

120. (c)

121. (d) : Case I : solution : Let $z = x + iy$

$$\text{Then } z^2 + z + 1 = a \Rightarrow (x + iy)^2 + (x + iy) + 1 = a$$

$$\Rightarrow x^2 - y^2 + i(2xy + y) + x + 1 = a$$

$$\Rightarrow (x^2 - y^2 + x + 1) + iy(2x + 1) - a = 0$$

Now $y = 0$ or $x = -\frac{1}{2}$ for imaginary part to be zero

$$x^2 + x + 1 - a = 0 \Rightarrow D \geq 0 \Rightarrow 1 - 4(1 - a) \geq 0$$

$$\Rightarrow -3 + 4a \geq 0 \Rightarrow 4a \geq 3 \Rightarrow a \geq \frac{3}{4}$$

Similarly one can investigate the other possibility.

$$\text{Case II : } z^2 + z + 1 - a = 0$$

$$\Rightarrow z^2 + z + (1 - a) = 0$$

As imaginary part of z is non-zero, we must have

$$1 - 4(1 - a) < 0$$

$$\text{Then } 4a - 3 < 0 \Rightarrow a < \frac{3}{4}$$

Hence a cannot take the value $\frac{3}{4}$.

122. (b) 123. (b)

$$124. (b) : \lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{x^2 + x + 1 - ax^2 - ax - bx - b}{x + 1} = 4$$

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{(1 - a)x^2 + (1 - a - b)x + 1 - b}{x + 1} = 4$$

$$1 - a = 0 \therefore a = 1$$

$$1 - a - b = 4 \therefore b = -4$$

$$125. (b) : f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$\text{We have } f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6)$$

$$= 6(x - 3)(x - 2)$$

$$f(0) = 1$$

$$f(3) = 54 - 135 + 108 + 1 = 28$$

$$f(2) = 16 - 60 + 72 + 1 = 29$$

Then the range = (1, 29)

Hence the given function is onto. But it is not one-one, as f' takes both positive and negative values.

$$\frac{+}{2} \quad \frac{-}{3} \quad \frac{+}{+}$$

126. (a) 127. (d) 128. (b) 129. (b) 130. (d)

131. (b) 132. (a) 133. (a) 134. (c) 135. (d)

136. (d) 137. (b) 138. (d) 139. (a) 140. (d)

141. (a) 142. (a) 143. (d) 144. (b) 145. (b)

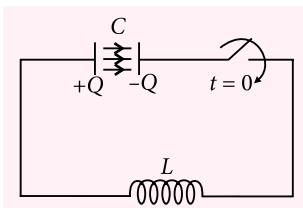
146. (d) 147. (b) 148. (b) 149. (c) 150. (d)



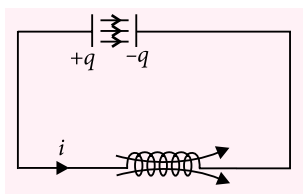
CORE CONCEPT on

LC Oscillations

A capacitor is charged and connected to an inductor at $t = 0$, as shown.



The entire system is lossless, since there are no resistors in the circuit. t seconds later, let us assume, the charge on the capacitor reduces to q and current through inductor is i .



The electric field energy gets converted to electric plus magnetic field energy.

The total energy at any instant can be written as

$$U_C + U_L = \frac{Q^2}{2C} = \text{constant}$$

where U_C = energy stored in capacitor in the form of electric field energy

U_L = energy stored in inductor in the form of magnetic field energy

$$\frac{q^2}{2C} + \frac{1}{2}Li^2 = \frac{Q^2}{2C}$$

Differentiating this equation with respect to time, we get

$$\begin{aligned} \frac{2q}{C} \frac{dq}{dt} + 2Li \frac{di}{dt} &= 0 \\ \Rightarrow \frac{q}{C} &= -L \frac{di}{dt} \quad \left[\because \frac{dq}{dt} = i \right] \\ \Rightarrow \frac{d^2q}{dt^2} &= -\left(\frac{1}{LC} \right) q \end{aligned}$$

Comparing this with a standard simple harmonic equation of a particle in simple harmonic motion,

$$\frac{d^2x}{dt^2} = -\omega^2 x,$$

we conclude that the charge on the capacitor oscillates simple harmonically. The angular frequency of oscillation of charge, hence becomes

$$\omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

Hence, the time period of oscillation of charge is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$$

\therefore The charge on the capacitor can be expressed in terms of a sinusoidal function (either sine or a cosine).

$$\therefore q(t) = Q \sin(\omega t + \phi)$$

But as at $t = 0$, $q = Q$, hence $\phi = \pi/2$

$$\therefore q(t) = Q \cos(\omega t) \quad \dots(i) \quad [\because \sin(90^\circ + \theta) = \cos\theta]$$

Remember this as a thumb rule, that, if a physical quantity varies simple harmonically and it starts

(i) from 0 (zero) initial value, use sine function

$$\text{i.e. } x(t) = A \sin(\omega t)$$

(ii) from maximum value, use cosine function

$$\text{i.e. } x(t) = A \cos(\omega t)$$

(iii) from any value in between 0 and maximum value (A), use $x(t) = A \sin(\omega t + \phi)$

where ϕ can be found out using initial conditions.

From equation (i), we can also find out the current in the circuit.

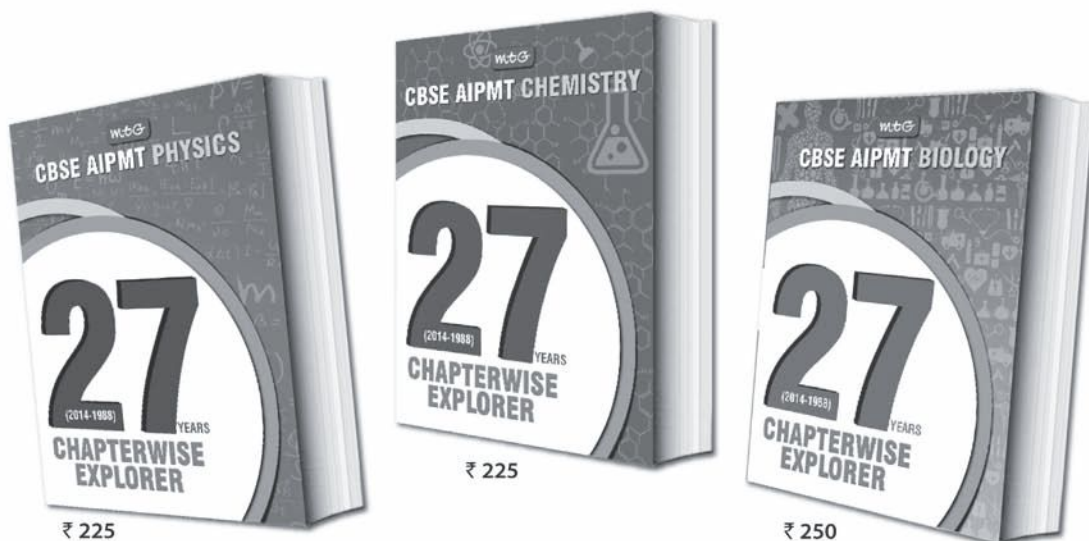
$$i = \frac{dq}{dt} = Q(-\sin \omega t) \omega \quad \therefore i(t) = -\frac{Q}{\sqrt{LC}} \sin(\omega t)$$

where $\frac{Q}{\sqrt{LC}}$ clearly indicates maximum current in the circuit.

This electrical system can be compared to the mechanical system, i.e. simple harmonic motion of a particle in motion.

Contributed By: Bishwajit Barnwal, Aakash Institute, Kolkata

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The comparison is shown below.

Simple harmonic motion of a particle	LC oscillation
x (instantaneous displacement of particle)	q (instantaneous charge on capacitor)
A (amplitude of oscillation)	Q (maximum charge on capacitor)
$v = \frac{dx}{dt}$ (velocity of particle)	$i = \frac{dq}{dt}$ (current through terminals of capacitor)
$v_0 = v_{\max} = \omega A$ (maximum speed of particle)	$i_0 = i_{\max} = \omega Q$ (maximum current)
$\frac{1}{2}kx^2$ (potential energy of oscillation)	$\frac{q^2}{2C}$ (energy stored in capacitor)
k (spring constant)	$\frac{1}{C}$ (inverse of capacitance)
$\frac{1}{2}mv^2$ (kinetic energy of oscillation)	$\frac{1}{2}Li^2$ (energy stored in inductor)
m (mass)	L (inductance, hence inductors are also known as electrical inertia)
$v = \omega\sqrt{A^2 - x^2}$ (speed of the particle as a function of position)	$i = \omega\sqrt{Q^2 - q^2}$ (current in the circuit as a function of charge on capacitor)

From this analogy, we learn that, if we can understand this comparison, we can straight forward use the standard results of simple harmonic motion here. For example,

1. Time taken for charge on capacitor to reach from

$q = 0$ to $q = \frac{Q}{2}$ and $q = \frac{Q}{2}$ to $q = Q$ are $\frac{T}{12}$ and $\frac{T}{6}$ respectively, because these are equivalent to phase changes of $\pi/6$ and $\pi/3$ radians respectively.

2. If q is cosine function, i is sine function, and sine-cosine being complimentary pairs, we can use $\sin^2\theta + \cos^2\theta = 1$

which means if $q = \frac{Q}{2}$, i.e., $\cos\theta = \frac{1}{2}$, we get $\sin\theta = \frac{\sqrt{3}}{2}$ hence $i = \frac{\sqrt{3}}{2}Q\omega$

3. If the charge/current is to be found when the initial energy of capacitor is equally shared between the capacitor and inductor, this effectively means the case when $\sin\theta = \cos\theta$. Hence $\theta = 45^\circ$

$$\therefore q = \frac{Q}{\sqrt{2}} \quad \text{and} \quad i = \frac{Q\omega}{\sqrt{2}}$$

In short, just by drawing the comparison, solve the question as if it was a question of simple harmonic motion, since we are already comfortable with simple harmonic motion.



Steps to Deal With Exam Stress

Exams are just to assess your preparation. So just relax, it is not the end of the world if you do not perform well in them.

Organize : Make sure you have all the things necessary for the exam, your stationery, your identity card, your watch etc. Last minute searching for things can stress your mind further and create panic during the exam which can be disastrous.

Diet : Before you go to the exam, eat foods that are energy producing and at the same time not too heavy on your stomach that make you sleepy in the examination hall. Never go in on an empty stomach as you can end up concentrating more on your hunger than your exam paper.

Relax : One hour before the exam, relax!! Don't stress yourself feeding more information to your already worked-up brain. Whatever you have learned, be confident of it and try to picture a calm stream, or take some deep breaths. You have done your preparation and now you should prepare yourself to give your best. A tired mind will do no good, so it is necessary to go into the hall with a refreshed mind. You have worked hard for it and nobody can take away your hard work. Remember nature's law, "What you give always comes back to you". If you not prepared well, let your mind accept the fact. It is not possible to go unprepared and expect no stress and good results. Instead of trying to revise that topic you forgot to learn, try and briefly go over the different topics in your head so that way you keep an organised and open mind without putting pressure on yourself to learn that other topic.

Plan : Once you get the question paper in your hand, read all the questions and make a quick rough plan how you are going to invest your time for doing your best. Attempt the questions which you know the best first. In this manner, you will increase your confidence further. Appreciate yourself for remembering the solutions and answers; your brain will work better. Never curse yourself if you forgot or didn't study something which you thought about studying. Remember it is too late now, and that your focus should be on the present moment.

Cross-check : The last 15 minutes should be to review your paper. It is very important to check your answers again in the end. Recheck every answer with patience and you will be surprised by how many careless faults you come across. Make the necessary corrections.

Forget : Most of the time after the exam is over, we worry about the results or waste time discussing what our friends have written. Realize that the time to do something about the results passed when you handed the answer sheet to the examiner. Knowing how your friend did on the paper will only add more worry. It is better to concentrate on how you will face your next exam or how you are going to spend your time efficiently.



1. (d) : As $\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} + \frac{1}{R_2} \right]$

$$\frac{1}{10} = (1.5 - 1) \left(\frac{1}{R} + \frac{1}{R} \right)$$

$$\Rightarrow \frac{1}{10} = 0.5 \times \frac{2}{R} \Rightarrow R = 10 \text{ cm}$$

Refraction from first surface,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v_1} - \frac{1}{-20} = \frac{1.5 - 1}{+10} \Rightarrow v_1 = \infty$$

For the second surface,

$$\frac{2}{v} - \frac{1.5}{\infty} = \frac{2 - 1.5}{-10} \Rightarrow v = -40 \text{ cm}$$

2. (b) : $\hat{R} = \hat{i} - 2(\hat{i} \cdot \hat{N})\hat{N}$

$$= \hat{i} - 2 \left[\hat{i} \cdot \left(-\frac{\hat{i}}{2} - \frac{\sqrt{3}}{2} \hat{j} \right) \right] \left(-\frac{\hat{i}}{2} - \frac{\sqrt{3}}{2} \hat{j} \right)$$

$$= \frac{\hat{i}}{2} - \frac{\sqrt{3}}{2} \hat{j}$$

3. (d) : $-(\hat{i} - 2\hat{j}) \cdot \hat{j} = |\hat{i} - 2\hat{j}| |\hat{j}| \cos \theta \Rightarrow \cos \theta = \frac{2}{\sqrt{5}}$

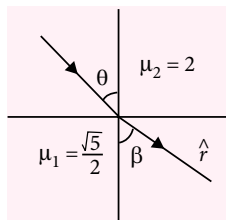
$$\therefore \sin \theta = \frac{1}{\sqrt{5}}$$

Using Snell's law at the interface

$$2 \times \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{2} \sin \beta$$

$$\Rightarrow \sin \beta = \frac{4}{5} \therefore \cos \beta = \frac{3}{5}$$

$$\text{So, } \hat{r} = \frac{4}{5} \hat{i} - \frac{3}{5} \hat{j}$$



4. (d) : Surface area of bubble of radius, $r = 4\pi r^2$.

Surface area of bubble of radius $2r = 4\pi(2r)^2 = 16\pi r^2$

Therefore, increase in surface area

$$= 16\pi r^2 - 4\pi r^2 = 12\pi r^2$$

Since a bubble has two surfaces, the total increase in surface area $= 24\pi r^2$.

$$\therefore \text{Energy spent} = \text{work done} = \text{surface tension} \times \text{increase in surface area} = 24\pi \sigma r^2$$

5. (d) : $F_{\text{req}} = mg + 2[T(2\pi R)]$

$$= 0.1 + 2(75 \times 10^{-3}(0.2)) = 0.130 \text{ N}$$

6. (a) : Let v is the speed of the centre of the ring then,

Loss in PE = Gain in KE

$$mgR = \frac{1}{2}m(v^2 + v^2) + \frac{1}{2}mv^2 + \frac{1}{2}(mR^2)\left(\frac{v}{R}\right)^2$$

$$mgR = 2mv^2 \text{ or } v = \sqrt{\frac{gR}{2}} \Rightarrow v\sqrt{2} = \sqrt{gR}$$

7. (c): Charge, $q = C(Blv_0) = \text{constant}$

$$\Rightarrow I = \frac{dq}{dt} = 0 \therefore \text{Current is zero.}$$

8. (c): Here, $\nu = 1000 \text{ Hz}$

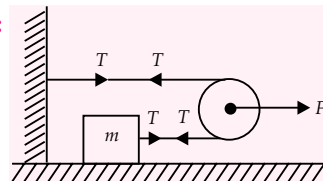
$$1000 = \frac{\nu}{4l_1} = \frac{3\nu}{4l_2}$$

Using $\nu = 320 \text{ m s}^{-1}$, we get,

$$l_1 = 8 \text{ cm and } l_2 = 24 \text{ cm}$$

$$\therefore \text{Minimum volume} = 16 \times 100 = 1600 \text{ cm}^3.$$

9. (c):



Equation of motion for pulley, $F - 2T = m_p \times a$

Since pulley is massless i.e., $m_p = 0$

$$F = 2T, \therefore T = \frac{F}{2}$$

10. (c): $F = \frac{dp}{dt} = \frac{m\Delta v}{\Delta t}$

For quarter of a circle, $\Delta v = v\sqrt{2}$ and $\Delta t = \frac{\pi r}{2v}$

$$\therefore F = \frac{2\sqrt{2}mv^2}{\pi r}$$

Solution Senders of Physics Musing

SET-20

1. Sandeep Kumar Rana (Kangra)
2. Anubhav (Punjab)
3. Anubhab Banerjee (Kolkata)
4. Soumya Mukherjee

SET-19

1. Rohan Hore (Kolkata)
2. Shivam Gupta
3. Shashi Kant (Rajasthan)
4. Shreyam Maity (West Bengal)
5. Himan Kumar Kundu (West Bengal)

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

Q1. When a bubble on the surface of a liquid, such as water, bursts, why does it throw tiny water drops up into the air?

– Anuj Gupta (U.P.)

Ans. A bubble on a liquid surface bursts because the liquid in the thin layer forming its top surface drains until it ruptures. As the rupture opens up the full top of the bubble, the sides of the bubble are pulled down to its bottom by surface tension, that is, by the mutual attraction of the molecules along the bubble wall. Descending liquid from opposite sides of the bubble collide at the bottom and shoot upward, forming a jet (column of water). The jet is unstable and surface tension quickly pinches it off into drops, which are the drops thrown up into the air by a bursting bubble.

Q2. When we keep refrigerator open in a closed room, the temperature of room increases, why?

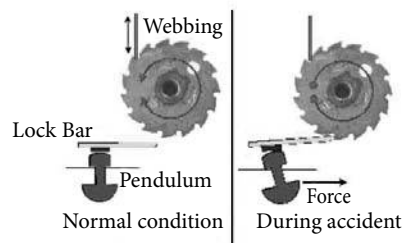
– Sumon Ghosh

Ans. A refrigerator is a heat engine that operates in the backward direction *i.e.*, it extracts heat from a low temperature reservoir and transfers it to a high temperature reservoir. No refrigerator is completely efficient. Thus, it exhausts more heat into the room than it extracts from it. Obviously, the net effect is to increase the temperature of the room. Thus, a room cannot be cooled by keeping the door of a refrigerator open. A refrigerator can remove heat from the inside air and reject it into the room.

Q3. In an automobile seat belt, a sudden jerk to the belt results in heightened tension and an arrest of forward motion, but if it is stretched gently, the belt offers no resistance, why?

– Aparna Sharma (New Delhi)

Ans. Automobile seat belts unwind freely when pulled gently, so they can be buckled. But in an



accident, they hold you safely in place. A seat belt mechanism consists of a ratchet wheel, a locking bar, and a pendulum. The belts are wound around a spool mounted on the ratchet wheel. While the car is at rest or moving at a constant velocity, the pendulum hangs straight down, and the locking bar rests horizontally. Consequently, nothing prevents the ratchet wheel from turning, and the seat belt can be pulled out easily. However, when the car suddenly slows down in an accident, the relatively massive lower part of the pendulum keeps moving forward because of its inertia. The pendulum swings on its pivot into the position shown in the figure and causes the locking bar to block the rotation of the ratchet wheel, thus preventing the seat belt from unwinding.

Q4. Why will a used tennis ball generally reach the receiver faster than a new tennis ball that is hit exactly the same way?

– Kajal Sirohi (Bhopal)

Ans. The flight time of a tennis ball is determined by air drag on the ball. If a given shot (speed and angle) is repeated many times, starting with a new ball, the air drag first increases and then gradually decreases to some minimum value. Presumably the reason lies in the nap (the surface fuzz). Initial play raises the nap, which then “catches” more air and thus increases the air drag. However, eventually the nap becomes torn away or flattened, and the air drag decreases. Thus, the server has a slight advantage when playing with a well-worn ball because the ball encounters less drag than a new one and reaches the receiver in less time, making a return more difficult.



“The ideals which have lighted my way, and time after time have given me new courage to face life cheerfully, have been Kindness, Beauty and Truth.”

– Albert Einstein

C R O S S W O R D

Readers can send their answer with complete address before 15th of every month to win exciting prizes.

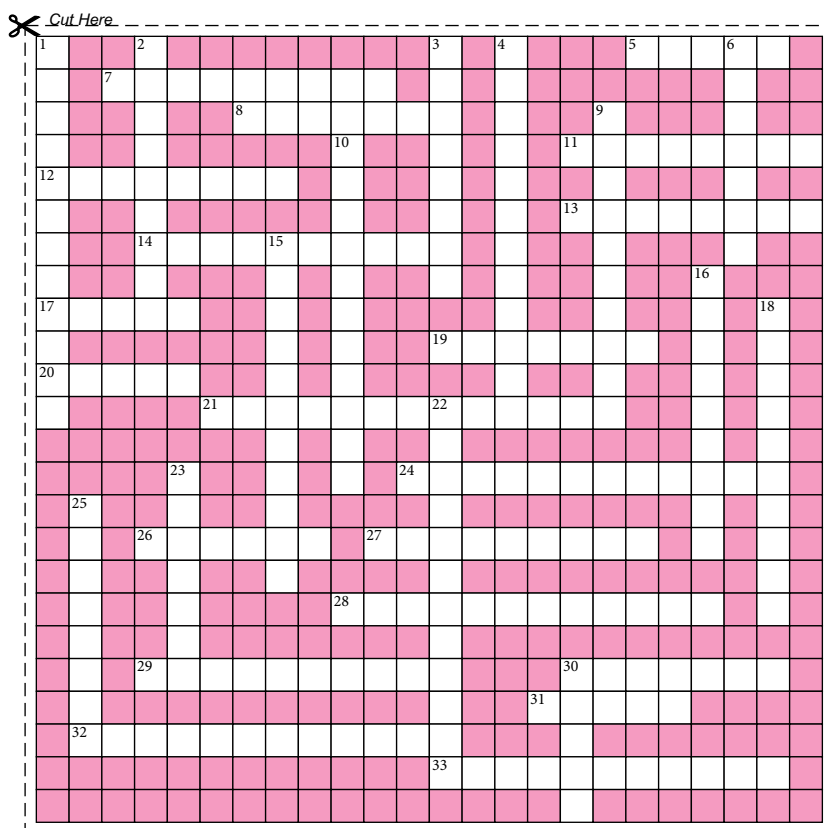
Winners' name with their valuable feedback will be published in next issue.

ACROSS

5. The degree of highness or lowness of a tone. (5)
7. The reciprocal of reluctance. (9)
8. Pulling force exerted by each end of a string, wire or rod. (7)
11. A wave whose frequency is an integral multiple of the frequency of some reference level. (8)
12. The degree of agreement between a measured and the standard value for a quantity. (8)
13. A curve on a P - V diagram for process whose temperature is constant. (8)
14. Phenomenon of melting under pressure and freezing again when pressure is reduced. (10)
17. SI unit of physical quantity represented by magnetic flux per unit area. (5)
19. Pattern of dark and bright bands produced by interference or diffraction. (7)
20. The path traced by an electron revolving around the nucleus of an atom. (5)
21. One of the two points defined on stress-strain curve to decide whether the material is brittle or ductile. (8, 5)
24. A device used for measuring the rate of flow of a liquid flowing through a pipe. (7, 5)
26. A U-shaped pipe that uses atmospheric pressure to draw liquid from one level to another. (6)
27. A decay process in which neutrino or anti-neutrino is emitted. (4, 5)
28. The unattainable lower limit to temperature. (8, 4)
29. Table used to compute the values of logical expressions. (5, 5)
30. The fidelity of reproduction of a sound. (7)
31. A single vibration of current, light or other wave. (5)
32. The most efficient heat engine. (6, 6)
33. Negative acceleration. (11)

DOWN

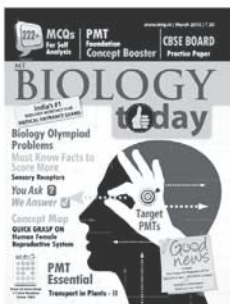
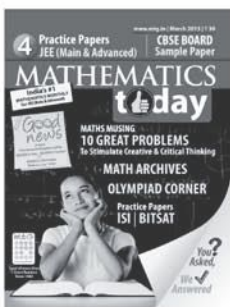
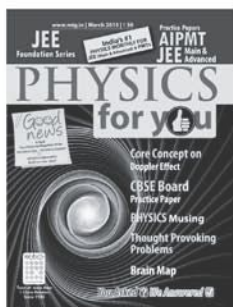
1. Name given to the electromagnetic radiation emitted during radioactive decay. (5)
2. SI unit of activity or decay rate. (9)
3. An instrument for showing the direction of the wind. (4, 4)
4. Material having different physical properties in different directions. (11)



6. A route along which information can be sent in a communication system. (7)
9. Mass equivalent of binding energy. (4, 6)
10. The densest and smallest star known to exist in the universe. (7, 4)
12. A length unit equal to 0.1 nm. (8)
15. Steady flow in which the fluid moves in parallel layers. (7, 4)
16. Any substance used in nuclear reactor to decrease the speed of fast neutrons. (9)
18. An application of Doppler effect in medical sciences. (10)
22. An instrument used for measuring potential difference. (12)
23. Any stimulus that initiates operation of an electronic circuit or device. (7)
25. A mixture of substances having a minimum melting point. (8)
30. A fundamental constituent of particles that take part in strong interactions. (5)



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